Supersymmetry breaking in the three-dimensional nonlinear sigma model

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In this work, we discuss the phase structure of a deformed $\mathcal{N}=1$ supersymmetric nonlinear sigma model in a three-dimensional space-time. The deformation is introduced by a term that breaks supersymmetry explicitly, through imposing a slightly different constraint to the fundamental superfields of the model. Using the tadpole method, we compute the effective potential at leading order in a $1/N$ expansion. From the gap equations, i.e., conditions that minimize the effective potential, we observe that this model presents two phases as the ordinary model, with two remarkable differences: 1) the fundamental fermionic field becomes massive in both phases of the model, which is closely related to the supersymmetry breaking term; 2) the $O(N)$ symmetric phase presents a metastable vacuum.

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I. INTRODUCTION

The nonlinear sigma model (NLSM) was first proposed to investigate the interaction between pions and nucleons [1]. In lower-dimensional systems, it is used to describe several aspects of condensed matter physics, for example, applications to ferromagnets [2–5]. In addition, this model provides a very good theoretical laboratory containing an interesting phase structure and at same time shares the wealth of more realistic theories, being a simple example of an asymptotically free theory [6,7]. Recently, it was conjectured that the $O(6)$ sigma model emerges as a scaling function in AdS/CFT correspondence [8,9].

The $O(N)$ NLSM can be defined through the action

$$S=\int d^Dx\left\{\frac{1}{2}\phi_a\Box\phi_a\right\}, \tag{1}$$

where the fields ϕ_a are constrained to satisfy $\phi_a^2=\frac{N}{g}$, D is the dimension of the space-time, and the index a assumes the values $1,2,\ldots,N$.

It is useful rewrite the $O(N)$ NLSM action implementing the constraint over ϕ_a by the use of Lagrange multiplier,

$$S=\int d^Dx\left\{\frac{1}{2}\phi_a\Box\phi_a+\sigma\left(\phi_a^2-\frac{N}{g}\right)\right\}, \tag{2}$$

where the field σ is the Lagrange multiplier that constrains $\phi_a^2=\frac{N}{g}$.

In the late 1970s, the phase structure and the renomalizability of the three-dimensional NLSM were established, showing that this model possesses two phases [10,11]. One phase is $O(N)$ symmetric and exhibits a spontaneous generation of mass due to a nonvanishing vacuum expectation value (VEV) of the Lagrange multiplier field σ, i.e., $\langle\sigma\rangle\neq 0$. On the other hand, if the fundamental bosonic field ϕ acquires a nonvanishing VEV, the $O(N)$ symmetry is spontaneously broken to $O(N-1)$, without any generation of mass. Several extensions of this model were studied afterward showing no changing in its phase structure [12–19].

The three-dimensional supersymmetric (SUSY) NLSM, in components [14], using the superfield formalism [15] and their noncommutative extensions [16,17], was shown to be renormalizable to all orders in the $1/N$ expansion. The phase structure of this model was also studied in Ref. [18]. In all these papers, a similar conclusion was achieved: no supersymmetry breaking is detected at leading order in the $1/N$ expansion.

The aim of this work is to show that, imposing a more general constraint on the SUSY NLSM, the solutions that minimize the effective potential present broken supersymmetry at leading order in the $1/N$ expansion. Moreover, the $O(N)$ symmetric phase presents a metastable vacuum.

II. SUPERSYMMETRIC NONLINEAR SIGMA MODEL

The usual three-dimensional $\mathcal{N}=1$ SUSY NLSM is defined through the action

$$S=\int d^5z\left\{\frac{1}{2}\Phi_a(z)D^2\Phi_a(z)+\Sigma(z)\left[\Phi_a(z)^2-\frac{N}{g}\right]\right\}, \tag{3}$$

where Σ is the Lagrange multiplier superfield that constrains Φ_a to satisfy $\Phi_a^2(z)=\frac{N}{g}$. With signature $(-,+,+)$, we are using notations and conventions as in Ref. [20]. Such definitions and some useful identities can be found in the Supplemental Material [21].

The superfields appearing in this model possess the following θ expansion:

$$\begin{aligned}\Phi_a(x,\theta)&=\phi_a(x)+\theta^\beta\psi_{a\beta}(x)-\theta^2F_a(x);\\ \Sigma(x,\theta)&=\rho(x)+\theta^\beta\chi_\beta(x)-\theta^2\sigma(x).\end{aligned} \tag{4}$$

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We can see that the SUSY NLSM possesses more constraints than the nonsupersymmetric one. Once the equation of motion of Σ constrains

$$\Phi_a^2(z) = \left[\phi_a^2 + 2\theta^\beta\phi_a\psi_{a\beta} - 2\theta^2\left(\phi_a F_a - \frac{1}{2}\psi_a^\beta\psi_{a\beta}\right)\right] = \frac{N}{g},$$

it is easy to see that the component fields ϕ_a, ψ_a^α, and F_a must satisfy

$$\phi_a^2 = \frac{N}{g}, \qquad \psi_a^\alpha\phi_a = 0, \qquad F_a\phi_a = \frac{1}{2}\psi_a^\beta\psi_{a\beta}. \quad (5)$$

Beyond the usual constraint $\phi_a^2 = N/g$, the SUSY NLSM also exhibits the constraints $\psi_a^\alpha\phi_a = 0$ and $F_a\phi_a = \frac{1}{2}\psi_a^\beta\psi_{a\beta}$.

Integrating the Eq. (3) over $d^2\theta$, the action of the model can be cast as

$$S = \int d^3x\left\{\frac{1}{2}\phi_a\Box\phi_a + \frac{1}{2}\psi_a^\alpha i\partial_\alpha{}^\beta\psi_{a\beta} + \frac{1}{2}F_a^2 + \sigma\left(\phi_a^2 - \frac{N}{g}\right) + 2\rho\left(F_a\phi_a + \frac{1}{2}\psi_a^\beta\psi_{a\beta}\right) + 2\chi^\beta\psi_{a\beta}\phi_a\right\}. \quad (6)$$

Notice that the usual model is obtained setting $\psi = \rho = \chi = 0$, and the auxiliary field σ must be nonvanishing.

We can eliminate the auxiliary field F_a using its equation of motion, $F_a = -2\rho\phi_a$. This way, the action

$$S = \int d^3x\left\{\frac{1}{2}\phi_a\Box\phi_a + \frac{1}{2}\psi_a^\alpha i\partial_\alpha{}^\beta\psi_{a\beta} + \sigma\left(\phi_a^2 - \frac{N}{g}\right) - 2\rho^2\phi_a^2 + \rho\psi_a^\beta\psi_{a\beta} + 2\chi^\beta\psi_{a\beta}\phi_a\right\} \quad (7)$$

describes the physical content of the model. It is easy to see that, if there exists a phase during which mass is generated to the fundamental fields ϕ and ψ, their masses will be given by the VEV of the fields ρ and σ as

$$M_\phi^2 = 4\langle\rho\rangle^2 - 2\langle\sigma\rangle, \qquad M_\psi^2 = 4\langle\rho\rangle^2, \quad (8)$$

from which we observe that SUSY should be spontaneously broken if $\langle\sigma\rangle \neq 0$, as commented before. For $\langle\sigma\rangle = 0$ and for a nonvanishing VEV of ρ, the fundamental bosonic and fermionic fields acquire the same squared mass $4\langle\rho\rangle^2$, indicating the generation of mass in a supersymmetric phase as is well known [14–18]. Here, we find an intriguing point. While in the non-SUSY model the spontaneous generation of mass occurs due to σ acquiring a nonvanishing vacuum expectation value, in the SUSY version, the field that acts like a "mass generator" to the fundamental fields is ρ, which is not present in the non-SUSY model. There is no soft transition or anything that we can interpret as a non-SUSY limit of the spontaneous generation of mass from the SUSY model.

Now, let us define a slightly deformed SUSY NLSM by

$$S = \int d^5z\left\{\frac{1}{2}\Phi_a(z)D^2\Phi_a(z) + \Sigma(z)\left[\Phi_a(z)^2 - \frac{N}{g}\delta(z)\right]\right\}, \quad (9)$$

with the single difference that Σ is a Lagrange multiplier superfield that constrains Φ_a to satisfy $\Phi_a^2(z) = \frac{N}{g}\delta(z)$, where $\delta(z)$ is a constant superfield that possesses the θ expansion $\delta(z) = \delta_1 - \theta^2 g\delta_2$. Doing $\delta_2 = 0$ and $\delta_1 = 1$, we obtain the usual supersymmetric action for the SUSY NLSM, Eq. (3).

The equation of motion of the Lagrange multiplier superfield Σ obtained from Eq. (9) generates new constraints to the components of the fundamental superfields Φ_a, namely,

$$\phi_a^2 = \frac{N}{g}\delta_1, \quad \psi_a^\alpha\phi_a = 0, \quad F_a\phi_a = \frac{1}{2}\psi_a^\beta\psi_{a\beta} + g\delta_2. \quad (10)$$

To study the phase structure of the model, let us assume that the Σ and the Nth component $\Phi_N(x,\theta)$ have a constant nontrivial VEV given by

$$\langle\Sigma\rangle = \Sigma_{\rm cl} = \rho_{\rm cl} - \theta^2\sigma_{\rm cl}, \quad \langle\Phi_N\rangle = \sqrt{N}\Phi_{\rm cl} = \sqrt{N}(\phi_{\rm cl} - \theta^2 F_{\rm cl}). \quad (11)$$

Therefore, let us dislocate these superfields by $\Sigma \to (\Sigma + \Sigma_{\rm cl})$ and $\Phi_N \to \sqrt{N}(\Phi_N + \Phi_{\rm cl})$. So, we can rewrite the action, Eq. (3), in terms of the new fields as

$$S = \int d^5z\left\{\frac{1}{2}\Phi_a(D^2 + 2\Sigma_{\rm cl})\Phi_a + \Sigma\left(\Phi_a^2 + N\Phi_{\rm cl}^2 + 2N\Phi_{\rm cl}\Phi_N - \frac{N}{g}\delta\right) + N\Phi_N(D^2\Phi_{\rm cl} + 2\Phi_{\rm cl}\Sigma_{\rm cl}) + \frac{N}{2}\Phi_{\rm cl}D^2\Phi_{\rm cl} + N\Sigma_{\rm cl}\left(\Phi_{\rm cl}^2 - \frac{1}{g}\right)\right\}. \quad (12)$$

We can note that the VEV of the superfield Σ, $\Sigma_{\rm cl}$, gives mass to the fundamental superfields Φ_a. This "mass" is θ dependent, generating different masses to the bosonic and fermionic components of the superfield Φ_a, showing a possible phase during which supersymmetry is broken.

At leading order, the propagator of the Φ_a superfield must satisfy the following equation:

$$[D^2(z_1) + 2\Sigma_{\rm cl}]\Delta(z_1 - z_2) = i\delta^{(5)}(z_1 - z_2), \quad (13)$$

where $\delta^{(5)}(z_1 - z_2) \equiv \delta^{(3)}(x_1 - x_2)\delta^{(2)}(\theta_1 - \theta_2)$, and $\delta^{(2)}(\theta) = -\theta^2$.

The solution to the above equation can be obtained from the ansatz

$$\Delta(z_1 - z_2) = (C_1 - \theta_1^2 C_2 - \theta_2^2 C_3 + \theta_1^\alpha\theta_2^\beta\Delta_{\alpha\beta} + \theta_1^2\theta_2^2 C_4)\delta^{(3)}(x_1 - x_2), \quad (14)$$

where, after some algebraic manipulations, we can write the propagator of the Φ_a superfield as

$$\Delta(k) = -i\frac{D_1^2 - 2\rho_{cl}}{k^2 + 4\rho_{cl}^2}\left\{1 + 2\sigma_{cl}\frac{\delta^{(2)}(\theta_1)(D_1^2 + 2\rho_{cl})}{k^2 + (4\rho_{cl}^2 - 2\sigma_{cl})}\right\} \times \delta^{(2)}(\theta_1 - \theta_2). \tag{15}$$

Notice that for $\sigma_{cl} = 0$, the above propagator reduces to the usual propagator of a massive scalar superfield. A propagator presenting a similar form was obtained in [22]. See Supplemental Material [21] for details in obtaining the superfield propagator.

From Eq. (12), we can see that there exists a mixing between Φ_N and Σ, but this mixing only contributes to next-to-leading order in the $1/N$ expansion. For now, we can neglect this mixing, since we will deal with the SUSY NLSM at leading order in $1/N$.

With the propagator of the Φ_a superfield, let us evaluate the effective potential through the tadpole method [23–25]. At leading order, the tadpole equation for the Φ_N superfield can be cast as

$$N[D^2\phi_{\rm cl} + 2\Phi_{\rm cl}\Sigma_{\rm cl}] = N[F_{\rm cl} + 2\phi_{\rm cl}\rho_{\rm cl} - 2\theta^2(\phi_{\rm cl}\sigma_{\rm cl} + F_{\rm cl}\rho_{\rm cl})]. \tag{16}$$

On the other hand, the tadpole equation for Σ, Fig. 1, is $[N\Phi_{\rm cl}^2 - \frac{N}{g}\delta + N\int\frac{d^3k}{(2\pi)^3}\Delta(k)]$. Substituting the expression for $\Delta(k)$, and using the fact that $D^2\delta^{(2)}(\theta - \theta) = 1$ and $\delta^{(2)}(\theta - \theta) = 0$, we obtain

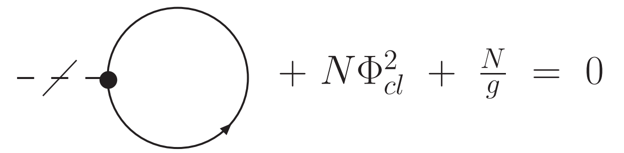

FIG. 1. Tadpole equation of Σ at leading order. Continuous lines represent the Φ_a superfield propagator, while the cut dashed line represents a removed external Σ propagator.

$$\begin{aligned} N\Phi_{\rm cl}^2 - \frac{N}{g}\delta - iN\int\frac{d^3k}{(2\pi)^3}&\left\{\frac{1}{k^2 + (4\rho_{\rm cl}^2 - 2\sigma_{\rm cl})} + \frac{8\sigma_{\rm cl}\rho_{\rm cl}\theta^2}{[k^2 + (4\rho_{\rm cl}^2 - 2\sigma_{\rm cl})](k^2 + 4\rho_{\rm cl}^2)}\right\} \\ &= N\left[\phi_{\rm cl}^2 - \left(\frac{\delta_1}{g} - \frac{1}{g_c}\right) - \frac{\sqrt{4\rho_{\rm cl}^2 - 2\sigma_{\rm cl}}}{4\pi} - \theta^2\left(2\phi_{\rm cl}F_{\rm cl} - \frac{2}{\pi}\rho_{\rm cl}^{3/2} + \frac{\rho_{\rm cl}}{\pi}\sqrt{4\rho_{\rm cl}^2 - 2\sigma_{\rm cl}} + \delta_2\right)\right], \end{aligned} \tag{17}$$

where $\frac{1}{g_c}$ is defined as usual, $\int_\Lambda\frac{d^3k}{(2\pi)^3}\frac{1}{k^2}$. The coupling g_c is the critical value of g so that the NLSM exhibits the phase transition.

With the tadpole equations in hand, the effective potential is obtained integrating Eq. (16) over Φ_N and Eq. (17) over Σ as

$$\begin{aligned} \frac{V_{\rm eff}}{N} = &-\int d^2\theta\left\{\int d\Phi_N[F_{\rm cl} + 2\phi_{\rm cl}\rho_{\rm cl} - 2\theta^2(\phi_{\rm cl}\sigma_{\rm cl} + F_{\rm cl}\rho_{\rm cl})]\right. \\ &\left. + \int d\Sigma\left[\phi_{\rm cl}^2 - \lambda - \frac{\sqrt{4\rho_{\rm cl}^2 - 2\sigma_{\rm cl}}}{4\pi} - \theta^2\left(2\phi_{\rm cl}F_{\rm cl} - \frac{2}{\pi}\rho_{\rm cl}|\rho_{\rm cl}| + \frac{\rho_{\rm cl}}{\pi}\sqrt{4\rho_{\rm cl}^2 - 2\sigma_{\rm cl}} + \delta_2\right)\right]\right\} \\ = &-\frac{F_{\rm cl}^2}{2} - \sigma_{\rm cl}(2\phi_{\rm cl}^2 - \lambda) - 6F_{\rm cl}\rho_{\rm cl}\phi_{\rm cl} + \frac{2}{3\pi}(\rho_{\rm cl}^2)^{3/2} - \frac{4}{3\pi}\left(\rho_{\rm cl}^2 - \frac{\sigma_{\rm cl}}{2}\right)^{3/2} - \delta_2\rho_{\rm cl} + C, \end{aligned} \tag{18}$$

where C is a constant of integration to be adjusted through the conditions that minimize the effective potential, the gap equations, and $\lambda \equiv (\frac{\delta_1}{g} - \frac{1}{g_c})$ is a parameter that can be positive, negative, or zero. In the thermodynamics of the NLSM, λ is interpreted as a quantity proportional to magnetization of the system [13].

Looking to the tadpole equations in Eqs. (16) and (17), we observe that the VEVs must to satisfy the following conditions:

$$\begin{aligned} &F_{\rm cl} + 2\phi_{\rm cl}\rho_{\rm cl} = 0, \qquad F_{\rm cl}\rho_{\rm cl} + \phi_{\rm cl}\sigma_{\rm cl} = 0, \\ &\phi_{\rm cl}^2 - \lambda - \frac{1}{2\pi}\sqrt{\rho_{\rm cl}^2 - \frac{\sigma_{\rm cl}}{2}} = 0, \\ &\phi_{\rm cl}F_{\rm cl} + \frac{\rho_{\rm cl}}{\pi}\left(\sqrt{\rho_{\rm cl}^2 - \frac{\sigma_{\rm cl}}{2}} - |\rho_{\rm cl}|\right) + \frac{\delta_2}{2} = 0. \end{aligned} \tag{19}$$

Therefore, setting $C = [\sigma_{\rm cl}\phi_{\rm cl}^2 + 4F_{\rm cl}\rho_{\rm cl}\phi_{\rm cl} + \frac{2}{3\pi} \times (\rho_{\rm cl}^2 - \frac{\sigma_{\rm cl}}{2})^{3/2}]$, the effective potential can be cast as

$$\frac{V_{\rm eff}}{N} = -\frac{F_{\rm cl}^2}{2} - \sigma_{\rm cl}(\phi_{\rm cl}^2 - \lambda) - 2F_{\rm cl}\rho_{\rm cl}\phi_{\rm cl} + \frac{2}{3\pi}(\rho_{\rm cl}^2)^{3/2} - \frac{2}{3\pi}\left(\rho_{\rm cl}^2 - \frac{\sigma_{\rm cl}}{2}\right)^{3/2} - \delta_2\rho_{\rm cl}. \tag{20}$$

As we did for the classical action, we can eliminate the auxiliary field $F_{\rm cl}$ using its equation of motion,

$$F_{\rm cl} = -2\rho_{\rm cl}\phi_{\rm cl}, \tag{21}$$

allowing us to write the effective potential as

$$\frac{V_{\rm eff}}{N} = -\sigma_{\rm cl}(\phi_{\rm cl}^2 - \lambda) + 2\rho_{\rm cl}^2\phi_{\rm cl}^2 + \frac{2}{3\pi}\left[(\rho_{\rm cl}^2)^{3/2} - \left(\rho_{\rm cl}^2 - \frac{\sigma_{\rm cl}}{2}\right)^{3/2}\right] - \delta_2\rho_{\rm cl}. \tag{22}$$

From the effective potential, Eq. (22), the conditions that extremize the effective potential are given by

$$\phi_{\rm cl}\left(\rho_{\rm cl}^2-\frac{\sigma_{\rm cl}}{2}\right)=0,\qquad \phi_{\rm cl}^2-\lambda-\frac{1}{2\pi}\sqrt{\rho_{\rm cl}^2-\frac{\sigma_{\rm cl}}{2}}=0,$$
$$\rho_{\rm cl}\left(2\pi\phi_{\rm cl}^2+|\rho_{\rm cl}|-\sqrt{\rho_{\rm cl}^2-\frac{\sigma_{\rm cl}}{2}}\right)=\frac{\pi}{2}\delta_2. \quad (23)$$

Solving these equations, we determine the field configurations that extremize the effective potential. Such solutions are presented in two phases, one $O(N)$ symmetric phase and another $O(N)$ broken to $O(N-1)$. In the $O(N)$ symmetric phase, $\lambda<0$ or $g>g_c$, the solutions are given by

$$\phi_{\rm cl}=0,\qquad \rho_{\rm cl}=\pi|\lambda|+\frac{1}{2}\sqrt{2\pi(2\pi\lambda^2-\delta_2)},$$
$$\sigma_{\rm cl}=\frac{1}{2}\left[2\pi|\lambda|+\sqrt{2\pi(2\pi\lambda^2-\delta_2)}\right]^2-8\pi^2\lambda^2; \quad (24)$$

$$\phi_{\rm cl}=0,\qquad \rho_{\rm cl}=-\pi|\lambda|-\frac{1}{2}\sqrt{2\pi(2\pi\lambda^2+\delta_2)},$$
$$\sigma_{\rm cl}=\frac{1}{2}\left[2\pi|\lambda|+\sqrt{2\pi(2\pi\lambda^2+\delta_2)}\right]^2-8\pi^2\lambda^2. \quad (25)$$

Note for *real* solutions, the parameter δ_2 is constrained to be $|\delta_2|\le 2\pi\lambda^2$. Moreover, as we will see, there exists a $\delta_2\neq 0$, which $V_{\rm eff}$ assumes its minimum value. Setting $\delta_2=0$, we have the well-known solutions [14–18]

$$\rho_{\rm cl}=\pm 2\pi|\lambda|,\qquad \phi_{\rm cl}=F_{\rm cl}=\sigma_{\rm cl}=0. \quad (26)$$

The solution, Eq. (24), is the global minimum of the effective potential, while Eq. (25) is a local one. The effective potential is plotted in Fig. 2 as a function of $\rho_{\rm cl}$ and $\phi_{\rm cl}$, where it is possible to see the true and the false vacua.

In the minimum, $V_{\rm eff}$ is negative; this is because we are dealing with an explicit breaking of supersymmetry. The generated masses for the fundamental fields ϕ and ψ in the $O(N)$ symmetric phase are given by

$$M_\phi^2=4\langle\rho\rangle^2-2\langle\sigma\rangle=16\pi^2\lambda^2 \quad (27)$$

$$M_\psi^2=4\langle\rho\rangle^2=8\pi^2\lambda^2+4\pi|\lambda|\sqrt{2\pi(2\pi\lambda^2-\delta_2)}-2\pi\delta_2. \quad (28)$$

In the limit $\delta_2\to 0$, the masses $M_\phi^2=M_\psi^2$, and supersymmetry is restored.

The second phase, $O(N)$ symmetry is broken to $O(N-1)$, $\lambda>0$ or $g<g_c$, and the solutions that minimize the effective potential are given by

$$\phi_{\rm cl}=\pm\sqrt{\lambda},\qquad \rho_{\rm cl}=\pi\lambda-\frac{1}{2}\sqrt{2\pi(2\pi\lambda^2-\delta_2)},$$
$$\sigma_{\rm cl}=\frac{1}{2}[2\pi\lambda-\sqrt{2\pi(2\pi\lambda^2-\delta_2)}]^2; \quad (29)$$

$$\phi_{\rm cl}=\pm\sqrt{\lambda},\qquad \rho_{\rm cl}=-\pi\lambda+\frac{1}{2}\sqrt{2\pi(2\pi\lambda^2+\delta_2)},$$
$$\sigma_{\rm cl}=\frac{1}{2}\left[2\pi\lambda-\sqrt{2\pi(2\pi\lambda^2+\delta_2)}\right]^2, \quad (30)$$

where, just as for the $O(N)$ symmetric phase discussed before, for $\delta_2\to 0$, the above solutions collapse to

$$\phi_{\rm cl}=\pm\sqrt{\lambda},\qquad F_{\rm cl}=\sigma_{\rm cl}=\rho_{\rm cl}=0. \quad (31)$$

Just as in the supersymmetric and nonsupersymmetric cases, in the $O(N)$ symmetric phase, the scalar field ϕ is kept massless, i.e., $M_\phi^2=0$. But, due to the parameter that breaks supersymmetry, δ_2, the fundamental fermion of the model acquires the mass

$$M_\psi^2=4\langle\rho\rangle^2=\left[2\pi\lambda-\sqrt{2\pi(2\pi\lambda^2-\delta_2)}\right]^2. \quad (32)$$

It is easy to see that if $\delta_2\to 0$, so $M_\psi^2\to 0$.

Finally, let us deal with the optimal value of the SUSY-breaking parameter δ_2. Eliminating, from Eq. (22), all fields by the use of their equations of motion, except the fundamental field ϕ, to $\lambda>0$, we find

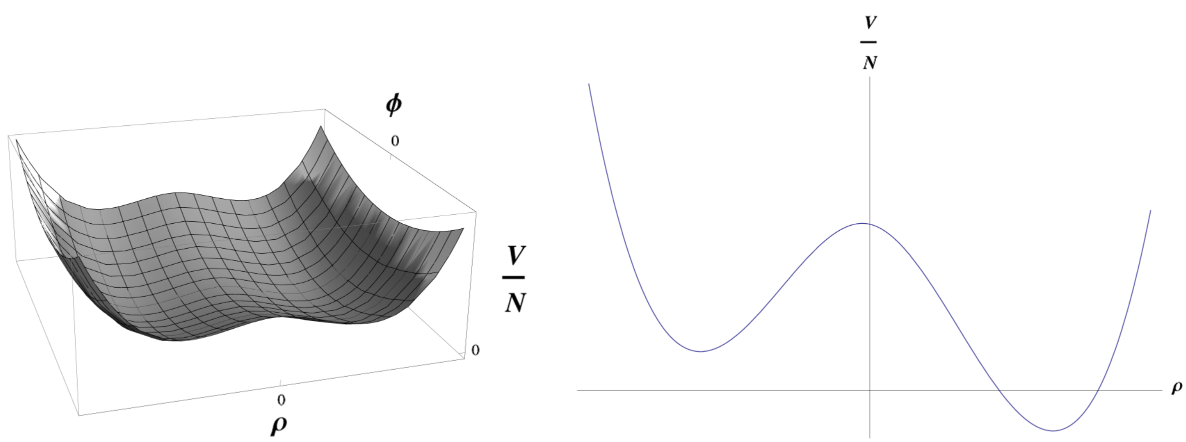

FIG. 2 (color online). Effective potential in the $O(N)$ symmetric phase as a function of $\rho_{\rm cl}$ and $\phi_{\rm cl}$. The plot in the right side of the figure is a slice of the $V_{\rm eff}$ at $\phi_{\rm cl}=0$, evidencing the presence of a metastable vacuum.

$$\frac{V_{\text{eff}}}{N}=\frac{1}{6}\Big\{-12\pi\lambda(\delta_2+2\pi\lambda^2)-3(4\pi\lambda^2-\delta_2)\times\sqrt{2\pi(2\pi\lambda^2-\delta_2)}+\Big[32\pi^4\lambda^2-8\pi^3\delta_2-16\pi^3\lambda\sqrt{2\pi(2\pi\lambda^2-\delta_2)}\Big]^{3/2}+144\pi^2\lambda\phi_{\text{cl}}^2(\lambda-\phi_{\text{cl}}^2)+48\pi^2\phi_{\text{cl}}^6-32\pi^2|\lambda-\phi_{\text{cl}}^2|\Big\}. \quad (33)$$

Minimizing Eq. (33) for δ_2, we obtain the solution

$$\delta_2=\frac{3\pi}{2}\lambda^2. \quad (34)$$

The effective potential, Eq. (22), evaluated for $\delta_2=\frac{3\pi}{2}\lambda^2$ is given by

$$\frac{V_{\text{eff}}}{N}=-\sigma_{\text{cl}}(\phi_{\text{cl}}^2-\lambda)+2\rho_{\text{cl}}^2\phi_{\text{cl}}^2+\frac{2}{3\pi}\Big[(\rho_{\text{cl}}^2)^{3/2}-\Big(\rho_{\text{cl}}^2-\frac{\sigma_{\text{cl}}}{2}\Big)^{3/2}\Big]-\frac{3\pi}{2}\lambda^2\rho_{\text{cl}}. \quad (35)$$

One interesting note is that $\delta_2=0$ becomes a local maximum in this model. Once the SUSY-breaking parameter has been introduced, the supersymmetric solutions are not the solutions that minimize the effective potential anymore.

III. FINAL REMARKS

Summarizing, the three-dimensional supersymmetric nonlinear sigma model, deformed by a nonsupersymmetric constraint, possess two phases. In the first one is the $O(N)$ symmetric phase, $\lambda<0$ or $g>g_c$, which possess the remarkable characteristic of the presence of a metastable vacuum. In this phase, all fields acquire a nonvanishing vacuum expectation value, generating masses to the fundamental fields ϕ and ψ. These masses are different for nonvanishing δ_2 coupling responsible for supersymmetry breaking. In the limit $\delta_2\rightarrow 0$, the masses of ϕ and ψ tend to be equal, restoring the supersymmetry. In the $O(N)$ broken phase, only the components of the Lagrange multiplier superfield acquire a nonvanishing vacuum expectation value, generating the mass to the fermonic field ψ and keeping ϕ massless. Also in this phase, the limit $\delta_2\rightarrow 0$ can be taken to restore the supersymmetric solutions. An important note is the fact that δ_2 cannot be chosen arbitrarily. It possesses an optimal value that minimizes the effective potential.

Finally, we think that gauge and noncommutative extensions (with constant noncommutative parameter; see, for example the SUSY $CP^{(N-1)}$ model presented in Ref. [26]) of this model should present a similar structure, including the presence of the metastable vacuum, since, in general, the tadpole diagrams in noncommutative models are the same as the commutative ones.

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