Magnetic-field-induced superconductivity and superfluidity of W and Z bosons: In tandem transport and kaleidoscopic vortex states

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We show that in a background of a sufficiently strong magnetic field the electroweak sector of the quantum vacuum exhibits superconducting and, unexpectedly, superfluid properties due to the magnetic-field-induced condensation of, respectively, W and Z bosons. The phase transition to the "tandem" superconductor-superfluid phase—which is weakly sensitive to the Higgs sector of the Standard Model—occurs at the critical magnetic field of 10^{20} T. The superconductor-superfluid phase of the electroweak vacuum has anisotropic transport properties as both charged and neutral superflows may propagate only along the magnetic field axis. The ground state possesses an unusual "kaleidoscopic" structure made of a hexagonal lattice of superfluid vortices superimposed on a triangular lattice of superconductor vortices.

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I. INTRODUCTION

It is known that an extremely high magnetic field of hadronic scale may lead to plenty of unusual effects both in (dense) matter and in the quantum vacuum. The chiral magnetic effect [1] provides a particularly interesting example: charge-parity-odd matter may generate an electric current along the axis of the magnetic field [2]. The corresponding conditions may be realized in noncentral heavy-ion collisions [3] in which hot quark matter is created along with a background of extremely high magnetic fields [4,5]. Similar conditions may have existed in the very early moments of our Universe [6]. The strong magnetic field also affects phases of the cold dense matter in the cores of strongly magnetized neutron stars [7].

Because of quantum effects an empty space may also exhibit quite unusual properties in a sufficiently strong magnetic background. In the background of a relatively low magnetic field of QED scale the vacuum should become optically birefringent [8]. The hadron-scale magnetic field should lead to magnetic catalysis [9], which implies, in particular, a steady enhancement of the chiral symmetry breaking in the QCD vacuum as the external magnetic field strengthens.

More recently it was found that the vacuum becomes an *electromagnetic* superconductor in sufficiently strong external magnetic fields [10,11]. The superconductivity of, basically, empty space is mediated via the spontaneous creation of a (charged) ρ-meson condensate if the magnetic field exceeds the critical value of $B_c^{\rm QCD} \simeq 1.0 \times 10^{16}$ T. The ground state of the vacuum superconductor is characterized by an inhomogeneous ground state of a very particular geometric structure [12], possessing intriguing metamaterial ("perfect lens") properties [13]. The magnetic fields of the required strength may be created on Earth in heavy-ion collisions at the Large Hadron Collider at CERN [5].

We show that as the background magnetic field strengthens further, the Standard Model experiences a second superconducting, and, simultaneously, superfluid transition associated with a condensation of the W and Z bosons at a larger critical magnetic field,

$$B_c^{\rm EW} = \frac{M_W^2}{e} \simeq 1.1 \times 10^{20}\ {\rm T}, \tag{1}$$

where $M_W = 80.4$ GeV is the mass of the W boson. The onset of the condensation of the W bosons at the magnetic field (1) was predicted by Ambjørn and Olesen in Ref. [14]. The key idea here is that the vacuum of charged vector particles (i.e., of the W mesons) is unstable in the background of a sufficiently strong magnetic field provided these particles have an anomalously large gyromagnetic ratio $g_m = 2$. The large value of g_m guarantees that the magnetic moment of such particles is too large to withstand a spontaneous condensation at sufficiently strong external magnetic fields. In this article we show that the inhomogeneous W condensation induces an inhomogeneous condensation of the Z bosons and leads to new superconducting and superfluid effects at the electroweak scale.

The electroweak sector possesses another phase transition which lifts off the electroweak symmetry breaking at a second critical magnetic field $B_{c2}^{\rm EW}$ which is stronger than the critical magnetic field of the electroweak superconducting transition (1), $B_{c2}^{\rm EW} > B_{c1}^{\rm EW} \equiv B_c^{\rm EW}$ [15]. A recent study of the second phase transition can be found in Ref. [16]. In this paper we concentrate on the W-meson condensed phase realized at $B_{c1}^{\rm EW} < B < B_{c2}^{\rm EW}$.

The structure of this paper is as follows. In Sec. II we solve the classical equations at the W-condensed phase

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and we show that the W condensate, originally found in Ref. [14], is accompanied by the condensation of the electrically neutral Z bosons. We point out that both condensates possess vortex defects which are aligned with the magnetic field axis forming a complicated regular structure in the transversal plane. In Sec. III we demonstrate that these condensates lead to the new transport phenomena of the ground state, which correspond to a dissipationless transfer of an electric current and a neutral Z-boson current. We associate these phenomena with superconductivity and superfluidity, respectively. The last section is devoted to our conclusions.

II. STRUCTURE OF THE GROUND STATE

A. Equations of motion

The bosonic part of the electroweak sector of the Standard Model is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda(|\Phi|^2 - v^2/2)^2, \tag{2}$$

where Φ is the complex Higgs doublet which interacts with $SU(2)L$ and $U(1)X$ gauge fields (W^a_μ and X_μ, respectively) via the covariant derivative

$$D_\mu = \partial_\mu - ig\tau^a W^a_\mu/2 - ig' X_\mu/2,$$

and τ^a are the Pauli matrices. The corresponding field strengths are $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$ and $X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$.

The Mexican-hat potential in Eq. (2) breaks the electroweak symmetry down to the electromagnetic subgroup, $SU(2)_L \times U(1)_X \to U(1)_{em}$ because the Higgs field Φ acquires a quantum expectation value, $\langle \Phi \rangle \neq 0$. In the unitary gauge, $\langle \Phi \rangle = (0, v)^T$, the third component of the non-Abelian gauge field W^3_μ mixes with the Abelian gauge field X_μ providing us with the massive Z_μ boson and the massless electromagnetic field A_μ,

$$W^3_\mu = \sin\theta A_\mu + \cos\theta Z_\mu, \tag{3}$$

$$X_\mu = \cos\theta A_\mu - \sin\theta Z_\mu, \tag{4}$$

where θ is the electroweak mixing (Weinberg) angle with $e = g\sin\theta = g'\cos\theta$ being the electric charge.

The classical equations of motion are as follows:

$$\begin{aligned} 0 &= \partial^\mu W^a_{\mu\nu} + g\epsilon^{abc} W^{b\mu} W^c_{\mu\nu} - ig[(D_\mu\Phi)^\dagger \tau^a \Phi - \text{H.c.}]/2, \\ 0 &= \partial^\mu X_{\mu\nu} - ig'(D_\mu\Phi)^\dagger \Phi - \text{H.c.})/2, \\ 0 &= -D_\mu D^\mu \Phi + 2\lambda\Phi(|\Phi|^2 - v^2/2). \end{aligned} \tag{5}$$

The instability of the vacuum in the presence of the sufficiently strong magnetic field was first demonstrated by Ambjørn and Olesen in Ref. [14], and we briefly repeat their arguments here. We restrict ourselves to the classical dynamics of the electroweak fields and ignore quantum corrections following the original approach of Ref. [14], which is justified in a sufficiently strong classical background.

Consider a uniform time-independent magnetic field directed along the third axis, $B_{\text{ext},i} = B_{\text{ext}}\delta_{i3}$ (for the sake of convenience, we always take $eB_{\text{ext}} > 0$). Then the quadratic part of the transverse (with respect to the magnetic field axis) components of the $W_\mu \equiv W^-_\mu$ field in Eq. (2) reads

$$\delta\mathcal{L}^{(2)}_{W_\perp} = (W_1^\dagger, W_2^\dagger)\begin{pmatrix} M_W^2 & -ieB_{\text{ext}} \\ ieB_{\text{ext}} & M_W^2 \end{pmatrix}\begin{pmatrix} W_1 \\ W_2 \end{pmatrix}, \tag{6}$$

(with $M_W = gv/2$), while the mass terms of the longitudinal components W_3 and W_0 and of other vector particles are not affected at the classical level. The mass eigenvalues of Eq. (6) are $\mu^2_\pm = M_W^2 \pm eB$. One of the masses, μ_-, vanishes at the critical value B_c of the magnetic field (1). This mass becomes purely imaginary at $B > B_c$, thus signaling a tachyonic instability towards condensation of the transverse components of the W_μ field. The unstable eigenvector is $(W_1, W_2) = (W, -iW)/2$, where W is a scalar field.

Since we consider the solutions in the transverse (x_1, x_2) plane for the transverse components of the fields, it is natural to use complex notation for the coordinates, $z = x_1 + ix_2$, and for the vectors $\mathcal{O}_\mu = \partial_\mu, A_\mu, Z_\mu, W_\mu$ ($\mathcal{O} = \mathcal{O}_1 + i\mathcal{O}_2$, $\bar{\mathcal{O}} = \mathcal{O}_1 - i\mathcal{O}_2$) and their field strengths, $\mathcal{O}_{12} = -\frac{i}{2}(\bar{\partial}\mathcal{O} - \partial\bar{\mathcal{O}})$. Notice that $W^\dagger \neq \bar{W}$.

We use a symmetric gauge for the external magnetic field, $A_{\text{ext},1} = -Bx_2/2$ and $A_{\text{ext},2} = Bx_1/2$, so that the corresponding covariant derivative is

$$\mathcal{D}_{\text{ext}} \equiv \partial - ieA_{\text{ext}} = \partial + ezB_{\text{ext}}/2.$$

The $\bar{W}$ component of the W^- corresponds to the μ_+ eigenvalue of the operator in Eq. (6), so that it is not condensed. Thus, we put $\bar{W} = 0$ as it does not lower the energy, and continue to work in the unitary gauge $\Phi = (0, \phi)^T$, where ϕ is a real-valued field.

In complex notation the energy density is

$$\begin{aligned} E = {}& \frac{1}{2}|(\bar{\mathcal{D}} + ig\cos\theta\bar{Z})W|^2 + \frac{1}{2}Z_{12}^2 + \frac{1}{2}B^2 + \frac{g^2}{8}|W|^4 \\ &+ \frac{1}{2}\Big(-eB - g\cos\theta Z_{12} + \frac{g^2}{2}\phi^2\Big)|W|^2 \\ &+ \frac{g^2}{4\cos^2\theta}|Z|^2\phi^2 + \bar{\partial}\phi\partial\phi + \lambda(\phi - v^2/2)^2, \end{aligned} \tag{7}$$

and the equations of motion then become as follows:

$$\mathfrak{D}\bar{\mathfrak{D}}W = \Big(\frac{g^2}{2}|W|^2 - g\cos\theta Z_{12} - eF_{12} + \frac{g^2}{2}\phi^2\Big)W, \tag{8}$$

$$\bar{\mathfrak{D}}^2 W = 0, \tag{9}$$

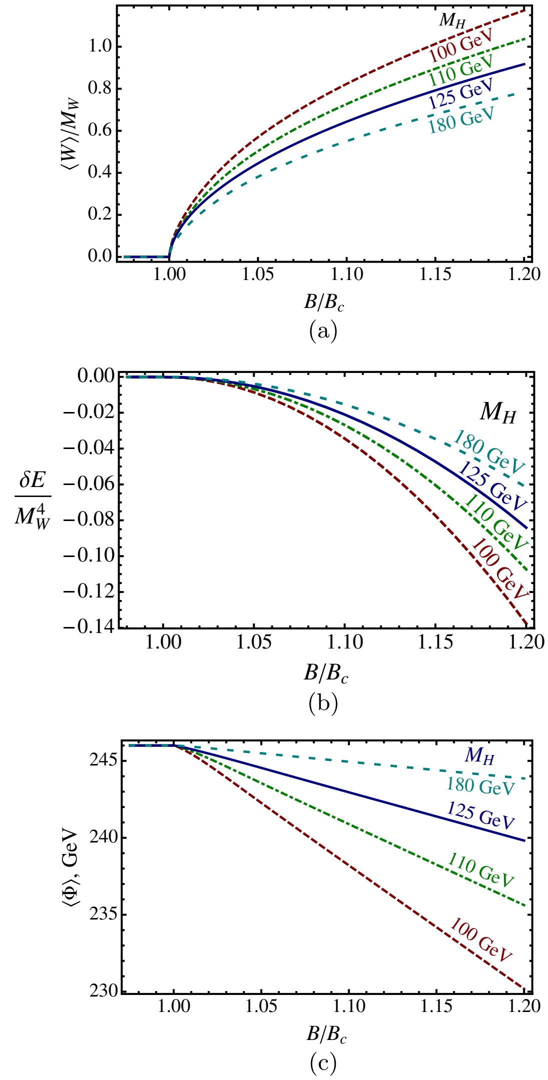

FIG. 1 (color online). (a) The cell-averaged W condensate $\langle |W|^2\rangle^{1/2}$; (b) the condensation energy density (7), $\delta E = \langle E\rangle_W - \langle E\rangle_{W=0}$; (c) the cell-averaged Higgs expectation value $\langle \Phi^\dagger \Phi\rangle^{1/2}$, Eq. (20), vs the strength of the magnetic field B (in units of the critical magnetic field $B \equiv B_c^{\mathrm{EW}}$). The plots are given for various Higgs masses M_H including the physical value of the Higgs mass (the latter are shown by the solid lines).

$$\bar\partial F_{12} = \frac{e}{2}\bar\partial |W|^2 + \frac{e}{2} W^\dagger \bar{\mathfrak{D}} W, \tag{10}$$

$$0 = \cos\theta \bar\partial F_{12} - \sin\theta \bar\partial Z_{12} + i g^2 \frac{\sin\theta}{2\cos^2\theta} \bar Z \phi^2 \tag{11}$$

$$\partial\bar\partial\phi = \frac{g^2}{4\cos^2\theta}|Z|^2\phi + \frac{g^2}{4}|W|^2\phi + 2\lambda\phi\left(\phi^2 - \frac{v^2}{2}\right), \tag{12}$$

$$0 = \phi\bar{\mathfrak{D}}W + 2W\left(\bar\partial + i\frac{g}{2\cos\theta}\bar Z\right)\phi, \tag{13}$$

where $\mathfrak{D} = \mathcal{D} + ig\cos\theta Z$ is a covariant derivative.

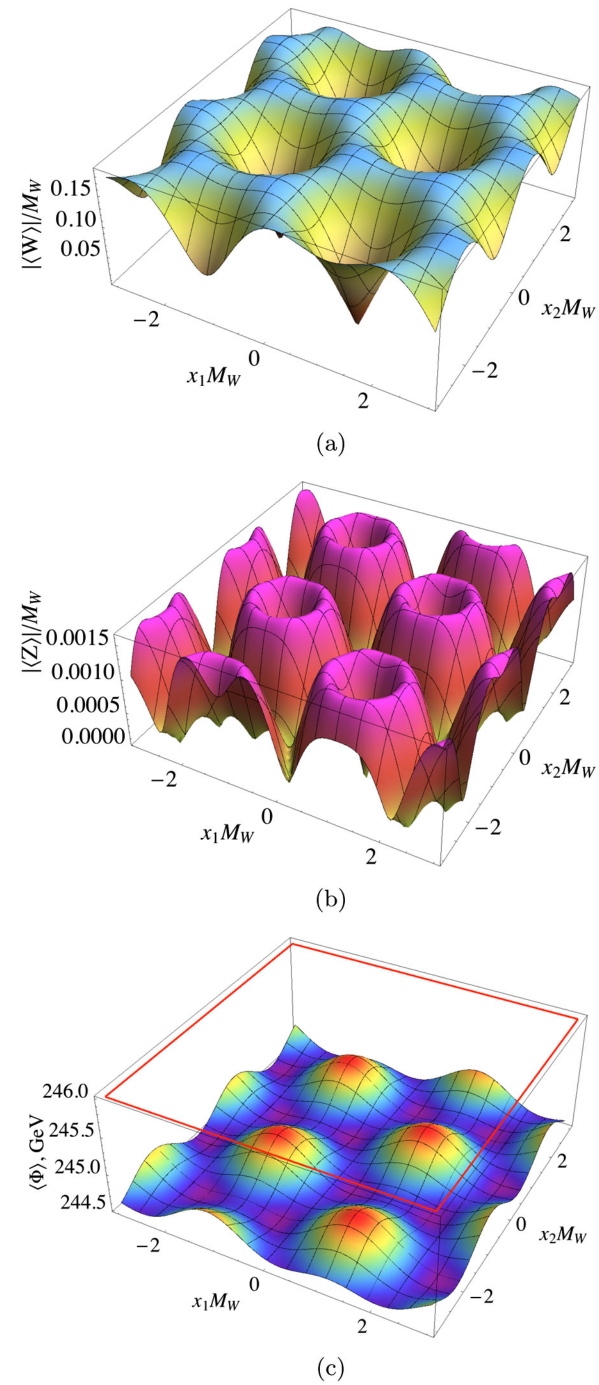

FIG. 2 (color online). (a) The superconducting W condensate (16); (b) the superfluid Z condensate (19); (c) the Higgs expectation value (20) as a function of the transverse plane coordinates x_1 and x_2 at the physical Higgs mass $M_H = 125$ GeV [21] in the background magnetic field $B = 1.01 B_c^{\mathrm{EW}}$ directed along the x_3 axis. The red line in plot (c) corresponds to the standard (coordinate-independent) Higgs expectation value, $\phi = v/\sqrt{2}$, at zero magnetic field $B = 0$.

In Ref. [14] the equations of motion were treated in the Bogomolny limit, $M_Z = M_H$, where $M_H = \sqrt{2\lambda} v$ and $M_Z = gv/(2\cos\theta)$ are the masses of the Higgs and Z bosons, respectively. Here we solve—partially following

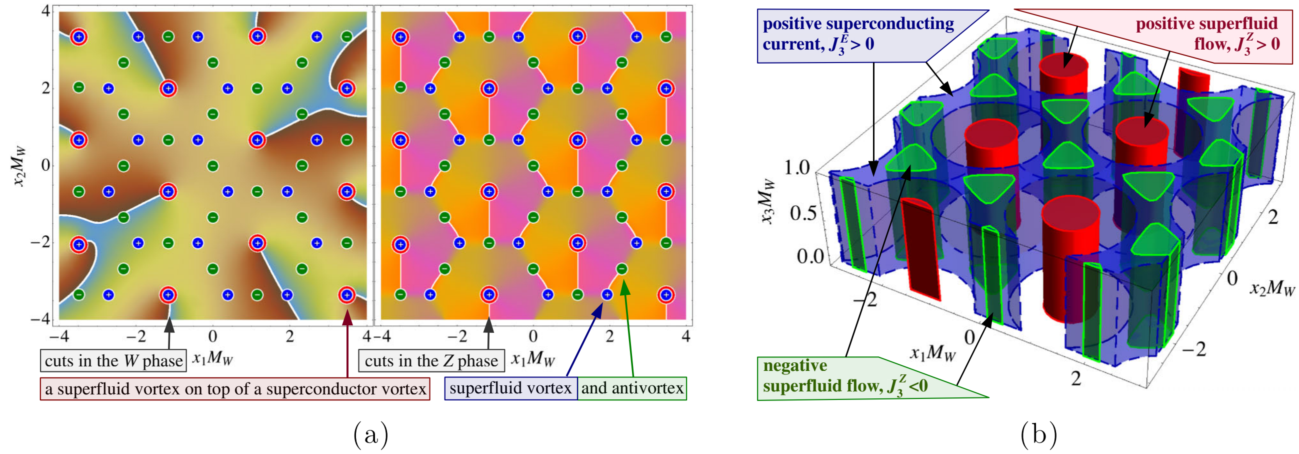

FIG. 3 (color online). Kaleidoscopic ground state. (a) The density plots of (left panel) the phases of the W condensate (16) and (right panel) the Z condensate (19) in the transversal (x_1, x_2) plane at $B = 1.01B_c^{\rm EW}$. The end points of the cuts in the phases (shown by the circles) are the superconductor vortices and superfluid (anti)vortices, respectively. (b) The three-dimensional regions of space predominantly occupied by the superconducting electric current J_3^E of the W bosons and the superfluid neutral flows J_3^Z of the Z bosons generated by a weak test electric field $E_{\rm ext} > 0$ parallel to the strong magnetic field $B = 1.01B_c^{\rm EW}$.

Ref. [17]—the equations of motion (8)–(13) for an arbitrary mass of the Higgs in the region $B \geq B_c$ near the phase transition point, $|B - B_c| \ll B_c$. The latter condition implies that the quantity

$$\epsilon = \frac{|W|}{M_W} \ll 1 \tag{14}$$

can serve as a small expansion parameter.

B. Vector-meson condensates and vortices

The combination of the two equations of motion, Eqs. (8) and (9), with the requirement of the minimization of the energy density (7), lead us to a simple Abrikosov equation,

$$\bar{\mathcal{D}}W \approx \bar{\mathfrak{D}}W = 0, \tag{15}$$

which is valid up to corrections of the order of $O(\epsilon^2)$. This equation has nontrivial periodic solutions known as Abrikosov lattices. Following Abrikosov [18], we choose a general solution of Eq. (15) as a sum over lowest Landau levels:

$$W(z) = \sum_{n\in\mathbb{Z}} C_n e^{-\frac{\pi}{2}(|z|^2+\bar{z}^2)-\pi\nu^2n^2+2\pi\nu n\bar{z}}, \tag{16}$$

where $L_B = \sqrt{2\pi/(eB)}$ is the magnetic length and ν is an arbitrary real-valued parameter. In order to ensure a regular structure of the lattice, the complex coefficients C_n are usually chosen in a periodic manner, $C_{n+N} = C_n$, where $N = 1, 2, \ldots$ is an integer number which has to be chosen using energy-minimization arguments.

The solution with $N = 1$ and $\nu = 1$ defines the square lattice of the original Abrikosov's solution [18]. However, the global energy minimum is reached for the equilateral triangular lattice at $N = 2$ with $C_1 = \pm iC_0$ and $\nu = \sqrt[4]{3}/\sqrt{2} \approx 0.9306$ in agreement with earlier studies devoted to the W condensation [14,17]. The energy density (7) is then minimized numerically with respect to the value of C_0 for fixed values of the magnetic field B and the Higgs mass M_H. This procedure allows us to determine the W condensate (16) and other interesting quantities.

At $B \geqslant B_c^{\rm EW}$ the condensation of the W bosons [Fig. 1(a)] makes the energy density smaller compared to its value in the trivial ground state [Fig. 1(b)]. Thus, the W-boson condensation is an energetically favorable state. Notice that the heavier the Higgs boson the weaker the effect of the magnetic field on the W condensate.

Equations (10) and (15) imply that the magnetic field $B = B(z) \equiv B(x_1, x_2)$ is related to the W condensate as follows:

$$\partial(B - e|W|^2/2) = 0. \tag{17}$$

This relation is valid up to $O(\epsilon^2)$ terms. The solution of Eq. (17) carrying a finite energy per unit cell $\mathcal{A}$ of the Abrikosov lattice is

$$B(z) = B_{\rm ext} + \frac{e}{2}|W(z)|^2 - \frac{e}{2}\frac{1}{{\rm Area}(\mathcal{A})}\int_{\mathcal{A}} dz d\bar{z}|W|^2, \tag{18}$$

where the integration constant in the last term (given by the integral over the unit lattice cell $\mathcal{A}$) guarantees the conservation of the magnetic flux,

$$\int_{\mathcal{A}} dz d\bar{z} B(z) = {\rm Area}(\mathcal{A}) \cdot B_{\rm ext}.$$

Thus, the magnetic field (18) becomes transversally nonuniform due to the backreaction of the inhomogeneous W condensate (16).

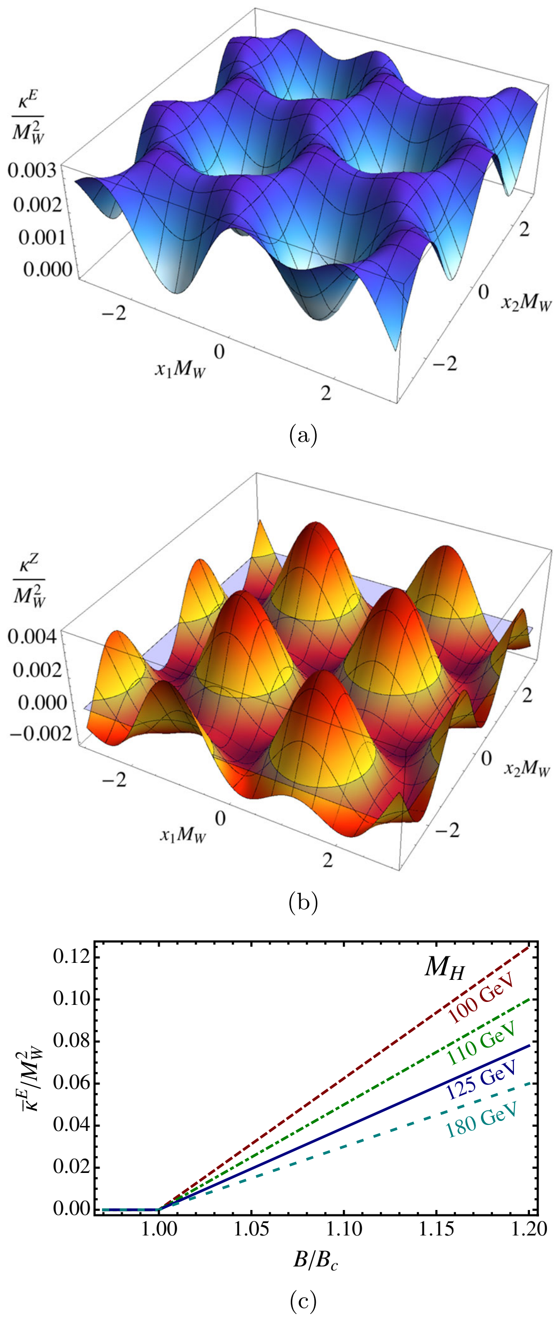

FIG. 4 (color online). (a) The superconducting (25) and (b) superfluid (26) transport coefficients as functions of the transverse plane coordinates x_1 and x_2 at the physical Higgs mass $M_H = 125$ GeV in the background magnetic field $B = 1.01 B_c^{\mathrm{EW}}$ directed along the x_3 axis. (c) The cell-averaged superconductivity transport coefficient (25) vs the magnetic field B at fixed values of the Higgs masses. The cell-averaged superfluidity coefficient is always zero.

Using the solution (18) for the magnetic field $B \equiv F_{12}$, one can solve Eqs. (11) and (12) and obtain the following nonlocal expressions for the Z and Higgs condensates, respectively:

$$Z \equiv Z_1 + iZ_2 = -i\frac{g\cos\theta}{2}\frac{\partial_1 + i\partial_2}{-\Delta + M_Z^2}|W|^2, \quad (19)$$

$$\phi = \frac{v}{\sqrt{2}}\left(1 - \frac{g^2}{4}\frac{1}{-\Delta + M_H^2}|W|^2\right). \quad (20)$$

Here $\Delta \equiv \bar{\partial}\partial = \partial_1^2 + \partial_2^2$ is the two-dimensional Laplacian in the transverse plane. The remaining equation (13) is satisfied automatically up to $O(\epsilon^2)$.

In the ground state at $B > B_c^{\mathrm{EW}}$, the W condensate (16), the Z condensate (19) and the Higgs condensate (20) are functions of the transversal coordinates x_1 and x_2, as visualized in Figs. 2(a)–2(c), respectively. The expectation value of the Higgs field falls down as the magnetic field rises, with a slope that becomes weaker as the Higgs mass increases [Fig. 2(c)].

It is known that the ground state of the vacuum at $B > B_c^{\mathrm{EW}}$ is an equilateral triangular lattice of the vortex defects in the W field [14,17] (we call these vortices the "superconductor vortices"). At the vortex positions the field $W \propto W_1^- + iW_2^-$ vanishes [Fig. 2(a)] and its phase, $\arg(W)$, winds around each vortex position. We find that the ground state has a much more complicated structure in the neutral sector: the state has an equilateral triangular lattice of the "superfluid" vortices, characterized by the vanishing field $Z \equiv Z_1 + iZ_2$ field [Fig. 2(b)] and by winding numbers in its phase.[1] The combined "kaleidoscopic" pattern of the vortex lattices, superimposed on the density plots of the phases of the superconducting W and superfluid Z fields, are shown in Fig. 3(a). Notice that certain superfluid vortices are located at the superconductor vortices.

III. NONDISSIPATIVE TRANSPORT

We point out that the ground state of the vacuum at $B > B_c^{\mathrm{EW}}$ is a "tandem" phase which is, simultaneously, an electromagnetic superconductor and a neutral superfluid. Indeed, by introducing an infinitesimally weak test electric field E^{ext} one can prove—with the use of Eq. (5)—the following transport laws for the electromagnetic and neutral Z-boson currents,

$$J_\mu^E = \partial^\nu F_{\nu\mu} \propto \frac{\delta\mathcal{L}}{\delta A^\mu}, \quad (21)$$

$$J_\mu^Z = \partial^\nu Z_{\nu\mu} \propto \frac{\delta\mathcal{L}}{\delta Z^\mu}, \quad (22)$$

respectively:

$$\partial_{[0}J_{3]}^E(x) = -\kappa^E(x_1, x_2)\cdot E_3^{\mathrm{ext}}, \qquad \partial_{[0}J_{i]}^E = 0, \quad (23)$$

$$\partial_{[0}J_{3]}^Z(x) = -\kappa^Z(x_1, x_2)\cdot E_3^{\mathrm{ext}}, \qquad \partial_{[0}J_{i]}^Z = 0, \quad (24)$$

[1] The superconductor and superfluid vortices, which are discussed in this paper, should be distinguished from the existing W and Z electroweak vortex solutions [19], including known solutions which carry electric currents along vortex cores [20].

where $i = 1, 2$. The transport parameters for the electromagnetic κ^E and neutral κ^Z currents,

$$\kappa^E(x_1, x_2) = e^2 |W|^2(x_1, x_2), \tag{25}$$

$$\kappa^Z(x_1, x_2) = -e^2 \cot\theta \frac{\Delta}{-\Delta + M_Z^2} |W|^2(x_1, x_2), \tag{26}$$

are functions of the transverse coordinates x_1 and x_2. These transport coefficients are shown in Figs. 4(a) and 4(b), respectively.

Equation (23) implies anisotropic superconductivity of the ground state at $B > B_c^{\mathrm{EW}}$ similarly to an analogous phenomenon in QCD [10,11]: a weak electric field introduces a resistance-free growth of electric current which continues streaming after the field is switched off. Equation (24) implies an anisotropic superfluidity of the neutral Z currents, and it illustrates a very unusual physical effect: an external electric field induces a current of neutral particles which are flowing frictionlessly along the magnetic field axis.

From the point of view of the electric conductivity properties, a ground state of the vacuum can either be a superconductor or an insulator due to Lorentz symmetry (indeed, a dissipative behavior, like in Ohm's Law, is inconsistent with the Lorentz symmetry of the vacuum). Thus, the absence of the electric resistance (and vanishing shear and bulk viscosities) in the $B > B_c^{\mathrm{EW}}$ phase are protected by a remnant Lorentz symmetry in the (x_0, x_3) plane. Similar Lorentz-protection arguments apply to the superfluid property as well.

The superconductivity coefficient (25), averaged over the transversal (x_1, x_2) plane,

$$\bar{\kappa}^E = \frac{1}{\mathrm{Area}(\mathcal{A})} \int_{\mathcal{A}} dx_1 dx_2 \kappa^E(x_1, x_2), \tag{27}$$

is a linearly growing function of the magnetic field B [Fig. 4(c)] at $B > B_c^{\mathrm{EW}}$. The superfluid coefficient (26) is a sign-changing function [Fig. 4(b)] of the transversal coordinates $x_{1,2}$ which has a vanishing mean value if averaged over the transversal plane ($\bar{\kappa}^Z \equiv 0$).

Thus, we conclude that a weak external electric field E_3^{ext} applied along the magnetic field in the condensed phase gives rise to

(i) a growing nonzero net electric current along the magnetic field axis, and

(ii) a neutral superfluid inhomogeneous flow in both directions with vanishing net current.

The spatial distribution of the electric and neutral currents flowing along the magnetic field axis can be read off from the corresponding superconducting coefficients in Figs. 4(a) and 4(b), respectively. The distribution of the currents in the transverse place is visualized in Fig. 3(b).

Notice that the transverse electric field $E_{1,2}^{\mathrm{ext}}$ induces neither superconducting nor superfluid currents.

IV. CONCLUSION

We have shown for the first time that the electroweak sector of the vacuum exhibits superconducting and superfluid properties due to the magnetic-field-induced condensation of, respectively, W and Z bosons provided the magnetic field exceeds the critical value (1). The superconductor-superfluid phase is characterized by the anisotropic and inhomogeneous ground state. Both charged and neutral currents may propagate nondissipatively only along the direction of the magnetic field. In the transverse directions the ground state has an unusual "kaleidoscopic" structure made of a hexagonal lattice of superfluid vortices superimposed on an equilateral triangular (hexagonal) lattice of superconductor vortices. Thus, in a strong enough magnetic field the electroweak sector of the quantum vacuum enters a superconductor-superfluid phase.

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