Auxiliary fields in open gauge theories

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We show that for open gauge theories, it is possible to build an off-shell Becchi-Rouet-Stora-Tyutin algebra together with an invariant extension of the classical action. This is based on the introduction of auxiliary fields, after having defined an on-shell invariant quantum action, where the gauge-fixing action is written as in Yang–Mills type theories up to a modified Becchi-Rouet-Stora-Tyutin operator. An application to simple supergravity is performed.

I. INTRODUCTION

The Batalin–Vilkovisky (BV) formalism [1] currently appears to be the most powerful method for quantizing general gauge theories, i.e., gauge theories which are reducible and/or with an open algebra (for a review, see Ref. [2]). It leads to the construction of the quantum theory in which effective BRST transformations are nilpotent on shell. This is realized by doubling the total number of gauge fields and ghost fields by introducing corresponding antifields, which are then eliminated by means of a gauge fermion functional containing the gauge-fixing conditions associated to all the invariances of the classical action defining the gauge theory.

On the other hand, it is well known that in supersymmetric theories, in particular for supergravity theories, which represent prototypes of open gauge theories, the superspace formalism enables one to close the gauge algebra through the introduction of auxiliary fields and then gives the possibility to quantize such theories by using the standard BRST formalism (for a review, see Ref. [3]). This is also the case of topological antisymmetric tensor gauge (BF) theories, which represent prototypes of reducible gauge theories, where auxiliary fields can be introduced in terms of a BRST superspace [4]. Let us note that the same geometrical approach has been considered in order to give another possibility leading to the standard minimal set of auxiliary fields in simple supergravity [5]. In an alternative way, it was shown that an off-shell formulation of simple supergravity in terms of a principal superfiber bundle can also be performed [6]. Moreover, still in the context of simple supergravity, the introduction of auxiliary fields can be realized via the BV formalism [7] (see also Ref. [8] for the case of reducible gauge theories).

Furthermore, we have shown for BF theories how the structure of auxiliary fields with nonvanishing ghost numbers as well as the invariant extension of the classical action come from an on-shell BRST invariant quantum action [9]. The latter was simply constructed by writing the gauge-fixing action as in Yang–Mills theories by modifying the classical BRST operator.

It would be worthwhile to extend the analysis developed in Ref. [9] in order to discuss general gauge theories, independent of the underlying classical action. In this paper we will be interested to build an on-shell invariant gauge-fixed action for irreducible open gauge theories, which permits us to introduce auxiliary fields and to determine an invariant extension of the classical action in such theories. The obtained auxiliary fields are of ghost number (-1) and thus cannot be considered classical fields. But even if these fields are objectively, at least at a formal level, different from those that appear in known open algebra theories, they have the crucial advantage that they can be introduced via a systematic procedure applied to any given gauge theory plagued with a symmetry algebra that closes only on shell. It is worth noting that a formal approach for introducing standard auxiliary fields (of ghost number zero) was also proposed in Ref. [10].

As an application we focus on the case of simple supergravity for which the introduced method to build up an off-shell BRST operator leads to a set of nonclassical auxiliary bosonic spinor fields of ghost number -1.

II. ON-SHELL FORMULATION

Let us start from a gauge system described by a classical action S^{cl} depending on the gauge fields ϕ^i with parity ϵ_i. The invariance of S^{cl} under gauge transformations, $\delta\phi^i = R^i_\alpha \varepsilon^\alpha$, leads to the Noether identities, $S^{\text{cl}}_{,i} R^i_\alpha = 0$, where $X_{,i}$ denotes the variation of X with respect to ϕ^i. The generators R^i_α are operators acting on the gauge parameters ε^α with parity ϵ_α. The gauge functions and their associated gauge equations which characterize the gauge algebra depend on the nature of the gauge theory [1,2]. In the following we restrict ourselves to irreducible open gauge theories, i.e., all generators R^i_α are independent, and the commutator of two gauge transformations leads to the definition of two gauge functions $G^\alpha_{\beta\gamma}$ and $G^{ij}_{\alpha\beta}$, satisfying $R^i_{\alpha,j} R^j_\beta - (-1)^{\epsilon_\alpha \epsilon_\beta} R^i_{\beta,j} R^j_\alpha = R^i_\gamma G^\gamma_{\alpha\beta} + S^{\text{cl}}_{,j} G^{ij}_{\alpha\beta}$, which means that the commutator is closed up to equations of motion. Upon introducing the ghost c^α with parity $(\epsilon_\alpha + 1)$, the above commutator can be put in the following form:

$$G^i_{,j}G^j + G^i_{,\alpha}G^\alpha + S^{cl}_{,j}G^{ij} = 0, \qquad (1)$$

where $G^i = R^i_\alpha c^\alpha$, $G^\alpha = \frac{1}{2}(-1)^{\epsilon_\beta}G^\alpha_{\beta\gamma}c^\gamma c^\beta$, $G^{ij} = \frac{1}{2}(-1)^{\epsilon_\alpha}G^{ij}_{\alpha\beta}c^\beta c^\alpha$, and $X_{,\alpha}$ denotes the variation of X with respect to c^α.

The classical BRST transformations of the fields ϕ^i are simply obtained as usual by replacing ε^α by c^α; we have

$$Q\phi^i = G^i. \qquad (2)$$

In view of Eq. (1), the on-shell nilpotency of Q acting on ϕ^i, i.e., $Q^2\phi^i = -S^{cl}_{,j}G^{ij}$, is ensured provided that

$$Qc^\alpha = G^\alpha. \qquad (3)$$

Furthermore, to express the on-shell nilpotency of Q acting on c^α, a new gauge function $G^{\alpha i}$ is also needed, in order to write $Q^2 c^\alpha = -S^{cl}_{,i}G^{\alpha i}$, and according to Eq. (3), we obtain the following gauge equation:

$$G^\alpha_{,i}G^i + G^\alpha_{,\beta}G^\beta + S^{cl}_{,i}G^{\alpha i} = 0. \qquad (4)$$

It is the Jacobi identity which leads to the definition of $G^{\alpha i}$ as well as of a second new gauge function G^{ijk}. Besides G^α, G^{ij} ($G^{\alpha i}$, G^{ijk}), which are quadratic (cubic) in c^α, it is possible to introduce other gauge functions G^{ijkl}, $G^{\alpha ij}, \ldots$, which are higher-order polynomials in c^α, by using higher-order commutators of the generators R^i_α [2]. Let us note that in the realm of the BV formalism, the gauge algebra is also generated by the classical master equation [1,2].

We shall mention that the known open gauge theories (e.g., supergravity theories) are described by a gauge algebra in which the set of gauge functions contains only G^α, G^{ij}, and $G^{\alpha i}$, and all the remaining gauge functions $G^{ijk}, G^{ijkl}G^{\alpha ij}, \ldots$, vanish. Thus, for simplicity and to present computations leading to insight in the generalization of the analysis in Ref. [9] (see also Ref. [6]) to open gauge theories, we consider an open gauge algebra that is characterized by the three nonvanishing gauge functions G^α, G^{ij}, and $G^{\alpha i}$. In addition to Eqs. (1) and (4), new identities need to be satisfied. The latter follow from the higher-order gauge equations [1,2], in which we take off the vanishing gauge functions; we find the following nontrivial identities:

$$G^{ij}_{,k}G^k + G^{ij}_{,\alpha}G^\alpha - (-1)^{\epsilon_j}\{G^i_{,k}G^{kj} + G^i_{,\alpha}G^{\alpha j}\}$$
$$+ (-1)^{\epsilon_j(\epsilon_i+1)}\{G^j_{,k}G^{ki} + G^j_{,\alpha}G^{\alpha i}\} = 0, \qquad (5)$$

$$G^{ij}_{,l}G^{lk} + G^{ij}_{,\alpha}G^{\alpha k} + (-1)^{\epsilon_k(\epsilon_i+\epsilon_j)}\{G^{jk}_{,l}G^{li} + G^{jk}_{,\alpha}G^{\alpha i}\}$$
$$+ (-1)^{\epsilon_j(\epsilon_i+\epsilon_k)}\{G^{ki}_{,l}G^{lj} + G^{ki}_{,\alpha}G^{\alpha j}\} = 0. \qquad (6)$$

In what follows we turn to discuss how to construct the quantum theory of an open gauge theory characterized by a classical BRST algebra given by Eqs. (1)–(6). It is obvious that a Q-exact form of the gauge-fixing action cannot be suitable to build the full invariant quantum action because of the on-shell nilpotency of the classical BRST operator

Q. To this end, we generalize the prescription discussed in Ref. [9] by simply modifying the classical BRST operator Q. Accordingly, the gauge-fixing action written as in Yang-Mills type theories must also be modified so that the complete quantum action becomes invariant on shell. We first introduce a gauge fermion ψ to implement gauge constraints, $F^\alpha = 0$, associated to all the invariances of the classical action S^{cl}; we have

$$\psi = \bar{c}_\alpha F^\alpha, \qquad (7)$$

where $\bar{c}_\alpha$ represent the antighosts, which allow us as usual to define the Stueckelberg auxiliary fields b_α through the action of Q, so that

$$Q\bar{c}_\alpha = b_\alpha, \qquad Qb_\alpha = 0. \qquad (8)$$

Let us note that the gauge-fixing functions F^α may depend only on the gauge fields ϕ^i, since the gauge symmetries are considered to be irreducible.

So, at the quantum level, we define a modified BRST operator Δ,

$$\Delta = Q + \tilde{Q}, \qquad (9)$$

satisfying $\Delta^2 = 0$, up to equations of motion, and $\Delta S^q = 0$, where S^q is the quantum action. As discussed above, the gauge-fixing action S^{gf} in S^q cannot be cast in the form $S^{gf} = Q\psi$. To rectify this we modify S^{gf} so that Q is replaced by $(Q + x\tilde{Q})$; i.e., we have

$$S^q = S^{cl} + (Q + x\tilde{Q})\psi. \qquad (10)$$

We remark that $\tilde{Q}$ has vanishing action on the pairs $(\bar{c}_\alpha, b_\alpha)$. This simply follows from Eq. (8), which says that the nilpotency on those fields is already guaranteed. To derive the action of $\tilde{Q}$ on the fields ϕ^i and c^α, we use the structure of the open gauge algebra together with the invariance of the quantum action S^q, as written in Eq. (10), under the on-shell nilpotent quantum BRST operator, as defined in Eq. (9).

In view of the on-shell nilpotency of the classical BRST operator Q, the variation of S^q under the quantum BRST Δ can be written as

$$\Delta S^q = S^{cl}_{,i}(\tilde{Q}\phi^i - (-1)^{\epsilon_i(\epsilon_j+1)}\psi_{,j}G^{ji})$$
$$+ (\tilde{Q}Q + xQ\tilde{Q} + x\tilde{Q}^2)\psi. \qquad (11)$$

To guarantee the invariance of S^q under Δ, we note that by choosing

$$\tilde{Q}\phi^i = -(-1)^{\epsilon_i}\psi_{,j}G^{ij}, \qquad (12)$$

the first term on the right-hand side of Eq. (11) vanishes. Substituting Eq. (12) into Eq. (11) and using the identity given by Eq. (5), we get

$$\Delta S^q = \psi_{,j} R_\alpha^j \{\tilde{Q} c^\alpha - (-1)^{(\epsilon_\alpha+1)(\epsilon_i+1)} \psi_{,i} G^{\alpha i}\}$$
$$+ (-1)^{\epsilon_j} x \psi_{,j} \psi_{,i} \{(-1)^{\epsilon_k+1} G^{ji}_{,k} \psi_{,l} G^{lk}$$
$$+ G^{ji}_{,\alpha} \tilde{Q} c^\alpha\} + (-1)^{\epsilon_i} (1-2x)\{(-1)^{\epsilon_\alpha} b_\alpha F^\alpha_{,j} \psi_{,i} G^{ji}$$
$$+ (-1)^{\epsilon_k} \psi_{,i} \psi_{,j} \psi_{,k} G^{jk} G^i\}$$
$$+ (-1)^{\epsilon_j} \left(x - \frac{1}{2}\right) \psi_{,j} \psi_{,i} \{G^{ji}_{,k} G^k + G^{ji}_{,\alpha} G^\alpha\}. \quad (13)$$

Further, we learn from Eq. (13) that by taking

$$\tilde{Q} c^\alpha = (-1)^{(\epsilon_\alpha+1)(\epsilon_i+1)} \psi_{,i} G^{\alpha i}, \quad (14)$$

$$x = \frac{1}{2}, \quad (15)$$

the Δ-invariance of S^q is completely ensured. We remark that the vanishing of the second term on the right-hand side of Eq. (13) follows from the use of the identity given by Eq. (6).

Thus, we have obtained the full quantum action S^q,

$$S^q = S^{\text{cl}} + \frac{1}{2} (-1)^{\epsilon_i} \psi_{,i} \psi_{,j} G^{ij} + \Delta \psi, \quad (16)$$

invariant under the BRST operator Δ determined by Eqs. (9), (12), and (14) together with Eqs. (2), (3), and (8), which is nilpotent on shell. In fact, after a similar straightforward computation, we get

$$\Delta^2 \phi^i = -S^q_{,j} G^{ij} - S^q_{,\alpha} G^{\alpha i}, \quad (17)$$

$$\Delta^2 c^\alpha = -S^q_{,i} G^{\alpha i}. \quad (18)$$

It is remarkable that the used prescription, which simply consists in the modification of the classical BRST operator and of the gauge-fixing action written as in Yang–Mills theories, provides an on-shell quantization, where in particular the quantum action contains four-ghost couplings. The latter are characteristic for open gauge theories like supergravity theories [3].

III. AUXILIARY FIELDS

Let us now discuss how we can introduce auxiliary fields, as a generalization of the approach developed in Ref. [9], so that we end up with an off-shell structure for open gauge theories. For this purpose, we start with the following BRST transformations:

$$\Delta \phi^i = G^i - (-1)^{\epsilon_i \epsilon_j} G^{ij} \eta_j, \quad (19)$$

$$\Delta c^\alpha = G^\alpha + G^{\alpha i} \eta_i, \quad (20)$$

$$\Delta \bar{c}_\alpha = b_\alpha, \qquad \Delta b_\alpha = 0. \quad (21)$$

These follow from those which are nilpotent on shell by replacing $\psi_{,j}$ by η_j. Making the same replacement in Eq. (16), we put the quantum action S^q in the form

$$S^q = S^{\text{cl}} + \frac{1}{2} (-1)^{\epsilon_j} G^{ij} \eta_i \eta_j + \Delta \psi. \quad (22)$$

At this point, by assuming that the η_i are now true fields of parity $(\epsilon_i + 1)$ and ghost number (-1), it is worth noting that the quantum action S^q allows us to see that they are auxiliary, nondynamical fields as their equations of motion are constraints,

$$(-1)^{\epsilon_j(\epsilon_i+1)} G^{ij} (\psi_{,j} - \eta_j) = 0. \quad (23)$$

Indeed, the only terms of the quantum action contributing to the equations of motion of the fields η_i are $(\frac{1}{2}(-1)^{\epsilon_j(\epsilon_i+1)} G^{ij} \eta_i \eta_j)$ and $(-(-1)^{\epsilon_j(\epsilon_i+1)} G^{ij} \psi_{,i} \eta_j)$. The last term follows from the gauge-fixing action $\Delta \psi$ by using the transformations of Eq. (19). However, substituting Eq. (23) into Eqs. (19)–(22), which is equivalent to replace η_i by $\psi_{,i}$, again we obtain the quantum action and its on-shell BRST symmetry.

Further, we shall determine the action of the BRST operator on these auxiliary fields, so that the BRST algebra closes off shell. This is simply realized by imposing the off-shell nilpotency condition $\Delta^2 = 0$. So, we obtain

$$\Delta \eta_i = -(-1)^{\epsilon_j(\epsilon_i+1)} G^j_{,i} \eta_j - (-1)^{\epsilon_i} S^{\text{cl}}_{,i}. \quad (24)$$

It is easy to check the off-shell nilpotency of Eqs. (19) and (20) by an explicit calculation. We note, in particular, that in deriving Eq. (24), we have used the identities given by Eqs. (5) and (6), which can be cast in the following form:

$$(-1)^{\epsilon_j(\epsilon_i+1)} \eta_j \eta_i \left(\frac{1}{2} G^{ij}_{,k} G^k + \frac{1}{2} G^{ij}_{,\alpha} G^\alpha - (-1)^{\epsilon_j} G^i_{,k} G^{kj}\right.$$
$$\left. - (-1)^{\epsilon_j} G^i_{,\alpha} G^{\alpha j}\right) = 0, \quad (25)$$

$$(-1)^{\epsilon_j(\epsilon_i+1)} \eta_k \eta_j \eta_i \left(\frac{1}{2} G^{ij}_{,l} G^{lk} + \frac{1}{2} G^{ij}_{,\alpha} G^{\alpha k}\right) = 0. \quad (26)$$

Moreover, after a similar straightforward calculation, we find that

$$S_{\text{inv}} = S^{\text{cl}} + \frac{1}{2} (-1)^{\epsilon_j} G^{ij} \eta_i \eta_j \quad (27)$$

represents the Δ-invariant extension of the classical action S^{cl}.

IV. CASE OF $D = 4$, $N = 1$ SUPERGRAVITY

The physical fields content of simple supergravity [3] is given by the vierbein e_μ^a, and the gravitino ψ_μ^A with $a = 1, \ldots, 4$ label the flat Minkowski space, $\mu = 1, \ldots, 4$ labels the curved Riemannian space, and $A = 1, \ldots, 4$ is related to the $N = 1$ supersymmetry. The classical action of the model is given by

$$S^{\text{cl}} = \frac{1}{2} e e_a^\mu e_b^\nu R_{\mu\nu}^{ab} - \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu S_{\rho\sigma}, \quad (28)$$

where $e = \det(e_\mu^a)$, $R_{\mu\nu}^{ab} = \partial_\mu \omega_\nu^{ab} + \frac{1}{2}(\omega_\mu^{ad}\omega_{\nu d}^b - \omega_\mu^{bd}\omega_{\nu d}^a) - (\mu \leftrightarrow \nu)$ is the Lorentz curvature, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, $S_{\rho\sigma} = \partial_\rho \psi_\sigma + \frac{1}{2}\omega_\rho^{ab}\sigma_{ab}\psi_\sigma - (\rho \leftrightarrow \sigma)$ with $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ is the Fermi curvature, $\bar{\psi}_\nu = \psi_\nu^T C$, C is the charge conjugation matrix, $\gamma_\nu = e_\nu^a \gamma_a$, γ_a, are the Dirac matrices. One recalls that the theory admits a vanishing torsion leading to a nonpropagating spin connection ω_μ^{ab}, which therefore can be expressed in terms of e_μ^a and ψ_μ^A, i.e., $\omega_\mu^{ab} = \frac{1}{2}e^{\nu b}[\{\partial_\nu e_\mu^a - \frac{1}{4}\bar{\psi}_\nu\gamma^a\psi_\mu - (\mu \leftrightarrow \nu)\} + e_\mu^d e^{\rho a}\{\partial_\nu e_{\rho a} - \frac{1}{4}\bar{\psi}_\nu\gamma_d\psi_\rho - (\nu \leftrightarrow \rho)\}]$; e_a^μ is the inverse of the vierbein defined by $e_a^\mu e_\nu^a = \delta_\nu^\mu$ and $e_a^\mu e_\mu^b = \delta_a^b$.

The symmetries of the model are diffeomorphism (general coordinates transformations), Lorentz rotations, and supersymmetry. So the action (28) is invariant under the transformation [3] (expressed *à la* BRST)

$$Q e_\mu^a = c^\nu \partial_\nu e_\mu^a + \partial_\mu c^\nu e_\nu^a - c^{ab}e_{\mu b} + \frac{1}{2}\bar{c}\gamma^a\psi_\mu,$$

$$Q\psi_\mu = c^\nu \partial_\nu \psi_\mu + \partial_\mu c^\nu \psi_\nu - \frac{1}{2}c^{ab}\sigma_{ab}\psi_\mu + \partial_\mu c$$

$$+ \frac{1}{2}\omega_\mu^{ab}\sigma_{ab}c, \tag{29}$$

where c^ν, c^{ab}, and c are, respectively, the ghost fields associated to local diffeomorphism, Lorentz rotations, and supersymmetry parameters. These ghost fields are all of ghost number $+1$; c^ν and c^{ab} are fermionic while c is bosonic. One can complete the action of the BRST operator (29) on the ghosts fields by

$$Q c^\mu = c^\nu \partial_\nu c^\mu - \frac{1}{4}\bar{c}\gamma^\mu c,$$

$$Q c^{ab} = c^\nu \partial_\nu c^{ab} - c^{ad}c_d^b + \frac{1}{4}\bar{c}\gamma^\mu c\omega_\mu^{ab}, \tag{30}$$

$$Q c = c^\nu \partial_\nu c - \frac{1}{2}c^{ab}\sigma_{ab}c + \frac{1}{4}\bar{c}\gamma^\mu c\psi_\mu,$$

leading to the following on-shell property of the BRST operator Q:

$$Q^2 \psi_\mu = -G_{\mu\nu}\frac{\delta S_0}{\delta\bar{\psi}_\nu}, \tag{31}$$

$$Q^2 c^{ab} = -G_\mu^{ab}\frac{\delta S_0}{\delta\bar{\psi}_\mu}, \tag{32}$$

$$Q^2 X = 0 \quad \text{for all others fields,} \tag{33}$$

where the equation of motion of the gravitino reads

$$\frac{\delta S_0}{\delta\bar{\psi}_\mu} = -\frac{1}{2e}\varepsilon^{\mu\nu\rho\lambda}\gamma_5\gamma_\nu S_{\rho\lambda}. \tag{34}$$

This on-shell structure follows easily from the open structure of the superalgebra of simple supergravity. The nonclosure gauge functions $G_{\mu\nu}$ and G_μ^{ab} can be

straightforwardly computed from Eqs. (29) and (30) and are given by [6]

$$G_{\mu\nu} = -\frac{1}{8}(\bar{c}\gamma_a c)\left(\frac{1}{4}e_\mu^b e_{\nu b}\gamma^a - \frac{1}{2}ee^{\rho a}\varepsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau\right)$$

$$-\frac{1}{8}(\bar{c}\sigma_{ab}c)\left(e_\mu^a e_\nu^b + \frac{1}{2}e_\mu^c e_{\nu c}\sigma^{ab}\right)$$

$$-\frac{1}{2}e\varepsilon_{\mu\nu\rho\tau}e^{\rho a}e^{\tau b}\gamma_5\Big), \tag{35}$$

$$G_\mu^{ab} = -\frac{1}{8}(\bar{c}\gamma_\mu\sigma^{ab}\gamma_5 c)\bar{c}\gamma_5. \tag{36}$$

This situation fits with the general framework presented in Sec. II. In this context one has to point out that these nonclosure gauge functions are related upon identities of type (5) and (6) (see Ref. [6]). Note that in the case of simple supergravity, the gauge functions $G_{\mu\nu}$ and G_μ^{ab} can be related linearly by the relation

$$G_\mu^{ab} = \frac{1}{2}e^{\rho a}e^{\nu b}\bar{c}\gamma_\rho G_{\nu\mu}, \tag{37}$$

which simply follows from the identity $G_{\mu\nu}\gamma_\rho c = \frac{1}{8}e\varepsilon_{\mu\nu\rho\tau}(\bar{c}\gamma^\tau c)\gamma_5 c$. Thus, all the nonclosure gauge functions and related equations can be written in terms of $G_{\mu\nu}$; for example, Eq. (5) can be recast as

$$QG_{\mu\nu} = c^\rho \partial_\rho G_{\mu\nu} + \partial_\mu c^\rho G_{\rho\nu} + \partial_\nu c^\rho G_{\mu\rho}$$

$$-\frac{1}{2}c^{ab}[\sigma_{ab}, G_{\mu\nu}] + \frac{1}{2}\bar{c}\gamma^\rho\psi_\rho G_{\mu\nu}$$

$$+\frac{1}{2}\bar{c}\gamma^\rho\psi_\mu G_{\rho\nu} + \frac{1}{2}\bar{c}\gamma^\rho\psi_\nu G_{\mu\rho}, \tag{38}$$

which gives, by the way, the BRST transformation of the gauge function $G_{\mu\nu}$. This latter equation holds of shell (without relying on the gravitino equation of motion) and thus indicates that no higher-order gauge functions exist.

We are now able to apply the prescription presented in Sec. III in order to introduce a suitable set of auxiliary fields allowing the construction of an off-shell BRST operator. We first observe that the only nonvanishing gauge functions are $G_{\mu\nu B}^A$ and $G_{\mu A}^{ab}$ (fermionic indices A and B are now exhibited). Thus, the off-shell BRST operator defined by Eqs. (19) and (20) as well as the definition of the extended invariant action (27) allow us to introduce the set of 16 bosonic auxiliary fields $\eta^{\mu A}$ of ghost number -1. In view of the prescription introduced in Sec. III, these auxiliary fields are related to the gravitino degrees of freedom. No auxiliary fields related to the vierbein (or graviton) may occur in this approach. The off-shell BRST transformations read then

$$\Delta e_\mu^a = Q e_\mu^a, \qquad \Delta\psi_\mu^A = Q\psi_\mu^A + G_{\mu\nu B}^A\eta^{\nu B}, \tag{39}$$

$$\Delta c^{\mu} = Q c^{\mu}, \qquad \Delta c^{ab} = Q c^{ab} + G^{ab}_{\mu A} \eta^{\mu A}, \qquad \Delta c = Q c, \tag{40}$$

$$\Delta \eta^{\mu A} = c^{\nu} \partial_{\nu} \eta^{\mu A} + \partial^{\mu} c_{\nu} \eta^{\nu A} - \frac{1}{2} c^{ab} \sigma^{A}_{abB} \eta^{\mu B}$$
$$- \frac{1}{2e} \varepsilon^{\mu \nu \rho \sigma} (\gamma_5 \gamma_{\nu})^{A}{}_{B} S^{B}_{\rho \sigma}, \tag{41}$$

ensuring that $\Delta^2 = 0$ on all fields. Moreover, the extended action

$$S_{\mathrm{inv}} = \frac{1}{2} e e^{\mu}_a e^{\nu}_b R^{ab}_{\mu \nu} - \frac{1}{4} \varepsilon^{\mu \nu \rho \sigma} \bar{\psi}_{\mu} \gamma_5 \gamma_{\nu} S_{\rho \sigma} - \frac{e}{2} \bar{\eta}^{\mu} G_{\mu \nu} \eta^{\nu} \tag{42}$$

is, up to a divergence term, invariant upon the action of Δ. One can also see that the fields η^{ν} are clearly nonpropagating fields since their equations of motion are purely algebraic, i.e., $\delta S_{\mathrm{inv}} / \delta \bar{\eta}^{\mu} = 0$ leads to $G_{\mu \nu} \eta^{\nu} = 0$.

V. CONCLUSION

To conclude, in the present paper, we have given a prescription leading to the construction of an off-shell BRST invariant quantum action for irreducible open gauge theories [1,2] described by a gauge algebra with vanishing higher-order gauge functions. We first obtained an on-shell BRST invariant quantum action containing four-ghost interaction terms typical for open gauge theories as in supergravity theories. This follows from a gauge -fixing action written as in Yang–Mills-type theories by modifying the classical BRST operator . We then used a trick that permits us to introduce auxiliary fields through the variation of the gauge-fixing fermion with respect to the gauge fields, as turned out to be possible in BF theory [9]. Thus, we arrived at a closed BRST algebra together with an off-shell invariant full quantum action in which, particularly, the invariant extension of the classical action arose from the quartic ghost interaction terms.

As an application we show what our proposed construction gives in the case of simple supergravity. The main result is that an off-shell realization can be achieved upon the introduction of 16 auxiliary bosonic spinor variables of ghost number -1. However, it is worthwhile to mention that the obtained set of auxiliary fields are nonclassical fields, not only in the sense that they are of nonzero ghost number but also because they do not balance the bosonic and fermionic degrees of freedom of shell. Indeed, with 16 bosonic auxiliary fields $\eta^{\mu A}$, one ends up at the off-shell level with 10 extra bosonic degrees of freedom of ghost number -1, while, as usual, the bosonic and fermionic degrees of freedom balance on shell. This can be compared with the already-known off-shell formulations of simple supergravity, which are, namely, minimal, new minimal, and non minimal (for a review, see Ref. [11]). Even if these formulations differ in their auxiliary fields structures, the total number of fermionic and bosonic degrees of freedom balances at both on-shell and off-shell levels. One may note that it is conceivable that the auxiliary fields introduced in this paper may be turned into a set of ''classical'' zero ghost bosonic fields by taking advantage of a kind of twist redefinition [12]; see also Ref. [13] and references therein. This kind of technique is (for instance) used in the context of superstring theory, where auxiliary bosonic spinor variables can be treated as ghosts (or antighosts) in the frame of the pure spinor approach by N. Berkovits; see, e.g., Ref. [14] and references therein. If such an approach might work, one should find some relations between the obtained auxiliary fields in order to reduce them to the usual auxiliary fields sets of the known off-shell formulations of simple supergravity. Such a construction will be analyzed in details elsewhere.

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