PHYSICAL REVIEW D **88,** 064038 (2013)

New synthesis of matter and gravity: A nongravitating scalar field

Inyong Cho[1,*] and Hyeong-Chan Kim[2,†]

[1]*Institute of Convergence Fundamental Studies and School of Liberal Arts, Seoul National University of Science and Technology, Seoul 139-743, Korea*

[2]*School of Liberal Arts and Sciences, Korea National University of Transportation, Chungju 380-702, Korea*

(Received 14 February 2013; revised manuscript received 2 April 2013; published 19 September 2013)

We present a new manifestation of the nonlinearity of gravity-matter interactions. We show explicitly that there exists a nongravitating dynamical scalar field solution in Eddington-inspired Born-Infeld gravity. This kind of solution has not been found in previous literature based on general relativity or other modified gravity theories. The perturbation analysis shows that the solution is a late-time attractor if the scalar field rolls down the potential. This indicates that there are two weak-gravity regimes in the theory—one in the general relativity regime, and the other in the Eddington regime.

DOI: 10.1103/PhysRevD.88.064038 PACS numbers: 04.50.−h, 98.80.−k

Matter interacts with gravity closely. In this interaction, the mutual roles of matter and gravity can be summarized by the words of J. A. Wheeler: ‘‘Spacetime tells matter how to move; matter tells spacetime how to curve.’’ However, looking closely into the solution space of general relativity (GR), one may find that the role played by gravity is not exactly the same as that played by matter as a generator of gravity, especially in the vacuum sector. In GR, the nonlinearity of gravity allows curved spacetimes in the absence of matter such as the gravitational soliton, gravitational instanton, Bianchi solutions, etc. These solutions play a crucial role in understanding the Einstein gravity. Contrary to that, GR does not allow matter dynamics in the absence of gravity. Several generalized gravity theories appear in the literature such as the scalar-tensor theory, $f(R)$ gravity, massive gravity, and Gauss-Bonnet gravity. However until now, no gravity theory is known to have the nonlinear interaction allowing a dynamical matter field without developing gravity. Therefore, it is worthwhile to investigate nongravitating matter distributions (nGMDs) via nonlinear interactions between the matter and gravity in newly suggested gravity theories.

In gravity theories modified from the Einstein-Hilbert action by adding higher-curvature terms, one can easily notice that the nGMDs may not exist. The equation of motion (EOM) for the action will take the form

$$\sum_{n=0} c_n(\mathcal{R}^n)_{\mu\nu} = T_{\mu\nu},$$

where $\mathcal{R}^n$ represents the every-nth-order polynomial of curvatures. Let us consider the regular situations when the curvatures vanish. In the case of $c_0 = 0$, one may easily notice that the zero curvature simply leads to zero stress tensor $T_{\mu\nu} = 0$. The stress tensor corresponding to c_0 plays the role of the cosmological constant, which implies the matter field is homogeneous and nondynamical. Therefore, we may conclude that the gravity theory allowing nGMDs should be based on a different starting point rather than simply including higher-curvature terms.

In this paper, we show that the Eddington-inspired Born-Infeld (EiBI) theory, which was suggested as an alternative theory of gravity recently [1], allows nGMDs. The EiBI action is given by

$$S_{\rm EiBI} = \frac{1}{\kappa}\int d^4x\left[\sqrt{-|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{-|g_{\mu\nu}|}\right] + S_M(g,\Phi), \tag{1}$$

where $|\mathcal{G}_{\mu\nu}|$ denotes the determinant of $\mathcal{G}_{\mu\nu}$, λ is a dimensionless parameter related with the cosmological constant by $\Lambda = (\lambda - 1)/\kappa$, and we set $8\pi G = 1$. In this theory, the metric $g_{\mu\nu}$ and the connection $\Gamma^\rho_{\mu\nu}$ are treated as independent fields (Palatini formalism). The Ricci tensor $R_{\mu\nu}(\Gamma)$ is evaluated solely by the connection, and the matter field Φ is coupled only to the gravitational field $g_{\mu\nu}$. The merits of this theory are that it requires only one more theory parameter κ, and that it is equivalent to the theory of GR in vacuum.

In Refs. [1,2], the evolution of the Universe driven by barotropic fluid is investigated in EiBI gravity. For the equation-of-state parameter, $w \equiv P/\rho > 0$, the Universe starts from a nonsingular initial state of a finite size for $\kappa > 0$. More interestingly, the initial state of the Universe driven by pressureless dust ($w = 0$) approaches the de Sitter state with the effective cosmological constant $\Lambda_{\rm eff} = 8/\kappa$ [2]. Subsequent works in EiBI gravity have been performed on the subjects of the cosmological and astrophysical constraints on the EiBI theory [3,4], the constraint on the value of κ by using the solar model [5], the tensor perturbation [6], bouncing cosmology [7], the five-dimensional brane model [8], the effective stress tensor and energy conditions [9], cosmology with scalar fields [10], the instability of compact stars [11], the surface singularity of the compact star [12], etc. Recently, the chaotic inflation [13] and the metric perturbations [14] based on EiBI gravity have been studied.

*iycho@seoultech.ac.kr
†hckim@ut.ac.kr

The equations of motion are obtained by varying the action in Eq. (1) with respect to the fields $g_{\mu\nu}$ and $\Gamma^{\rho}_{\mu\nu}$, respectively:

$$\frac{\sqrt{-|q|}}{\sqrt{-|g|}} q^{\mu\nu} = \lambda g^{\mu\nu} - \kappa T^{\mu\nu} \tag{2}$$

and

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \tag{3}$$

where $q_{\mu\nu}$ is the auxiliary metric by which the connection $\Gamma^{\rho}_{\mu\nu}$ is defined, and $q^{\mu\nu}$ is the matrix inverse of $q_{\mu\nu}$. The energy-momentum tensor is given by the usual sense, $T^{\mu\nu} = (2/\sqrt{-|g|})\delta L_M/\delta g_{\mu\nu}$.

In this paper, we investigate the dynamics of a homogeneous scalar field in EiBI gravity. The action for the matter field is then given by

$$S_M = \int d^4x \sqrt{-|g|}\left[-\frac{1}{2} g_{\mu\nu}\partial^{\mu}\phi\partial^{\nu}\phi - V(\phi)\right], \tag{4}$$

where the scalar field depends only on time, $\phi(t)$, because of the spatial homogeneity. The homogeneous and isotropic *Ansätze* for the metric and auxiliary metric are

$$\begin{aligned} g_{\mu\nu}dx^{\mu}dx^{\nu} &= -dt^2 + a^2(t)d\mathbf{x}^2, \\ q_{\mu\nu}dx^{\mu}dx^{\nu} &= -X^2(t)dt^2 + Y^2(t)d\mathbf{x}^2. \end{aligned} \tag{5}$$

From the EOM of the first kind [Eq. (2)], we obtain the auxiliary metric in terms of physical parameters,

$$\begin{aligned} X &= (\lambda - \kappa p)^{3/4}(\lambda + \kappa\rho)^{-1/4}, \\ Y &= [(\lambda + \kappa\rho)(\lambda - \kappa p)]^{1/4} a, \end{aligned} \tag{6}$$

where $\rho = \dot{\phi}^2/2 + V$ and $p = \dot{\phi}^2/2 - V$. From the components of the EOM of the second kind [Eq. (3)], we obtain the Hubble parameter,

$$\begin{aligned} H \equiv \frac{\dot{a}}{a} = \frac{1}{\sqrt{3\kappa}} \frac{1}{(\lambda/\kappa + V)^2 + \dot{\phi}^4/2} \Bigg\{ &-\frac{\sqrt{3\kappa}}{2}\left(\frac{\lambda}{\kappa} + V + \frac{\dot{\phi}^2}{2}\right)V'(\phi)\dot{\phi} \pm \left(\frac{\lambda}{\kappa} + V - \frac{\dot{\phi}^2}{2}\right) \\ &\times \left[\kappa\left(\frac{\lambda}{\kappa} + V + \frac{1}{2}\dot{\phi}^2\right)^{3/2}\left(\frac{\lambda}{\kappa} + V - \frac{\dot{\phi}^2}{2}\right)^{3/2} - \left(\frac{\lambda}{\kappa} + V + \frac{1}{2}\dot{\phi}^2\right)\left(\frac{\lambda}{\kappa} + V - \dot{\phi}^2\right)\right]^{1/2}\Bigg\}. \end{aligned} \tag{7}$$

This equation was first obtained in Eq. (46) in Ref. [10] with $\lambda = 1$. There, it was also shown that GR is recovered in the leading order at later times in the expanding Universe. Varying the action in Eq. (4) with respect to ϕ, the equation of motion for the scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{8}$$

Now let us investigate the zero-curvature solution, i.e., the flat solution in terms of the metric $g_{\mu\nu}$. We consider the flat spacetime, $a(t) =$ constant, with the scalar field $\phi(t)$ being dynamical. This corresponds to $H = 0$. Equation (7) allows a nontrivial solution for this, contrary to the case of GR in which $H = 0$ at all times directly implies $\rho = 0 = p$. The scalar field equation [Eq. (8)] can be integrated as follows:

$$\ddot{\phi} = -V'(\phi) \Rightarrow \frac{1}{2}\dot{\phi}^2 = E - V(\phi), \tag{9}$$

where E is the integration constant which plays the role of a conserved energy density. If we introduce an effective potential $\mathcal{V} \equiv (\lambda - \kappa E + 2\kappa V)/c^2$, where $c^2 \equiv \lambda + \kappa E$, $H = 0$ reduces to a simple form,

$$\frac{d\mathcal{V}}{d\phi} = \pm\sqrt{\frac{8}{3}} \frac{\mathcal{V}\sqrt{1 - 3\mathcal{V} + 2c^2\mathcal{V}^{3/2}}}{c\sqrt{1 - \mathcal{V}}}. \tag{10}$$

Equation (9) can be recast into

$$\frac{\kappa}{c^2}\dot{\phi}^2 + \mathcal{V}(\phi) = 1. \tag{11}$$

The rescaled dynamical field $(\sqrt{\kappa}/c)\phi$ with a fixed energy scale $\mathcal{E} = 1$ subject to the potential $\mathcal{V}(\phi)$ which is a solution to Eq. (10) produces the nGMD, a *flat spacetime with a dynamical field.*[1]

Let us comment on the values of $c = \sqrt{\lambda + \kappa E}$. We shall consider the case of $\kappa > 0$ in this work, and assume that the energy density is non-negative, $\rho = E \geq 0$. The cosmological constant becomes $\Lambda \gtreqless 0$ for $\lambda \gtreqless 1$. Then one can have $c > 1$ for all types of the cosmological constant by tuning the energy density $\rho = E$. One can have $c < 1$ only for a negative cosmological constant. One can have $c = 1$ for a negative cosmological constant with $E > 0$, or for a zero cosmological constant with $E = 0$.

The shape of $d\mathcal{V}/d\phi$ for $c > 0$ is qualitatively different from that for $c \leq 1$. The domain of $\mathcal{V}$ is [0, 1]. For all the values of c, $d\mathcal{V}/d\phi = 0$ at $\mathcal{V} = 0$. For $c > 1$, $d\mathcal{V}/d\phi$ diverges to infinity at $\mathcal{V} = 1$. For $c \leq 1$, $d\mathcal{V}/d\phi$ possesses another zero at $\mathcal{V}_c$ where $1 - 3\mathcal{V}_c + 2c^2\mathcal{V}_c^{3/2} = 0$. [See Fig. 1(a).]

Now let us analyze the field dynamics and the effective potential from Eq. (10).

[1] From Eq. (11), one gets $\dot{\phi} = \pm(c/\sqrt{\kappa})\sqrt{1 - \mathcal{V}}$. Without loss of generality, one can take $c > 0$. Here, the signature change $+ \rightarrow -$ is equivalent to the transformation $V(\phi) \rightarrow V(-\phi)$ accompanying $\phi \rightarrow -\phi$. Therefore, in this work, we consider only the positive signature, which describes the positive velocity, $\dot{\phi} > 0$.

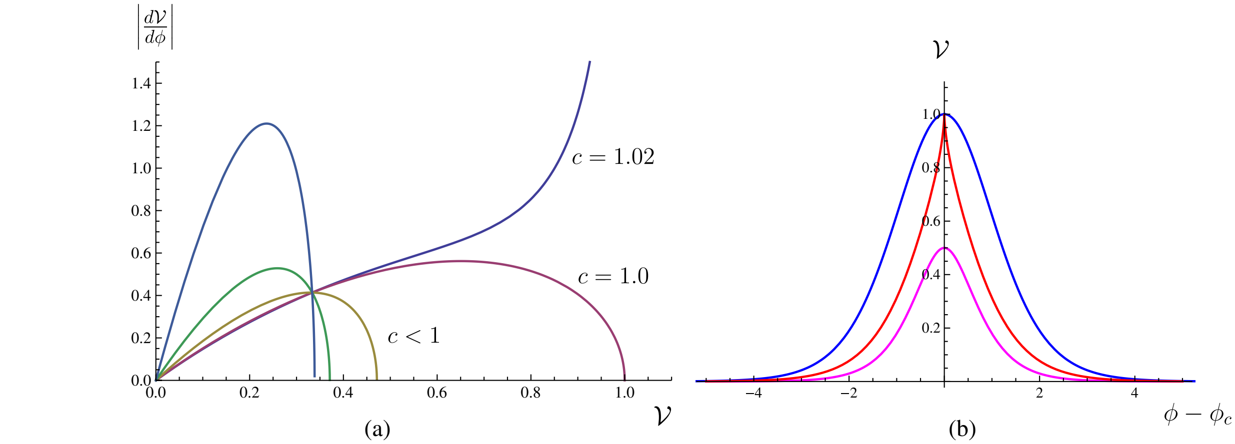

FIG. 1 (color online). (a) Plot of $|d\mathcal{V}/d\phi|$ vs $\mathcal{V}$ for $c = 1.02, 1.0, 0.8, 0.5, 0.2$. (b) Plot of $\mathcal{V}$ vs ϕ for $c = 1, 1.02, 0.84$, respectively from the top. For the motion of the field ϕ subject to $\mathcal{V}$, the energy level is fixed to $\mathcal{E} = 1$.

(i) For $c = 1$, Eq. (10) is integrated in terms of elliptic functions,

$$2i\left[F\left(\arcsin\left(\sqrt{\frac{1+\mathcal{V}^{1/2}}{2}}\right)\middle|4\right) - \Pi\left(2;\arcsin\left(\sqrt{\frac{1+\mathcal{V}^{1/2}}{2}}\right)\middle|4\right) - F\left(\frac{\pi}{2}\middle|4\right) + \Pi\left(2;\frac{\pi}{2}\middle|4\right)\right] = \pm\sqrt{\frac{2}{3}}(\phi - \phi_c), \tag{12}$$

where F and Π are the elliptic integral of the first kind and the incomplete elliptic integral. The solution $\mathcal{V}(\phi)$ is plotted in Fig. 1(b). The field ϕ evolves in the positive direction with the energy level $\mathcal{E} = 1$, while the spacetime remains flat. The field velocity becomes zero at $\phi = \phi_c$, and it takes infinite time in arriving there. This point is an unstable extremum.

(ii) Near $\mathcal{V} = 0$ for all values of c, we have

$$\frac{d\mathcal{V}}{d\phi} \approx \pm\sqrt{\frac{8}{3}}\frac{\mathcal{V}}{c} \Rightarrow \mathcal{V} \approx \mathcal{V}_0 e^{\pm\sqrt{\frac{8}{3}}\frac{\phi}{c}}, \tag{13}$$

where $\mathcal{V}_0$ is an integration constant. Note that this solution is valid for $\mathcal{V} \approx 0$, and so for $|\phi| \gg c$. The positive/negative signature corresponds to the left/right side of $\mathcal{V}$ in Fig. 1(b). From $\dot{\phi} = (c/\sqrt{\kappa})\sqrt{1-\mathcal{V}}$, the scalar field becomes

$$\phi(t) \approx \frac{c}{\sqrt{\kappa}}t \mp \sqrt{\frac{3}{32}}c\mathcal{V}_0 e^{\pm\sqrt{\frac{8}{3\kappa}}t} \quad \text{for } t \to \mp\infty. \tag{14}$$

The scalar field rolls up the exponential potential [Eq. (13)] for $t \ll -\sqrt{\kappa}/c$, and rolls down for $t \gg \sqrt{\kappa}/c$.

(iii) Near $\mathcal{V} = \mathcal{V}_c \leq 1$ for $c \leq 1$, we have

$$\frac{d\mathcal{V}}{d\phi} \approx \pm\frac{\sqrt{\mathcal{V}_c - \mathcal{V}}}{c_1} \Rightarrow \mathcal{V} \approx \mathcal{V}_c - \frac{(\phi - \phi_c)^2}{4c_1^2} \quad \text{with} \quad \pm\phi \leq \pm\phi_c, \tag{15}$$

where $c_1 = (c/\mathcal{V}_c)[(1-\mathcal{V}_c)/8/(1+c^2\mathcal{V}_c^{1/2})]^{1/2}$. The scalar field becomes then

$$\phi(t) \approx \phi_c + 2c_1\sqrt{1-\mathcal{V}_c}\sinh\left[\frac{c(t-t_c)}{2c_1\sqrt{\kappa}}\right]. \tag{16}$$

The top of the potential $\mathcal{V}_c$ is reached at $\phi(t = t_c) = \phi_c$. The field passes this point with nonvanishing velocity except for $c = 1$.

(iv) Near $\mathcal{V} = 1$ for $c > 1$, we have

$$\frac{\partial\mathcal{V}}{\partial\phi} \approx \pm\frac{1}{c_2\sqrt{1-\mathcal{V}}}, \Rightarrow \mathcal{V} \approx 1 - \left[\frac{3(\phi - \phi_c)}{2c_2}\right]^{2/3} \quad \text{with} \quad \pm\phi \leq \pm\phi_c, \tag{17}$$

where $c_2 = \sqrt{3}c/(4\sqrt{c^2-1})$. The scalar field evolves as

$$\phi \approx \phi_c \mp \frac{2}{3\sqrt{c_2}}\left(\frac{c}{\sqrt{\kappa}}|t - t_c|\right)^{3/2}. \tag{18}$$

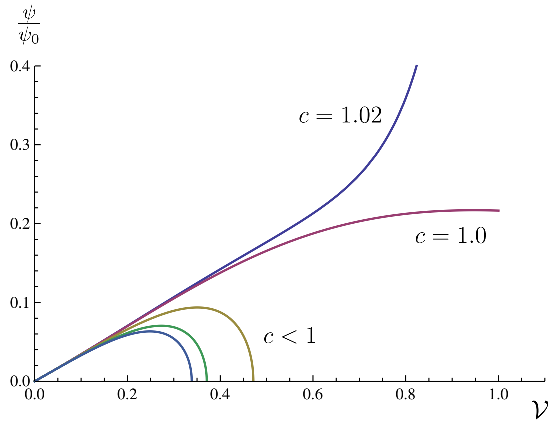

FIG. 2 (color online). The perturbation $\psi(t)$ for the same values of c as in Fig. 1(a). The perturbation vanishes at $\mathcal{V} = 0$ and $\mathcal{V}_c(<1)$.

At the top of the potential, $\mathcal{V}[\phi(t=t_c)=\phi_c]=1$, the field velocity becomes zero ($\dot{\phi} = 0$) and the acceleration becomes infinite ($\ddot{\phi} = -d\mathcal{V}/d\phi = \mp\infty$). This point is unstable.

The dynamical scalar field solutions that we have obtained here produce the flat spacetime. Since EiBI gravity is equivalent to GR in vacuum, the flat spacetime is also achieved in vacuum without a scalar field. This means that there exist two branches of the flat spacetime. Therefore, it is worthwhile to check the stability of the flat spacetime originated from the dynamical scalar field.

In order to investigate the linear perturbation, we introduce the linear perturbations $h(t)$ and $\psi(t)$ for the velocities of the metric and the scalar field,

$$H(t) = 0 + \epsilon h(t), \qquad \dot{\phi}(t) = \frac{c}{\sqrt{\kappa}}\sqrt{1-\mathcal{V}}[1 + \epsilon\psi(t)], \tag{19}$$

and consider the field equations in the linear order in ϵ. The scalar field equation [Eq. (8)] in the linear order is recast as

$$\frac{\dot{\psi}}{\psi} + \frac{3h}{\psi} - \frac{\dot{\mathcal{V}}}{1-\mathcal{V}} = 0. \tag{20}$$

The Hubble parameter [Eq. (7)] in the linear order is given by

$$\frac{h}{\psi} = \frac{\dot{\mathcal{V}}}{3(1+\mathcal{V}^2) - 2\mathcal{V}}\left[\frac{(1-\mathcal{V})(5 - 3\mathcal{V} - 3c^2\mathcal{V}^{1/2}(1-\mathcal{V}))}{2(1 - 3\mathcal{V} + 2c^2\mathcal{V}^{3/2})} + \mathcal{V} - 1 - \frac{1}{\mathcal{V}}\right]. \tag{21}$$

Then Eq. (20) can be integrated as

$$\psi = \psi_0 \frac{\mathcal{V}(1 - 3\mathcal{V} + 2c^2\mathcal{V}^{3/2})^{1/2}}{(1-\mathcal{V})(3 - 2\mathcal{V} + 3\mathcal{V}^2)}. \tag{22}$$

The solution is plotted in Fig. 2. The perturbation of the gravitational field velocity $h(t)$ in Eq. (21) becomes

$$h = \pm\sqrt{\frac{2}{3\kappa}}\psi_0 \frac{\mathcal{V}(-2 + 9\mathcal{V} - 7c^2\mathcal{V}^{3/2} + 2c^2\mathcal{V}^{5/2} - 3\mathcal{V}^3 + c^2\mathcal{V}^{7/2})}{(1-\mathcal{V})(3 - 2\mathcal{V} + 3\mathcal{V}^2)^2}. \tag{23}$$

Stability requires that both ψ and h approach zero in time. From Eq. (22), $\psi = 0$ when $\mathcal{V} = 0$ and $\mathcal{V}_c(<1)$. From Eq. (23), $h = 0$ when $\mathcal{V} = 0$, $\mathcal{V}_*$ (the value of $\mathcal{V}$ which makes the numerator zero). Therefore, the stability is achieved only when $\mathcal{V} = 0$. There are two regions of ϕ for $\mathcal{V} \to 0$:

(i) At $\phi \ll \phi_c - c$, $\mathcal{V} \to 0$. There, the potential is approximated by $\mathcal{V} \approx \mathcal{V}_0 e^{\sqrt{\frac{8\phi}{3c}}}$ from Eq. (13). As ϕ increases with time, ψ and h increase exponentially. Therefore, the nGMD is unstable in this region.

(ii) At $\phi \gg \phi_c + c$, $\mathcal{V} \to 0$. There, the potential is approximated by $\mathcal{V} \approx \mathcal{V}_0 e^{-\sqrt{\frac{8\phi}{3c}}}$. As ϕ increases with time, ψ and h decrease exponentially. Therefore, the nGMD becomes an attractor as $\phi \to \infty$.

As a whole, when the scalar field climbs up the potential at $\phi < \phi_c$, the nGMD is difficult to produce by the dynamical scalar field because it is unstable under perturbations. On the other hand, when the scalar field rolls down the potential at $\phi > \phi_c$, the perturbations decrease and go to zero as $\phi \to \infty$. Especially for $0 < c \leq 1$, the perturbations ψ and h always remain finite. Therefore, although the nGMD is perturbed after it is formed, it comes back to the nGMD configuration when the scalar field enters the region rolling down the potential at late times. Note that the scalar field is still dynamical ($\dot{\phi} \neq 0$), so this type of flat spacetime is different from the ordinary flat one produced in vacuum.

Owing to the stability analysis discussed above, the nGMD will be achieved at late times during the cosmological evolution of the scalar field if the potential asymptotes to $\mathcal{V} = \mathcal{V}_0 e^{-\sqrt{\frac{8\phi}{3c}}}$. For example, consider the scalar field subject to the potential,

$$V_{\exp}(\phi) = E - \frac{c^2}{2\kappa} + \frac{c^2\mathcal{V}_0}{2\kappa}e^{-\sqrt{\frac{8\phi}{3c}}}, \tag{24}$$

which is defined for the whole range of ϕ. When the scalar field slides down this potential, there could exist a solution which approaches the one in Eq. (14). At late times, this solution will asymptote to the attractor solution which is the nGMD and is stable under perturbations for $\phi > c$. Therefore, this system gives rise to a stable, nongravitating dynamical scalar field.

In summary, we have investigated the possibility that the nonlinearity of matter-gravity interactions allows nongravitating matter distributions (nGMDs). We showed that nGMDs are not possible in gravity theories such as general relativity, $f(R)$ gravity, and higher-derivative gravity, if they are regular in the zero-curvature limit. On the other hand, the Eddington-inspired Born-Infeld gravity allows nGMDs. Explicitly, we found that a flat zero-curvature spacetime exists with nontrivial scalar field configurations subject to the potential $V(\phi)$ which satisfies the differential equation (10). This potential is a runaway type as in Fig. 1(b). For this configuration, the metric $g_{\mu\nu}$ is flat, while the auxiliary metric $q_{\mu\nu}$ is nontrivial as in Eq. (6). The spacetime curvature vanishes while the Ricci tensor evaluated by $q_{\mu\nu}$ is not trivial. The modification of GR through the connection term by the Palatini formulation makes the EiBI theory allow nGMDs. The general gravity theories will be divided into two classes: the collections of gravity theories with and without nGMDs. The EiBI gravity will be the first example belonging to the class with nGMDs.

Since EiBI gravity is equivalent to GR in vacuum, a flat spacetime is also achieved when there is *no matter*. This implies that there are two stable branches of the flat spacetime. One is the nGMD solution in the Eddington regime[2] given above, and the other is the usual flat spacetime in the GR regime. In other words, there are two weak-gravity regimes in the EiBI theory.

During the evolution of the Universe driven by a scalar field rolling down the potential [Eq. (24)], it is very interesting to ask when it goes to the GR regime and when to the Eddington regime. At the moment, what we know is as follows. As the Universe expands, both the Hubble parameter H and the field velocity $\dot{\phi}$ decrease. If H becomes zero with finite $\dot{\phi}$, the Eddington regime appears. On the other hand, if $\dot{\phi} \rightarrow 0$ with $H \geq 0$, the GR regime will be realized. Another interesting question is whether or not the EiBI gravity allows a nongravitating localized object such as a boson star.

This work was supported by the Korea Research Foundation (KRF) funded by the Korea government (MEST) under Grants No. NRF-2012R1A1A2006136 (I. C.) and No. NRF-2013R1A1A2006548 (H. K.). The authors are grateful to the hospitality of the Asia Pacific Center for Theoretical Physics (APCTP).

[2]We use this wording to denote that it is distinct from the GR regime.

[1] M. Banados and P. G. Ferreira, Phys. Rev. Lett. **105**, 011101 (2010).
[2] I. Cho, H.-C. Kim, and T. Moon, Phys. Rev. D **86**, 084018 (2012).
[3] A. De Felice, B. Gumjudpai, and S. Jhingan, Phys. Rev. D **86**, 043525 (2012).
[4] P. P. Avelino, Phys. Rev. D **85**, 104053 (2012); J. Cosmol. Astropart. Phys. 11 (2012) 022.
[5] J. Casanellas, P. Pani, I. Lopes, and V. Cardoso, Astrophys. J. **745**, 15 (2012).
[6] C. Escamilla-Rivera, M. Banados, and P. G. Ferreira, Phys. Rev. D **85**, 087302 (2012).
[7] P. P. Avelino and R. Z. Ferreira, Phys. Rev. D **86**, 041501 (2012).
[8] Y.-X. Liu, K. Yang, H. Guo, and Y. Zhong, Phys. Rev. D **85**, 124053 (2012).
[9] T. Delsate and J. Steinhoff, Phys. Rev. Lett. **109**, 021101 (2012).
[10] J. H. C. Scargill, M. Banados, and P. G. Ferreira, Phys. Rev. D **86**, 103533 (2012).
[11] Y.-H. Sham, L.-M. Lin, and P. T. Leung, Phys. Rev. D **86**, 064015 (2012).
[12] P. Pani and T. P. Sotiriou, Phys. Rev. Lett. **109**, 251102 (2012).
[13] I. Cho, H.-C. Kim, and T. Moon, Phys. Rev. Lett. **111**, 071301 (2013).
[14] K. Yang, X.-L. Du, and Y.-X. Liu, arXiv:1307.2969.