Noncommutative effects in entropic gravity

C. M. Gregory[1,*] and A. Pinzul[1,2,†]

[1]*Instituto de Física, Universidade de Brasília, 70910-900 Brasília, DF, Brasil*
[2]*International Center of Condensed Matter Physics, C.P. 04667, 70919-970 Brasília, DF, Brasil*
(Received 8 November 2012; published 16 September 2013)

We analyze the question of possible quantum corrections in the entropic scenario of emergent gravity. Using a fuzzy sphere as a natural quasiclassical approximation for the spherical holographic screen, we analyze whether it is possible to observe such corrections to Newton's law in principle. The main outcome of our analysis is that without the complete knowledge of the quantum dynamics of the microscopic degrees of freedom, any Plank-scale correction cannot be trusted. Some perturbative corrections might produce reliable predictions well below the Plank scale.

DOI: 10.1103/PhysRevD.88.064030 PACS numbers: 04.60.−m, 02.40.Gh, 04.50.Kd

I. INTRODUCTION

The final form of quantum gravity (QG) is yet to be found. Either of the two main candidates for such a theory—(super)string theory and loop quantum gravity (LQG)—despite much progress, still cannot be taken as the final answer (the very existence of two seriously different theories of QG means that the problem of the quantization of gravity is far from being settled). In this situation, any effort in this direction should be welcomed. In particular, given a model for QG, it is very important to understand how the classical limit of Einstein's general relativity (GR) emerges, as well as to learn how to calculate possible quantum-gravitational corrections. Concerning the classical limit, the recently proposed entropic gravity [1] might turn out to be quite important. In this model, E. Verlinde proposes that gravity, instead of being a fundamental force, has an emergent, entropic origin. Roughly speaking, one can think of the gravitational force as being caused by the change of the entropy of the system—a holographic screen plus a test mass. Under some assumptions, this model uniquely leads to the classical GR for quite general quantum dynamics. E.g., in Ref. [2], it was shown that by using this approach one can get Newton's law in the framework of LQG. Due to this universality, the question of quantum-gravitational corrections becomes very important. The hope is that this will eventually allow us to tell the difference between different models of QG. In this regard, the following observation is of great importance [3]: independently of specific details of the final theory of QG, the quasiclassical regime—i.e. when the typical energy scale is close to the Plank scale—should be described by a field theory on some noncommutative space-time. Of course this is true only if one can use *unmodified* GR all the way up to the relevant scale. The more accurate conclusion seems to be that the gravity (or geometry) should somehow be modified close to this scale. Noncommutative deformation is a suggestive possible candidate for such a modification. It should capture some nonperturbative effects of QG. The details of this noncommutativity do depend on the QG model, i.e. on the quantum dynamics. The naturalness of the noncommutativity as some QG residue was explicitly demonstrated in the three-dimensional case for the Ponzano-Regge model[4]. (See also earlier papers [5,6] where similar conclusions were reached for the case of particles coupled to three-dimensional gravity.)

The main goal of our work is to analyze possible effects of the underlying noncommutativity on the entropic gravity. Ever since its publication, the entropic scenario has generated a series of comments and criticisms. One of the main objections is that the obtained results are a consequence more of the dimensional analysis rather then some fundamental physical reasons (this especially applies to Newton's law). This makes it very important to analyze the process of the "interaction" of a test particle with the holographic screen, i.e. how this particle becomes a part of the screen. From our point of view, here one has one of the major problems of the model: a well-defined smooth holographic screen, e.g. a sphere, is an adequate model to reproduce the GR limit and is too restrictive if one wants to go beyond the classical approximation. Taking into account the above observation about the universality of noncommutativity in QG, we address this point by considering a fuzzy sphere as a natural candidate for a holographic screen, which "remembers" its quantum-gravitational origin.

As the detailed answer to the above question on how a test particle becomes a part of the screen depends on our knowledge of the quantum dynamics of the microscopic degrees of freedom, we will be interested in a less ambitious problem: how does a test particle "see" the holographic screen (which is taken to be a fuzzy sphere)? As the main tool of our analysis, we use some methods of spectral geometry. Our main results are the following:

(1) To actually discuss quantum corrections close to the Plank scale, one needs to know the details of the

*carogregory@gmail.com
†apinzul@unb.br

 © 2013 American Physical Society

quantum dynamics. Any attempt to obtain such corrections without knowing how the test particle "sees" the holographic screen will be destroyed by the uncertainties due to our ignorance about this process. On the other hand, away from the Plank scale, some universal corrections might be well defined, confirming their perturbative origin.

(2) A somewhat related and more important point is that a test particle is a very important ingredient of the whole construction. One cannot remove it from the picture in principle. We will see that there is a regime (though quite beyond any experimental reach) when different test particles will see the holographic screen quite differently. Hence, one can speculate about a possible violation of the equivalence principle by quantum gravity (in this scenario).

The plan of the paper is as follows. In Sec. II we briefly discuss Verlinde's approach to entropic gravity, its consistency and some possible corrections. After arguing that to go beyond the classical limit one has to abandon the notion of a smooth holographic screen in favor of a noncommutative one, in Sec. III we introduce the fuzzy sphere and review some properties of its Dirac operator. In Sec. IV, we apply (the generalization of) Weyl's theorem to calculate the area of a fuzzy holographic screen as seen by a test particle. We analyze possible corrections to the classical area in the regimes of weak and full noncommutativity. Section V contains a discussion and interpretation of the obtained results. We conclude with a summary and some final remarks.

II. CLASSICAL ENTROPIC GRAVITY AND BEYOND

Here we briefly summarize the main steps and inputs leading to the entropic scenario [1]. We stress the points that need—from our point of view—more justification or more careful analysis.

(1) One starts by assigning to any surface some entropy, which scales as the area, A. In such a way, any surface—not just black hole horizons—plays the role of a "holographic screen."

(2) When a test particle (which is assumed to be elementary, i.e. pointlike) of mass m approaches such a holographic screen at a distance of the order of the Compton wavelength, $\lambda_m = \frac{\hbar}{mc}$, the entropy of the screen is increased by $\Delta S = 2\pi k_B$. To simulate a more continuous change in entropy, one assumes that when the particle is at a distance Δx from the screen, the change is given by

$$\Delta S = 2\pi k_B \frac{\Delta x}{\lambda_m}. \tag{1}$$

One immediately sees tension between the first two assumptions [7]: while in (1) entropy scales like the area, according to Eq. (1) it scales as the distance. Already this suggests that one should have a better understanding of how exactly a test particle becomes a part of the holographic screen.

(3) The energy associated to the screen, E_S, is given by the total energy inside the screen. In the nonrelativistic limit, $E_S = Mc^2$, where M is the total mass encircled by the surface.

(4) This energy is equally distributed between N quanta of the surface, which leads to the temperature of the screen,

$$E_S = \frac{1}{2} N k_B T, \tag{2}$$

where $N = \frac{A}{l_P^2}$ and $l_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Plank length.

(5) The last assumption is that the resulting entropic force

$$F = T\frac{\Delta S}{\Delta x} \tag{3}$$

is gravity.

Combining assumptions (1)–(5), one immediately arrives (for a spherically symmetric configuration) at Newton's law [1]. One of the most attractive features of this scenario is its universality: independently of the actual microscopic dynamics, as long as the fundamental theory satisfies (1)–(5), it will lead to general relativity. This has already been used as a possible way to get Newton's law from LQG [2].

We have already mentioned one potential tension between some of the assumptions (1)–(5). Another one was raised by many critiques of the entropic scenario: with this setup, Newton's law is a mere consequence of the dimensional analysis. This makes it crucial to check the emergence of gravity in this way from some fundamental theory or at least to go beyond the classical limit and try to calculate quantum corrections. The major effort in this direction has been based on calculating corrections to the entropy and then using this quantum-corrected entropy in the derivation of a corrected Newton's law. Here we mention just a couple of works, which are relevant for our consideration. (For some other approaches see, e.g. Refs. [8,9].)

In Ref. [7], LQG-inspired corrections were considered in the form

$$S = \frac{Ak_B}{4l_P^2} - ak_B \ln\left(\frac{A}{l_P^2}\right) + bk_B\left(\frac{A}{l_P^2}\right)^{3/2}, \tag{4}$$

where a and b are some constants of order one. While the first term in Eq. (4) is the usual Bekenstein entropy, the others represent corrections. The logarithmic correction is quite universal, while the volume correction is motivated by LQG. The use of Eq. (4) leads to the following corrections to Newton's law:

$$F = -\frac{GMm}{R^2}\left(1 - a\frac{l_P^2}{\pi R^2} + 12b\sqrt{\pi}\frac{R}{l_P}\right). \quad (5)$$

Due to the universality of the logarithmic correction in Eq. (4), the first correction to Newton's law is also quite universal and follows from different models for quantum corrections [10–12]. The volume correction to entropy was advocated in Ref. [13] and it is interesting because it leads to much stronger gravity at large distances, which has the potential of explaining anomalous galactic rotational curves [7], though a quick inspection shows that this is the case for quite an unnatural value of the parameter b.[1]

The approach taken in Ref. [15] uses some specific model to calculate corrections to entropy due to the noncommutativity of space-time. One arrives at the corrected Newton's law,

$$F = -\frac{GMm}{R^2}\left(1 + \frac{Re^{-R^2/(4\alpha l_P^2)}}{\sqrt{\pi\alpha}l_P} + \frac{R^2 e^{-R^2/(2\alpha l_P^2)}}{2\alpha\sqrt{\pi}l_P^2}\right). \quad (6)$$

Note that in this case the corrections are exponentially suppressed by $\frac{R^2}{l_P^2}$. This means that even the hypothetical possibility to measure such corrections becomes even more evasive.

We mention these two works because, from our point of view, they are trying to address two very important issues of the original entropic scenario: while Ref. [7] has something to say about the process of a test particle becoming a part of the holographic screen, Ref. [15] considers noncommutative space-time, which is more natural from the quantum gravity point of view [3].

In this paper, we also take this "noncommutative" point of view: we model our spherical holographic screen by a fuzzy sphere S_F [16] in place of a smooth S_2. But instead of considering corrections to entropy (which, anyway, is beyond our reach without knowing something about quantum degrees of freedom), we ask the following question: how does a test particle see this sphere? In particular, we will try to analyze the area of the holographic screen as seen by a test particle. It should be clear that this is closely related to the question of how a test particle "interacts" with the screen, though the complete answer to this question once again requires information about microscopic dynamics.

[1]Though in Ref. [7] it is said that b is of order 1 or less, we can see that it should be *much* less to pass the Solar System tests. This is due to the huge factor $\frac{R}{l_P}$ in Eq. (5). Even more strange restrictions should be imposed on b if one wants to use Eq. (5) to explain galactic rotational curves. As it was shown in Ref. [7], one should take $b = \frac{l_P}{3}\sqrt{\frac{a_0}{16\pi GM}}$, where $a_0 \simeq 1.2 \times 10^{-10}$ m/s^2 [14]. Then $b = 10^{-36}\sqrt{\frac{1}{M}}$, where M is in kilograms. So, b should not only be tiny but should also nontrivially depend on mass, i.e. not appear to be universal. So, unless one provides serious arguments for such behavior of b, the last term in Eq. (5) should be treated with great care if trusted at all.

Before we discuss some properties of the fuzzy sphere and its Dirac operator, we would like to give some arguments on why we think a fuzzy sphere should be a natural candidate for a spherical screen as well as why we need its Dirac operator.

(1) As we have already stressed several times, the prediction of the noncommutativity of the space-time at some quasiclassical regime is model independent [3].
(2) For a spherically symmetric classical distribution of a mass M, we should expect that some notion of spherical symmetry is left even in this quasiclassical regime. Recalling that a fuzzy sphere is essentially a unique deformation of the commutative one such that it carries the usual, undeformed action of SO(3), we conclude that if a spherical holographic screen should be deformed, a fuzzy sphere is the most natural candidate for this.
(3) A fuzzy sphere has a natural, built-in discreteness, which should be compared with the assumed discreteness of a holographic screen.
(4) Using the same kind of arguments, it was speculated (see e.g. Ref. [17]) that a fuzzy sphere can be quite useful and natural in black hole physics.
(5) Why do we need a Dirac operator? Below we will discuss this in more detail, and here we just give some motivation. In our previous work [18], we showed the effectiveness of a physically relevant Dirac operator for calculations of such geometrical characteristics as area and dimension in the case of deformed geometries (in the example of the Horava-Lifshitz deformation of GR). Here we would like to adopt the same procedure for the calculation of the "physical" area of a fuzzy sphere as seen by a test particle.

III. FUZZY SPHERE AND ITS DIRAC OPERATOR

A fuzzy sphere [16] provides a very important example of a noncommutative space, such that the commutative isometry, SO(3), remains as a symmetry on the noncommutative level. The algebra of the $(N+1)\times(N+1)$ fuzzy sphere of radius R is generated by noncommutative coordinates,

$$[\mathbf{x_i}, \mathbf{x_j}] = i\lambda\epsilon_{ijk}\mathbf{x_k}, \quad (7)$$

where $\lambda = \frac{2R}{\sqrt{N(N+2)}}$. It is clear that Eq. (7) is invariant under the usual SO(3) action, i.e. when $\mathbf{x_i}$ transforms as a vector. The presence of two parameters, R and N, allows one to recover as special limits not only a commutative sphere but also the Moyal and commutative planes [19]. It is clear from the definition that the noncommutative coordinates, rescaled by λ, are given by the $(N+1)$-dimensional irreducible representation of the SU(2) generators, $\mathbf{L_i}$. Then one immediately recognizes R^2 as the value of the Casimir, $\sum_i \mathbf{x_i}\mathbf{x_i}$, in this $(N+1)\times(N+1)$

representation. This is why we still can speak of a well-defined radius.

At this point, R and N are independent. Depending on the model at hand, they might remain independent or there could be some relation between them. The typical example of the first situation is given by the different field theory models on a fuzzy sphere (see, e.g. Ref. [20]). In this case, N serves as a UV regulator to be removed at the end, while keeping R fixed. In this way one can study the development of UV divergencies as well as the appearance of the famous UV/IR mixing [21]. The examples of the second type, i.e. when N is not independent, should come from some fundamental physics, which does not contain additional free parameters. E.g. in Ref. [17], it was argued that in the case when a fuzzy sphere is used to model a black hole horizon, N should be proportional to the area of the horizon, i.e. to R^2.[2] This is what we adopt in this paper due to the obvious analogy between horizons and holographic screens in entropic gravity (see also the discussion in Sec. IV B).

To proceed with our goal, we will need a Dirac operator. This is one of two of the most important operators in noncommutative geometry (the second one being the chirality operator). It is essential for the construction of the differential and integral calculus. As such, it is subject to several natural conditions (see Ref. [23] on the role of the Dirac operator in noncommutative geometry). In the case of a fuzzy sphere, we have one extra condition: because this noncommutative space is rotationally invariant, it would be quite desirable that the corresponding Dirac operator respects this symmetry. There are essentially two slightly different proposals for such an operator [24,25] (see also Ref. [26] for the treatment of a more general case of a q-deformed sphere, which reduces to that in Ref. [24] in a special limit). In this paper, we will work with the operator defined in Ref. [24],[3]

$$\not{D} = \frac{1}{R}\sqrt{\frac{N(N+2)}{(N+1)^2}}\left((\Lambda + 1) - \frac{1}{2R^2}\{\chi, \mathbf{x_i L_i}\} - \frac{\lambda}{R^2}\mathbf{x_i L_i}\right), \tag{8}$$

where $\Lambda = \mathbf{L_i} \otimes \sigma_i$, $\chi = \mathbf{x_i} \otimes \sigma_i$, σ_i are the Pauli matrices and the rest of the operators are understood as, e.g., $\mathbf{x_i} = \mathbf{x_i} \otimes \mathbb{1}$, etc. It is clear that Eq. (8) respects the SU(2) symmetry of the fuzzy sphere. In addition, it anticommutes with the natural chirality operator, which is given by a linear function of χ [24]. From the definition (8), it should be clear, that the Dirac operator is acting, as in the commutative case, on the space of the two-component spinors.

So, we still have the commutative relation between the dimension of the space and the dimension of the spinor bundle. This will be important in our further discussion. Using the rotational invariance and the above-mentioned fact that the $\mathbf{x_i}$'s are proportional to the $\mathbf{L_i}$'s, after some standard algebra, one arrives at the spectrum of the Dirac operator [24],

$$\omega_{j\pm} = \pm\frac{1}{R}\left(j+\frac{1}{2}\right)\left\{1 - \frac{1}{N(N+2)}\left[\left(j+\frac{1}{2}\right)^2 - 1\right]\right\}^{\frac{1}{2}}, \tag{9}$$

where the $j(j+1)$'s are the eigenvalues of the square of the total angular momentum $\mathbf{J_i}^2$, where $\mathbf{J_i} := \mathbf{L_i} + \frac{1}{2}\sigma_i$. Then, from the fact that we are working with an $(N+1)$-dimensional irreducible representation, it easy to see that $j \in \mathbb{N}$ or $j \in \mathbb{N} + \frac{1}{2}$ depending on whether N is odd or even and $0 \le j \le \frac{N+1}{2}$.[4]

IV. SPECTRAL AREA OF A FUZZY SPHERE

In this section, we show how the Dirac operator (8) with the help of Weyl's theorem can be used to calculate the corrections to the area of a fuzzy sphere. We start with the classical formulation of Weyl's theorem for the case of commutative geometry, and then we will argue that it still makes sense to use this theorem (but now as the definition) for the analysis of some properties of noncommutative or other generalized geometries.

Weyl's Theorem: Let Δ be the Laplace operator on a closed Riemannian manifold $\mathcal{M}$ of dimension n. Let $N_\Delta(\omega)$ be the number of eigenvalues of Δ, counting multiplicities, less then ω, i.e. $N_\Delta(\omega)$ is the counting function

$$N_\Delta(\omega) := \#\{\omega_k(\Delta):\omega_k(\Delta) \le \omega\}. \tag{10}$$

Then

$$\lim_{\omega\to\infty}\frac{N_\Delta(\omega)}{\omega^{\frac{n}{2}}} = \frac{\mathrm{Vol}(\mathcal{M})}{(4\pi)^{\frac{n}{2}}\Gamma(\frac{n}{2}+1)}, \tag{11}$$

where $\mathrm{Vol}(\mathcal{M})$ is the total volume of the manifold $\mathcal{M}$.

Though the theorem is given in terms of the Laplace operator, with the help of the Lichnerowicz formula, $\not{D}^2 = \Delta + \frac{1}{4}\mathcal{R}$, $\mathcal{R}$ being the curvature, it could be easily rewritten in terms of the Dirac operator, $\not{D}$. In this case, one should take care of the dimension of the spinor bundle, which is, in the commutative case, equal to 2^m, where $n = 2m$ or $n = 2m + 1$. So, we can see that in the commutative case Weyl's theorem provides a way of simultaneously calculating both the volume and the dimension of a manifold. The advantage of this method is in the fact that it is purely algebraic, which allows for immediate generalizations to

[2] To get the standard area-entropy relation, one still has to use Wheeler's "it from bit" argument [22], i.e. that every elementary area carries several bits of information. Of course, the fundamental theory should provide an explanation for this (e.g. it has a more or less natural explanation in LQG).

[3] The other choices should not seriously change our conclusions; see Sec. V.

[4] In Ref. [24], it was also argued that to have a correct commutative limit, we should keep only the case of the even N, but this will not be important for our calculations. Moreover, we believe that if a fuzzy sphere should come from some theory of quantum gravity, both representations should be allowed.

the cases when the usual geometrical techniques do not exist. Before we proceed with the application to the fuzzy-sphere case, we would like to give some justifications of such an application.

At first sight, the naive application of Weyl's theorem to a geometry given by the finite matrix algebra (as in our case) may not seem correct. Nevertheless, we will argue that using this theorem—but now as the *definition*—still makes sense even in this case. But now, as we will see, one has to clearly distinguish between the formal mathematical and applied physical approaches.

(i) The mathematically meaningful application of Weyl's theorem to finite matrix models seems quite doubtful. This could be understood as follows: when the spectrum is unbounded [in particular, $N_\Delta(\omega) \to \infty$], the requirement that the right-hand side of Eq. (11) makes sense (i.e. finite for compact geometries) fixes uniquely the dimension n, which, in turn, allows one to determine $\mathrm{Vol}(\mathcal{M})$. But for the case of a finite model, the spectrum is finite, as in Eq. (9), and $N_\Delta(\omega)$ is finite too. As a result, we do not have any requirement that could fix either n or $\mathrm{Vol}(\mathcal{M})$. Here it is crucial that ω can (and should) be taken arbitrarily large. We will see how the situation changes in the presence of the physically motivated cutoff.

(ii) Let us now use Weyl's theorem as a physical tool to *measure* the dimension and area of some (possibly noncommutative) space. For this we are going to use the experimental spectrum of the corresponding Dirac operator. Clearly, this spectrum could be measured only up to some cutoff, $\omega_{\mathbf{co}}$. Typically, even in the case of a finite model, this cutoff is below the maximal eigenvalue of the Dirac operator. So, the apparatus used to probe the geometry will not know whether the spectrum is finite or not. Then we can continue to use Weyl's theorem, but now instead of the mathematical limit, $\omega \to \infty$, we should take the "physical" one, $\omega \le \omega_{\mathbf{co}}$. (See further discussion below.) Now, in general, both the volume and dimension will depend nontrivially on the cutoff and without some further (physical) input it is impossible to determine both of them. If we assume the classical value for the dimension, as we will do in this paper, then we can derive the cutoff-dependent corrections to the classical volume. In Ref. [18], this approach was successfully used to analyze the UV/IR behavior of the spectral dimension in the Hořava-Lifshitz models of gravity. (See also Ref. [18], especially the concluding section, for the discussion and physical interpretation of this approach.)

After these comments, let us apply this approach to the case of the fuzzy sphere. First of all, we would like to give two arguments in favor of why we want to keep the dimension of the fuzzy sphere equal to the classical one, n = 2. Firstly, as we commented after Eq. (8), the noncommutative Dirac operator acts in the space of the two-component spinors. This means that the passage between the formulation of Weyl's theorem in terms of the Laplace operator Δ and the one in terms of the dirac operator $\not{D}$ is the same as in the case of n = 2. Secondly, if we look at this from the physical point of view, then during the process of measuring the spectrum of $\not{D}$ (or Δ), we are already assuming that we are measuring the spectrum of some operator defined on some two-dimensional surface. We then treat any deviations from the commutative result as the quantum-geometrical corrections to the area. Keeping this in mind, let us proceed.

As a first step, we need to calculate the counting function (10). For this, we need to calculate j as a function of ω. Inverting Eq. (9), we obtain

$$\left(j+\frac{1}{2}\right)^2 = \frac{(N+1)^2 \pm [(N+1)^4 - 4\omega^2 R^2 N(N+2)]^{1/2}}{2}. \tag{12}$$

To choose the correct sign in Eq. (12), we note that $(j+\frac{1}{2})^2 \le \frac{(N+2)^2}{4}$. This leads to the choice of the minus sign. Then we have the maximal value of j corresponding to a cutoff scale $\omega_{\mathbf{co}}$,

$$\left(j_{\max}+\frac{1}{2}\right)^2 = \frac{(N+1)^2 - [(N+1)^4 - 4\omega_{\mathbf{co}}^2 R^2 N(N+2)]^{1/2}}{2}. \tag{13}$$

Taking into account that the degeneracy of each eigenvalue is equal to $(2j+1)$, we can write the counting function as

$$N_{|\not{D}|}(\omega_{\mathbf{co}}) = 2\sum^{j_{\max}}(2j+1). \tag{14}$$

The coefficient of 2 comes from the plus/minus sign in Eq. (9). Taking (for definiteness) j to be half-integer, i.e. N to be even (see the footnote [4]), we obtain the following exact expression for the counting function:

$$\begin{aligned} N_{|\not{D}|}(\omega_{\mathbf{co}}) &= 2\left(j_{\max}+\frac{1}{2}\right)^2 + 2\left(j_{\max}+\frac{1}{2}\right) \\ &= (N+1)^2\left[1-\left(1-\frac{4\omega_{\mathbf{co}}^2 R^2(N+2)N}{(N+1)^4}\right)^{\frac{1}{2}}\right] \\ &\quad + \sqrt{2}(N+1)\left[1-\left(1-\frac{4\omega_{\mathbf{co}}^2 R^2(N+2)N}{(N+1)^4}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}. \end{aligned} \tag{15}$$

A. Commutative Limit, $N \to \infty$

Let us first use Eq. (15) to reproduce the commutative result for the area of a sphere. This will later help us clarify some points about the applicability of the method, as well as provide the example of the effectiveness of Weyl's theorem.

The commutative limit corresponds to sending the dimension of the representation, N, to infinity, while keeping $\omega_{\mathbf{co}}$ finite. (In the end, it will also be sent to infinity or, rather, made "big enough.") In this limit, we have

$$\left(j_{\max}+\frac{1}{2}\right)^2 = R^2\omega_{\mathbf{co}}^2. \quad (16)$$

So, the counting function (15) becomes

$$N_{|\not{D}|}(\omega_{\mathbf{co}}) = 2R^2\omega_{\mathbf{co}}^2 + 2R\omega_{\mathbf{co}}. \quad (17)$$

Now we would like to use Weyl's theorem (11) in the form suited for the Dirac operator, $\not{D}$ (setting n = 2),

$$\lim_{\omega_{\mathbf{co}}\to\infty}\frac{N_{|\not{D}|}(\omega_{\mathbf{co}})}{\omega_{\mathbf{co}}^2} = \frac{2\mathrm{Area}(\mathcal{M})}{4\pi\Gamma(2)}, \quad (18)$$

where the factor of 2 is the dimension of the spinors [see the discussion after Eq. (11)]. Using Eqs. (17) and (18), one immediately obtains the well-known result for the area of a commutative sphere S_2,

$$\mathrm{Area}(S_2) = 4\pi R^2. \quad (19)$$

What happens if—instead of the exact limit in Eq. (18)—we take just a "physical" limit, i.e. take $\omega_{\mathbf{co}}$ very large but finite? How big should $\omega_{\mathbf{co}}$ be so that we could still conclude that the area is given, within experimental uncertainty, by the formula (19)? From Eq. (17), we have

$$\frac{N_{|\not{D}|}(\omega_{\mathbf{co}})}{\omega_{\mathbf{co}}^2} = 2R^2 + \frac{2R}{\omega_{\mathbf{co}}}. \quad (20)$$

Then if $\omega_{\mathbf{co}} \gg 1/R$, or $j_{\max} \gg 1$, the "physical" area will be given exactly by Eq. (19). The second term in Eq. (20), which is the correction to the commutative answer, is nothing but the physical uncertainty in measuring R using a test particle of mass m as a device. Really, we have $\Delta R \sim \lambda_m \Rightarrow \Delta S \sim \lambda_m R$ or, assuming that $\lambda_m \sim \frac{1}{\omega_{\mathbf{co}}}$, we get the needed correction (see also the discussion at the end of the next section). This should make it clear that our definition is a physical one: the outcome really depends on what particle is used to probe our screen. This will become even more important after we move to the discussion of possible corrections.

B. Case of Weak Noncommutativity

Now we pass to the calculation of the noncommutative corrections to the formula (19) for the case when noncommutativity is not too strong. To begin with, we would like to discuss the range of the applicability of our method. From the previous section, we already know that the cutoff scale, $\omega_{\mathbf{co}}$, should be much bigger then $1/R$ if we want to see any correction to the commutative result (otherwise any correction will just be masked by the experimental error). But there is another bound on $\omega_{\mathbf{co}}$ coming from the fact that this cutoff should still be well below N/R. This comes about due to the following reason: we still want the fuzzy sphere, S_F, to not be too fuzzy, i.e. N still should be much bigger then 1. Then N/R is just the order of the largest eigenvalue [see Eq. (9)], and to be in the regime of corrections we need $\omega_{\mathbf{co}}$ to be much smaller then this largest eigenvalue. So, combining these reasons, we have the following range for the cutoff, where we expect to see corrections due to noncommutativity:

$$1 \ll R\omega_{\mathbf{co}} \ll N. \quad (21)$$

Assuming that Eq. (21) holds, we have the following leading correction to the classical result (16):

$$\left(j_{\max}+\frac{1}{2}\right)^2 = R^2\omega_{\mathbf{co}}^2\left(1+\frac{R^2\omega_{\mathbf{co}}^2}{N^2}+\mathcal{O}\left(\frac{1}{N^2},\frac{R^2\omega_{\mathbf{co}}^2}{N^3}\right)\right). \quad (22)$$

Using this result in Eq. (15), we arrive at the corrections to Eq. (19),

$$\mathrm{Area}(S_F) \approx 4\pi R^2\left(1+\frac{R^2\omega_{\mathbf{co}}^2}{N^2}\right). \quad (23)$$

Let us analyze the result (23). First of all, we have to make sure that the noncommutative correction is seen on the background of the classical error; see Eq. (20) and the discussion thereafter. This is equivalent to neglecting the second term in Eq. (15). This means that while satisfying Eq. (21), $\omega_{\mathbf{co}}$ should also respect the following:

$$\frac{R^2\omega_{\mathbf{co}}^2}{N^2} \gg \frac{1}{R\omega_{\mathbf{co}}}. \quad (24)$$

Combined with Eq. (21), this puts quite unrealistic restrictions on the possibility to observe these noncommutative corrections even in principle. To see this, we should answer the following question: what are $\omega_{\mathbf{co}}$ and N? We assume that $\omega_{\mathbf{co}}$ should be of the order of the inverse Compton wavelength of the test particle, which is used to probe the fuzzy sphere. This assumption seems very natural in view of the fact that this particle is the "device" used to probe the holographic screen in the entropic approach. As for N, we make another natural assumption that this is the same N that is used in the formulation of the entropic scenario, i.e. the number of quanta of area, $N \sim A/l_P^2$. Now, if we require that the quantum corrections (23) are seen on top of the classical "experimental" error (20), we immediately see that this is equivalent to

$$\left(\frac{l_P}{\lambda_m}\right)^3 \gg \frac{R}{l_P}. \quad (25)$$

It is clear that Eq. (25) could be satisfied only in the regime of strong noncommutativity, which is well beyond the assumed weak noncommutativity. So, we have to analyze

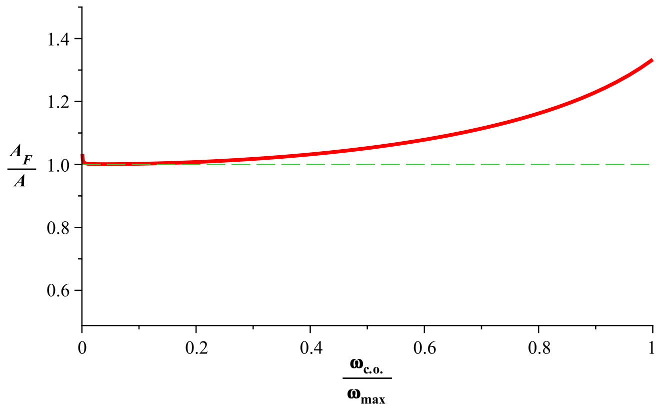

FIG. 1 (color online). The example of A_F/A vs $\omega_{\mathbf{co}}$, where $A = 4\pi R^2$, and $\omega_{\max}$ is determined from Eq. (13) and $N = 100\,000$.

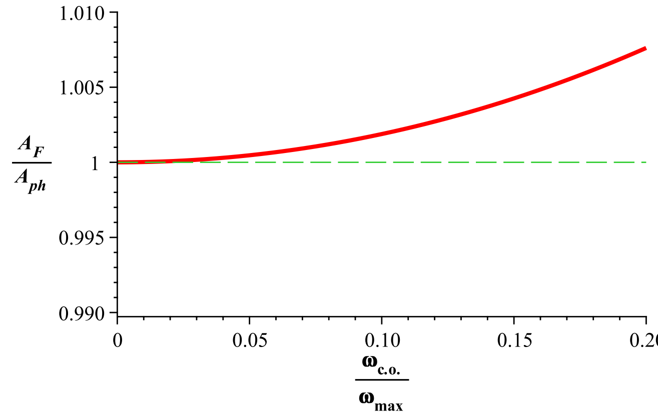

FIG. 2 (color online). Noncommutative area [normalized to A_{ph}, the classical "physical" area, which includes the "experimental" error (20)] as a function of the cutoff scale. A significant deviation is seen well below the cutoff.

the fully noncommutative model, i.e. Eq. (15), without assuming Eq. (21).

C. Strong Noncommutativity

Because in this regime it is hard to expect that our model will correctly describe quantum-gravitational effects, we will just perform a qualitative analysis.[5] For this, let us plot the behavior of the area as seen by the particle, A_F, versus the cutoff scale, $\omega_{\mathbf{co}}$. The result is shown in Fig. 1. What exactly does this picture mean?

First of all, one should not be deceived by a "big enough" N: $N = 100\,000$ corresponds to a highly noncommutative (i.e. quantum) regime. This is because for this N, the radius of the screen (using our assumption, $N \sim \frac{A}{l_P^2}$) is just three orders below the Plank scale.

Secondly, where is the Plank scale, $\omega_P \sim \frac{1}{l_P}$, in this figure? With the same assumptions as above, we can easily see that

$$\frac{\omega_P}{\omega_{\max}} \sim \frac{1}{\sqrt{\omega_{\max} R}} \sim \frac{1}{\sqrt{N}}. \tag{26}$$

So, even for such a highly quantum regime $\frac{\omega_P}{\omega_{\max}} \sim 10^{-3}$, i.e. the Plank scale is well below $\omega_{\max}$.

Thirdly, we can see in Fig. 2 that deviations from the classical area (plus experimental error) defined by Eq. (20) start at quite a high cutoff, which in this specific case is well above the Planck scale. This is in complete agreement with Eq. (21)—which could be rewritten as $\frac{1}{N} \ll \frac{\omega_{\mathrm{co}}}{\omega_{\max}} \ll 1$—and the conclusion at the end of the previous section that for this range noncommutative corrections are not seen on the background of the experimental uncertainty. So, we can say that even if a test particle could probe the Plank scale[6] it will see almost the classical area (for this value of N). We can see that, due to Eq. (26), the situation will drastically change if one goes deeper into the quantum regime, i.e. smaller N. (See the discussion in the next section.)

All of the above seems to indicate that there are no significant corrections to the physical (i.e. as seen by a test particle) area of the holographic screen due to the noncommutativity of this screen. As a result, one might conclude that the only corrections to Newton's law in the entropic scenario are due to the corrections to the entropy, as in Eqs. (4)–(6). In the next section we will discuss whether this is true or not.

V. DISCUSSION AND INTERPRETATION

Let us analyze what we have obtained. We will start by confronting our results with the result (6), which was also obtained in the noncommutative framework. Looking at Eq. (6), we immediately notice that the only way to see any sizable corrections is to approach the source mass M to a distance of order of the Plank scale; otherwise, any such correction will be exponentially suppressed. What is this regime in our picture? It is not hard to see that it corresponds to $N \sim 1$. But, according to Eq. (26), this is exactly where the maximal cutoff scale $\omega_{\max}$ is of order of the Plank scale! Then it is pretty obvious that the quantum corrections to the area will be very significant. E.g., if $N = 100$, the corrections in Eq. (6) will be suppressed by a factor of the order of e^{-100}, while by looking at Fig. 2 (which looks pretty much the same for $N = 100$, but now the Plank scale is around $0.2\omega_{\max}$) we see that noncommutative corrections to the area will be of order of 1%. In this

[5]This still should make sense because, as we mentioned before, noncommutativity is a nonperturbative residue of QG, so it should capture QG effects up to $\lambda_m \leq l_P$.

[6]One can imagine using as a test particle the so-called maximon [27], i.e. a speculated elementary particle such that its Compton wavelength is equal to its gravitational radius.

regard, it is important to understand that to see these corrections one needs a test particle that can probe the Plank scale. But while Eq. (6) produces corrections only within the Plank distance of the source, the noncommutative corrections to area are nonzero even away from the origin. This leads to the conclusion that these noncommutative corrections can completely shadow the effects due to the corrections to the entropy [given by Eq. (6)].

Let us now look at Eq. (5). This type of corrections looks much more reliable. Namely, let us consider the first correction [due to the logarithmic term in Eq. (4)]. It is pretty obvious from our analysis that there exists some range where this term will clearly dominate any correction to the area. This is because this term behaves as $1/N$ and does not depend on the Compton wavelength of a test particle. So, when the test particle has a large Compton wavelength compared to the Plank scale [but still small enough to probe such distances, i.e. $\lambda_m \ll R$; see Eq. (21) and the discussion after Eq. (20)], one will have sizable corrections to Newton's law while having almost negligible corrections to the area [see Fig. 2 and Eq. (26)]. However, closer to the Plank scale this will be completely masked by the area correction. This could be interpreted in the following way: it is well known that the same corrections also come from perturbative quantum gravity [10,11], so it is reasonable to believe that they should be trusted well above the Plank scale. Closer to the Plank scale perturbative calculations would clearly fail, and this is where noncommutative effects, which are nonperturbative traces of QG, will start to matter.[7]

This consideration brings our attention to the very important point of the nontrivial dependence of the possible corrections on a test particle. A test particle now becomes the essential part of the definition of gravity. It is needed not just to reveal already-existing gravity (in the form of curvature of space-time as in GR), but to "produce" it by changing the entropy of the holographic screen, which leads to the entropic force. By looking at the holographic screen from the point of view of noncommutative geometry, it makes this special role of a test particle even more obvious: now, the same holographic screen will look different for different test particles. This looks quite like the violation of the equivalence principle by the quantum-gravitational effects. In our approach, this is reflected in the result that the area corrections depend on the physical cutoff, which is a function of the mass of a test particle, $\omega_{\mathbf{co}} \sim m$.

Before we close this section, we would like to support the assumption made at the very beginning: the cutoff scale is always below the "end" of the spectrum. Using considerations such as those in Ref. [28], one can argue that the cutoff scale should be less than or equal to the Plank scale. But we have seen that the Plank scale is always below the maximal eigenvalue (at least in our model). Thus, from the point of view of any experiment (i.e. from the operational point of view) one could never tell whether the spectrum is finite or not. What one can only do is to measure possible deviations in geometrical quantities based on the deviation of the *observed* part of the spectrum from the classical one.

VI. SUMMARY AND CONCLUSIONS

In this paper we analyzed the possible effects of noncommutativity in the entropic scenario by using a fuzzy sphere as a holographic screen. In contrast with the other efforts in this direction, which deal with the corrections to entropy (and, as a consequence, to the apparent gravitational force), we concentrated our attention on the question of the interaction of a test particle and a holographic screen. That this is very important follows from the special role played by a test particle in the entopic scenario, which is rather different from its role in GR, based on the equivalence principle.[8]

In the absence of the necessary apparatus to directly study the process during which a test particle "becomes a part of the screen," we made an attempt to study how this screen is seen by the particle. For this we adopted the model of a noncommutative screen that, as we argued, should capture some nonperturbative QG effects. As the main tool, we used the generalization of Weyl's theorem, which has proven to be quite efficient in the study of deformed geometries.

The main conclusion of this paper could be formulated as follows:

While perturbative corrections [such as the second term in Eq. (5)] can be trusted well below the Plank scale (in conformity with their universal model independence), neither correction should be trusted close to the Plank scale. In particular, it seems that the corrections given by Eq. (6) will be washed away by the uncertainties due to our

[7]It is worth remembering that Eq. (20) was calculated for a very specific choice of the Dirac operator. As a result, the absence of $1/N$ corrections in Eq. (23) is very much accidental. Other choices of the Dirac operator might easily produce these corrections, leading to a much stronger deviation from the classical area. In this case, even the term in Newton's law due to the logarithmic correction could be overshadowed by the area corrections.

[8]To demonstrate a very special role of a test particle in this scenario as well as the importance of the knowledge of the microscopic dynamics of the interaction of the particle with a screen, we could imagine the following Gedankenexperiment. Let us consider a test particle with some Compton wavelength λ_m. Typically, there will be several holographic screens on this length (in our case the number of the screens can be estimated as $\frac{\lambda_m R}{l_P^2}$, which is huge away from the Plank scale). Then the question is to the entropy of which of these screens does this test particle contribute? We can answer this question only if we know the details of the microscopic dynamics of the screen-particle system.

ignorance about the details of the interaction between a screen and a test particle.

Our discussion opens up the way to speculations about the possible violations of the equivalence principle. But this could happen only on the Plank scale, so it is hard to believe in any possibility of experimental confrontation.

To conclude, our study showed that no effects of noncommutativity (which encodes at least some nonperturbative QG effects) will be seen below the Plank scale, but when one approaches this scale they will start to dominate over the perturbative corrections. To get further control over these effects, one needs to use the full quantum gravity in the form of strings, loops or any other theory.

ACKNOWLEDGMENTS

A. P. acknowledges the partial support of CNPq under Grant No. 308911/2009-1. C. M. G. is supported by CNPq master's scholarship.

[1] E. P. Verlinde, J. High Energy Phys. 04 (2011) 029.
[2] L. Smolin, arXiv:1001.3668.
[3] S. Doplicher, K. Fredenhagen, and J. E. Roberts, Commun. Math. Phys. **172**, 187 (1995).
[4] L. Freidel and E. R. Livine, Classical Quantum Gravity **23**, 2021 (2006).
[5] H.-J. Matschull and M. Welling, Classical Quantum Gravity **15**, 2981 (1998).
[6] F. A. Bais, N. M. Muller, and B. J. Schroers, Nucl. Phys. **B640**, 3 (2002).
[7] L. Modesto and A. Randono, arXiv:1003.1998.
[8] I. V. Vancea and M. A. Santos, Mod. Phys. Lett. A **27**, 1250012 (2012).
[9] S. Ghosh, arXiv:1003.0285.
[10] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).
[11] H. W. Hamber and S. Liu, Phys. Lett. B **357**, 51 (1995).
[12] E. G. Floratos and G. K. Leontaris, Phys. Lett. B **465**, 95 (1999).
[13] E. R. Livine and D. R. Terno, Nucl. Phys. **B794**, 138 (2008).
[14] M. Milgrom, Proc. Sci., HRMS (2010) 033.
[15] P. Nicolini, Phys. Rev. D **82**, 044030 (2010).
[16] J. Madore, Classical Quantum Gravity **9**, 69 (1992).
[17] B. P. Dolan, J. High Energy Phys. 02 (2005) 008.
[18] A. Pinzul, Classical Quantum Gravity **28**, 195005 (2011).
[19] G. Alexanian, A. Pinzul, and A. Stern, Nucl. Phys. **B600**, 531 (2001).
[20] S. Vaidya, Phys. Lett. B **512**, 403 (2001).
[21] S. Vaidya and B. Ydri, Nucl. Phys. **B671**, 401 (2003).
[22] J. A. Wheeler, in *Complexity, Entropy and the Physics of Information*, edited by W. H. Zurek (Addison-Wesley, Boston, 1991).
[23] A. Connes, *Noncommutative geometry* (Academic Press, Amsterdam, 1994).
[24] U. Carow-Watamura and S. Watamura, Commun. Math. Phys. **183**, 365 (1997).
[25] H. Grosse and P. Presnajder, Lett. Math. Phys. **33**, 171 (1995).
[26] A. Pinzul and A. Stern, Phys. Lett. B **512**, 217 (2001).
[27] M. A. Markov, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 115 (1987) [JETP Lett. **45**, 141 (1987)].
[28] G. Dvali, G. F. Giudice, C. Gomez, and A. Kehagias, J. High Energy Phys. 08 (2011) 108.