Note on gauge invariance and causal propagation

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Interactions of gauge-invariant systems are severely constrained by several consistency requirements. One is the preservation of the number of gauge symmetries, another is causal propagation. For lower-spin fields, the emphasis is usually put on gauge invariance that happens to be very selective by itself. We demonstrate with an explicit example, however, that gauge invariance, albeit indispensable for constructing interactions, may not suffice as a consistency condition. The chosen example that exhibits this feature is the theory of a massless spin-3/2 field coupled to electromagnetism. We show that this system admits an electromagnetic background in which the spin-3/2 gauge field may move faster than light. Requiring causal propagation rules out otherwise allowed gauge-invariant couplings. This emphasizes the importance of causality analysis as an independent test for a system of interacting gauge fields. We comment on the implications of allowing new degrees of freedom and nonlocality in a theory, on higher-derivative gravity and Vasiliev's higher-spin theories.

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I. INTRODUCTION

Interacting theories of gauge fields are severely constrained. Powerful no-go theorems [1] prohibit minimal coupling to gravity when the massless particle has spin $s \geq \frac{5}{2}$, as well as to electromagnetism (EM) in flat space when $s \geq \frac{3}{2}$. Nonminimal couplings are still allowed. In fact, trilinear vertices involving gauge fields of arbitrary spins can be classified by using the light-cone formulation [2] and their covariant forms can be obtained by employing either the Noether procedure [3] or the Becchi-Rouet-Stora-Tyutin (BRST) deformation scheme [4–6]. These are higher-derivative interactions that result solely from the requirement of gauge invariance. Is there anything that ensures that these terms respect causality and do not give rise to superluminal modes?

For low-spin systems the situation is somewhat different. In Yang-Mills theory, for example, potentially bad interaction terms containing second time derivatives are eliminated by gauge symmetry itself [7]. Thus gauge invariance implies causality, which is nonetheless an independent consistency check. For higher spins this is no longer true, so that requiring causality along with gauge invariance becomes essential. The purpose of this letter is to highlight this point by proving it for some specific example.

Indeed, gauge invariance does not suffice as a consistency condition. For massive higher-spin particles one can introduce the Stückelberg fields to invent a fake gauge invariance, and then exploit this symmetry to find deformations of the free theory [8]. While this approach enables us to find possible interactions for massive fields, it may leave the coupling constants free. The requirement of causal propagation then fixes some, if not all, of these couplings [9]. The results of Ref. [9] reaffirm, among others, the fact that the "Velo-Zwanziger acausality" [10] for a massive charged spin-2 particle in an EM background can be cured not for an arbitrary magnetic dipole term as the gauge-invariant description might suggest, but precisely when the gyromagnetic ratio is fixed to $g = 2$ [11].

The organization of this letter is as follows. In Sec. II we take the free system of a massless Rarita-Schwinger field and a photon, and consider its gauge deformations. All but one of the cubic couplings are eliminated either by the lack of higher-order consistency or by the potential presence of propagating ghosts. The only remaining vertex, however, leads to acausal propagation for the spin-$\frac{3}{2}$ field in a nontrivial EM background, and we demonstrate this in Sec. III. Finally, we make some remarks in Sec. IV on the implications of adding new degrees of freedom and admitting nonlocality in a theory, and also on higher-derivative (super)gravity and Vasiliev's higher-spin theories as opposed to string theory.

II. THE SYSTEM OF SPIN-3/2 AND SPIN-1 FIELDS

Let us consider the free theory containing a massless Rarita-Schwinger field ψ_μ and a photon A_μ. It is described by the action[1]

[1]We work in Minkowski space-time with mostly positive metric. The Clifford algebra is $\{\gamma^\mu, \gamma^\nu\} = +2\eta^{\mu\nu}$, and $\gamma^{\mu\dagger} = \eta^{\mu\mu}\gamma^\mu$. The Dirac adjoint is defined as $\bar{\psi}_\mu = \psi_\mu^\dagger\gamma^0$. The D-dimensional Levi-Civita tensor, $\varepsilon_{\mu_1\mu_2\ldots\mu_D}$, is normalized as $\varepsilon_{01\ldots(D-1)} = +1$. We define $\gamma^{\mu_1\cdots\mu_n} = \gamma^{[\mu_1}\gamma^{\mu_2}\ldots\gamma^{\mu_n]}$, where the notation $[i_1\ldots i_n]$ means totally antisymmetric expression in all the indices $i_1, \ldots, i_n$ with a normalization factor $\frac{1}{n!}$.

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$$\mathcal{L}_{\rm free} = -i\bar{\psi}_\mu\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho - \frac{1}{4}F^2_{\mu\nu}, \tag{2.1}$$

which enjoys two Abelian gauge invariances:

$$\delta_\lambda A_\mu = \partial_\mu\lambda, \qquad \delta_\varepsilon\psi_\mu = \partial_\mu\varepsilon. \tag{2.2}$$

To construct covariant cubic vertices one may employ, for example, the BRST deformation scheme for irreducible gauge theories [4]. The possible couplings are all nonminimal and may contain 1, 2 or 3 derivatives [2,6,12]. The parity-preserving covariant vertices are [6]

$$\mathcal{L}_{\rm cubic} = g_1\bar{\psi}_\mu\left(F^{\mu\nu}+\frac{1}{2}\gamma^{\mu\nu\rho\sigma}F_{\rho\sigma}\right)\psi_\nu + g_2(\bar{\Psi}_{\mu\nu}\gamma^{\mu\nu\alpha\beta\lambda}\Psi_{\alpha\beta})A_\lambda + g_3\bar{\Psi}_{\mu\alpha}\Psi^{\alpha}{}_{\nu}F^{\mu\nu}, \tag{2.3}$$

where the coupling constants g_i's are all real by Hermiticity. The EM and spin-$\frac{3}{2}$ field strengths are respectively given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\Psi_{\mu\nu} = \partial_\mu\psi_\nu - \partial_\nu\psi_\mu$.

The 1-derivative Pauli term is a non-Abelian deformation, i.e., it deforms the gauge algebra. The other two Abelian pieces do not deform the gauge transformations. The 2-derivative vertex, which exists in $D \geq 5$, is gauge invariant up to a total derivative, while the 3-derivative one is just a 3-curvature term (Born-Infeld type). The non-Abelian piece faces obstruction in a local theory beyond the cubic order [6]. In other words, if $g_1 \neq 0$, the cubic couplings (2.3) are killed by the quartic-order consistency unless one adds new degrees of freedom and/or admits nonlocality. If we demand locality, the original system (2.1) of a spin-$\frac{3}{2}$ gauge field and a photon has consistent deformation up to all orders if and only if

$$g_1 = 0. \tag{2.4}$$

Given this, the spin-$\frac{3}{2}$ equations of motion (EoM) become

$$\gamma^{\mu\nu\rho}\partial_\nu\psi_\rho + ig_2\gamma^{\mu\nu\rho\alpha\beta}F_{\alpha\beta}\Psi_{\nu\rho} - 2ig_3\partial_\nu(F^{\alpha[\mu}\Psi^{\nu]}{}_{\alpha}) = 0. \tag{2.5}$$

These EoMs necessarily contain second time derivatives for generic $F_{\mu\nu}$; this is due to the presence of the last term in Eq. (2.5), which can be made explicit by writing

$$-2\partial_\nu(F^{\alpha[\mu}\Psi^{\nu]}{}_{\alpha}) = F^{\mu\nu}(\Box\psi_\nu - \partial_\nu\partial\cdot\psi) - F^{\alpha\beta}\partial^\mu\partial_\alpha\psi_\beta + \text{lower time derivatives}. \tag{2.6}$$

As a result, the system may have propagating ghosts since the gauge symmetry no longer guarantees the removal of all but the physical polarizations. The remedy is simply to remove the 3-derivative coupling, i.e., to set

$$g_3 = 0. \tag{2.7}$$

With the only nonzero coupling constant $g_2 \equiv g$, the EoMs now reduce to

$$[\gamma^{\mu\nu\rho} + 2ig\gamma^{\mu\nu\rho\alpha\beta}F_{\alpha\beta}]\partial_\nu\psi_\rho = 0. \tag{2.8}$$

On the other hand, the spin-1 field obeys the EoMs

$$\partial_\mu F^{\mu\nu} = J^\nu, \tag{2.9}$$

where the current J^μ comprises some spin-$\frac{3}{2}$ bilinears. Below we will study a possible solution of the system of equations (2.8) and (2.9).

III. CAUSALITY ANALYSIS

Let us consider small fluctuations of the spin-$\frac{3}{2}$ field. The right-hand side of Eq. (2.9) can therefore be neglected, so that the photon EoMs have the solution

$$F_{\mu\nu} = \text{constant}. \tag{3.1}$$

In this EM background, we would like to investigate the propagation of the spin-$\frac{3}{2}$ field as a probe. Its dynamics is governed by the Lagrangian equation (2.8), which has the same number of components as the vector-spinor ψ_μ, i.e., $D \times 2^{[D]/2}$ components in D space-time dimensions, with $[D] \equiv D + \frac{1}{2}[(-1)^D - 1]$. Now the $\mu = 0$ component of Eq. (2.8) does not contain any time derivative and hence constitutes a constraint, which renders $2^{[D]/2}$ of the components nondynamical. Because $\dot{\psi}_0$ never appears in Eq. (2.8), ψ_0 is just a Lagrange multiplier, and thus one gets rid of additional $2^{[D]/2}$ components. Finally, one can do a complete gauge fixing by setting, for example,[2]

$$\gamma^i\psi_i = 0, \tag{3.2}$$

to end up having a correct total of $(D-3)\times 2^{[D]/2}$ propagating degrees of freedom (fields and momenta) for a massless spin-$\frac{3}{2}$ field in D dimensions.

In order to see if the propagation of the physical components is inside the light cone, we take recourse of the shock-wave formalism [13]. The method relies on the fact that characteristic surfaces for wave propagation are those that support discontinuities in the highest-order derivative terms in the EoMs. Let us denote the discontinuity across the characteristic as

$$[\partial_\mu\psi_\nu] = \zeta_\mu\tilde{\psi}_\nu, \tag{3.3}$$

where ζ_μ is a vector normal to the characteristic surface and $\tilde{\psi}_\mu$ is some vector-spinor defined on the same. Thus Eq. (2.8) yields

$$[\gamma^{\mu\nu\rho} + 2ig\gamma^{\mu\nu\rho\alpha\beta}F_{\alpha\beta}]\zeta_\nu\tilde{\psi}_\rho = 0. \tag{3.4}$$

[2]Here $i = 1, 2, \ldots, D-1$ corresponds to the spatial components. To see that this is indeed a complete gauge fixing, suppose this is not the case. Then the residual gauge parameter must satisfy the constraint $\gamma^i\partial_i\varepsilon = 0$, and hence, in particular, the Laplace equation: $\nabla^2\varepsilon = 0$. Given that the gauge parameter should vanish at spatial infinity, the only possible solution is $\varepsilon = 0$. Therefore, there is no residual gauge symmetry.

If the wave propagation is causal, any component of $\tilde{\psi}_\mu$ must vanish for a timelike ζ_μ unless it corresponds to an unphysical or a nondynamical mode. Without loss of generality, let us choose the timelike vector $\zeta_\mu = (1, 0, \ldots, 0)$. While the $\mu = 0$ component of Eq. (3.4) is trivially satisfied, the spacelike components give

$$[\gamma^{ij} + 2ig\gamma^{ijkl}F_{kl}]\tilde{\psi}_j = 0. \tag{3.5}$$

On the other hand, the discontinuity of the time derivative of the gauge choice (3.2) across the characteristic sets

$$\gamma^i\tilde{\psi}_i = 0. \tag{3.6}$$

Let us first implement this consequence of the gauge choice in Eq. (3.5) to write

$$[\mathbf{1}\delta^{ij} - 2ig\gamma^{ijkl}F_{kl}]\tilde{\psi}_j = 0. \tag{3.7}$$

If g vanishes, clearly all $\tilde{\psi}_i = 0$, so that the wave propagation is causal as expected.[3] When $g \neq 0$, we would like to see if Eq. (3.7) could admit nontrivial solutions for $\tilde{\psi}_i$.

For the rest of the analysis, let us consider $D = 5$. The components of the electric field $\vec{E}$ and the magnetic field $\mathbb{B}$, which is an antisymmetric rank-2 spatial tensor, are given by

$$F^{0i} = E^i, \qquad F^{ij} = \frac{1}{2}\varepsilon^{ijkl}B_{kl}. \tag{3.8}$$

In $D = 5$, there are two independent EM field invariants:

$$\begin{aligned} \mathrm{Tr}F^2 &= 2\vec{E}^2 + \mathrm{tr}\mathbb{B}^2, \\ \mathrm{Tr}F^4 &= 2(\vec{E}^2)^2 + \mathrm{tr}\mathbb{B}^4 - (\vec{E}\times\mathbb{B})^2, \end{aligned} \tag{3.9}$$

where Tr and tr denote traces in 5-dimensional Minkowski space-time and 4-dimensional space respectively, and $\vec{E}\times\mathbb{B}$ is the spatial vector with the ith component $\varepsilon^{ijkl}E_jB_{kl}$. With a nonzero $\mathbb{B}$ field, one finds that $\mathrm{tr}\mathbb{B}^2$ is always negative whereas $\mathrm{tr}\mathbb{B}^4$ is always positive.

Let us now consider Eq. (3.7) to see if there exist nonzero solutions for $\tilde{\psi}_\mu$. These equations involve only the magnetic field, which we can choose to be

$$\mathbb{B} = \left(\begin{array}{cc|cc} 0 & B_{12} & 0 & 0 \\ -B_{12} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & B_{34} \\ 0 & 0 & -B_{34} & 0 \end{array}\right), \tag{3.10}$$

thanks to the spatial-isotropy-preserving gauge choice (3.2). For simplicity, we stick to the special case $B_{12} = B \neq 0$, and $B_{34} = 0$. The only nonzero components of the EM field strength are then $F_{34} = -F_{43} = B$. In this case, Eq. (3.7) reduces to

[3]Because $\tilde{\psi}_0$ corresponds to a Lagrange multiplier, not a dynamical field, it is irrelevant for our discussion.

$$\left(\begin{array}{cc|cc} \mathbf{1} & -i\mathbf{\Gamma} & \mathbf{0} & \mathbf{0} \\ i\mathbf{\Gamma} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{array}\right)\begin{pmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \\ \tilde{\psi}_3 \\ \tilde{\psi}_4 \end{pmatrix} = \begin{pmatrix} \underline{0} \\ \underline{0} \\ \underline{0} \\ \underline{0} \end{pmatrix}, \tag{3.11}$$

where $\mathbf{\Gamma} = 4gB\gamma^{1234}$. From the block diagonal form of the matrix it is clear that

$$\tilde{\psi}_3 = 0, \qquad \tilde{\psi}_4 = 0. \tag{3.12}$$

Then the consequence (3.6) of the gauge choice (3.2) reduces to

$$\gamma^1\tilde{\psi}_1 + \gamma^2\tilde{\psi}_2 = 0. \tag{3.13}$$

This enables us to write $\tilde{\psi}_2 = \gamma^{12}\tilde{\psi}_1$, so that Eq. (3.11) gives

$$(\mathbf{1} + 4igB\gamma^{34})\tilde{\psi}_1 = 0. \tag{3.14}$$

The determinant of this coefficient matrix is

$$\det(\mathbf{1} + 4igB\gamma^{34}) = [(4gB)^2 - 1]^2. \tag{3.15}$$

If $g \neq 0$, this will vanish if the magnetic field is

$$g^2B^2 = \frac{1}{16}. \tag{3.16}$$

Thus, indeed $\tilde{\psi}_{1,2}$ may have nontrivial solutions.

Note that we are interested only in small values of the EM field invariants:

$$g^2|\mathrm{Tr}F^2| \ll 1, \qquad g^4|\mathrm{Tr}F^4| \ll 1. \tag{3.17}$$

Otherwise, various instabilities appear [14] and the concept of long-lived propagating particles ceases to make sense. But the nontrivial solutions for $\tilde{\psi}_\mu$ show up even for infinitesimally small values of the EM field invariants if the electric field $\vec{E}$ is such that

$$\vec{E}^2 = -\frac{1}{2}\mathrm{tr}\mathbb{B}^2 + \epsilon_1, \qquad (\vec{E}\times\mathbb{B})^2 = \frac{1}{2}(\mathrm{tr}\mathbb{B}^2)^2 + \mathrm{tr}\mathbb{B}^4 + \epsilon_2, \tag{3.18}$$

where $|\epsilon_1| \ll 1/g^2$ and $|\epsilon_2| \ll 1/g^4$. We conclude that superluminal propagation takes place within the regime of physical interest. To cure this pathology, one must set

$$g \equiv g_2 = 0. \tag{3.19}$$

Therefore, causal propagation in the absence of additional degrees of freedom admits no cubic couplings at all, although they are allowed by gauge invariance alone.

IV. REMARKS

In this paper, we presented an explicit example to make the point that gauge invariance alone does not guarantee the consistency of a theory of interacting gauge fields and that causality must be added as an independent requirement. To the best of our knowledge, this is the first time such an analysis has been done for massless fields.

As pointed out in Ref. [6], the g_2 term analyzed here is of Chern-Simons (C-S) type. Now, for bosonic fields with free EoMs containing second-order derivatives, a C-S term cannot lead to shock-wave acausal behavior because its contribution to the EoMs contains only first-order derivatives. The reason why the g_2 term plays a critical role here is that the free fermionic EoMs are first order in derivatives, and so the C-S term competes with the unperturbed Lagrangian in the shock-wave causality analysis.

The three cubic couplings appearing in Eq. (2.3) were considered piecemeal in the subsequent discussion and it is legitimate to do so from a gauge-theoretic point of view. Note that the addition of a dynamical graviton in the theory removes the obstruction of the non-Abelian piece at higher orders while keeping locality intact. Indeed, this vertex is present in $\mathcal{N} = 2$ gauged supergravity [15]. Decoupling gravity by taking $M_{\rm P} \to \infty$ kills this Pauli term because the dimensionful coupling constant goes like $1/M_{\rm P}$ [15]. Given this, one may require that the consistency of the gauge deformation itself should define the degrees of freedom to begin with. With the inclusion of gravity, all the pieces in the vertex (2.3) may pass the quartic-order consistency to give rise to some higher-derivative counterpart of $\mathcal{N} = 2$ supergravity. One would like to know if this higher-derivative supergravity theory is causal. On the other hand, integrating out the massless graviton gives a nonlocal theory of spin-$\frac{3}{2}$ and spin-1 fields with the Pauli term. Thus one can forgo locality for the sake of higher-order consistency. In the resulting theory, however, the issue of causality becomes very obscure.

Similarly, one can take ghost-free higher-derivative gravity theories, i.e., Lanczos-Lovelock gravities [16], to see if they suffer from superluminality. A canonical analysis of this is intricate [17], but the existence of such a pathology may not come as a surprise. After all, AdS/CFT analysis shows that generic values of the higher-derivative couplings in the bulk afflict the boundary theory with superluminal modes [18]. While such higher-derivative terms do exist in the α' expansion of string theory,[4] they show up with a plethora of other terms; in the end, the theory contains infinitely many derivatives to become essentially nonlocal. Ghost-free nonlinear theories also exist for massive gravity [19], but they are plagued by superluminal propagation [20]. The lack of contradiction between ghostlessness and acausality in this case is very similar in spirit to the fact that gauge invariance for massless models and causality are independent requirements, which must be checked separately.[5]

Neither does gauge invariance imply the consistency of the S matrix. This issue is addressed in [21], and the analysis uses cubic couplings in Minkowski space, just like ours does. The conclusion of [21] is that essentially all local higher-spin cubic vertices are ruled out in flat space, even with an infinite tower of fields. Higher-spin gauge theories on a flat background may still make sense if they incorporate extended and possibly nonlocal objects, like the stringy Pomerons [22]. It remains to be seen how causality works in this case.

In view of our results, it would be interesting to see whether Vasiliev's higher-spin theories [23] pass the test of causal propagation. One might, however, argue that in these theories the metric itself and hence the light cone have no gauge-invariant meaning. Therefore, the issue of causality becomes tricky. Yet if the (infinite tower of) higher-spin excitations are treated as perturbations, it would still make sense to ask if they travel within the light cone. Requiring causality for Vasiliev's theories may shed important light on string theory by telling us whether consistent gauge theories for higher-spin particles necessarily call for the full tower of string states in the tensionless limit.

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[4] The smallness of α' saves the day because "small" couplings are still consistent [18].

[5] We thank S. Deser for comments stressing this point.

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