

## Conformal stealth for any standard cosmology

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It is shown that *any* homogeneous and isotropic universe, independently of its spatial topology and matter content, allows for the presence of a conformal stealth, i.e., a nontrivial conformally invariant scalar field with vanishing energy-momentum tensor, which evolves along with the universe without causing even the smallest backreaction. Surprisingly, this gravitationally invisible universal witness is inhomogeneous with zero consequences for the underlying cosmology. Additionally, it is shown that these results are not exclusive of a four-dimensional universe by generalizing them to higher dimensions.

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### I. INTRODUCTION

Today, cosmologists are living in exciting times because of the continuous arrival of data from highly accurate satellite and ground-based observations (see, for instance, Refs. [1–6]). These data are already able to tightly constrain the theoretical description of the evolution of our Universe, arguably pointing out the so-called  $\Lambda$ CDM model as the best framework for this description [6]. This model is a realization of the standard cosmology, i.e., a homogeneous and isotropic universe,

$$ds^2 = a(\tau)^2 \left( -d\tau^2 + \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right), \quad (1)$$

which for a given matter content  $\varphi_m$  extremizes the action

$$S[g, \varphi_m] = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa} (R - 2\Lambda) + L_m \right) \quad (2)$$

by solving the Einstein equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \kappa T_{\mu\nu} = 0. \quad (3)$$

As is common practice in general relativity, in this framework, it is always considered that any component of the matter content will necessarily leave a trace in the spacetime geometry. Nevertheless, there are special nontrivial matter configurations with no backreaction on the gravitational field. Scalar fields with this property have been found for the static Bañados-Teitelboim-Zanelli (BTZ) black hole [7], Minkowski flat space [8], and (anti-)de Sitter [(A)dS] space [9]. They were coined gravitational *stealths*. In the cosmological context, the existence of stealths has been shown for the de Sitter cosmology [10]. The nontrivial role they play in the probability creation of these universes has been emphasized in Ref. [11]. In this

paper, it is shown that not only de Sitter universes but *any* homogeneous and isotropic universe, without regard to its spatial topology and matter content, allow for the presence of a conformal stealth that evolves along with the universe without exhibiting its gravitational fingerprints.

We will consider the general metric (1), without any assumption on the allowed spatial topology ( $k = 0, \pm 1$ ), and supplement action (2) with an additional term describing a conformally invariant self-interacting scalar field,

$$S[g, \varphi_m] - \frac{1}{2} \int d^4x \sqrt{-g} \left( \partial_\mu \Psi \partial^\mu \Psi + \frac{1}{6} R \Psi^2 + \lambda \Psi^4 \right). \quad (4)$$

Our aim is not to find the whole spectrum of configurations described by the new action but just those critical ones with the special property that both sides of Einstein equations vanish independently,

$$0 = G_{\mu\nu} + \Lambda g_{\mu\nu} - \kappa T_{\mu\nu} = \kappa T_{\mu\nu}^s = 0, \quad (5)$$

where

$$T_{\mu\nu}^s = \partial_\mu \Psi \partial_\nu \Psi - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \Psi \partial^\alpha \Psi + \lambda \Psi^4) + \frac{1}{6} (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu + G_{\mu\nu}) \Psi^2. \quad (6)$$

The vanishing of the left-hand side gets us back to the starting Friedman–Robertson–Walker (FRW) universe with its corresponding matter source. To prove the existence of a stealth in any standard cosmology, we need to show that imposing the vanishing of the energy-momentum tensor (6) evaluated in the Friedman–Robertson–Walker background (1) is compatible with a nontrivial scalar behavior: the stealth.

In Sec. II, we discuss how new stealth configurations can be derived exploiting symmetries of the stealth action (in the present case, the conformal symmetry) and highlight how in some cases this argument can fail to give the correct answer due to its local nature. In Sec. III, the general derivation is done explicitly for the four-dimensional case, which is the most interesting from a

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cosmologist point of view. In future work, this will be also useful as a methodology for finding stealth configurations for more general nonminimal couplings not necessarily of a conformal nature and for which no obvious conformal arguments can be exploited. In the next two sections, Secs. IV and V, we show the explicit results for flat and curved universes, respectively; show how they are generalized to any number of dimensions; analyze the impact of spacetime symmetries on the corresponding solutions; and establish the explicit comparison with the previously discussed local conformal arguments. In the last section, we briefly summarize our conclusions.

## II. STEALTHS FROM ACTION SYMMETRIES: LOCAL VS GLOBAL

A useful way to approach the problem of the existence of stealths is the following. The equations determining the stealth [the right-hand side of Eqs. (5)] can be interpreted as demanding the gravitational background to be an extremal of the related stealth action only [the second term in the total action (4)]. Under this interpretation, a *full* stealth configuration, i.e., a given background together with its allowed stealth, can be linked to another potentially different full configuration via any symmetry transformation of the stealth action. In particular, having a concrete example of stealth for a conformally invariant action implies that its whole conformal class allows also a stealth interpretation. The above argument works well for the case of a conformal stealth on (A)dS space since, (A)dS being conformally flat, the related configuration [9] is conformally related to that of Minkowski flat space [8].

It is well-known that the Weyl tensor vanishes for Friedman–Robertson–Walker metric (1), which implies that it is also conformally flat. That is obvious for flat universes,  $k = 0$ , for which the conformal factor is just the scale factor  $a$ . It is less trivial for curved universes,  $k = \pm 1$ , but explicit conformal relations exist also in these cases [12]. Following the above-described approach to the problem, one may wonder whether we can use these conformal maps to find the stealths for FRW spacetimes starting from the already-known configurations of Minkowski spacetime [8]. However, it should be emphasized that the most general conformal transformations mapping Minkowski spacetime into the FRW spacetimes with curved spatial topology are only defined locally [13]. Just the flat case  $k = 0$  is globally conformal to the Minkowski spacetime.

All the above implies that we cannot ensure that the stealth found by the corresponding conformal transformation of the one for Minkowski spacetime will actually be well-defined in the whole FRW spacetime. An illustrative example of how this mapping can fail is given by the stealth of the BTZ black hole [7]. This background is a rotating black hole solution of vacuum AdS<sub>3</sub> gravity [14] and therefore has zero Cotton tensor, implying that it is

conformally flat. On the other hand, it has constant negative curvature, meaning that it is locally diffeomorphic to AdS<sub>3</sub> (in fact, it is a proper identification of AdS<sub>3</sub> [15]). As we mentioned previously, (A)dS space is conformally flat in any dimension and by this property supports a stealth conformally related to the one of Minkowski spacetime. Nevertheless, if one then makes use of the diffeomorphism between the BTZ black hole and AdS<sub>3</sub> to find the corresponding stealth over BTZ, after imposing the global boundary condition that defines this solution, i.e., identifying the rotation angle by  $\phi = \phi + 2\pi$ , the resulting expression is found to be multivalued. Requiring it to be single-valued imposes the vanishing of the black hole angular momentum. Therefore, there is not a stealth configuration for the rotating case, even if the local conformal transformation does exist.

Taking this discussion into account, in this paper, we proceed first to search for the global stealth configurations of FRW models by solving in general the defining stealth constraints  $T_{\mu\nu}^s = 0$ . Later, for each curvature, we compare with the results obtained from the corresponding local conformal transformation. Fortunately, no discrepancy is found in the cosmological context, as opposed to the above-mentioned three-dimensional example.

## III. COSMOLOGY WITH STEALTHS

For the purpose of accomplishing our task, we find it useful to use the redefinition

$$\Psi = \frac{1}{a\sigma}, \quad (7)$$

where the function  $\sigma = \sigma(x^\mu)$  inherits the full spacetime dependence of the scalar field.

We start by writing the off-diagonal constraints determining the stealth, i.e.,  $T_{\mu\nu}^s = 0$  for  $\mu \neq \nu$ . Let us consider first the ones involving the conformal time,

$$T_{\tau i}^s = \frac{1}{3a^2\sigma^3} \partial_{\tau i}^2 \sigma = 0, \quad (8)$$

where we label the coordinates as  $\{x^\mu\} = \{\tau, x^i\}$ , with  $\{x^i\} = \{r, \Omega^b\}$  denoting the full spatial coordinates and  $\{\Omega^b\} = \{\theta, \phi\}$  just the angular ones. The above equations imply that  $\sigma$  is separable as a sum of functions for the conformal time and the spatial coordinates. Next, we consider the off-diagonal spatial components,

$$T_{rb}^s = \frac{r}{3a^2\sigma^3} \partial_{rb}^2 \left( \frac{\sigma}{r} \right) = 0, \quad (9)$$

and note that the separable spatial dependence of  $\sigma$  divided by  $r$  is itself separable as a sum of functions for  $r$  and the angles. The remaining off-diagonal component is

$$T_{\theta\phi}^s = \frac{\sin\theta}{3a^2\sigma^3} \partial_{\theta\phi}^2 \left( \frac{\sigma}{\sin\theta} \right) = 0; \quad (10)$$

that is, the angular dependence of  $\sigma$  divided by  $\sin\theta$  turns out to be separable as a sum for the angles. Summarizing,

the study of the off-diagonal components leads to the following separable form for the function  $\sigma$ ,

$$\sigma(x^\mu) = T(\tau) + R(r) + r[\Theta(\theta) + \sin\theta\Phi(\phi)]. \quad (11)$$

There exists a freedom in the election of the above functions; concretely, homogeneous terms in  $\Phi$ ,  $\Theta$ , and  $R$  can be compensated by a sinusoidal dependence in  $\Theta$ , a linear one in  $R$ , and another homogeneous term in  $T$ , respectively.

Let us study now the diagonal components; the mixed combinations

$$3ra^4\sigma^3(T^s_{\theta^{\theta}} - T^s_{r^r}) = \frac{d^2\Theta}{d\theta^2} + \Theta - (1 - kr^2)r\frac{d^2R}{dr^2} + \frac{dR}{dr} = 0,$$

$$3r\sin\theta a^4\sigma^3(T^s_{\phi^{\phi}} - T^s_{\theta^{\theta}}) = \frac{d^2\Phi}{d\phi^2} + \Phi - \sin\theta\frac{d^2\Theta}{d\theta^2} + \cos\theta\frac{d\Theta}{d\theta} = 0$$

give rise to the separable equations

$$\frac{d^2\Theta}{d\theta^2} + \Theta = (1 - kr^2)r\frac{d^2R}{dr^2} - \frac{dR}{dr}, \quad (12)$$

$$\frac{d^2\Phi}{d\phi^2} + \Phi = \sin\theta\frac{d^2\Theta}{d\theta^2} - \cos\theta\frac{d\Theta}{d\theta}, \quad (13)$$

which integrate as

$$\Phi(\phi) = A_1 \cos\phi + A_2 \sin\phi, \quad (14a)$$

$$\Theta(\theta) = A_3 \cos\theta, \quad (14b)$$

$$R(r) = \begin{cases} B_- \sqrt{1 - kr^2}, & k \neq 0, \\ \frac{1}{2}\alpha r^2, & k = 0, \end{cases} \quad (14c)$$

modulo the previously mentioned freedom in the election of the above functions. Here,  $A_i$  (with  $i = 1, 2, 3$ ),  $\alpha$ , and  $B_-$  are integration constants.

Using now the combination

$$3a^4\sigma^3(T^s_{\theta^{\theta}} - T^s_{t^t}) = \begin{cases} \frac{d^2}{d\tau^2}T + kT = 0, & k \neq 0, \\ \frac{d^2}{d\tau^2}T + \alpha = 0, & k = 0, \end{cases} \quad (15)$$

we have that the dependence on the conformal time is

$$T(\tau) = \begin{cases} -\frac{A_0}{\sqrt{k}} \sin(\sqrt{k}\tau) + B_+ \cos(\sqrt{k}\tau), & k \neq 0, \\ -\frac{1}{2}\alpha\tau^2 - A_0\tau + \sigma_0, & k = 0, \end{cases} \quad (16)$$

where  $A_0$ ,  $B_+$ , and  $\sigma_0$  are integration constants. The above temporal dependences of the stealth make clear why the use of the conformal time is essential; using the comoving time  $t = \int d\tau a(\tau)$ , the solution is expressed in terms of quadratures of the scale factor.

Only one equation remains in our study and is independent of the value of the curvature if one makes the following redefinitions of the integration constants of the curved cases  $B_{\pm} = \sigma_0/2 \pm \alpha/k$ :

$$-2a^4\sigma^4 T^s_{t^t} = \lambda + A_0^2 - \vec{A}^2 + 2\alpha\sigma_0 = 0, \quad (17)$$

which shows that one of the integration constants is fixed in terms of the coupling constant of the conformal potential and the other integration constants.

Therefore, for any standard cosmology, independently of its spatial topology and matter content, there exists a conformal stealth generally described by Eq. (7) with  $\sigma$  written as in Eq. (11) and functions  $R$ ,  $\Theta$ ,  $\Phi$ , and  $T$  given by Eqs. (14) and (16). Finally, one of the involved integration constants is not independent and is determined by the coupling constant  $\lambda$  and the remaining integration constants from Eqs. (17).

Next, we analyze separately the explicit form of the conformal stealth for flat and curved universes, generalize these results to any number of dimensions, and study how many integration constants can be eliminated using space-time symmetries in each case.

#### IV. FLAT UNIVERSES

For universes with flat spatial topology,  $k = 0$ , the FRW metric (1) becomes manifestly conformally flat:

$$ds^2 = a(\tau)^2 \eta_{\mu\nu} dx^\mu dx^\nu. \quad (18)$$

Therefore, in this case, we indeed expect the expression for the stealth (7) to be exactly a conformal transformation of the stealth for Minkowski flat spacetime:

$$\Psi = \frac{1}{a} \Psi_{\text{flat}}. \quad (19)$$

To check this, we combine the results for  $k = 0$  of Eqs. (11), (14), and (16) of the previous section to obtain

$$\sigma(x^\mu) = \frac{\alpha}{2} x_\mu x^\mu + A_\mu x^\mu + \sigma_0, \quad (20)$$

where we raise and lower indices with the flat metric. This is precisely the result for flat spacetime found in Ref. [8].

The above results are easy to generalize to any number of dimensions using the following recipe. The metric and the auxiliary function  $\sigma$  are still given by expressions (18) and (20) but with  $\mu$  and  $\nu$  now running from 0 to  $D - 1$ . The conformal stealth must be written now as

$$\Psi = \frac{1}{(a\sigma)^{(D-2)/2}}. \quad (21)$$

Then, in the  $D$ -dimensional version of action (4), the conformal coupling must be generalized to

$$\frac{1}{6} \rightarrow \frac{D-2}{4(D-1)}, \quad (22)$$

and similarly for the conformal potential:

$$\frac{1}{2} \lambda \Psi^4 \rightarrow \frac{(D-2)^2}{8} \lambda \Psi^{2D/(D-2)}. \quad (23)$$

The relation between the coupling constant  $\lambda$  and the integration constants is again determined by Eq. (17), where vectorial quantities have now  $D - 1$  components.

Finally, it is worthy to emphasize an important difference between the conformally related versions of the stealths in Minkowski spacetime and flat universes. In Minkowski spacetime, due to translational invariance, it is possible to fix the constants  $A_\mu$  in expression (20) to zero; hence, to describe the conformal stealth, only one integration constant is required, and the related solution is manifestly Lorentz invariant [8]. Flat universes (18) have translation invariance only along spatial directions, which allows us to choose as vanishing only the spatial constants  $A_i$  in Eq. (20). Consequently, the conformal stealth of flat universes allows for two integration constants and is only manifestly isotropic.

## V. CURVED UNIVERSES

For universes with curved spatial topology,  $k = \pm 1$ , the FRW metric (1) can be rewritten as

$$ds^2 = a(\tau)^2 \left( -d\tau^2 + d\vec{x}^2 + \frac{k(\vec{x} \cdot d\vec{x})^2}{1 - k\vec{x}^2} \right), \quad (24)$$

using standard Euclidean coordinates. Combining again Eqs. (11), (14), and (16), the function  $\sigma$  characterizing the corresponding stealth is given by

$$\begin{aligned} \sigma(x^\mu) = & -\frac{A_0}{\sqrt{k}} \sin(\sqrt{k}\tau) + \left( \frac{\sigma_0}{2} + \frac{\alpha}{k} \right) \cos(\sqrt{k}\tau) \\ & + \vec{A} \cdot \vec{x} + \left( \frac{\sigma_0}{2} - \frac{\alpha}{k} \right) \sqrt{1 - k\vec{x}^2}, \end{aligned} \quad (25)$$

where we have used the redefinition on the integration constants giving the universal relation (17) to the coupling constant  $\lambda$ . Notice that, as a byproduct, these redefinitions allow us to recover consistently the flat case (20) by taking the limit  $k \rightarrow 0$ .

Expressions (24) and (25) allow obvious generalizations to higher dimensions, which define a higher-dimensional conformal stealth also for the curved cases. It is only needed to follow the outlines given in the previous section.

Here, the standard spatial translation invariance is broken due to the presence of spatial curvature; however, a generalization of spatial translations still remains as symmetry. These quasitranslations [16] can be understood as follows. As is well-known, the constant curvature spatial sections can be isometrically embedded in a flat space with one extra dimension whose rotations induce the constant curvature isometries. The spatial coordinates of metric (24) are just the embedding coordinates, and rotations along the planes orthogonal to the extra dimension in the ambient space induce just the isotropy of metric (24). Moreover, rotations along the planes formed with the extra dimension induce the quasitranslations, which have the following explicit form [16]:

$$\vec{x} \mapsto \vec{x} + \vec{a} \left( \sqrt{1 - k\vec{x}^2} - \frac{(1 - \sqrt{1 - k\vec{x}^2})}{\vec{a}^2} \vec{a} \cdot \vec{x} \right). \quad (26)$$

For  $k = 0$ , they become just standard translations. For any curvature, these transformations map the origin  $\vec{x} = 0$  to any arbitrary point  $\vec{x} = \vec{a}$ , which is an explicit realization of the homogeneous character of spacetime (24). Under quasitranslations, metric (24) is invariant, but the stealth is just form invariant; i.e., its local dependence after the transformation is the same but with transformed integration constants. This allows us to choose specific values for the quasitranslations parameters  $\vec{a}$  such that the transformed integration constants  $\vec{A}$  acquire vanishing values. This is achieved choosing the parameters as

$$\vec{a} = \frac{\vec{A}}{\sqrt{(\alpha - \frac{k}{2}\sigma_0)^2 + k\vec{A}^2}}. \quad (27)$$

Notice that, consequently, in the limit  $k \rightarrow 0$ , these become just the standard translations annihilating the vector  $\vec{A}$  in the flat case. Because of the above argument, after considering the relation to the coupling constant (17), the conformal stealth of curved universes has also only two integration constants and is manifestly isotropic as in the case of flat universes.

### A. Conformal transformation from Minkowski spacetime

According to our discussion of Sec. II, in a proper limit, it should be possible to reduce expression (25) to the result obtained by conformally mapping the stealth from Minkowski spacetime. We will take that into account to crosscheck (25) and also to gain a deeper insight into the role of the symmetries in stealth configurations. With these aims we use the following map between Minkowski and FRW spacetimes [12],

$$\tau = \frac{1}{\sqrt{k}} \arctan \left( \frac{\sqrt{k}t}{1 - \frac{k}{4}(t^2 - \rho^2)} \right), \quad (28a)$$

$$r = \frac{1}{\sqrt{k}} \sin \left[ \arctan \left( \frac{\sqrt{k}\rho}{1 + \frac{k}{4}(t^2 - \rho^2)} \right) \right], \quad (28b)$$

with inverse given by

$$t = \frac{2 \sin(\sqrt{k}\tau)}{\sqrt{k}(\cos(\sqrt{k}\tau) + \sqrt{1 - kr^2})}, \quad (29a)$$

$$\rho = \frac{2r}{\cos(\sqrt{k}\tau) + \sqrt{1 - kr^2}}. \quad (29b)$$

It is important to notice here that these expressions provide only a local map. For  $k = 1$ , the whole Minkowski spacetime is mapped into just a patch of FRW, while, conversely, for  $k = -1$ , just a patch of the Minkowski spacetime is mapped into the whole FRW.

We develop the conformal argument in any dimension, since there is nothing particular in the tetradimensional case. This way, the Friedman–Robertson–Walker metric in  $D$  dimensions can be written as [12]

$$\begin{aligned} ds_{\text{FRW}}^2 &= a(\tau)^2 \left( -d\tau^2 + \frac{dr^2}{1-kr^2} + r^2 d\Omega_{D-2}^2 \right) \\ &= \frac{a(\tau(t, \rho))^2 (-dt^2 + d\rho^2 + \rho^2 d\Omega_{D-2}^2)}{[\frac{k}{4}(\rho+t)^2 + 1][\frac{k}{4}(\rho-t)^2 + 1]} \\ &= \Omega^2 ds_{\text{M}}^2. \end{aligned} \quad (30)$$

Consequently, the conformal transformation for the stealth will be

$$\Psi_{\text{FRW}} = \frac{1}{\Omega^{(D-2)/2}} \Psi_{\text{M}} = \frac{1}{(\Omega \sigma_{\text{M}})^{(D-2)/2}}, \quad (31)$$

where the conformal factor  $\Omega$  can be drawn from Eq. (30) and the auxiliary function  $\sigma_{\text{M}}$  of Minkowski spacetime (20) is rewritten now as

$$\sigma_{\text{M}} = \frac{\alpha}{2} (-t^2 + \rho^2) - A_0 t + \rho A_m \pi^m + \sigma_0, \quad (32)$$

where  $\pi^m$  are the polar coordinates of the  $S^{D-2}$  unit sphere. Using the inverse transformations (29), the conformal factor reduces to

$$\Omega = \frac{a(\tau)}{2} \left[ \cos(\sqrt{k}\tau) + \sqrt{1-kr^2} \right],$$

while, for  $\sigma_{\text{M}}$ , we obtain

$$\begin{aligned} \sigma_{\text{M}} &= 2 \left( -\frac{\alpha \cos(\sqrt{k}\tau) - \sqrt{1-kr^2}}{k \cos(\sqrt{k}\tau) + \sqrt{1-kr^2}} \right. \\ &\quad - \frac{A_0 \sin(\sqrt{k}\tau)}{\sqrt{k} \cos(\sqrt{k}\tau) + \sqrt{1-kr^2}} \\ &\quad \left. + \frac{r A_m \pi^m}{\cos(\sqrt{k}\tau) + \sqrt{1-kr^2}} + \frac{\sigma_0}{2} \right). \end{aligned} \quad (33)$$

Combining these last two expressions leads us finally to Eq. (25), i.e.,  $\Omega \sigma_{\text{M}} = a \sigma_{\text{FRW}}$ , and the local expression for the stealth obtained from the conformal transformation coincides exactly with the general previously found solution.

## VI. CONCLUSIONS

We have proven that any homogeneous and isotropic universe, independently of its spatial topology and matter

content, allows for the existence of a conformal stealth. Surprisingly, although the stealth is isotropic, it is not homogeneous. Nevertheless, its presence leaves no trace in the cosmological evolution of the given universe. Additionally, we have shown that these results are not exclusive of our four-dimensional Universe but are also valid for higher-dimensional generalizations of the FRW spacetime. We prove all the above by solving explicitly the related constraints, but we also discuss to some extension the local conformal arguments that alternatively allow us to build such configurations from the ones of Minkowski spacetime. After making the explicit construction, we show that the potential problems that are known to occur in other contexts, due to the local nature of these arguments, are not present in the cosmological framework. However, due to the fact that the involved conformal factors break in general the maximally symmetric character of Minkowski spacetime, the resulting conformally generated stealth configurations allow more integration constants than its seeds from Minkowski spacetime. For FRW spacetimes, the conformal factors break time-translation invariance, and, as a consequence, its stealths allow one additional integration constant in comparison to the single one allowed by its conformally related cousins of Minkowski spacetime.

Last, but not least, it is important to understand the observational consequences of the existence of cosmological stealths. In this sense, we note that its fluctuations are not expected to be stealth themselves. This way, the corresponding stealth perturbations may have an imprint on the spectra of the cosmic microwave background radiation as well as in the statistics of the cosmological large scale structures. Exploring these consequences is the subject of our current research program.

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