

Universal two-body physics in dark matter near an S -wave resonance

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The dark matter annihilation rate at small relative velocities can be amplified by a large boost factor using various mechanisms, including Sommerfeld enhancement, resonance enhancement, and Breit-Wigner enhancement. These mechanisms all involve a resonance near the threshold for a pair of dark matter particles. We point out that if the resonance is in the S -wave channel, the mechanisms are equivalent sufficiently near the resonance and they are constrained by universal two-body physics. The amplified annihilation rate requires a corresponding amplification of the elastic scattering cross section. If the resonance is a bound state below the threshold, it has an increased lifetime that is inversely proportional to the square root of the binding energy. Its spatial structure is that of two dark matter particles whose mean separation is also inversely proportional to the square root of the binding energy.

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I. INTRODUCTION

One of the greatest mysteries in physics today is the nature of the dark matter that comprises most of the mass of the Universe. A possibility that is strongly motivated by elementary particle physics is weakly interacting massive particles that are relics of a thermal distribution in the early universe [1]. One possible signature for dark matter particles is their annihilation into ordinary particles, which could be detected through observations of the annihilation products. In the standard scenario, the annihilation rate in the present era is completely determined by the mass of the dark matter particles and their velocity-independent annihilation rate $v\sigma_{\text{ann}}$. Limits on dark matter annihilation can provide constraints on physics beyond the Standard Model for elementary particles.

Observations of high-energy electrons and positrons in cosmic rays at rates larger than expected have motivated models for dark matter in which the present annihilation rate is boosted above that in the standard scenario by orders of magnitude. In these models, the low-energy annihilation rate is enhanced by powers of $1/v$, where v is the relative velocity of the dark matter particles. Sommerfeld enhancement gives a factor of $1/v$ from the exchange of a light mediator between the dark matter particles. Resonance enhancement gives a factor of $1/v^2$ from a bound state of two dark matter particles near their scattering threshold. Breit-Wigner enhancement gives a factor of $1/v^4$ from an elementary particle near the scattering threshold.

In this paper, we point out that these enhancement mechanisms are actually equivalent very near the

resonance, provided the resonance is in the S -wave channel. An S -wave resonance is the most interesting case, because it can provide more dramatic enhancement for a given degree of fine tuning than a resonance with higher angular momentum. For a P -wave or higher angular momentum resonance, the fine tuning must also compensate for the angular momentum suppression. An S -wave resonance near the scattering threshold produces a scattering length that is much larger than the range of interactions. The two-body physics of dark matter particles with an S -wave resonance near threshold has universal behavior that depends only on their large scattering length and not on the enhancement mechanism [2]. Universal two-body physics produces constraints on the behavior of dark matter that have not been taken into account in previous calculations of its properties.

II. DARK MATTER SCATTERING

Conventional dark matter consistent with cosmological constraints consists of particles that are nonrelativistic and have weak short-range interactions with ordinary particles. The dark matter particles also have short-range self-interactions, which could be the weak interactions of the Standard Model and/or a new interaction mediated by a particle from a hidden sector. To be concise, we will refer to the dark matter particles as wimps, regardless of their interactions. A bound state of two wimps will be called “wimponium.” We denote the mass of the wimps by M . Their behavior in the low-energy limit is described by a complex scattering length a that has a small negative imaginary part due to the annihilation channel or, equivalently, by the inverse scattering length $\gamma = 1/a$. For a conventional wimp, such as a neutralino in a supersymmetric extension of the Standard Model, the real and

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imaginary parts of the scattering length $a_w = 1/\gamma_w$ are of order $\alpha_w M/m_w^2$ and $\alpha_w^2 M^3/m_w^4$, respectively, where $\alpha_w \sim 10^{-2}$ and $m_w \sim 100$ GeV are the coupling constant and mass scale of the weak interactions. If the wimps interact through the exchange of a mediator particle with mass m_y and small coupling constant α_y , their interaction in the nonrelativistic limit can be approximated by a Yukawa potential $-\alpha_y \exp(-m_y r)/r$.

The effect on the relic abundance of dark matter from a resonance whose mass M_R is close to the threshold $2M$ for a pair of wimps was considered long ago [3,4]. The resonance is typically an elementary particle with a weak coupling to dark matter. The use of such a resonance to boost the low-energy annihilation rate was proposed by Ibe *et al.* [5] and is called ‘‘Breit-Wigner enhancement.’’ The effect on the kinetic decoupling temperature of the dark matter from elastic scattering with ordinary particles through t -channel exchange of the same resonance was studied by Bi *et al.* [6].

In Refs. [3–6], the annihilation cross section was assumed to be that of a Breit-Wigner resonance. However, when a resonance is sufficiently close to the threshold for a pair of particles, the cross section can be significantly modified by rescattering of those particles. The effects can be particularly dramatic when the resonance is in the S -wave channel. In the limit of a weak coupling α_R of the resonance to the wimps, the resonance has a well-defined mass M_R and width Γ_R . The low-energy interactions of the wimps in this limit are described by a complex scattering length $a_w = 1/\gamma_w$. If α_R is nonzero and if the mass of the resonance is sufficiently close to the threshold $2M$, the effects of self-scattering of the wimps and their scattering through the resonance must both be summed up to all orders. The resulting scattering amplitude for wimps with nonrelativistic relative momentum k reduces to [7]

$$f(k) = \left[-\left(\frac{1}{\gamma_w} - \frac{\alpha_R}{\delta_R - i\Gamma_R/2 - k^2/M} \right)^{-1} - ik \right]^{-1}, \quad (1)$$

where $\delta_R = M_R - 2M$. The elastic cross section is $\sigma_{\text{el}} = 4\pi |f(k)|^2$, and the inelastic cross section σ_{in} , which includes the annihilation cross section, can be determined from the optical theorem, $\sigma_{\text{el}} + \sigma_{\text{in}} = (4\pi/k) \text{Im} f$. Cutting rules can be used to resolve σ_{in} into terms proportional to $\text{Im}(\gamma_w)$ and Γ_R , corresponding to dark matter annihilation and resonance decay, respectively. The conventional annihilation cross section with Breit-Wigner enhancement is $(4\pi/k) |f(k)|^2 \text{Im}(\gamma_w)$, with the $1/\gamma_w$ and $-ik$ terms in the expression for $f(k)$ in Eq. (1) omitted.

We now consider the behavior of the annihilation cross section σ_{ann} as a function of the relative velocity $v = 2k/M$ of the wimps. If M_R is far from the threshold, $v\sigma_{\text{ann}}$ approaches the constant $8\pi \text{Im}(-a_w)/M$ in the low-energy limit. If M_R is tuned to be very close to the threshold, $v\sigma_{\text{ann}}$ scales as $1/v^4$ in most of the region where

the resonance term in Eq. (1) dominates. For small velocities, this scaling behavior is cut off at $v \sim (2\Gamma_R/M)^{1/2}$ or at $v \sim (4|\delta_R|/M)^{1/2}$ by the width or energy of the resonance. In previous work on the Breit-Wigner enhancement of dark matter annihilation, it was not recognized that there can be another scaling region of v in which the rescattering term $-ik$ in Eq. (1) dominates. This can occur if $1/\gamma_w$ is sufficiently small and if the resonance is sufficiently narrow, $\Gamma_R \ll \alpha_R^2 M_R$. As v decreases, the scaling behavior $1/v^4$ for $v\sigma_{\text{ann}}$ crosses over to $1/v^2$ at $v \sim \alpha_R$ before it is ultimately cut off at $v \sim \Gamma_R/(\alpha_R M_R)$. If there was a massive Dirac neutrino whose threshold $2M$ was close to the mass of the Z^0 resonance, its coupling strength would be $\alpha_R \approx 0.0012$, and $\Gamma_Z/(\alpha_R^2 M_Z)$ would be 1.9×10^4 . Thus a $1/v^2$ scaling region requires a resonance for which $\Gamma_R/(\alpha_R^2 M_R)$ is more than 4 orders of magnitude smaller.

In the $1/v^2$ scaling region and below, the elastic and inelastic self-scattering cross sections reduce, if the particles are distinguishable, to

$$\sigma_{\text{el}}(k) = \frac{4\pi}{|-\gamma - ik|^2}, \quad (2a)$$

$$\sigma_{\text{in}}(k) = \frac{4\pi \text{Im}(\gamma)}{k|-\gamma - ik|^2}, \quad (2b)$$

where $\gamma = [1/\gamma_w - \alpha_R/(\delta_R - i\Gamma_R/2)]^{-1}$ is the inverse scattering length. The elastic cross section can be as large as $16\pi\alpha_R^2/\Gamma_R^2$ if M is near $M_R/2$. Using Eq. (2b), the annihilation rate reduces in the low-energy limit to

$$v\sigma_{\text{in}} \rightarrow \frac{8\pi}{M} \left(\text{Im}(-a_w) + \frac{\alpha_R \Gamma_R/2}{\delta_R^2 + \Gamma_R^2/4} \right). \quad (3)$$

The first term on the right side represents the conventional annihilation modes of the wimps. The second term represents their annihilation into decay products of the resonance. For conventional wimps, the first term in the parentheses is of order $\alpha_w^2 M^3/m_w^4$, while the second term can be at most $2\alpha_R/\Gamma_R$. The condition $\Gamma_R \ll \alpha_R^2 M_R$ for the $1/v^2$ scaling region does not exclude the second term from being larger than the first. Thus, the dark matter could annihilate predominantly into the decay products of the resonance.

There is an analogous phenomenon in cold atom physics called a ‘‘Feshbach resonance’’ [8]. A magnetic field can be used to tune a diatomic molecule to near the atom pair threshold. If the molecule has an S -wave coupling to the atoms, there is a scaling region of the magnetic field in which the elastic cross section of the atoms has the universal form in Eq. (2a). The magnetic field can be used to control the scattering length $a = 1/\gamma$ of the atoms, making it arbitrarily large or arbitrarily small.

The elastic and inelastic cross sections in Eq. (2) are appropriately called universal, because they apply to any particles with short-range interactions that have an S -wave resonance sufficiently close to threshold. More specifically,

if there is a parameter whose variation makes an S -wave bound state or resonance pass through the threshold while keeping the range of interactions fixed, generically there will be a scaling region of that parameter in which the cross sections have the universal forms in Eq. (2). More intricate low-energy behavior requires multiple fine tuning. In the dark matter context, the universal cross sections in Eqs. (2) were written down previously by March-Russel and West to describe resonance enhancement from an S -wave wimponium near the threshold $2M$ [9]. There is a scaling region of M in which the wimponium mass is sufficiently close to the threshold that the cross sections have the universal forms in Eqs. (2). If the wimps interact through a Yukawa potential with range $1/m_y$, the universal cross sections apply when the wimponium binding energy is much less than m_y^2/M . As an illustration, we consider an attractive potential between wimps that is mediated by Z^0 exchange. The resulting Yukawa potential is $-\alpha_Z \exp(-M_Z r)/r$, where $\alpha_Z = 0.011$ if the Z^0 couples to the wimps with the same strength with which it couples to neutrinos. The critical value M_* for the wimp mass for which the lowest bound state is at threshold is $M_* = 1.68M_Z/\alpha_Z \approx 14$ TeV. There is a universal $1/v^2$ scaling region if $M - M_*$ is less than about $M_Z^2/M_* \approx 0.6$ GeV. The universal region of M extends a similar distance below M_* , where the wimponium is unbound. The real part of γ can be calculated as a function of M , α_y , and m_y by solving the Schrödinger equation for zero-energy scattering from the Yukawa potential. The imaginary part of γ can be obtained by calculating σ_{in} in the scaling region, where it reduces to $4\pi\text{Im}(\gamma)/k^3$.

Sommerfeld enhancement occurs if the mediator mass m_y is orders of magnitude smaller than that of the wimp. If the Yukawa potential is attractive, the wimp annihilation rate is generically enhanced by a factor of approximately $\pi\alpha_y/v$ that is related to Sommerfeld's enhancement factor in Coulomb scattering. The Yukawa potential in this case supports many bound states. A further boost of the enhancement factor to order $1/v^2$ can be obtained by resonance enhancement, in which M is tuned to near a critical value for which one of the excited S -wave bound states is at the threshold. The Sommerfeld enhancement of the annihilation of neutralino dark matter from electroweak gauge boson exchange was first noticed by Hisano *et al.* [10]. The same authors noted that the enhancement is more dramatic when there is a wimponium near the wimp pair threshold [11]. In Ref. [12], Hisano *et al.* labeled both effects ‘‘Sommerfeld enhancement.’’ The enhancement from a wimponium near the wimp pair threshold is more properly referred to as ‘‘resonance enhancement,’’ because it has nothing to do with Coulomb or Yukawa potentials. It occurs for any short-range potential with an S -wave bound state near threshold. Arkani-Hamed *et al.* increased the interest in these enhancements when they pointed out that a light gauge boson from a hidden sector

could explain several possible anomalies in particle astrophysics [13].

III. UNIVERSAL RELATIONS

The universal cross sections in Eqs. (2) depend on a single complex parameter γ , whose imaginary part is positive. Any relation between observables that can be expressed in terms of γ is universal, because it will apply to all models in which there is an S -wave resonance sufficiently close to threshold. One simple universal relation can be obtained by taking the ratio of the universal cross sections in Eqs. (2),

$$\frac{k\sigma_{\text{in}}(k)}{\sigma_{\text{el}}(k)} = \text{Im}(\gamma). \quad (4)$$

The imaginary part of γ is insensitive to the fine-tuning parameters that are most commonly used to tune the resonance to the threshold, including the wimp mass M . The universal relation in Eq. (4) therefore implies that any mechanism that boosts the annihilation rate of the dark matter by orders of magnitude will inevitably also boost its elastic self-scattering cross section by a comparable amount. Feng *et al.* have considered the effect of wimp elastic scattering on the annihilation of dark matter after freeze-out and found that it can lead to chemical recoupling to ordinary matter [14]. For the elastic cross section, they used an empirical parametrization of a numerical cross section for a Yukawa potential with large α_y . It has a scaling region where it increases like $1/v^{0.7}$ before crossing over to its asymptotic behavior $\ln^2(1/v)$. The universal elastic cross section in Eq. (2a) could be used to calculate the effects in the resonance region accurately. Buckley and Fox have noted that the enhancement of σ_{ann} requires an enhancement of σ_{el} , but they only presented graphical results from the numerical solution of the Schrödinger equation for a Yukawa potential [15]. Tulin *et al.* have used analytic solutions for S -wave scattering in the Hulthén potential to approximate σ_{el} and σ_{ann} for particles interacting through a Yukawa potential [16]. They noted that exactly at the resonance, σ_{el} reduces to Eq. (2a) with $\gamma = 0$.

The limit $\gamma = 0$ in which the universal elastic cross section in Eq. (2a) reduces to $4\pi/k^2$ is called the ‘‘unitary limit,’’ because σ_{el} saturates the unitarity bound for S -wave scattering. Since this cross section does not depend on any interaction parameters, the interactions between the particles must be scale invariant. The particles can be described by a nonrelativistic conformal field theory with nontrivial scaling dimensions [17]. The two-body physics in this conformal field theory, which is encapsulated in the simple scattering amplitude $f(k) = i/k$, involves a nontrivial scaling dimension. The scaling behavior $1/v^2$ of the universal elastic cross section is a reflection of the anomalous scaling dimension -2 of the interaction energy operator [17]. Away from the unitary limit where γ is

nonzero, the particles can be described by a nonrelativistic field theory whose renormalization is governed by the conformal field theory.

Backović and Ralston have emphasized the importance of the widths of particles in dark matter annihilation [18]. The effects of widths are fully incorporated into the universal cross sections in Eqs. (2) through the imaginary part of γ . Backović and Ralston also argued that unitarity does not allow large enhancements in low-energy cross sections in a weakly coupled theory [18]. They did not recognize that the strong coupling required for a large enhancement can arise through small energy denominators instead of through a large coupling constant.

The nature of the resonance is determined by the sign of $\text{Re}(\gamma)$. If $\text{Re}(\gamma) < 0$, the resonance is a virtual state whose only physical manifestation is the enhancement of the cross sections. If $\text{Re}(\gamma) > 0$, the resonance is a bound state below the threshold $2M$. We will refer to it as resonant wimponium. Resonant wimponium has universal properties that are determined by γ . Its binding energy $E_X = 2M - M_X$ and its width Γ_X are determined by the pole in the analytic continuation of the scattering amplitude $f(k) = 1/(-\gamma - ik)$. Expressing the complex energy of the pole as $-E_X - i\Gamma_X/2$, we find

$$E_X = (\text{Re}(\gamma)^2 - \text{Im}(\gamma)^2)/M, \quad (5a)$$

$$\Gamma_X = 4 \text{Re}(\gamma)\text{Im}(\gamma)/M. \quad (5b)$$

Since $\text{Im}(\gamma)$ is insensitive to the fine tuning that changes $\text{Re}(\gamma)$, the universal relations in Eqs. (5) imply that $\Gamma_X/(ME_X)^{1/2}$ is approximately equal to the constant $4\text{Im}(\gamma)/M$ in the region where $\text{Re}(\gamma) \gg \text{Im}(\gamma)$. This implies that as the binding energy E_X is tuned towards 0, the width Γ_X decreases in proportion to $E_X^{1/2}$ until $\text{Re}(\gamma)$ is comparable to $\text{Im}(\gamma)$. Thus resonant wimponium has an enhanced lifetime that can be as large as $M/[\text{Im}(\gamma)]^2$ very near the resonance.

What is most remarkable about resonant wimponium is its spatial structure. It can be described by a Schrödinger wave function for a pair of wimps,

$$\psi(r) = [\text{Re}(\gamma)/2\pi]^{1/2} e^{-\gamma r}/r. \quad (6)$$

This universal wave function implies that the typical separation of the wimps is $1/\text{Re}(\gamma)$. This is larger than the range of the interactions between the wimps, and it becomes increasingly large as one approaches the resonance, ultimately reaching the size $1/\text{Im}(\gamma)$. This behavior is particularly surprising in the case of Breit-Wigner enhancement, where the resonance is a pointlike elementary particle when it is far from the threshold. Near the resonance, interactions with the dark matter transform it into an extended object consisting of two well-separated wimps.

Another interesting aspect of the universal wave function in Eq. (6) is that it diverges at the origin. This might

seem problematic for the production of resonant wimponium through processes that involve a large momentum transfer, because bound state effects are commonly absorbed into the square of the wave function at the origin, $|\psi(0)|^2$. The quadratic divergence of $|\psi(r)|^2$ as $r \rightarrow 0$ is a reflection of the anomalous dimension -2 of the interaction energy operator. A pragmatic way to deal with this problem is to note that $|\psi(0)|^2$ also appears in a naive calculation of the annihilation decay rate. By the optical theorem, the transition rate to annihilation channels is proportional to $\text{Im}(-a)$. The naive result for the inclusive decay width is therefore $\Gamma_X = 8\pi\text{Im}(-a)|\psi(0)|^2/M$, while the correct result is the universal expression in Eq. (5b). The correct result for the production rate can therefore be obtained from the naive calculation by making the substitution $|\psi(0)|^2 \rightarrow |\gamma|^2\text{Re}(\gamma)/4\pi$.

IV. RADIATIVE CAPTURE OF WIMPS

Another annihilation mechanism for dark matter is the formation of wimponium through the radiative capture of wimps. The radiated particle could be the light mediator responsible for the Yukawa potential between the wimps. Once it is formed, the wimponium will eventually decay through the annihilation of its constituents. It was pointed out by Pospelov and Ritz and by March-Russel and West that this indirect process could significantly enhance the dark matter annihilation rate [9,19]. Pospelov and Ritz calculated the radiative capture cross section for the case of true Sommerfeld enhancement [19]. March-Russel and West calculated the cross section for radiative capture into the most deeply bound S - and P -states for the case of an excited S -wave resonant wimponium and a very light mediator [9].

The cross section σ_{rc} for radiative capture into the resonant wimponium can be calculated in an effective field theory for nonrelativistic particles with large scattering length (see, e.g., Ref. [20] and references therein). In this theory, low-energy processes are described in an expansion in γR , where R is the range of the wimp-wimp interaction. It is convenient to use an auxiliary field, d , for the resonant S -wave wimponium. To leading order, the effective Lagrangian can then be written as

$$\begin{aligned} \mathcal{L} = & \sum_{j=1}^2 \psi_j^\dagger \left(i\partial_0 - \hat{Q}A_0 + \frac{(\nabla - i\hat{Q}\mathbf{A})^2}{2M} \right) \psi_j + \Delta d^\dagger d \\ & - g(\psi_1^\dagger \psi_2^\dagger d + \text{H.c.}) + \dots, \end{aligned} \quad (7)$$

where ψ_1, ψ_2 denote the wimp fields and $A^\mu = (A_0, \mathbf{A})$ is the field for the light mediator which we assume to be a vector particle with mass m_γ . Integrating out the auxiliary field d , \mathcal{L} is equivalent to a Lagrangian with two-body and higher contact interactions of the ψ_1 and ψ_2 fields. However, only the two-body interaction contributes in the capture process. Higher-order derivative interactions indicated by the ellipses are not considered here.



FIG. 1. Leading order Feynman diagrams for radiative capture of wimps into resonant wimponium. Solid, double, and wavy lines indicate wimps, wimponium and mediator, respectively.

We assume that the mediator is a vector particle that couples minimally and with opposite signs to the constituents of wimponium (\hat{Q} is the corresponding charge operator). To leading order in the expansion in γR , the capture process is then given by the two Feynman diagrams shown in Fig. 1. The relative minus sign is due to the opposite coupling of the mediator to ψ_1 and ψ_2 . In the center-of-mass frame, the incoming wimps each have energies $k^2/(2M)$ and momenta \mathbf{k} and $-\mathbf{k}$, respectively. The outgoing mediator has four-momentum $((q^2 + m_y^2)^{1/2}, \mathbf{q})$, while the outgoing wimponium has four-momentum $(q^2/(4M) - \gamma^2/M, -\mathbf{q})$, where γ^2/M is the wimponium binding energy. The matrix element for the capture process is easily obtained from the Feynman rules encoded in the Lagrangian (7).

Close to threshold, the E1 multipole dominates and the incoming wimps have to be in a relative P wave. Picking out the E1 contribution, squaring the matrix element, and summing over the polarizations of the mediator, we obtain the universal capture cross section in the center-of-mass frame,

$$\sigma_{\text{rc}}(k) = \frac{32\pi\alpha_y\gamma M k q(q^2 + m_y^2)}{(\gamma^2 + k^2)^4}, \quad (8)$$

with $\alpha_y = Q^2/(4\pi)$. The two incoming wimps are assumed to be spinless. If the wimps and wimponium carry spin, appropriate factors for the spin average of the wimps and the sum over wimponium spins have to be applied to Eq. (8).

In the limit $m_y \rightarrow 0$, the momenta of the outgoing particles close to threshold are particularly simple, $q \simeq (k^2 + \gamma^2)/M$, and the universal capture cross section

simplifies further. Near threshold, it scales as $\sigma_{\text{rc}}(k) \sim k/(\gamma^2 + k^2)$, which is in agreement with the E1 term in the radiative capture cross section for a proton (p) and a neutron (n) into a deuteron (d) calculated in a low-energy effective field theory for nucleons with large scattering lengths [21]. Note that the cross section for $np \rightarrow d\gamma$ is a factor 8 smaller than Eq. (8) in this limit. A factor of 4 arises since there is only one charged particle and the remaining factor of 2 is due to the spin projection. Finally, we note that a comparison of our result with the previous work of Pospelov and Ritz [19] is not possible. They calculate the expectation value of the recombination rate to an approximately Coulombic ground state, while we calculate the elementary cross section as a function of the momenta for radiative capture to a wimponium state very near the threshold.

V. CONCLUSION

We have pointed out that dark matter with an S -wave resonance close enough to threshold has universal behavior determined by the complex scattering length. The universal aspects include the elastic and inelastic self-scattering cross sections and, if the resonance is a bound state below the threshold, its binding energy and lifetime. The universal constraints have not been taken into account in previous calculations of the behavior of dark matter. The universality of the two-body problem with a large scattering length extends to the three-body sector and beyond [2]. This raises the question of whether any aspects of the beautiful universal physics in these sectors are relevant for dark matter despite its extremely low number density.

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