Low-scale quark-lepton unification

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We investigate the possibility that quarks and leptons are unified at a low energy scale much smaller than the grand unified scale. A simple theory for quark-lepton unification based on the gauge group $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$ is proposed. This theory predicts the existence of scalar leptoquarks which could be produced at the Large Hadron Collider. In order to have light neutrinos without fine-tuning, their masses are generated through the inverse seesaw mechanism.

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I. INTRODUCTION

In the Standard Model (SM) there are two types of matter fields: the lepton and quark fields. The SM with right-handed neutrinos describes all the measured properties of quarks and leptons. Grand unified theories based on gauge groups, such as SU(5) or $SO(10)$, provide one avenue for unifying the properties of quarks and leptons since quarks and leptons are part of the same representation of the gauge group. However, in this case the scale of unification is very high, $M_{\rm GUT} \sim 10^{15-16}$ GeV.

An appealing approach to quark and lepton unification was proposed by Pati and Salam in Ref. [1]. They used an $SU(4)_C$ gauge symmetry and the quarks and leptons are together in the fundamental representation of the gauge group. In this framework the leptons are the fermions with the fourth color. This idea also played a major role in grand unification since the Pati-Salam gauge group is the maximal subgroup of $SO(10)$. This theory also predicts the existence of right-handed neutrinos needed for the seesaw mechanism [2–6] of neutrino masses.

In this paper we revisit the idea of quark-lepton (QL) unification based on the Pati and Salam paper where the leptons have the fourth color. We find a very simple extension of the SM based on the gauge group $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$, where quarks and leptons are unified in the same representation. In addition to the SM fermions the model contains three right-handed singlet fermions needed to generate Majorana neutrino masses through the inverse seesaw mechanism.

Assuming near alignment of the quark and lepton generations the experimental limit on the branching ratio for $K_L^0 \to \mu^\pm e^\mp$ implies that the scale of $SU(4)_C$ breaking must be greater than 1000 TeV. See for example the studies in Refs. [7,8] for the constraints coming from meson decays. This theory predicts the existence of vector and scalar leptoquarks. While the vector leptoquarks must be heavy, the scalar leptoquarks could be at the TeV scale and give rise to exotic signatures at the Large Hadron Collider. In this article we discuss the spectrum of our model and outline its main phenomenological consequences.

In Sec. II we present the simplest model with quark-lepton unification at a low scale that is consistent with the experimental properties of quarks and leptons. In Sec. III we discuss the properties of the vector and scalar leptoquarks. Finally, we briefly summarize our main results in Sec. IV.

II. QUARK-LEPTON UNIFICATION

In models with QL unification based on the idea that leptons have the fourth color [1] the SM quarks and leptons can be unified in the same multiplets: (Q_L, ℓ_L), (u_R, ν_R), and (d_R, e_R). Therefore, naively one finds the following relations between quark and lepton masses:

$$M_u = M_\nu^D \quad \text{and} \quad M_e = M_d, \tag{1}$$

where M_u, M_ν^D, M_e and M_d are the up-quark, Dirac neutrino, charged lepton and down-quark masses, respectively. As is well known, these relations are not consistent with experiment.

We now construct the simplest model of quark-lepton unification based on the gauge group

$$G_{\rm QL} = SU(4)_C \otimes SU(2)_L \otimes U(1)_R, \tag{2}$$

which is consistent with experimental results on the properties of quarks and leptons evading the unacceptable mass relations discussed above, and which allows the scale for the symmetry breaking $G_{\rm QL} \to G_{\rm SM} = SU(3) \otimes SU(2)_L \otimes U(1)_Y$ to be much smaller than the grand unification scale.

The fermion matter fields are in the representations

$$F_{\rm QL} = \begin{pmatrix} u & \nu \\ d & e \end{pmatrix} \sim (4, 2, 0), \tag{3}$$

$$F_u = \begin{pmatrix} u^c & \nu^c \end{pmatrix} \sim (\bar{4}, 1, -1/2), \tag{4}$$

$$F_d = \begin{pmatrix} d^c & e^c \end{pmatrix} \sim (\bar{4}, 1, 1/2). \tag{5}$$

Here we have chosen to work only with left-handed fermion fields, as is common in discussions of grand unified theories.

The gauge group $G_{\rm QL}$ is spontaneously broken to $G_{\rm SM}$ by the vacuum expectation value of the scalar field,

$$\chi = (\chi_u \quad \chi_R^0) \sim (4, 1, 1/2). \tag{6}$$

Without loss of generality the vacuum expectation value can be taken to be only in the fourth component, $\langle \chi_R^0 \rangle = v_\chi/\sqrt{2}$. The SM hypercharge Y is given by

$$Y = R + \frac{\sqrt{6}}{3} T_4, \tag{7}$$

where T_4 is the properly normalized $\mathrm{SU}(4)_C$ generator, that when acting on the fundamental 4 representation is the diagonal matrix

$$T_4 = \frac{1}{2\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}. \tag{8}$$

To break the gauge group down to the low-energy $\mathrm{SU}(3)_C \otimes \mathrm{U}(1)_Y$ gauge group in a way that can give acceptable fermion masses we add two more scalar representations, a Higgs doublet

$$H^T = (H^+ \quad H^0) \sim (1, 2, 1/2), \tag{9}$$

and the scalar $\Phi \sim (15, 2, 1/2)$,

$$\Phi = \begin{pmatrix} \Phi_8 & \Phi_3 \\ \Phi_4 & 0 \end{pmatrix} + T_4 H_2, \tag{10}$$

which contains a second Higgs doublet H_2. The new scalars in Φ are the second Higgs doublet, the color octet with the same weak quantum numbers as the Higgs doublet, $\Phi_8 \sim (8, 2, 1/2)_{\rm SM}$, studied by Manohar and Wise in Ref. [9], and the scalar leptoquarks $\Phi_3 \sim (\bar{3}, 2, -1/6)_{\rm SM}$ and $\Phi_4 \sim (3, 2, 7/6)_{\rm SM}$. These scalar leptoquarks do not give rise to proton decay [10] at the renormalizable level since they do not couple to a quark pair. Proton decay occurs at the dimension-six level.

The Yukawa interactions in this theory are given by

$$\mathcal{L}_{\rm QL}^Y = Y_1 F_{\rm QL} F_u H + Y_2 F_{\rm QL} F_u \Phi + Y_3 H^\dagger F_{\rm QL} F_d + Y_4 \Phi^\dagger F_{\rm QL} F_d + \text{H.c.}, \tag{11}$$

which after symmetry breaking give rise to the following mass matrices for the SM fermions:

$$M_u = Y_1 \frac{v_1}{\sqrt{2}} + \frac{1}{2\sqrt{6}} Y_2 \frac{v_2}{\sqrt{2}}, \tag{12}$$

$$M_\nu^D = Y_1 \frac{v_1}{\sqrt{2}} - \frac{3}{2\sqrt{6}} Y_2 \frac{v_2}{\sqrt{2}}, \tag{13}$$

$$M_d = Y_3 \frac{v_1}{\sqrt{2}} + \frac{1}{2\sqrt{6}} Y_4 \frac{v_2}{\sqrt{2}}, \tag{14}$$

$$M_e = Y_3 \frac{v_1}{\sqrt{2}} - \frac{3}{2\sqrt{6}} Y_4 \frac{v_2}{\sqrt{2}}. \tag{15}$$

Here the vacuum expectation values that break $G_{\rm SM}$ are

$$\langle H^0 \rangle = \frac{v_1}{\sqrt{2}} \quad \text{and} \quad \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}}. \tag{16}$$

Since there are four independent Yukawa coupling matrices in the above equations we can generate acceptable masses for all the quarks and leptons. However, in order to achieve light Dirac neutrino masses one needs a *severe fine-tuning* between the two terms contributing to M_ν^D in Eq. (13). See Ref. [11] for an alternative model using the Pati-Salam symmetry.

It is useful to note that the renormalizable couplings of the model contain an automatic global U(1) fermion matter symmetry $\mathrm{U}(1)_F$ where the matter charges of the fermion fields $F_{F_{\rm QL}} = 1$, $F_{F_u} = F_{F_d} = -1$. The scalar fields do not transform under this symmetry.

Although this model can be consistent with experiment the fine-tuning needed to get very light Dirac neutrinos is not attractive. A modest extension of the fermion content of the model allows us to avoid this fine-tuning.

We can generate small Majorana masses for the light neutrinos if we add three new singlet left-handed fermionic fields N and use the following interaction terms:

$$\mathcal{L}_{\rm QL}^\nu = Y_5 F_u \chi N + \frac{1}{2} \mu N N + \text{H.c.} \tag{17}$$

For simplicity, we now discuss the neutrino sector in the one-generation case. The discussion generalizes easily to the three-generation case. Then, there are three left-handed neutrino fields (one each of ν, ν^c and N) and the neutrino mass matrix reads as

$$(\nu \quad \nu^c \quad N) \begin{pmatrix} 0 & M_\nu^D & 0 \\ (M_\nu^D)^T & 0 & M_\chi^D \\ 0 & (M_\chi^D)^T & \mu \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \\ N \end{pmatrix}. \tag{18}$$

Here M_ν^D is given by Eq. (13) and

$$M_\chi^D = Y_5 \frac{v_\chi}{\sqrt{2}}. \tag{19}$$

By assigning the N fermion charge $F_N = 1$ only the term proportional to μ breaks the matter symmetry. Hence it is natural to take this parameter to be much smaller than the other entries in the mass matrix.

In the limit when $\mu \ll M_\nu^D, M_\chi^D$, the model has a heavy Dirac neutrino with mass $\sqrt{(M_\chi^D)^2 + (M_\nu^D)^2}$, where ν^c is paired with the linear combination

$$\nu_h = (M_\chi^D N + M_\nu^D \nu)/\sqrt{(M_\chi^D)^2 + (M_\nu^D)^2}. \tag{20}$$

The orthogonal linear combination is the light Majorana neutrino

$$\nu_l = (M_\chi^D \nu - M_\nu^D N)/\sqrt{(M_\chi^D)^2 + (M_\nu^D)^2}. \quad (21)$$

It would be massless in the limit $\mu \to 0$ since then Majorana masses are forbidden by the fermion matter symmetry.

So for this neutrino to have the same properties as in the SM we need $M_\nu^D \ll M_\chi^D$, which is reasonable when the scale of SM symmetry breaking is much smaller than the scale of $SU(4)_C \otimes U(1)_R$ symmetry breaking. When the parameters in the neutrino mass matrix follow the relation $\mu \ll M_\nu^D \ll M_\chi$, the Majorana light neutrino masses are given by

$$m_\nu = \mu (M_\nu^D)^2/(M_\chi^D)^2, \quad (22)$$

which is the usual relation in the inverse seesaw mechanism [12,13]. Therefore, we can have light neutrinos without fine-tuning. If $M_\nu^D \sim 10^2$ GeV and $M_\chi^D \sim 10^6$ GeV, the neutrino mass satisfies $m_\nu \sim \mu \times 10^{-8}$. Thus, μ has to be very small—smaller than 0.1 GeV—but it is protected by the matter symmetry.

If we use the usual seesaw mechanism the scale for QL unification has to be close to the seesaw scale $\sim 10^{14}$ GeV. See for example Ref. [14]. However, the inverse seesaw mechanism allows us to have a low scale for QL unification as we have discussed above.

The $SU(4)_C$ gauge boson, $A_\mu \sim (15, 1, 0)$, can be written as

$$A_\mu = \begin{pmatrix} G_\mu & X_\mu/\sqrt{2} \\ X_\mu^*/\sqrt{2} & 0 \end{pmatrix} + T_4 B'_\mu. \quad (23)$$

Here $G_\mu \sim (8, 1, 0)_{SM}$ are the gluons and $X_\mu \sim (3, 1, 2/3)_{SM}$ are the new massive vector leptoquarks. The different transformation properties under color of X_μ and Φ_3 arise from our conventions for how the two 15's A_μ and Φ transform under $SU(4)_C$ gauge transformations U. Neglecting the space-time dependence of the transformations, $A_\mu \to U A_\mu U^\dagger$, while $\Phi \to U^* \Phi U^T$.

We have mentioned before that the Higgs sector is composed of three Higgses: $H \sim (1, 2, 1/2)$, $\chi \sim (4, 1, 1/2)$ and $\Phi \sim (15, 2, 1/2)$. Therefore, the scalar potential can be written as

$$\begin{aligned} V = {} & m_H^2 H^\dagger H + m_\chi^2 \chi^\dagger \chi + m_\Phi^2 \mathrm{Tr}(\Phi^\dagger \Phi) + \lambda_1 H^\dagger H \chi^\dagger \chi \\ & + \lambda_2 H^\dagger H \, \mathrm{Tr}(\Phi^\dagger \Phi) + \lambda_3 \chi^\dagger \chi \, \mathrm{Tr}(\Phi^\dagger \Phi) \\ & + (\lambda_4 H^\dagger \chi^\dagger \Phi \chi + \mathrm{H.c.}) + \lambda_5 H^\dagger \, \mathrm{Tr}(\Phi^\dagger \Phi) H \\ & + \lambda_6 \chi^\dagger \Phi \Phi^\dagger \chi + \lambda_7 (H^\dagger H)^2 + \lambda_8 (\chi^\dagger \chi)^2 \\ & + \lambda_9 \, \mathrm{Tr}(\Phi^\dagger \Phi)^2 + \lambda_{10} (\mathrm{Tr}\Phi^\dagger \Phi)^2. \end{aligned} \quad (24)$$

Here the trace is only in the $SU(4)_C$ space. We assume that the parameters of the potential can be chosen so that these additional scalars get vacuum expectation values that leave color and the electromagnetic charge $Q = T_L^3 + Y$ unbroken, i.e., only the neutral components of χ, H and H_2 get expectation values.

III. VECTOR AND SCALAR LEPTOQUARKS

This theory predicts the existence of vector and scalar leptoquarks. The vector leptoquarks, $X_\mu \sim (3, 1, 2/3)_{SM}$, have the following interactions:

$$\begin{aligned} \mathcal{L} \supset {} & \frac{g_4}{\sqrt{2}} X_\mu (\bar{Q}_L \gamma^\mu \ell_L + \bar{u}_R \gamma^\mu \nu_R + \bar{d}_R \gamma^\mu e_R + \bar{u}_R \gamma^\mu \nu_R) \\ & + \mathrm{H.c.} \end{aligned} \quad (25)$$

The gauge coupling g_4 is equal to the strong coupling constant evaluated at the $SU(4)_C$ scale. The vector leptoquarks contribute to the rare meson decays, $K_L^0 \to e^\mp \mu^\pm$, which give a lower bound on the $SU(4)_C$ scale. This issue has been studied by several groups (see for example Refs. [7,8]), and the bound is $M_X \geq 10^3$ TeV. Notice that there is freedom in the unknown mixings between quarks and leptons and one can have a lower scale. For simplicity we assume that there is no suppression mechanism.

The scalar leptoquarks $\Phi_3 \sim (\bar{3}, 2, -1/6)_{SM}$ and $\Phi_4 \sim (3, 2, 7/6)_{SM}$ present in the field $\Phi \sim (15, 2, 1/2)$ have the following interactions:

$$\begin{aligned} \mathcal{L} \supset {} & Y_2 Q_L \Phi_3 \nu^c + Y_2 \ell_L \Phi_4 u^c + Y_4 Q_L \Phi_4^\dagger e^c \\ & + Y_4 \ell_L \Phi_3^\dagger d^c + \mathrm{H.c.} \end{aligned} \quad (26)$$

It is important to mention that the only coupling constrained by $K_L^0 \to e^\mp \mu^\pm$ is Y_4, but this coupling is small—below 10^{-2}—to be in agreement with fermion masses. The coupling Y_2 is less constrained and can be small as well. Therefore, the scalar leptoquarks Φ_3 and Φ_4 can be at the TeV scale and can be produced at the Large Hadron Collider. As is well known, one can have the QCD pair production of leptoquarks and the decays into a jet and lepton can give a unique signal. For a recent discussion of the leptoquark signatures at the LHC see Refs. [15–18]. A detailed analysis of the constraints coming from meson decays, the lepton-number-violating decays, and the collider signatures is beyond the scope of this paper.

IV. CONCLUDING REMARKS

In this article we have proposed a simple theory where the Standard Model quarks and leptons are unified using the gauge symmetry $SU(4)_C \otimes SU(2)_L \otimes U(1)_R$. The neutrinos are Majorana fermions and their masses are generated through the inverse seesaw mechanism. The quark and lepton unification scale can be as low as 10^3 TeV. The main constraints on the QL breaking scale are coming from the rare meson decays mediated by the vector leptoquark. This theory predicts the existence of scalar leptoquarks, $\Phi_3 \sim (\bar{3}, 2, -1/6)_{SM}$ and $\Phi_4 \sim (3, 2, 7/6)_{SM}$, which could be at the TeV scale and give rise to exotic signatures at the

Large Hadron Collider. Subjects for further work include studying the correlation between the collider signals and the different constraints coming from flavor violation, and constructing the supersymmetric version of the theory to solve the hierarchy problem. We would like to mention that this theory could have a UV completion based on the Pati-Salam gauge group or it could arise from a grand unified theory based on SU(6).

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