Rotating away proton decay in flipped unification

S. M. Barr

Department of Physics Astronomy and The Bartol Research Institute University of Delaware Newark, Delaware 19716, USA

(Received 29 July 2013; published 26 September 2013)

It is shown that by a simple extension of the fermion sector of flipped $SU(5)$ models and other flipped models, proton decay coming from dimension-six operators can be suppressed by fermion mixing angles by an arbitrary amount in a natural way.

DOI: 10.1103/PhysRevD.88.057702 PACS numbers: 12.10.Dm

In unified theories there are mixing angles that describe how the Standard Model quark and lepton multiplets are arranged within the multiplets of the unified group. These unification mixing angles come into proton decay amplitudes [1], and if some of these angles are small, it is possible that proton decay could be greatly suppressed.

It is an interesting question whether proton decay might be *completely* "rotated away" in unified theories by a suitable choice of these mixing angles. For such a choice of angles, what would happen is that the lighter quarks and leptons would be partnered within the irreducible multiplets of the unification group with fermions that are too heavy to appear in the final state of proton decay (which we will call "heavy fermions"). The tree-level amplitudes for proton decay would therefore vanish. [At higher-loop level, these heavier fermions might convert into lighter flavors, but the loop factors of $(16\pi^2)^{-2}$, not to mention suppressions of the Glashow-Iliopoulos-Maiani type, would then effectively kill proton decay anyway.]

There are two ways that the rotating away of proton decay could happen: (a) The "heavy fermions" might be the heavier flavors of the Standard Model (τ, c, b, t), or (b) they might be new, non-Standard Model fermions.

If only the Standard Model quarks and leptons exist, it was shown already in [2] that proton decay cannot be rotated away in $SU(5)$. If the gauge group is flipped $SU(5)$ [3], however, it was shown in [4] that proton decay can be rotated away, although it requires an artificial choice of unification mixing angles. On the other hand, if there exist new fermions, then it is possible to rotate away proton decay even in $SU(5)$. An elegant (though no longer viable) model in which this happens was proposed in [5]. The idea was to impose a conserved Abelian charge that absolutely forbids proton decay. This required a doubling of the fermion content of the model. The new, heavy fermions were partnered with the known quarks and leptons in $SU(5)$ multiplets in such a way that proton decay could not occur.

The purpose of this paper is to point out that dimension-six proton-decay operators can be rotated away in a simple manner in unification schemes based on flipped $SU(5)$ [3], or larger unitary groups in which flipped $SU(5)$ is embedded [6], if there exist extra $\mathbf{5} + \bar{\mathbf{5}}$ fermions. The limits in which the proton decay is completely rotated away are not artificial but correspond to points in parameter space, where the coefficient of a certain type of term in the Lagrangian becomes small. We shall first show this in flipped $SU(5)$ and then in flipped $SU(6) \times SU(2)$.

Consider a model in which the gauge group is $SU(5) \times U(1)_X$, with each family consisting of

$$[\mathbf{10}^{(1)} + \bar{\mathbf{5}}^{(-3)} + \mathbf{1}^{(5)}] + [\mathbf{5}^{(-2)} + \bar{\mathbf{5}}^{(2)}]. \tag{1}$$

If $SU(5) \times U(1)_X$ were embedded in $SO(10)$, the multiplets in the first square brackets would make up a **16**, while those in the second square brackets would make up a **10**. [However, if the model is embedded in $SO(10)$, it turns out that proton decay cannot be "rotated away" in the manner that we shall discuss.] Let us denote by $Y_5/2$ the $SU(5)$ generator $\mathrm{diag}(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3})$ and by $Y/2$ the weak hypercharge generator of the Standard Model. Then, as is well known, the flipped embedding of the Standard Model group in $SU(5) \times U(1)_X$ gives $Y/2 = \frac{1}{5}(-Y_5/2 + X)$. Let us, for the sake of simplicity, consider a model with a single family. The generalization to three families is trivial. The $SU(5) \times U(1)_X$ fermion multiplets then decompose as follows:

$$\begin{array}{lcccccccl}
\mathbf{10}^{(1)} & = & \psi^{[\alpha\beta]} & = & \psi^{[12]} & = & N^c & = & (1,1,0) \\
 & & & = & \psi^{[1a]} & = & u & = & \left(3,2,\frac{1}{6}\right) \\
 & & & = & \psi^{[2a]} & = & d & = & \left(3,2,\frac{1}{6}\right) \\
 & & & = & \psi^{[ab]} & = & d'^c & = & \left(\bar{3},1,\frac{1}{3}\right), \\
\bar{\mathbf{5}}^{(-3)} & = & \psi_\alpha & = & \psi_1 & = & e' & = & \left(1,2,-\frac{1}{2}\right) \\
 & & & = & \psi_2 & = & \nu' & = & \left(1,2,-\frac{1}{2}\right) \\
 & & & = & \psi_a & = & u^c & = & \left(\bar{3},1,-\frac{2}{3}\right), \\
\mathbf{1}^{(5)} & & & = & \psi & = & e^c & = & (1,1,+1), \\
\bar{\mathbf{5}}^{(2)} & = & \tilde{\psi}_\alpha & = & \tilde{\psi}_1 & = & -\bar{N} & = & \left(1,2,+\frac{1}{2}\right) \\
 & & & = & \tilde{\psi}_2 & = & \bar{E} & = & \left(1,2,+\frac{1}{2}\right) \\
 & & & = & \tilde{\psi}_a & = & D'^c & = & \left(\bar{3},1,\frac{1}{3}\right), \\
\mathbf{5}^{(-2)} & = & \tilde{\psi}^\alpha & = & \tilde{\psi}^1 & = & -N' & = & \left(1,2,-\frac{1}{2}\right) \\
 & & & = & \tilde{\psi}^2 & = & E' & = & \left(1,2,-\frac{1}{2}\right) \\
 & & & = & \tilde{\psi}^a & = & \overline{D^c} & = & \left(3,1,-\frac{1}{3}\right).
\end{array} \tag{2}$$

The primes on certain multiplets refer to the fact that they are not mass eigenstates. As we shall see, d'^c and D'^c mix, as do e' and E', and ν' and N'.

Several Higgs multiplets are required to do the symmetry breaking and give mass to the fermions. A $\mathbf{10}_h^{(1)}$ of Higgs fields, which we denote as $\Omega^{[\alpha\beta]}$, obtains a superheavy VEV: $\langle\Omega^{[12]}\rangle = \Omega$. This breaks $SU(5)\times U(1)_X$ down to the Standard Model group in the well-known way. Two Higgs multiplets are needed to do the breaking of the electroweak group and give weak-scale masses to the quarks and leptons, namely a $\mathbf{5}_h^{(-2)}$ and a $\mathbf{5}_h^{(3)}$, which we denote respectively as h^α and H^α. These have the nonzero VEVs $\langle h^1\rangle = v$ and $\langle H^2\rangle = V$.

There are three terms that give fermions superheavy masses:

$$\begin{aligned}
&M\bar{\mathbf{5}}^{(2)}\mathbf{5}^{(-2)} + Y_d\mathbf{10}^{(1)}\mathbf{5}^{(-2)}\langle\mathbf{10}_h^{(1)}\rangle + Y_L\bar{\mathbf{5}}^{(-3)}\bar{\mathbf{5}}^{(2)}\langle\mathbf{10}_h^{(1)}\rangle \\
&\quad = M\tilde{\psi}_\alpha\tilde{\psi}^\alpha + Y_d\psi^{\alpha\beta}\tilde{\psi}^\gamma\langle\Omega^{\delta\eta}\rangle\epsilon_{\alpha\beta\gamma\delta\eta} + Y_L\psi_\alpha\tilde{\psi}_\beta\langle\Omega^{\alpha\beta}\rangle \\
&\quad \to M[\overline{D^c}D'^c + E'\bar{E} + N'\bar{N}] + Y_d(d'^c\overline{D^c})\Omega \\
&\qquad + Y_L[e'\bar{E} + \nu'\bar{N}]\Omega. \qquad (3)
\end{aligned}$$

Collecting terms, this gives $\overline{D^c}(MD'^c + Y_d\Omega d'^c)$, $\bar{E}(ME' + Y_L\Omega e')$, and $\bar{N}(MN' + Y_L\Omega\nu')$. Defining $\tan\theta_d = Y_d\Omega/M$ and $\tan\theta_L = Y_L\Omega/M$, we see that these terms give superheavy masses to the linear combinations $D^c \equiv \cos\theta_d D'^c + \sin\theta_d d'^c$, $E \equiv \cos\theta_L E' + \sin\theta_L e'$, and $N \equiv \cos\theta_L N' + \sin\theta_L \nu'$. The orthogonal linear combinations remain light (and acquire weak-scale masses from the Yukawa terms to be discussed below). We denote these light linear combinations by $d^c \equiv -\sin\theta_d D'^c + \cos\theta_d d'^c$, $e \equiv -\sin\theta_L E' + \cos\theta_L e'$, and $\nu \equiv -\sin\theta_L N' + \cos\theta_L \nu'$.

The $SU(5)\times U(1)_X$ fermion multiplets written in terms of the light and heavy mass eigenstates thus become

$$\begin{aligned}
\mathbf{10}^{(1)} &= \left(N^c, \begin{pmatrix} u \\ d \end{pmatrix}, [\cos\theta_d d^c + \sin\theta_d D^c]\right), \\
\bar{\mathbf{5}}^{(-3)} &= \left(\begin{pmatrix} \cos\theta_L e + \sin\theta_L E \\ \cos\theta_L \nu + \sin\theta_L N \end{pmatrix}, u^c\right), \\
\mathbf{1}^{(5)} &= (e^c), \qquad (4) \\
\bar{\mathbf{5}}^{(2)} &= \left(\begin{pmatrix} -\bar{N} \\ \bar{E} \end{pmatrix}, [-\sin\theta_d d^c + \cos\theta_d D^c]\right), \\
\mathbf{5}^{(-2)} &= \left(\begin{pmatrix} \sin\theta_L \nu - \cos\theta_L N \\ -\sin\theta_L e + \cos\theta_L E \end{pmatrix}, \overline{D^c}\right).
\end{aligned}$$

One sees that in the limit $\cos\theta_d = 0$ and $\cos\theta_L = 0$, the Standard Model fermion multiplets (u, d), u^c, d^c, (ν, e), and e^c are all in different $SU(5)$ multiplets. Thus the superheavy $SU(5)$ gauge bosons cannot cause transitions among the light fermions but always turn a light fermion into a heavy one. Therefore, in this limit, there is no proton decay mediated by superheavy gauge bosons. This limit corresponds to $M \ll Y_d\Omega$, $Y_L\Omega$.

Note that if this model were embedded in $SO(10)$, then there would be gauge bosons that mediated proton decay even in the limit $\cos\theta_d = \cos\theta_L = 0$. In particular, in $SO(10)$ there are gauge bosons that transform as $\mathbf{10}^{(-4)}$ under $SU(5)\times U(1)_X$ and that make transitions $\mathbf{1}^{(5)} \to \mathbf{10}^{(1)}$ and $\mathbf{10}^{(1)} \to \bar{\mathbf{5}}^{(-3)}$. One of these couples to $(\bar{u}\gamma^\mu e^c)$ and $(\overline{u^c}\gamma^\mu d)$ and thus gives the dimension-six operator $(\bar{u}\gamma^\mu e^c)(\bar{d}\gamma_\mu u^c)$, which violates baryon number and contains only light fermions and thus produces proton decay.

In the limit $\cos\theta_d = \cos\theta_L = 0$, proton decay mediated by colored scalars can also be completely suppressed, as we now show. In this limit, the mass of the u quark must come from $\lambda_u\mathbf{10}^{(1)}\bar{\mathbf{5}}^{(-3)}\langle\bar{\mathbf{5}}_h^{(2)}\rangle = \lambda_u\psi^{[1a]}\psi_a\langle h_1^*\rangle$. The d quark mass must come from $\lambda_d\mathbf{10}^{(1)}\bar{\mathbf{5}}^{(2)}\langle\bar{\mathbf{5}}_h^{(-3)}\rangle = \lambda_u\psi^{[2a]}\tilde{\psi}_a\langle H_2^*\rangle$. And the mass of the e must come from $\lambda_\ell\mathbf{5}^{(-2)}\mathbf{1}^{(5)}\langle\bar{\mathbf{5}}_h^{(-3)}\rangle = \lambda_\ell\tilde{\psi}^2\psi\langle H_2^*\rangle$. It is easily seen that the interactions of the colored scalars h^a and H^a produced by these terms only couple a light fermion to a heavy fermion in the limit $\cos\theta_d = \cos\theta_L = 0$. Therefore, they do not produce proton decay. Of course, there could be other Yukawa terms that do produce proton decay, but they are not needed to give mass to the light fermions.

The same analysis applies to supersymmetric versions of such models. The dangerous operators in supersymmetric unification arise from exchange of superheavy colored Higgsinos. These colored Higgsinos couple to the quarks and leptons and their scalar partners through the Yukawa terms in the superpotential. As we have just seen, in the limit that $\cos\theta_d = \cos\theta_L = 0$ the only Yukawa terms required to generate masses for the known quarks and leptons lead to interactions of colored Higgs that couple a light fermion to a heavy fermion. For the same reason, the colored Higgsinos would only couple a light fermion to a heavy sfermion or light sfermion to a heavy fermion. Thus dangerous $d = 5$ proton decay operators of supersymmetric unified models can also be "rotated away."

In the limit $\cos\theta_d = \cos\theta_L = 0$ in this $SU(5)\times U(1)$ model there is quark-lepton unification, but not of the usual kind, since Standard Model quarks (leptons) are only unified in $SU(5)$ multiplets with "heavy" rather than Standard Model leptons (quarks). In models based on higher unitary groups, however, such as the $SU(6)\times SU(2)$ case discussed below, Standard Model quarks do get placed together in multiplets with Standard Model leptons.

As we have seen, if flipped $SU(5)$ is embedded in $SO(10)$, it is no longer possible to rotate away all the dimension-six proton-decay operators. However, they can be rotated away if flipped $SU(5)$ is embedded in larger unitary groups. We will illustrate this for the case of $SU(6)\times SU(2)$. [This is a maximal subgroup of E_6, but if $SU(6)\times SU(2)$ is embedded in E_6, the dimension-six proton-decay operators can no longer be rotated away.]

The smallest chiral and anomaly free set of fermions in $SU(6) \times SU(2)$ consists of $(15, 1) + (\bar{6}, 2)$, which we shall denote as $\psi^{[AB]} + \psi^I_A$, where $A, B, \ldots = 1, 2, \ldots, 6$ are $SU(6)$ indices and $I, J, \ldots = 1, 2$ are $SU(2)$ indices. This set contains one family. As before, we shall consider a one-family model. The generalization to three families is trivial.

Two Higgs multiplets with superheavy VEVs are required to break to the Standard Model group, a (6, 2) that we shall denote as $\omega^{A,I}$ and a (15, 1) that we shall denote as $\Omega^{[AB]}$. The first of these obtains a VEV $\langle \omega^{6,1} \rangle = \omega$, which has the effect of breaking $SU(6) \times SU(2)$ down to $SU(5) \times U(1)_X$, where $X = \frac{1}{2} T_{35} + 5 I_3$. Here X is the generator of $U(1)_X$, $T_{35} = \mathrm{diag}(1, 1, 1, 1, 1, -5)$ is a generator of $SU(6)$, and $I_3 = \mathrm{diag}(\frac{1}{2}, -\frac{1}{2})$ is a generator of $SU(2)$.

It is easy to verify that the fermion multiplets decompose under $SU(5) \times U(1)_X$ into exactly the set given in Eq. (1): $\psi^{[\alpha\beta]} = \mathbf{10}^{(1)}$, $\psi^{[\alpha 6]} = \mathbf{5}^{(-2)} \equiv \tilde{\psi}^\alpha$, $\psi^1_\alpha = \bar{\mathbf{5}}^{(2)} \equiv \tilde{\psi}_\alpha$, $\psi^1_6 = \mathbf{1}^{(5)} \equiv \psi$, $\psi^2_\alpha = \bar{\mathbf{5}}^{(-3)} \equiv \psi_\alpha$, and $\psi^2_6 = \mathbf{1}^{(0)}$. The superheavy mass terms in Eq. (3) arise from the following Yukawa terms:

$$\begin{aligned} Y \psi^{[AB]} \psi^I_A \langle \omega_{A,I} \rangle &= Y \psi^{(\alpha 6)} \psi^1_\alpha \langle \omega_{6,1} \rangle = (Y\omega) \tilde{\psi}^\alpha \tilde{\psi}_\alpha, \\ \frac{1}{4} Y_d \psi^{[AB]} \psi^{[CD]} \langle \Omega^{[EF]} \rangle \epsilon_{ABCDEF} &= Y_d \psi^{[\alpha\beta]} \psi^{[\gamma 6]} \langle \Omega^{[\delta\eta]} \rangle \epsilon_{\alpha\beta\gamma\delta\eta} = Y_d \psi^{[\alpha\beta]} \tilde{\psi}^\gamma \langle \Omega^{[\delta\eta]} \rangle \epsilon_{\alpha\beta\gamma\delta\eta}, \\ \frac{1}{2} Y_L \psi^I_A \psi^J_B \langle \Omega^{[AB]} \rangle \epsilon_{IJ} &= Y_L \psi^1_\alpha \psi^2_\beta \langle \Omega^{\alpha\beta} \rangle = Y_L \tilde{\psi}_\alpha \psi_\beta \langle \Omega^{\alpha\beta} \rangle. \end{aligned} \tag{5}$$

The analysis that led to Eq. (4) is unchanged, and therefore the conclusion still holds that the superheavy gauge bosons of $SU(5) \times U(1)_X$ do not lead to proton decay in the limit that $\cos\theta_d = \cos\theta_L = 0$. Now, however, there are additional superheavy gauge bosons to consider, namely those of $(SU(6) \times SU(2))/(SU(5) \times U(1)_X)$. One of these, an $SU(6)$ gauge boson that can be denoted as W^6_a, has the following couplings involving only light fermions: $(\overline{\psi^{i6}} \not{W}^6_a \psi^{ia}) + (\overline{\psi^1_a} \not{W}^6_a \psi^1_6)$, which can be written $(\bar{L} \not{W} Q) + (\overline{d^c} \not{W} e^c)$. In both of these terms W^6_a acts as though it has $B = -1/3$ and $L = 1$. Therefore, its exchange between light fermions does not violate baryon and lepton number.

There is also a gauge boson in $SU(2)$ that violates baryon number, but it is easy to see that its couplings always involve the heavy fermions, so it too does not lead to proton decay. Finally, one can show that the Yukawa terms that are needed to give weak-scale mass to the light fermions do not couple colored Higgs fields in such a way as to produce proton decay, as they always couple light fermions to heavy fermions, as we saw in the flipped $SU(5)$ example.

This work was supported by U.S. DOE under Contract No. DE-FG02-12ER41808.

[1] A. DeRujula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **45**, 413 (1980).

[2] C. Jarlskog, Phys. Lett. **82B**, 401 (1979); R. Mohapatra, Phys. Rev. Lett. **43**, 893 (1979); S. Nandi, A. Stern, and E. C. G. Sudarshan, Phys. Lett. **113B**, 165 (1982).

[3] A. DeRujula, H. Georgi, and S. L. Glashow; Phys. Rev. Lett. **45**, 413 (1980). H. Georgi, S. L. Glashow, and M. Machacek, Phys. Rev. D **23**, 783 (1981); S. M. Barr, Phys. Lett. **112B**, 219 (1982).

[4] I. Dorsner and P. Fileviez Perez, Phys. Lett. B **606**, 367 (2005).

[5] G. Segre and H. A. Weldon, Phys. Rev. Lett. **44**, 1737 (1980).

[6] S. M. Barr, Phys. Rev. D **40**, 2457 (1989).