Higgs boson of mass 125 GeV in gauge mediated supersymmetry breaking models with matter-messenger mixing

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We investigate the effects of messenger–matter mixing on the lightest CP–even Higgs boson mass m_h in gauge–mediated supersymmetry breaking models. It is shown that with such mixings m_h can be raised to about 125 GeV, even when the superparticles have sub–TeV masses, and when the gravitino has a cosmologically preferred sub-keV mass. In minimal gauge mediation without messenger-matter mixing, realizing $m_h \sim 125$ GeV would require multi-TeV supersymmetry spectrum. The increase in m_h due to messenger–matter mixing is maximal in the case of messengers belonging to $10 + \overline{10}$ of $SU(5)$ unification, while it is still significant when they belong to $5 + \bar{5}$ of $SU(5)$. Our results are compatible with gauge coupling unification, perturbativity, and the unification of messenger Yukawa couplings. We embed these models into a grand unification framework with a $U(1)$ flavor symmetry that addresses the fermion mass hierarchy and generates naturally large neutrino mixing angles. While supersymmetry mediated flavor changing processes are sufficiently suppressed in such an embedding, small new contributions to $K^0 - \overline{K^0}$ mixing can resolve the apparent discrepancy in the CP asymmetry parameters $\sin 2\beta$ and ϵ_K.

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I. INTRODUCTION

The Higgs boson continues to be a subject of intense scrutiny. The CMS [1] and ATLAS [2] collaborations have recently reported observation of a new particle with a mass near 125 GeV with properties that are consistent with the Standard Model Higgs boson. Each of these experiments has a statistical significance of 5 standard deviations. The observed mass of the particle is consistent with exclusions obtained previously for the SM Higgs boson, viz., $m_h < 114.4$ GeV excluded by the LEP experiments [3], $156\ \text{GeV} < m_h < 177$ GeV excluded by the Tevatron experiments [4], $131 < m_h < 237$ GeV and $251 < m_h < 453$ GeV excluded by ATLAS [5], and $127 < m_h < 600$ GeV excluded by CMS [6], all at 95% CL. A light Higgs boson is a characteristic prediction of supersymmetric theories. In view of the interest in $m_h \sim 125$ GeV, in this paper we investigate expectations for the lightest CP–even Higgs boson mass in a popular class of supersymmetric (SUSY) models, viz., gauge mediated supersymmetry breaking (GMSB) models [7,8].

Simple versions of GMSB models would predict $m_h < 118$ GeV, if the SUSY particle masses lie below 2 TeV or so [9,10]. In the general minimal supersymmetric standard model (MSSM) with arbitrary soft breaking, this mass can be as large as about 130 GeV, with sparticle masses below 2 TeV. GMSB models set a restriction on the soft SUSY breaking trilinear A_t parameter, $A_t = 0$ at the messenger scale, which disallows maximal stop mixing, leading to the reduced upper limit on m_h. If larger values of the sparticle masses are allowed within GMSB, the limit of 118 GeV can be raised somewhat, but masses in excess of 2 TeV, especially for the stops, would go against naturalness in the Higgs mass, and would also render SUSY untestable at the Large Hadron Collider. Here we address the general question: How large can m_h be in minimal gauge mediation without making sparticles beyond the reach of LHC? We find that m_h can be raised naturally to about (125–126) GeV, with SUSY particles all below 2 TeV, if the messengers of SUSY breaking are allowed to mix with the Standard Model fields. Such a scenario would make GMSB models compatible with the recent Higgs observations.

Along with the trilinear A-terms, the bilinear SUSY breaking B term ($\mathcal{L} \supset -\mu B H_u H_d$) also vanishes at the messenger scale in a class of minimal GMSB models. Upon renormalization, this condition would determine through the minimization of the Higgs potential the value of the parameter $\tan\beta$, which turns out to be typically large, $\tan\beta \approx (35\text{–}40)$ [11–13]. We show that much lower values, $\tan\beta \approx (2\text{–}8)$, can be realized in the presence of order one mixed messenger-matter Yukawa couplings. Thus, the entire range $\tan\beta = (2\text{–}40)$ can be realized with vanishing B term at the messenger scale. The SUSY spectroscopy of these models is different from those of GMSB without messenger-matter mixing, and leads to relatively light stops. The mixing of messenger fields with the MSSM fields has a cosmological advantage that it would break "messenger number" that would have led to a stable messenger particle, which is not an ideal candidate for dark matter. While SUSY flavor violation arising from

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messenger-matter mixing is not excessive, the proposed scenario predicts small but observable flavor effects. When the models presented are embedded in a unified $SU(5)$ framework with a flavor $U(1)$ symmetry, it is found that the CP asymmetry parameter ϵ_K in the K meson system is slightly modified, which can explain the apparent discrepancy in the extracted value of $\sin 2\beta$ in the B meson system. We also find that the rare decay $\mu \to e\gamma$ should be accessible to the next generation experiments.

This paper is organized as follows. In Sec. II we summarize the salient features of minimal gauge mediation. In Sec. III we discuss the upper limit on the lightest Higgs boson mass in GMSB models including messenger-matter mixing. Two models are studied, a $5 + \bar{5}$ messenger model, and a $10 + \overline{10}$ messenger model. In this section we also discuss the sparticle spectroscopy, allowing for messenger-matter mixing, and obtain limits on $\tan\beta$ with the boundary condition $B = 0$ at the messenger scale. In Sec. IV we discuss flavor violation arising from messenger–matter mixing in the two models. Here we embed these models in a grand unification framework based on $SU(5)$ along with a flavor $U(1)$ symmetry that addresses the quark and lepton mass and mixing hierarchies. Section V has our conclusions. The relevant renormalization group equations (RGE) for the two models are given in Appendix A, and the GMSB boundary conditions for the mass parameters are derived in Appendix B. Preliminary results of this work were presented at PHENO 2011 [14].

II. ESSENTIAL FEATURES OF MINIMAL GAUGE MEDIATION

GMSB models are well motivated, since SUSY solves the hierarchy problem, and gauge mediation of SUSY breaking solves the SUSY flavor problem. These models also predict correctly the unification of the three gauge couplings, leading to an eventual embedding in a grand unified theory (GUT) such as $SU(5)$. Gravity mediation of SUSY breaking (SUGRA) also shares these features, except that generically it would lead to excessive flavor changing processes mediated by the SUSY particles. Consistency of SUGRA models with experiments would typically require two assumptions [15]: (i) the soft masses of sparticles in any given sector are universal, and (ii) the trilinear A–terms are proportional to the corresponding Yukawa couplings. Such assumptions are not necessary in GMSB models, rather they are automatic consequences. GMSB models assume that SUSY is dynamically broken in a secluded sector, and that this breaking is communicated to the MSSM sector via the SM gauge interactions by a set of messenger fields which are charged under the SM. Owing to the universality of the gauge interactions, the soft SUSY breaking mass parameters would be flavor universal, and the induced A–terms would be proportional to the Yukawa couplings.

Minimal gauge mediation assumes that a gauge singlet superfield Z develops nonzero vacuum expectation values (VEVs) along its scalar component $\langle Z\rangle$ as well as along its auxiliary component $\langle F_Z\rangle$. This field couples to a set of messenger fields Φ_i and $\bar{\Phi}_i$ which transform vectorially under the SM gauge symmetry:

$$W = \lambda_i Z \Phi_i \bar{\Phi}_i. \tag{1}$$

$\langle F_Z\rangle \neq 0$ would split the masses of the scalars in Φ_i from the corresponding fermions. This breaking of SUSY is communicated to the SM sector via loops involving the SM gauge bosons. The gaugino masses and the scalar masses for the MSSM fields at the messenger scale are given by

$$\begin{aligned} M_a &= \frac{\alpha_a}{4\pi}\Lambda n_a(i) g(x_i) \quad (a = 1-3), \\ \tilde{m}^2 &= 2\Lambda^2 \sum_{a=1}^{3}\left(\frac{\alpha_a}{4\pi}\right)^2 C_a n_a(i) f(x_i). \end{aligned} \tag{2}$$

Here $\Lambda \equiv \langle F_Z\rangle/\langle Z\rangle$ and $n_a(i)$ is the Dynkin index of the messenger pair Φ_i with $n_a(i) = 1$ for $N + \bar{N}$ of $SU(N)$. C_a is the quadratic Casimir invariant of the relevant MSSM scalar with $C_a = (N^2 - 1)/(2N)$ for N–plet of $SU(N)$ and $C_a = (3/5)Y^2$ for $U(1)_Y$. The functions $f(x_i)$ and $g(x_i)$ can be found, for e.g., in Ref. [16], and are nearly equal to one for small values of x_i, defined as $x_i = |\langle F_Z\rangle/\lambda_i\langle Z\rangle^2|$ with $x_i < 1$ necessary for color and charge conservation. In addition, GMSB models impose the following boundary conditions at the messenger scale on the MSSM trilinear and bilinear soft SUSY breaking parameters:

$$A_f = 0 \quad \text{for all } f \qquad B = 0. \tag{3}$$

The second of these relations, $B = 0$, is sometimes ignored anticipating some mechanism that explains the magnitude of the μ parameter [17,18]. For example, in Ref. [18], a flavor symmetry is assumed in the singlet (Z) sector, so that $B\mu \ll \mu^2$ or $B\mu \sim \mu^2$ can be realized at the messenger scale ($M_{\rm mess}$), depending on the assignment of flavor charges. In our analysis we shall allow for $B = 0$ as well as $B \neq 0$ at $M_{\rm mess}$.

A few features are worth emphasizing in Eqs. (2) and (3). Sparticles of a given quantum number are all degenerate in mass, which is crucial in solving the SUSY flavor problem. The induced trilinear couplings would be proportional to the respective Yukawa couplings owing to the vanishing of the A terms, also crucial for solving the SUSY flavor problem. The minimal GMSB models also have only a small number of effective parameters.

The gravitino is the lightest supersymmetric particles in minimal GMSB. Its mass is given by $m_{3/2} = \langle F_Z\rangle/(\sqrt{3}kM_{\rm Pl})$ where $M_{\rm Pl}$ is the reduced Planck mass, and k is a typical Yukawa coupling of the type λ given in

Eq. (1). The cosmological requirement that the gravitinos do not overclose the universe requires $m_{3/2} < \mathrm{keV}$ [19], which in turn requires $\sqrt{\langle F_Z \rangle} < \sqrt{k} 2 \times 10^6$ GeV. In GMSB with a single set of messenger fields, $M_{\rm mess} = \lambda \langle Z \rangle$, so that this constraint would require $M_{\rm mess} = \lambda \langle F_Z \rangle / \Lambda$ to obey $M_{\rm mess} \leq \lambda^2$ (10^8 GeV), where $k = \lambda$ and $\Lambda = 3 \times 10^4$ GeV, its lowest allowed value, are used. Perturbativity would require $\lambda < 1$, so that cosmology prefers $M_{\rm mess} < 10^8$ GeV. Since there are ways around the gravitino overclosure problem, such as by late decays of particles, or other ways of entropy dumping, the cosmological limit is not absolute. In our analysis we find fully consistent solutions when this limit is satisfied. We also allow $M_{\rm mess}$ to be greater than 10^8 GeV, as large as 10^{14} GeV. Any larger value would lead to $m_{3/2} > 1$ GeV, and thus generate supergravity contributions to the scalar masses that can bring back the SUSY flavor problem.

The messenger fields, which are taken to be vectorlike under the SM gauge symmetry, are usually assumed to form complete multiplets of a grand unified group. This is motivated by the observed meeting of the three gauge couplings at a scale near 2×10^{16} GeV when extrapolated with the MSSM spectrum. Complete multiplets of a GUT symmetry group such as $SU(5)$ would preserve this successful unification (modulo small two–loop effects). Messenger fields belonging to $5 + \bar{5}$ of $SU(5)$ or $10 + \overline{10}$ of $SU(5)$ are then the simplest choices. One could introduce multiple copies of these fields, or one could introduce both of them simultaneously. We shall consider only two minimal choices in this paper, viz., having either one pair of $5 + \bar{5}$ or one pair of $10 + \overline{10}$ messenger fields.

Messenger fields belonging to $5 + \bar{5}$ of $SU(5)$ or $10 + \overline{10}$ of $SU(5)$ can mix with the MSSM superfields. If such mixings are written down arbitrarily, that would reintroduce SUSY flavor problem. However, in the context of an underlying flavor symmetry that addresses the mass and mixing hierarchy of quarks and leptons, it is not unreasonable to imagine that significant mixing of the messenger fields occurs only with the third family fermions. This is the situation we investigate in the next sections. Complete separation of messenger fields from the MSSM fields is in general problematic for cosmology, since this would lead to messenger number conservation and a stable messenger particle, which is not an ideal dark matter candidate [20]. Messenger–matter mixing avoids this difficulty. In the presence of such mixings, the expressions given in Eqs. (2) and (3) for the soft SUSY breaking parameters would receive new contributions [21–23]. This can help increase the lightest Higgs boson mass of GMSB, and can lead to significantly different SUSY spectrum. We also point out that such mixings can modify the derived value of $\tan\beta$, which can now be quite low, in the range of 2–8, with order one Yukawa couplings.

III. HIGGS BOSON MASS BOUND IN GMSB MODELS

Low energy supersymmetry characteristically predicts one light Higgs boson. In the MSSM, at the tree level, the lightest Higgs boson mass is bounded by $m_h \leq M_Z$. Radiative corrections proportional to the top quark Yukawa couplings shift this limit significantly [24,25]. Including the leading two loop corrections, this mass can be written approximately as [25]

$$m_h^2 = M_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} t\right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[\frac{1}{2} X_t + t + \frac{1}{16\pi^2} \left(\frac{3}{2} \frac{m_t^2}{v^2} - 32\pi\alpha_3\right)(X_t t + t^2)\right], \quad (4)$$

where

$$v^2 = v_d^2 + v_u^2, \qquad t = \log\left(\frac{M_s^2}{M_t^2}\right), \qquad X_t = \frac{2\tilde{A}_t^2}{M_s^2}\left(1 - \frac{\tilde{A}_t^2}{12 M_s^2}\right), \quad (5)$$

with the scale M_s defined in terms of the stop mass eigenvalues as $M_s^2 = \tilde{m}_{t_1} \tilde{m}_{t_2}$. Here $\tilde{A}_t = A_t - \mu \cot\beta$, with A_t being the trilinear soft term for the stop. Equation (4) is accurate to about 3 GeV, when compared with computational packages such as SUSPECT [9] which do not make certain simplifying assumptions made in obtaining Eq. (4). Since we find that the numerical package SOFTSUSY consistently gives 2 GeV larger Higgs mass compared to Eq. (4), we find it appropriate to add 2 GeV to m_h computed from Eq. (4) for interpretation. The upper bound on m_h depends crucially on M_s and the mixing parameter X_t. It is maximal in the case of maximal stop mixing (corresponding to $X_t = 6$), in which case $m_h = 130$ GeV can be realized with all SUSY particles below 2 TeV. The first boundary condition of Eq. (3) would, however, forbid realizing maximal stop mixing in minimal GMSB. The upper limit on m_h in this case is $m_h < 118$ GeV, with all sparticle masses below 2 TeV [9].

In the presence of messenger matter mixings, the boundary conditions Eqs. (2) and (3) will receive new contributions. In such cases, near maximal mixing of the stops can be realized, as we show here, and thus the upper limit on m_h can be raised to about (125–126) GeV. New contributions to the A–terms also would imply that the value of $\tan\beta$ derived with the condition $B = 0$ at $M_{\rm mess}$ (this condition is unaltered even with matter–messenger mixing) would be different. Lower values of $\tan\beta$ are found, which can be understood from the one–loop RGE for the B parameter below $M_{\rm mess}$:

$$\frac{dB}{dt} = \frac{1}{2\pi}\left(3\alpha_t A_t + 3\alpha_2 M_2 + \frac{3}{5}\alpha_1 M_1\right), \quad (6)$$

where $\alpha_t = \frac{\lambda_t^2}{4\pi}$, λ_t being the top quark Yukawa coupling. The nonzero initial value of A_t modifies the evolution of B, which is related to $\tan\beta$ by the electroweak symmetry breaking conditions given by

$$\frac{M_Z^2}{2} = -|\mu|^2 - \frac{m_{H_u}^2 \tan^2\beta - m_{H_d}^2}{\tan^2\beta - 1}, \tag{7}$$

$$\sin 2\beta = \frac{2B\mu}{2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2}. \tag{8}$$

The effect of nonzero A_t is to decrease the value of $\tan\beta$. For example, in the $10 + \overline{10}$ messenger model, we find the range $1.6 \le \tan\beta \le 7$ assuming $B = 0$ at $M_{\rm mess}$ with order one messenger Yukawa couplings, corresponding to 10^{14} GeV $\ge M_{\rm mess} \ge 10^5$ GeV.

A. Higgs mass bound in the $5 + \bar{5}$ messenger model

In this model, messenger fields belong to $5 + \bar{5}$ of $SU(5)$, with the content $5 = (\overline{d^c}_m + \bar{L}_m)$ and $\bar{5} = (d_m^c + L_m)$. Here d_m^c and L_m have the same quantum numbers as the d^c and L superfields of MSSM. We assume that these messenger fields have the same R–parity as the quarks and leptons of MSSM.[1] The following R–invariant superpotential can now be written, which mixes the MSSM fields with the messenger fields:

$$W_{5+\bar{5}} = f_d \overline{d^c}_m d_m^c Z + f_e \bar{L}_m L_m Z + \lambda_b' Q_3 d_m^c H_d + \lambda_{\tau^c}' L_m e_3^c H_d. \tag{9}$$

Here we have assumed that the messenger fields couple only with the third family MSSM fields. This will be justified based on a flavor symmetry discussed in Sec. IV, where the lighter family couplings to the messenger fields are suppressed by powers of a small parameter ϵ.[2] Equation (9) can arise in $SU(5)$ as $W = f_0 5_m \bar{5}_m Z + \lambda_0' 10_3 \bar{5}_m \bar{5}_H$ with only the H_d component kept from $\bar{5}_H$ (the color triplet from the 5_H and $\bar{5}_H$ acquire GUT scale masses and decouple at M_X). Thus, imposing $SU(5)$ symmetry, we see that at the GUT scale $M_X \simeq 2\times 10^{16}$ GeV, there are only two unified Yukawa couplings (f_0, λ_0') that involve the messenger fields. The RGE for the Yukawa couplings entering Eq. (9), along with those for the MSSM Yukawa couplings, are listed in Appendix A 1, valid for the momentum regime $M_{\rm mess} < \mu < M_X$. In Fig. 1, left panel, we plot the evolution of the couplings λ_b' and λ_{τ^c}' of Eq. (9) in this momentum regime, assuming unification of these couplings at the scale M_X, and taking $\lambda_0' = 1.6$ and $f_0 = 0.25$.

Without messenger–matter mixing, the scalar masses and the trilinear A–terms at $M_{\rm mess}$ are obtained from Eqs. (2) and (3). With such mixings allowed, as in Eq. (9), these relations are modified. It was shown in Ref. [21] that the mixed messenger–matter couplings would induce negative one-loop contributions to the supersymmetry-breaking masses. However, these one-loop contributions have additional $\langle F_Z\rangle/M_{\rm mess}^2$ suppression factors, and can be safely ignored compared to the two–loop induced terms which do not have such suppression, provided that $\langle F_Z\rangle/M_{\rm mess}^2 \le g_3/4\pi$. We shall assume that this condition is met in this paper. For $M_{\rm mess} > 10^7$ GeV, this condition is automatically satisfied. New contributions to the scalar masses and the A–terms arise at the two–loop and one–loop level respectively, proportional to the mixed messenger–matter Yukawa couplings. These contributions can be obtained from the general expressions given in Ref. [22,23]. The Yukawa couplings λ_b' and λ_{τ^c}' of Eq. (9) lead to a splitting in the mass of the $\tilde{Q}_3$ squark doublet from those of $\tilde{Q}_{1,2}$, and of the right-handed stau $\tilde{\tau}^c$ from those of $\tilde{e}_{1,2}^c$. These shifts, which add to the universal contributions of Eq. (2) at the messenger scale are (see Appendix B for the derivation):

$$\delta\tilde{m}_{Q_3}^2 = \frac{\alpha_b'\Lambda^2}{8\pi^2}\left(3\alpha_b' + \frac{1}{2}\alpha_{\tau^c}' - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{7}{30}\alpha_1\right), \tag{10}$$

$$\delta\tilde{m}_{\tau^c}^2 = \frac{2\alpha_{\tau^c}'\Lambda^2}{8\pi^2}\left(2\alpha_{\tau^c}' + \frac{3}{2}\alpha_b' - \frac{3}{2}\alpha_2 - \frac{9}{10}\alpha_1\right), \tag{11}$$

$$\delta\tilde{m}_{H_d}^2 = \frac{\delta\tilde{m}_{\tau^c}^2}{2} + 3\delta\tilde{m}_{Q_3}^2 + \frac{3\Lambda^2\alpha_b'\alpha_t}{16\pi^2}. \tag{12}$$

New contributions to the A-terms generated by messenger–matter mixing at the messenger scale are given by

$$\delta A_t = -\frac{1}{4\pi}\alpha_b'\Lambda, \tag{13}$$

$$\delta A_b = -\left(\frac{4\alpha_b' + \alpha_{\tau^c}'}{4\pi}\right)\Lambda, \tag{14}$$

$$\delta A_\tau = -\left(\frac{3\alpha_b' + 3\alpha_{\tau^c}'}{4\pi}\right)\Lambda, \tag{15}$$

where $\alpha_b' = \frac{\lambda_b'^2}{4\pi}$, and $\alpha_{\tau^c}' = \frac{\lambda_{\tau^c}'^2}{4\pi}$. Here we have followed the definition $\mathcal{L}_{\rm soft} \supset \lambda_{abc}A_{abc}\tilde{\Phi}_a\tilde{\Phi}_b\tilde{\Phi}_c$ for the trilinear soft terms, corresponding to the superpotential $W \supset \lambda_{abc}\Phi_a\Phi_b\Phi_c$. Since λ_b' and λ_{τ^c}' originate from one unified coupling λ_0' as shown in the left panel of Fig. 1, the scalar mass spectrum at the messenger scale depends on λ_0', the messenger scale $M_{\rm mess}$, and the effective SUSY breaking scale Λ. (There is also a mild dependence on f_0 via RGE, we fix $f_0 = 0.25$ in our analysis.) A range of λ_0' is excluded since it leads to negative squared masses for certain sparticles at the scale $M_{\rm mess}$. This range depends on the value of $M_{\rm mess}$. For the most part, we consider the range

[1] While this work was written up a related work appeared, which discusses messenger mixing with the MSSM Higgs fields, $W \supset Q_3 u_3^c \bar{L}_m$, along with $H_u - \bar{L}_m$ mixing [26].

[2] We shall in fact see that by field redefinitions the general messenger–matter mixing can be brought to the form of Eq. (9).

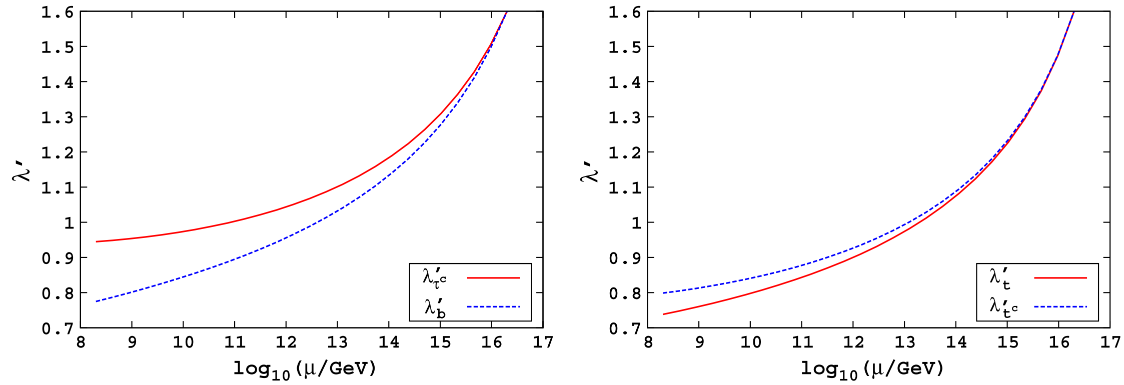

FIG. 1 (color online). The running of the mixed messenger–matter Yukawa couplings λ'_b and λ'_{τ^c} of Eq. (9) of the $5+\bar{5}$ messenger model (left panel). The right panel shows the evolution of λ'_t and λ'_{t^c} of Eq. (17) of the $10+\overline{10}$ messenger model from $M_X = 2\times 10^{16}$ GeV down to the messenger scale $M_{\rm mes} = 10^8$ GeV. In both cases the unified Yukawa couplings are taken to be $\lambda'_0 = 1.6$ and $f_0 = 0.25$. In the right panel, $\lambda'_{m0} = 0.1$ has been used.

10^7 GeV $\leq M_{\rm mess} \leq 10^{14}$ GeV, the lower value arising from the demand that the negative one–loop contributions to scalar masses remain small, and the upper value arising by requiring the supergravity contributions to be small. We plot the exclusion on λ'_0 from the positivity of the right-handed stau mass in Fig. 2, left panel. On the right panel, the right-handed stop mass is plotted, versus λ'_0. Figure 2 shows that the interval $0.2 < \lambda'_0 < 0.5$ ($0.1 < \lambda'_0 < 0.4$) is excluded, corresponding to $M_{\rm mess} = 10^{14}$ GeV (10^7 GeV), since that leads to negative $\tilde{m}^2_{\tau^c}$. We also see that both the $\tilde{\tau}^c$ and the $\tilde{t}^c$ can be relative light in this scenario.

Below the scale $M_{\rm mess}$, the theory is just the MSSM. We have solved the one–loop RGEs for the MSSM with the boundary conditions at $M_{\rm mess}$ given by Eqs. (2) and (3) and by Eqs. (10)–(15). The soft breaking mass–squared $m^2_{H_u}$ is driven to negative values at low energy scale, leading to the breaking of electroweak symmetry. In order to avoid driving $\tilde{m}^2_{t^c}$ to negative values at low energy scale, so that color and electric charge remain unbroken, a region of λ'_0 is forbidden. For example, the region of $\lambda'_0 > 1.3$ for $M_{\rm mess} = 10^{14}$ GeV is forbidden as shown in the right panel of Fig. 2. This exclusion arises because of the top quark Yukawa coupling contribution to the $\tilde{m}^2_{t^c}$, in conjunction with A_t in the RGE, which becomes large due to the large initial A_t value at $M_{\rm mess}$.

Since all the soft terms at the messenger scale are determined by the three parameters λ'_0, Λ and $M_{\rm mess}$ (with $f_0 = 0.25$ fixed for RGE evolution), the lightest Higgs mass is also determined by these three parameters. As we discussed previously, the maximal mixing condition $\tilde{A}_t = \sqrt{6}M_s$ (or $X_t = 6$) gives the largest value of the lightest Higgs boson mass. It is not possible to realize this maximal mixing condition in GMSB without messenger–matter mixing because A_t vanishes at the scale $M_{\rm mess}$ and the induced value at low energy scale through RGEs is not sufficient. On the other hand, allowing mixed messenger–matter couplings generates A_t as shown in Eq. (13). This leads to an enhancement of the Higgs mass. Choosing the parameters to lie in the range 4×10^4 GeV $< \Lambda < 2\times 10^5$ GeV, 10^7 GeV $< M_{\rm mess} < 10^{14}$ GeV and $0 < \lambda'_0 < 2$, we report the numerical values for the lightest Higgs boson mass m_h in Table I for different choices of these

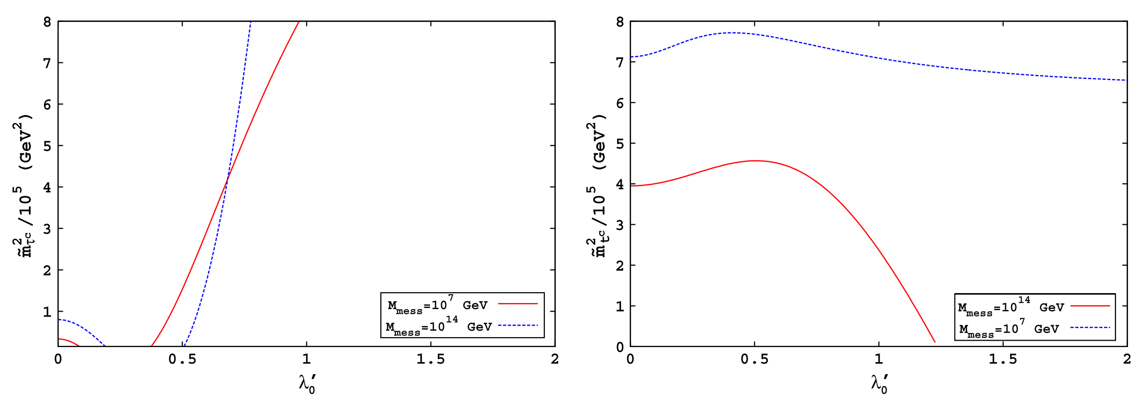

FIG. 2 (color online). $\tilde{m}^2_{\tau^c}$ versus λ'_0 at the scale $M_{\rm mess}$ for two different messenger scales $M_{\rm mess} = (10^7, 10^{14})$ GeV, in the $5+\bar{5}$ model (left panel). The right panel shows $\tilde{m}^2_{t^c}$ versus λ'_0 at $M_{\rm mess}$ for the same two messenger scales in this model. Here $f_0 = 0.25$ has been used.

TABLE I. The lightest Higgs boson mass m_h in the $5+\bar{5}$ model as functions of the GMSB input parameters, Λ, λ_0' and $M_{\rm mess}$ for $\tan\beta = 10$. Here we have fixed $f_0 = 0.25$.

λ_0'	m_h (GeV)	$\Lambda(10^5$ GeV)	$M(10^{13}$ GeV)	$\tilde{m}_{t_1}$ (GeV)	$\tilde{m}_{t_2}$ (GeV)
0	114	2	1.78	1249	1695
0.8	116	2	10	1212	1583
1.2	119	2	10	384	2613

parameters. In this table, we have excluded values of λ_0' that give negative values for $\tilde{m}^2_{\tau^c}$ and $\tilde{m}^2_{t^c}$. We have used Eq. (4) to compute m_h, and since SUSPECT gives m_h values systematically higher by 2 GeV, the quoted upper limit of $m_h = 114$ GeV for the case of $\lambda_0' = 0$ actually should be interpreted as $m_h = 116$ GeV. This value increases by about 5 GeV to 121 GeV in the case of large λ_0'. This limit is 118 GeV when the stops have masses <1.5 TeV, as indicated in Table I. While the increase in m_h is significant in this model with messenger–matter mixing, here maximal stop mixing is not realized, primarily due to the positivity conditions on $\tilde{m}^2_{t^c}$. Note that there is no contribution to $\tilde{m}^2_{t^c}$ from the mixed Yukawa coupling in this model, which implies that this parameter turns negative quickly below $M_{\rm mess}$ if λ_0' is large. This situation improves, enabling larger values for m_h when the messenger fields belong to $10+\overline{10}$, as discussed in the next subsection.

B. Higgs mass bound in the $10+\overline{10}$ messenger model

Here we consider messenger fields belonging to $10+\overline{10}$ of $SU(5)$. These fields decompose in terms of MSSM–like fields as:

$$10+\overline{10} = (Q_m + \bar{Q}_m) + (u^c_m + \overline{u^c}_m) + (e^c_m + \overline{e^c}_m). \quad (16)$$

As before, we assume that the messenger fields only couple with the third generation of MSSM fields, and that they have the same R–parity as the MSSM quarks and leptons. In this case the following superpotential couplings can be written.

$$W_{10+\overline{10}} = \lambda'_{t^c} Q_3 u^c_m H_u + \lambda'_t Q_m u^c_3 H_u + \lambda'_m Q_m u^c_m H_u + f_{e^c}\overline{e^c}_m e^c_m Z + f_{u^c}\overline{u^c}_m u^c_m Z + f_Q \bar{Q}_m Q_m Z. \quad (17)$$

Although the couplings $Q_m d^c_3 H_d + L_3 e^c_m H_d$ are allowed by gauge symmetry, we have not included them in the above superpotential because these terms will be suppressed by a small parameter ϵ when this model is embedded in a flavor $U(1)$ symmetric framework, as we shall see in the next section. The couplings of Eq. (17) can arise in $SU(5)$ theory from $W \supset \lambda_0' 10_3 10_m 5_H + \lambda'_{m0} 10_m 10_m 5_H + f_0 10_m \overline{10}_m Z$, with only the H_u component of 5_H, and not the color triplet component, kept below M_X. Thus we see that the Yukawa couplings λ'_{t^c} and λ'_t are equal to the unified coupling λ_0' at the GUT scale. Similarly, the three Yukawa couplings f_{e^c}, f_Q and f_{u^c} are equal to a single coupling f_0 at the GUT scale. In other words, the six Yukawa couplings appearing in the superpotential of Eq. (17) are reduced to three: λ_0', f_0 and λ'_{m0} at the GUT scale. We shall use these unification conditions and derive the couplings of Eq. (17) by using the RGE listed in Appendix A 2. The evolution of λ'_t and λ'_{t^c} below M_X is shown in the right panel of Fig. 1 with $f_0 = 0.25$ and $\lambda'_{m0} = 0.1$ fixed.

The Yukawa couplings λ'_{t^c}, λ'_t and λ'_m generate 2-loop (1-loop) scalar masses (A-terms) at the scale $M_{\rm mess}$, as derived in Appendix B 2. As a result, the universal scalar masses given by Eqs. (2) and (3) (with $N_{\rm mess} = 3$ corresponding to $10+\overline{10}$ messenger fields) would receive additional contributions at the scale $M_{\rm mess}$:

$$\delta\tilde{m}^2_{Q_3} = \frac{\Lambda^2}{8\pi^2}\Big[\alpha'_{t^c}\Big(3\alpha'_{t^c} + \frac{3}{2}\alpha'_t + \frac{5}{2}\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1\Big) - \alpha_t\Big(\frac{5}{2}\alpha'_t + \frac{3}{2}\alpha'_m\Big)\Big], \quad (18)$$

$$\delta\tilde{m}^2_{t^c} = \frac{2\Lambda^2}{8\pi^2}\Big[\alpha'_t\Big(3\alpha'_t + \frac{3}{2}\alpha'_{t^c} + 2\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1\Big) - \alpha_t\Big(2\alpha'_{t^c} + \frac{3}{2}\alpha'_m\Big)\Big], \quad (19)$$

$$\delta\tilde{m}^2_{H_u} = \frac{3\Lambda^2}{8\pi^2}\Big[\alpha'_{t^c}\Big(3\alpha'_{t^c} + \frac{3}{2}\alpha'_t + \frac{5}{2}\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1\Big) + \alpha'_t\Big(3\alpha'_t + \frac{3}{2}\alpha'_{t^c} + 2\alpha'_m - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1\Big) + \alpha'_m\Big(3\alpha'_m + 2\alpha'_t + \frac{5}{2}\alpha'_{t^c} - \frac{8}{3}\alpha_3 - \frac{3}{2}\alpha_2 - \frac{13}{30}\alpha_1\Big)\Big], \quad (20)$$

$$\delta A_t = -\Big[\frac{5\alpha'_t + 4\alpha'_{t^c} + 3\alpha'_m}{4\pi}\Big]\Lambda, \quad (21)$$

$$\delta A_b = -\frac{\alpha'_{t^c}}{4\pi}\Lambda, \quad (22)$$

where $\alpha'_{t^c} = \frac{\lambda'^2_{t^c}}{4\pi}$, $\alpha'_t = \frac{\lambda'^2_t}{4\pi}$, and $\alpha'_m = \frac{\lambda'^2_m}{4\pi}$. An interesting feature of the $10+\overline{10}$ model is that unlike the $5+\bar{5}$ model, here along with A_t, the $\tilde{m}^2_{t^c}$ also receives new contributions which can be positive. As a result, sufficiently large A_t can be generated without turning $\tilde{m}^2_{t^c}$ negative, and the maximal mixing condition $X_t = 6$ can be realized, leading to an increased upper limit on m_h, as large as (125–126) GeV.

In order to find the upper limit on m_h and the SUSY mass spectrum, we solve the MSSM RGE numerically from the messenger scale to the low scale with the boundary conditions given in Eqs. (18)–(22) and in Eqs. (2) and (3). These masses would depend on four parameters: Λ, $M_{\rm mess}$, λ'_0 and λ'_{m0}. (The value of f_0 is also relevant for RGE evolution, we fix $f_0 = 0.25$ in our analysis. m_h is not very sensitive to the choice of f_0.) In Table II we report the values of m_h for different values of Λ, $M_{\rm mess}$ and λ'_0 with a fixed value of $\lambda'_{m0} = 0$. In Table III we report the same, but now with $\lambda'_{m0} = 1.2$ fixed. In both cases $m_h = 125$ GeV can be obtained (once 2 GeV is added to the numbers quoted in these tables), with all SUSY particles below 1.5 TeV. For example, in the case of $\lambda'_{m0} = 0$ (Table II), without messenger–matter mixing, obtaining $m_h = 119$ GeV would require one of the stops to be heavier than 3 TeV, while with such mixings, $m_h = 125$ GeV is realized with both stops below 1.5 TeV.

In Fig. 3, left panel, we plot the Higgs mass as a function of Λ for two values of the unified Yukawa coupling $\lambda'_0 = (0, 1.2)$, where $\lambda'_0 = 0$ corresponds to minimal GMSB without messenger–matter mixing. We see that the Higgs mass is raised by 10 GeV in the case of $\lambda'_0 = 1.2$ compared to the case of $\lambda'_0 = 0$ for low values of $\Lambda = 4 \times 10^4$ GeV. This increase is about 6 GeV for larger Λ. Note that smaller values of Λ lead to lighter SUSY particles, with the stop mass around 500–600 GeV, which might be accessible to early run of LHC. In the right panel of Fig. 3 we have plotted m_h versus λ'_0 for various values of $M_{\rm mess}$, and for $\Lambda = 10^5$ GeV fixed. There is a nontrivial constraint on λ'_0 when $M_{\rm mess} > 10^{11}$ GeV, owing to the stop squared mass turning negative at low energies. Note that $m_h \simeq 125$ GeV is realized in this model, along with sub–TeV superparticles, even for low messenger scale, $M_{\rm mess} \leq 3 \times 10^8$ GeV, preferred by cosmology.

We present three different spectra for the superparticle masses in Table IV, two corresponding to the $10 + \overline{10}$ model, and one for the $5 + \bar{5}$ model of the previous subsection. In this table, the masses quoted in the last two columns correspond to $\tan\beta = 6.1$ (for the $10 + \overline{10}$ model) and $\tan\beta = 15.6$ (for the $5 + \bar{5}$ model). These values are derived by assuming the vanishing of the B–term at $M_{\rm mess}$, as in Eq. (3). The third column of Table IV lists the sparticle spectrum for an arbitrary value of $\tan\beta = 10$. The spectrum in the fourth column assumes a low messenger scale of $M_{\rm mess} = 4 \times 10^5$ GeV. The negative one–loop contributions to the scalar masses are $<5\%$ of the positive two–loop contributions arising from messenger–matter mixing for this value of $M_{\rm mess}$. For larger values of $M_{\rm mess}$, as in the third and fifth columns of Table IV, these one–loop contributions are even smaller. The mass values of Table IV show that light SUSY spectrum is possible in GMSB along with a Higgs boson mass around 125 GeV, if messenger–matter mixing is allowed. We shall use the mass values of Table IV in deriving flavor violation constraints on the model, which is addressed in the next section.

TABLE II. The lightest Higgs boson mass m_h, along with the stop masses, and the stop mixing parameter A_t/m_s for different values of the GMSB input parameters Λ, λ'_0 and $M_{\rm mess}$ in the $10 + \overline{10}$ model. Here we have fixed $\lambda'_{m0} = 0$, $f_0 = 0.25$, and set $\tan\beta = 10$.

λ'_0	m_h (GeV)	$\Lambda(10^5$ GeV)	$M_{\rm mess}$ (GeV)	$\tilde{m}_{t_1}$ (GeV)	$\tilde{m}_{t_2}$ (GeV)	A_t/M_s
0	117	1.6	3×10^{13}	2656	3284	−0.86
0.4	118	1.36	10^8	1795	2396	−1.27
0.8	122	0.912	10^{13}	1553	2143	−1.95
1.1	123	0.784	2×10^{11}	735	1429	−2.0
2	123	0.784	10^8	743	1426	−2.26

TABLE III. Same as in Table II, but now with $\lambda'_{m0} = 1.2$ fixed.

λ'_0	m_h (GeV)	$\Lambda(10^5$ GeV)	$M_{\rm mess}$ (GeV)	$\tilde{m}_{t_1}$ (GeV)	$\tilde{m}_{t_2}$ (GeV)	A_t/M_s
0	121	0.97	2×10^{13}	928	1636	−1.8
0.4	123	0.91	3×10^{13}	656	1612	−2.3
0.6	123	0.848	10^{12}	673	1512	−2.3
0.8	123	0.784	10^{11}	682	1509	−2.3
2	123	0.784	10^8	753	1425	−2.2

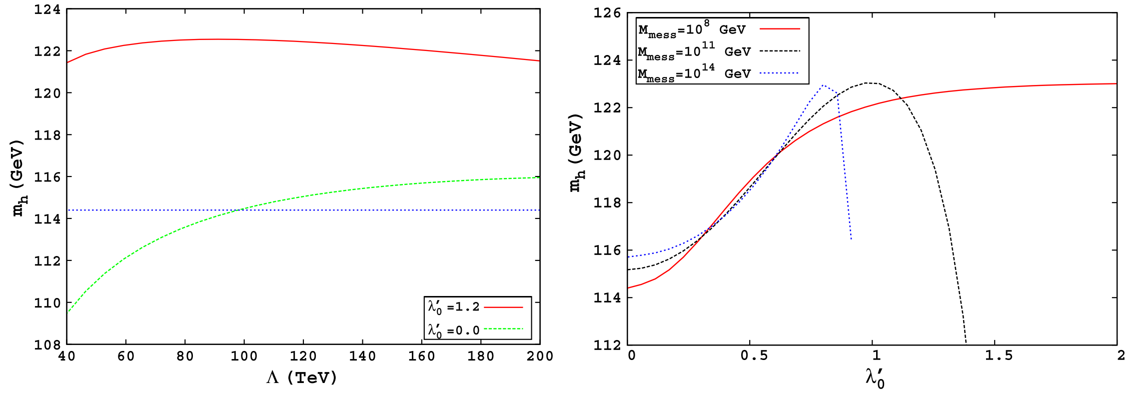

FIG. 3 (color online). m_h versus Λ for $\lambda'_0 = 0$ and $\lambda'_0 = 1.2$ (left panel). The horizontal line indicates the LEP lower limit $m_h > 114.4$ GeV. The right panel shows m_h versus λ'_0 for different messenger scales and with $\Lambda = 10^5$ GeV fixed.

TABLE IV. The SUSY spectrum corresponding to $10 + \overline{10}$ model and $5 + \bar{5}$ model for three choices of input parameters. All masses are in GeV. The values of $\tan\beta$ in the last two columns are derived from the condition that $B = 0$ at $M_{\rm mess}$. 2 GeV should be added to m_h quoted here to be consistent with results obtained from SUSPECT.

Particle		$10 + \overline{10}$	$10 + \overline{10}$	$5 + \bar{5}$
Inputs	$M_{\rm mess}$	10^8	4×10^5	10^8
	$N_{\rm mess}$	3	3	1
	$\Lambda(10^5$ GeV)	0.45	0.3	1.5
	$\tan\beta$	10	6.1	15.6
	f_0	0.25	0.25	0.25
	λ_0	1.3	1.2	1.2
Higgs:	m_h	122	118	114.5
	m_H^0	858	592	1690
	m_A	858	591	1690
	$m_{H^\pm}$	862	597	1689
Gluino:	$\tilde{m}_g$	980	667	1041
Neutralinos:	m_{χ_1}	186	124	208
	m_{χ_2}	346	225	408
	m_{χ_3}	800	557	781
	m_{χ_4}	807	569	790
Charginos:	χ_1^+	347	227	409
	χ_2^+	807	569	790
Squarks:	$\tilde{m}_{u_L,c_L}$	972	657	1480
	$\tilde{m}_{u_R,c_R}$	929	632	1377
	$\tilde{m}_{d_L,s_L}$	971	657	1480
	$\tilde{m}_{d_R,s_R}$	922	630	1365
	$\tilde{m}_{b_L}$	800	555	1315
	$\tilde{m}_{b_R}$	919	629	1294
	$\tilde{m}_{t_L}$	853	621	1315
	$\tilde{m}_{t_R}$	412	270	1123
Sleptons:	$\tilde{m}_{e_L,\mu_L}$	323	200	596
	$\tilde{m}_{\nu_{eL},\nu_{\mu L}}$	323	200	596
	$\tilde{m}_{e_R,\mu_R}$	152	92	290
	$\tilde{m}_{\tau_L}$	322	197	539
	$\tilde{m}_{\tau_R}$	151	92	1543

IV. FLAVOR VIOLATION INDUCED BY MESSENGER–MATTER MIXING

The main motivation for gauge mediation of SUSY breaking is that it naturally solves the SUSY flavor problem. This is possible because of the universality in the scalar masses induced by gauge mediation. This universality is however violated by messenger–matter mixing, as seen from Eq. (12) in the $5 + \bar{5}$ model, and from Eq. (18) in the $10 + \overline{10}$ model. In this section we show that flavor violation induced by such nonuniversal contributions to the soft scalar masses can be all within experimental limits, if we embed these models in a framework with a $U(1)$ flavor symmetry. This $U(1)$ symmetry also addresses the hierarchies in the fermion masses and mixings [27]. When embedded in $SU(5)$ unified theory, this framework would lead to a lopsided structure for the down quark and charged lepton mass matrices [28], which explains naturally why the quark mixing angles are small, while the leptonic mixing angles are large. Such matrices also explain other features of the fermion mass spectrum, such as why the charge 2/3 quark mass ratios exhibit a stronger hierarchy compared to the charge $-1/3$ quark mass ratios or the charged lepton mass ratios. We shall see that while dangerous flavor violation is suppressed by this flavor symmetry, a small amount of flavor violation is present in these models, which can have testable consequences.

The flavor $U(1)$ symmetry serves another important purpose. It forbids bare masses for the messenger fields, a requirement for successful gauge mediation. GMSB models usually assume these bare masses are zero, here there is a symmetry-based explanation for them to vanish.

In our construction, owing to the $U(1)$ flavor symmetry, renormalizable Yukawa couplings are allowed only for the third family fermions. The vacuum expectation value of a SM singlet field S, which breaks this $U(1)$ at a scale slightly below M_*, identified as the Planck scale or the string scale, generates masses for the first two families via nonrenormalizable operators which are suppressed by powers of a small parameter $\epsilon \equiv \langle S\rangle/M_*$. The power suppression arises because of the flavor–dependent $U(1)$ charges of the fermions. In such a framework, all fundamental Yukawa couplings can be of order one and still the hierarchy in the fermion masses and mixings can be explained [29]. This $U(1)$ can be naturally identified as the anomalous $U(1)$ symmetry of string theory [30].

We now turn to the embedding of the $5 + \bar{5}$ messenger model and the $10 + \overline{10}$ messenger model of the previous section into a unified $SU(5)$ framework along with a flavor $U(1)$ symmetry and discuss flavor violation mediated by SUSY particles in these models.

A. Flavor violation in the $5 + \bar{5}$ messenger model

Although we do not construct complete $SU(5)$ models, the assignment of $U(1)$ charges for the fields will be compatible with $SU(5)$ symmetry. So we can use the notation of SUSY $SU(5)$. The three families of quarks and leptons belong to $\bar{5}_i + 10_i$ under $SU(5)$, with $i =$ 1–3. Here $10_i \subset \{Q_i, u_i^c, e_i^c\}$ and $\bar{5}_i \subset \{d_i^c, L_i\}$. The Higgs doublets (H_u, H_d) of MSSM are contained in 5_H and $\bar{5}_H$ of $SU(5)$. It should be understood that from these Higgs fields, the color triplet components have been removed for our discussions which relate to momentum scales below M_X. The messenger fields are denoted as $5_m + \bar{5}_m$, and are assumed to have the same R–parity as quarks and leptons. The flavor $U(1)$ charges of these fields are listed in Table V. The charge assignment for the MSSM fields is the same as the one given in Ref. [31], but here we extend it to include the messenger fields.

In Table V the parameter p is an integer which can take values 0, 1 or 2, corresponding to large, medium or small $\tan\beta$ values. Although the value $p = 0$ can explain the fermion mass hierarchy, we will see that this choice is disfavored from flavor changing neutral currents (FCNC) constraints, while $p = 1, 2$ are both acceptable. The field S acquires a vacuum expectation value (VEV) just below M_* without breaking SUSY, while the field Z acquires a VEV along its scalar component $\langle Z\rangle \sim M_{\rm mess}$, which is much smaller than $\langle S\rangle$. The field Z also acquires an F–component, which breaks supersymmetry. If the $U(1)$ symmetry is identified as the anomalous $U(1)$ of string theory, even without writing any superpotential, $\langle S\rangle \neq 0$ can develop by the shift in fields required to set the gravity–induced Fayet–Iliopoulos D–term for the $U(1)$ to zero, so that SUSY remains unbroken [32]. In such schemes, typically one finds $\epsilon \equiv \langle S\rangle/M_* \sim 0.2$, which provides a small expansion parameter to explain the fermion mass hierarchy. The charge α in Table V is not specified for now, but it should be positive, and if it is an integer, $\alpha > p + 1$ should be satisfied. These conditions are needed to guarantee that bare masses for the messenger fields are forbidden, and that the 5_m messenger field does not acquire a mass by pairing with $\bar{5}_i$ fields through superpotential couplings such as $\bar{5}_1 5_m S^n$ for some positive integer n. (Successful gauge mediation requires that the masses of the messenger fields arise from the coupling $\bar{5}_m 5_m Z$, with Z acquiring VEVs along the scalar and F–components.) If the charge α is undetermined, that could lead to an additional global $U(1)$ symmetry which can result in an unwanted Goldstone boson. Since Z carries a charge α, and since it couples to the secluded sector where supersymmetry breaks dynamically, α may get determined from such couplings. We also note that for a rational value of α, $\alpha = a/b$ with a, b being positive integers, the superpotential coupling $S^a Z^b/M_*^{b-1}$ is allowed, which can fix α without upsetting the success of gauge mediation. For example, the superpotential coupling $S^4 Z^5/M_*^6$ would fix $\alpha = 4/5$, which should be harmless as far as the conditions $\langle F_Z\rangle \neq 0$, $\langle Z\rangle \neq 0$ are concerned.

TABLE V. The $U(1)$ charges of the MSSM fields, the messenger fields, and the singlets Z and S in the $5 + \bar{5}$ messenger model in the $SU(5)$ notation. p here is an integer which can take values $p = (0, 1, 2)$ corresponding to (large, medium, small) $\tan\beta$.

Particle	10_1	10_2	10_3	$\bar{5}_1$	$\bar{5}_2, \bar{5}_3$	$5_H, \bar{5}_H$	S	5_m	$\bar{5}_m$	Z
$U(1)$	4	2	0	$p+1$	p	0	-1	$-\alpha$	0	α

The superpotential of the model consistent with the flavor $U(1)$ symmetry of Table V (in the notation of MSSM fields) is

$$\begin{aligned} W = {} & y_{ij}^u \epsilon^{n_{ij}^u} u_i^c Q_j H_u + y_{ij}^d \epsilon^{n_{ij}^d} d_i^c Q_j H_d + y_{ij}^e \epsilon^{n_{ij}^e} e_i^c L_j H_d \\ & + f_d \overline{d^c}_m d_m^c Z + f_e \bar{L}_m L_m Z + \lambda_b' Q_3 d_m^c H_d \\ & + \lambda_{\tau^c}' L_m e_3^c H_d. \end{aligned} \tag{23}$$

Here $y_{ij}^{u,d,e}$ are order one Yukawa couplings. The powers of ϵ appear in Eq. (23) from $(\langle S\rangle/M_*)^{n_{ij}}$ factors, needed to preserve the $U(1)$ symmetry. Here $n_{ij}^u = Q(u_i^c) + Q(Q_j)$, $n_{ij}^d = Q(d_i^c) + Q(Q_j)$, and $n_{ij}^e = Q(e_i^c) + Q(L_j)$, where $Q(f)$ refers to the $U(1)$ charge of the field f. Thus $n_{12}^d = p + 3 = n_{21}^e$, etc.

The second line of Eq. (23) represents messenger–matter mixing allowed by the $U(1)$ symmetry. One can choose a basis where such mixings involve only the third family fermions Q_3 and e_3^c. The coupling $\bar{L}_m(f_e' L_m + f_3 \epsilon^p L_3 + f_2 \epsilon^p L_2 + f_1 \epsilon^{p+1} L_1)Z$ has been redefined simply as $f_e \bar{L}_m L_m Z$ by rotating the (L_i, L_m) fields. In the $L_\alpha e_j^c H_d$ couplings, terms with $\alpha, j =$ 1–3 are part of the first line of Eq. (23), while in the terms $L_m e_j^c H_d$, a redefinition of e_j^c fields can be made so that a single term $L_m e_3^c H_d$, the last term of Eq. (23), is necessary. Similar arguments apply for the Q fields, so that only Q_3 has mixed couplings with d_m^c. If $SU(5)$ boundary conditions are applied to the messenger Yukawa couplings, we would have $\lambda_{\tau^c}' = \lambda_b'$ at M_X. In this section we shall allow for the possibility that these couplings are not unified, and define a prameter

$$r = \frac{\lambda_{\tau^c}'(M_X)}{\lambda_b'(M_X)} \tag{24}$$

so that $r = 1$ corresponds to $SU(5)$ unification condition, while $r = 0$ would imply that $\lambda'_{\tau^c} = 0$ at M_X and below. The latter choice will turn out to be useful to satisfy FCNC constraints. Note that even when $r = 0$, the increase in m_h found in Sec. III will hold, since the initial condition for A_t is determined by λ'_b [see Eq. (13)].

The first line of Eq. (23) provides an explanation for the hierarchy in the masses and mixings of quarks and leptons. The mass matrices for the up–quarks, down–quarks and charged leptons arising from Eq. (23) have the form:

$$M^u = Y^u v_u = \begin{pmatrix} y^u_{11}\epsilon^8 & y^u_{12}\epsilon^6 & y^u_{13}\epsilon^4 \\ y^u_{21}\epsilon^6 & y^u_{22}\epsilon^4 & y^u_{23}\epsilon^2 \\ y^u_{31}\epsilon^4 & y^u_{32}\epsilon^2 & y^u_{33} \end{pmatrix} v_u, \quad (25)$$

$$M^d = Y^d v_d = \epsilon^p \begin{pmatrix} y^d_{11}\epsilon^5 & y^d_{12}\epsilon^3 & y^d_{13}\epsilon \\ y^d_{21}\epsilon^4 & y^d_{22}\epsilon^2 & y^d_{23} \\ y^d_{31}\epsilon^4 & y^d_{32}\epsilon^2 & y^d_{33} \end{pmatrix} v_d, \quad (26)$$

$$M^e = Y^e v_d = \epsilon^p \begin{pmatrix} y^e_{11}\epsilon^5 & y^e_{12}\epsilon^4 & y^e_{13}\epsilon^4 \\ y^e_{21}\epsilon^3 & y^e_{22}\epsilon^2 & y^e_{23}\epsilon^2 \\ y^e_{31}\epsilon & y^e_{32} & y^e_{33} \end{pmatrix} v_d. \quad (27)$$

These matrices have been written down with the left-handed antifermion fields multiplying on the left and the left-handed fermion fields multiplying on the right. With all the $y^{u,d,e}_{ij}$ factors being order one, we see that these matrices lead to the mass hierarchy m_u: m_c: $m_t \sim \epsilon^8$: ϵ^4: 1, m_d: m_s: $m_b \sim \epsilon^5$: ϵ^2: 1, and m_e: m_μ: $m_\tau \sim \epsilon^5$: ϵ^2: 1, in nice agreement with observations [28,29], with the choice $\epsilon \simeq 0.2$. This pattern explains why the up–type quarks exhibit stronger hierarchy compared to the down-type quarks, which have a similar hierarchy structure as the charged leptons. We also see from the (3, 3) entries of M^u and M^d that $\tan\beta \sim \epsilon^p(m_t/m_b)$, which suggests the values of $p = (0, 1, 2)$ for (large, medium, small) $\tan\beta$. Note that the rotations done in obtaining Eq. (23) so as to make only the third family couple to messenger fields do not upset the hierarchy factors of Eqs. (25)–(27). The mixed Yukawa couplings of Eq. (23) also do not affect these mass matrices, since these contributions are suppressed by $\lambda' v_d/M_{\rm mess}$.

One can diagonalize the mass matrices of Eqs. (25)–(27) via biunitary transformations defined as $(U^{u,d,e}_R)M^{u,d,e}(U^{u,d,e}_L)^\dagger = M^{u,d,e}_{\rm diag}$. Then the left-handed rotation matrices $U^{u,d,e}_L$ and the right-handed rotation matrices $U^{u,d,e}_R$ would be of the form

$$U^e_L \sim U^d_R \sim \begin{pmatrix} 1 & \epsilon & \epsilon \\ \epsilon & \omega & \omega \\ \epsilon & \omega & \omega \end{pmatrix}, \quad (28)$$

$$U^u_L \sim U^u_R \sim U^d_L \sim U^e_R \sim \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \quad (29)$$

where ω is a mixing angle of order one, and coefficients of order one multiplying ϵ terms are not exhibited. These matrices are of course subject to unitarity constraints. The CKM mixing matrix for the quarks is given by $V_{\rm CKM} = (U^u_L)(U^d_L)^\dagger$, which has small off-diagonal entries as in Eq. (29).[3] On the other hand, the leptonic mixing matrix, $U_{PMNS} = (U^e_L)(U^\nu_L)^\dagger$ will contain large off-diagonal entries, as in Eq. (28). This is true even when U^ν_L, the unitary matrix that diagonalizes the light neutrino mass matrix is identity. For $\epsilon \simeq 0.22$, a good fit to all the mixing angles in the quark and the lepton sector is obtained.[4] Note that the lopsided nature of M^d and M^e of Eqs. (26) and (27) [i.e., $(M^d)_{23} \gg (M^d)_{32}$, etc.] is crucial for this result, since large left-handed lepton mixing is correlated with large right-handed down quark mixing, which, however, is unobservable in the SM.

To investigate SUSY flavor violation, we introduce mass insertion parameters defined as

$$(\delta^{d,l}_{LL,RR})_{ij} = (U^{\dagger d,l}_{L,R}, \tilde{m}^2_{LL,RR} U^{d,l}_{L,R})_{ij}/\tilde{m}^2_{d,l}, \quad (30)$$

$$(\delta^{d,l}_{LR,RL})_{ij} = (U^{\dagger d,l}_{R,L} \tilde{m}^2_{LR,RL} U^{d,l}_{L,R})_{ij}/\tilde{m}^2_{d,l}, \quad (31)$$

where $\tilde{m}^2_{d,l}$ is the average of the diagonal entries of the scalar mass-squared matrix for the down quarks and charged leptons and the matrix $\tilde{m}^2_{LR,RL}$ is related to trilinear A-terms. In Table VII we list the leading contributions to various FCNC processes in powers of the small parameter $\epsilon \simeq 0.2$. Since the messenger superfields couple with left-handed down quarks and right-handed charged leptons, the flavor violating off-diagonal elements are only induced in the quadratic scalar mass matrices for the left-handed down quarks and right-handed charged leptons. These matrices are given in Appendix A 1. The experimental bounds of the mass insertion parameters δ_{LL}, δ_{RR} and $\delta_{LR,RL}$ that are presented in the table were obtained by comparing the hadronic and leptonic flavor changing processes to their experimental values [33,34]. We used the branching-ratio expressions of the decay rates $l_i \to l_j\gamma$ given in [34] in order to find the experimental upper bounds on the leptonic mass insertion parameters that are consistent with the spectra presented in Table IV. The numerical values of $\kappa^{d,l} = \frac{m_{b,\tau}A_{d,l}}{\tilde{m}^2_{d,\tau}}$ are given in Table VII. These values are based on the spectra given in Table IV. We can see from Table VII

[3]The Cabibbo angle is formally of order ϵ^2 from Eq. (29), but coefficients of order 2 can bring this value to 0.22 [31].

[4]Small neutrino masses can be incorporated via the seesaw mechanism by introducing right-handed neutrinos ν^c_i with $U(1)$ charges (1, 0, 0). This would lead to a mild mass hierarchy in the light neutrino sector, as shown in Ref. [31].

TABLE VI. The $U(1)$ charge assignments to the $10 + \overline{10}$ messenger, MSSM, Z and S superfields.

SU(5)	10_1	10_2	10_3	$\bar{5}_1$	$\bar{5}_2, \bar{5}_3$	$5_u, \bar{5}_d$	S	10_m	$\overline{10}_m$	Z
$U(1)$	4	2	0	$1+p$	p	0	-1	0	$-\alpha$	α

that the $5 + \bar{5}$ model is safe from flavor violation problems as long as $p \geq 2$, especially when $r \ll 1$.

B. Flavor violation in $10 + \overline{10}$ model

The $U(1)$ charge assignments for the messenger, MSSM, S, and Z are given in Table VI. The superpotential for this model, after field redefinitions, is

$$\begin{aligned} W_{10+\overline{10}} = {} & (\lambda'_{u^c}\epsilon^4 Q_1 + \lambda'_{c^c}\epsilon^2 Q_2 + \lambda'_{t^c} Q_3) u^c_m H_u \\ & + Q_m(\lambda'_u \epsilon^4 u^c_1 + \lambda'_c \epsilon^2 u^c_2 + \lambda'_t u^c_3) H_u + \lambda'_m Q_m u^c_m H_u \\ & + \lambda'_b \epsilon^p Q_m d^c_3 H_d + \lambda'_\tau \epsilon^p L_3 e^c_m H_d + f_{e^c} \overline{e^c}_m e^c_m Z \\ & + f_{u^c} \overline{u^c}_m u^c_m Z + f_Q \bar{Q}_m Q_m Z. \end{aligned} \quad (32)$$

In the $10 + \overline{10}$ model, the flavor violating off-diagonal elements are induced in the scalar matrices of the left-handed down quarks, right-hand down quarks, and left-handed charged leptons. These matrices are evaluated in Appendix B 2. Using Eqs. (30) and (31) and the unitary transformation given in Eqs. (28) and (29), the mass insertion parameters for the $10 + \overline{10}$ model are listed in Table VII. The stringent constraint comes from the $\mu \to e\gamma$ decay as shown in Table VII. The inequality $p \geq 1$, or $r \simeq 0$ should be satisfied in order to suppress the $\mu \to e\gamma$ decay process [35,36].

From Table VII, it is clear that all present experimental limits are satisfied. Setting the integer $p = 1$, we see that the values close to experimental limits are in CP violation in K^0 system, and in $\mu \to e\gamma$ decay. The latter is predicted to occur with an increased experimental sensitivity of 10 to 100. Using $\epsilon = 0.22$, we see that new SUSY contributions to ϵ_K can be about 30% of the SM value. Such new contributions can resolve the apparent discrepancy between the determinations of $\sin 2\beta$ in B_d system and ϵ_K [37].

TABLE VII. The calculated mass insertion parameters for the $5 + \bar{5}$ and $10 + \overline{10}$ models and their experimental upper bounds. The numerical values of κ's are $\kappa^d_5 = 0.0045$, $\kappa^l_5 = 0.019$, $\kappa^d_{10} = 0.002$ and $\kappa^l_{10} = 0.0014$. The spectrum corresponds to that of Table IV.

Mass insertion (δ)	$5 + \bar{5}$	$10 + \overline{10}$	Process	Experimental bounds
$(\delta^l_{12})_{LL}$	...	ϵ^{1+2p}		0.00028
$(\delta^l_{12})_{RR}$	$r\ \epsilon^6$	...	$\mu \to e\gamma$	0.0004
$(\delta^l_{12})_{RL,LR}$	$r\ \kappa^l_5(\epsilon^4, \epsilon^3)$	$\kappa^l_{10}(\epsilon^{4+2p}, \epsilon^{3+2p})$		1.3×10^{-6}
$(\delta^l_{13})_{LL}$	...	ϵ^{1+2p}		0.026
$(\delta^l_{13})_{RR}$	$r\ \epsilon^4$	...	$\tau \to e\gamma$	0.04
$(\delta^l_{13})_{RL,LR}$	$r\ \kappa^l_5(\epsilon^4, \epsilon^1)$	$\kappa^l_{10}(\epsilon^{4+2p}, \epsilon^{1+2p})$		0.002
$(\delta^l_{23})_{LL}$	...	ϵ^{2p}		0.02
$(\delta^l_{23})_{RR}$	$r\ \epsilon^2$	...	$\tau \to \mu\gamma$	0.03
$(\delta^l_{23})_{RL,LR}$	$r\ \kappa^l_5(\epsilon^2, 1)$	$\kappa^l_{10}(\epsilon^{2+2p}, \epsilon^{2p})$		0.0015
$\left(\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LL}\rvert}, \sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LL}\rvert}\right)$	ϵ^6	ϵ^6		(0.065, 0.0052)
$\left(\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{RR}\rvert}, \sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{RR}\rvert}\right)$	...	ϵ^{1+2p}		(0.065, 0.0052)
$\left(\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{LR}\rvert}, \sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{LR}\rvert}\right)$	$\kappa^d_5 \epsilon^3$	$\kappa^d_{10} \epsilon^3$	$K - \bar{K}$	$(0.007, 5.2 \times 10^{-5})$
$\left(\sqrt{\lvert \mathrm{Re}(\delta^d_{12})^2_{RL}\rvert}, \sqrt{\lvert \mathrm{Im}(\delta^d_{12})^2_{RL}\rvert}\right)$	$\kappa^d_5 \epsilon^4$	$\kappa^d_{10} \epsilon^4$		$(0.007, 5.2 \times 10^{-5})$
$\sqrt{\lvert \mathrm{Re}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$	...	$\epsilon^{3.5+p}$		0.00453
$\sqrt{\lvert \mathrm{Im}(\delta^d_{12})_{LL}(\delta^d_{12})_{RR}\rvert}$	...	$\epsilon^{3.5+p}$		0.00057
$(\mathrm{Re}\delta^d_{13}, \mathrm{Im}\delta^d_{13})_{LL}$	ϵ^4	ϵ^4		(0.238, 0.51)
$(\mathrm{Re}\delta^d_{13}, \mathrm{Im}\delta^d_{13})_{RR}$	...	ϵ^{1+2p}	$B_d - \bar{B}_d$	(0.238, 0.51)
$(\mathrm{Re}\delta^d_{13}, \mathrm{Im}\delta^d_{13})_{LR,RL}$	$\kappa^d_5(\epsilon^4, \epsilon)$	$\kappa^d_{10}(\epsilon, \epsilon^4)$		(0.0557, 0.125)
$(\delta^d_{23})_{LL}$	ϵ^2	ϵ^2		1.19
$(\delta^d_{23})_{RR}$	...	ϵ^{2p}	$B_s - \bar{B}_s$	1.19
$(\delta^d_{23})_{LR,RL}$	$\kappa^d_5(1, \epsilon^2)$	$\kappa^d_{10}(1, \epsilon^2)$	$b \to s\gamma$	0.04

V. CONCLUSION

In this paper we have investigated the upper limit on the lightest Higgs boson mass m_h in gauge mediated supersymmetry breaking models. In minimal GMSB models, with all the SUSY particle masses below 2 TeV, the upper limit on m_h is about 118 GeV. The vanishing of the trilinear soft term A_t that occurs in minimal GMSB models at the messenger scale sets this restriction on m_h, which could otherwise have been as large as 130 GeV. We have shown that the mixing of messenger fields with the MSSM quark and lepton fields can relax this constraint significantly, primarily because A_t receives new contributions from the mixed Yukawa couplings at the messenger scale. Mixing of the messenger fields with the MSSM fields would avoid potential problems in cosmology with having a stable messenger particle. We studied two models, one with messengers belonging to $5 + \bar{5}$ of $SU(5)$ unification, and one where they belong to $10 + \overline{10}$ of $SU(5)$. In the former case, m_h can be as large as about 121 GeV, while in the latter case $m_h \sim 125$ GeV is realized. These values of m_h are realized even for $M_{\rm mess} < 10^8$ GeV, which is preferred by cosmology, since the gravitino LSP mass would be sub–keV in this case, which avoids gravitino overclosure of the Universe. The mixed messenger–matter Yukawa couplings are restricted by the demand that $\tilde{m}^2_{\tau^c}$ and $\tilde{m}^2_{\tau^c}$ should not turn negative. We have delineated the allowed parameter space of these models and have computed the supersymmetric particle spectrum. Relatively light stops are realized, along with $m_h \simeq 125$ GeV, especially in the $10 + \overline{10}$ model.

Arbitrary mixing of messenger fields with the MSSM fields can open up the SUSY flavor problem even in GMSB models. The increase in m_h and the changes in the SUSY spectrum rely primarily on the mixing of the third family with the messenger fields. We have embedded the two models studied here in a unified framework based on $SU(5)$, along with a flavor $U(1)$ symmetry. This $U(1)$ symmetry provides an understanding of the mass and mixing angle hierarchies in the quark and lepton sectors via the Froggatt–Nielsen mechanism [27]. We have shown that the same $U(1)$ symmetry can prevent bare masses for the messenger fields, which is necessary for the consistency of gauge mediation. This $U(1)$ also forbids excessive SUSY flavor violation by suppressing the mixing of the first two families with the messenger fields. There could, however, be residual but small flavor violation arising from the SUSY exchange diagrams. We find that new contributions to the CP asymmetry parameter ϵ_K in the K meson system can be at the (10–30)% level, which can explain the apparent discrepancy between ϵ_K and $\sin 2\beta$ extracted from the B meson system. We also find that the branching ratio for the decay $\mu \to e\gamma$ is in the interesting range for next generation experiments.

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APPENDIX A

In this appendix, we present the RGE for the gauge and Yukawa couplings for the two models considered in the text.

1. RGE for the gauge and Yukawa couplings in the $5 + \bar{5}$ messenger model

Here we present the one–loop RGE for the gauge and Yukawa couplings for the $5 + \bar{5}$ model, with the superpotential given in Eq. (9), valid in the momentum regime $M_{\rm mess} \leq \mu \leq M_X$. We include the effects of the mixed messenger–matter Yukawa couplings, and ignore the Yukawa couplings of the first two families.

$$\frac{dg_3^2}{dt} = \frac{-g_3^4}{4\pi^2},$$

$$\frac{dg_2^2}{dt} = \frac{g_2^4}{4\pi^2},$$

$$\frac{dg_1^2}{dt} = \frac{19 g_1^4}{20\pi^2},$$

$$\frac{d\lambda_t^2}{dt} = \frac{\lambda_t^2}{8\pi^2}\left[6\lambda_t^2 + \lambda_b^2 + \lambda_b'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$

$$\frac{d\lambda_b^2}{dt} = \frac{\lambda_b^2}{8\pi^2}\left[6\lambda_b^2 + \lambda_t^2 + \lambda_\tau^2 + \lambda_{\tau^c}'^2 + 4\lambda_b'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right],$$

$$\frac{d\lambda_\tau^2}{dt} = \frac{\lambda_\tau^2}{8\pi^2}\left[4\lambda_\tau^2 + 3\lambda_b^2 + 3\lambda_{\tau^c}'^2 + 3\lambda_b'^2 - 3g_2^2 - \frac{9}{5}g_1^2\right],$$

$$\frac{d\lambda_b'^2}{dt} = \frac{\lambda_b'^2}{8\pi^2}\left[6\lambda_b'^2 + 4\lambda_b^2 + \lambda_{\tau^c}'^2 + \lambda_t^2 + \lambda_\tau^2 + f_d^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right],$$

$$\frac{d\lambda_{\tau^c}'^2}{dt} = \frac{\lambda_{\tau^c}'^2}{8\pi^2}\left[4\lambda_{\tau^c}'^2 + 3\lambda_b^2 + 3\lambda_b'^2 + 3\lambda_\tau^2 + f_e^2 - 3g_2^2 - \frac{9}{5}g_1^2\right],$$

$$\frac{df_d^2}{dt} = \frac{f_d^2}{8\pi^2}\left[5f_d^2 + 2f_e^2 + 2\lambda_b'^2 - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2\right],$$

$$\frac{df_e^2}{dt} = \frac{f_e^2}{8\pi^2}\left[4f_e^2 + 3f_d^2 + \lambda_{\tau^c}'^2 - 3g_2^2 - \frac{3}{5}g_1^2\right].$$

2. RGE for the $10 + \overline{10}$ messenger model

Here we present the one–loop RGE for the various parameters of the $10 + \overline{10}$ model, corresponding to the superpotential given in Eq. (17), valid in the momentum regime $M_{\rm mess} \leq \mu \leq M_X$.

$$\frac{dg_3^2}{dt} = 0,$$

$$\frac{dg_2^2}{dt} = \frac{g_2^4}{4\pi^2},$$

$$\frac{dg_1^2}{dt} = \frac{3g_1^4}{5\pi^2},$$

$$\frac{d\lambda_t^2}{dt} = \frac{\lambda_t^2}{8\pi^2}\left[6\lambda_t^2 + \lambda_b^2 + 4\lambda_{t^c}'^2 + 5\lambda_t'^2 + 3\lambda_m'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$

$$\frac{d\lambda_b^2}{dt} = \frac{\lambda_b^2}{8\pi^2}\left[6\lambda_b^2 + \lambda_t^2 + \lambda_\tau^2 + \lambda_{t^c}'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right],$$

$$\frac{d\lambda_\tau^2}{dt} = \frac{\lambda_\tau^2}{8\pi^2}\left[4\lambda_\tau^2 + 3\lambda_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right],$$

$$\frac{d\lambda_m'^2}{dt} = \frac{\lambda_m'^2}{8\pi^2}\left[6\lambda_m'^2 + 4\lambda_t'^2 + 5\lambda_{t^c}'^2 + 3\lambda_t^2 + f_Q^2 + f_{u^c}^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$

$$\frac{df_{e^c}^2}{dt} = \frac{f_{e^c}^2}{8\pi^2}\left[3f_{e^c}^2 + 6f_Q^2 + 3f_{u^c}^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{12}{5}g_1^2\right],$$

$$\frac{df_{u^c}^2}{dt} = \frac{f_{u^c}^2}{8\pi^2}\left[3f_{u^c}^2 + 6f_Q^2 + f_{e^c}^2 + 2\lambda_{t^c}'^2 + 2\lambda_m'^2 - \frac{16}{3}g_3^2 - \frac{16}{15}g_1^2\right],$$

$$\frac{df_Q^2}{dt} = \frac{f_Q^2}{8\pi^2}\left[8f_Q^2 + 3f_{u^c}^2 + f_{e^c}^2 + \lambda_t'^2 + \lambda_m'^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{1}{15}g_1^2\right],$$

$$\frac{d\lambda_t'^2}{dt} = \frac{\lambda_t'^2}{8\pi^2}\left[6\lambda_t'^2 + 3\lambda_{t^c}'^2 + 5\lambda_t^2 + 4\lambda_m'^2 + f_Q^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right],$$

$$\frac{d\lambda_{t^c}'^2}{dt} = \frac{\lambda_{t^c}'^2}{8\pi^2}\left[6\lambda_{t^c}'^2 + 3\lambda_t'^2 + 4\lambda_t^2 + 5\lambda_m'^2 + \lambda_b^2 + f_{u^c}^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right].$$

APPENDIX B

In this appendix, we present the new contributions to the scalar masses and the trilinear A–terms arising from messenger–matter mixing in the $5 + \bar{5}$ model and the $10 + \overline{10}$ model. We follow the method of Ref. [23] in our derivations. The general expressions for the SUSY breaking mass and trilinear parameters, valid in both the $5 + \bar{5}$ and the $10 + \overline{10}$ model, can be written down as [23]

$$\delta\tilde{m}_Q^2(M_{\rm mess}) = -\frac{1}{4}\left\{\sum_\lambda\left(\frac{d\Delta\gamma}{d\lambda}\beta_>[\lambda] - \frac{d\gamma_<}{d\lambda}\Delta\beta[\lambda]\right)\right\}\Lambda^2, \quad \text{(B1)}$$

$$\delta\tilde{A}_{abc}(M_{\rm mess}) = \frac{1}{2}(\lambda_{a'bc}\Delta\gamma_a^{a'} + \lambda_{ab'c}\Delta\gamma_b^{b'} + \lambda_{abc'}\Delta\gamma_c^{c'})\Lambda. \quad \text{(B2)}$$

Here the λ–summation is over the MSSM and mixed MSSM–messenger Yukawa couplings, $\Delta\beta[\lambda(M_{\rm mess})] = \beta_>[\lambda(M_{\rm mess})] - \beta_<[\lambda(M_{\rm mess})]$, and $\Delta\gamma(M_{\rm mess}) = \gamma_>(M_{\rm mess}) - \gamma_<(M_{\rm mess})$, where $\gamma_>(\gamma_<)$ is the anomalous dimension above (below) $M_{\rm mess}$ and $\beta[\lambda]$ is the beta function for the Yukawa coupling λ. Here $\tilde{A}_{abc}$ is defined through the soft term $V \supset \tilde{A}_{abc}\Phi_a\Phi_b\Phi_c$, and is related to A_{abc} given in Eqs. (13)–(15), (21), and (22) as $\tilde{A}_{abc} = \lambda_{abc}A_{abc}$.

1. Soft mass parameters in the $5 + \bar{5}$ model

The (3, 3) elements of the $\Delta\gamma(M_{\rm mess})$ matrix for the Q, e^c fields, and $\Delta\gamma(M_{\rm mess})$ for the H_d field in the $5 + \bar{5}$ model are

$$\Delta\gamma_{Q_{33}}(M_{\rm mess}) = -\frac{\lambda_b'^2}{8\pi^2}, \quad \text{(B3)}$$

$$\Delta\gamma_{e^c_{33}}(M_{\rm mess}) = -2\frac{\lambda_{\tau^c}'^2}{8\pi^2}, \quad \text{(B4)}$$

$$\Delta\gamma_{H_d}(M_{\rm mess}) = -\frac{3\lambda_b'^2 + \lambda_{\tau^c}'^2}{8\pi^2}. \quad \text{(B5)}$$

The anomalous dimension matrices for the Q and the e^c fields below $M_{\rm mess}$ are given by

$$\gamma_{Q_{ij}<}(M_{\rm mess}) = -\frac{1}{8\pi^2}\left[Y_{ki}^uY_{kj}^{*u} + Y_{ki}^dY_{kj}^{*d} - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2\right], \quad \text{(B6)}$$

$$\gamma_{e^c_{ij}<}(M_{\rm mess}) = -\frac{1}{8\pi^2}\left[2Y_{ik}^eY_{jk}^{*e} - \frac{6}{5}g_1^2\right]. \quad \text{(B7)}$$

With the flavor $U(1)$ symmetry, the MSSM Yukawa couplings take the hierarchical form

$$Y^u = \begin{pmatrix} Y_{11}^u\epsilon^8 & Y_{12}^u\epsilon^6 & Y_{13}^u\epsilon^4 \\ Y_{21}^u\epsilon^6 & Y_{22}^u\epsilon^4 & Y_{23}^u\epsilon^2 \\ Y_{31}^u\epsilon^4 & Y_{32}^u\epsilon^2 & Y_{33}^u \end{pmatrix}, \quad \text{(B8)}$$

$$Y^d = \epsilon^p\begin{pmatrix} Y_{11}^d\epsilon^5 & Y_{12}^d\epsilon^3 & Y_{13}^d\epsilon \\ Y_{21}^d\epsilon^4 & Y_{22}^d\epsilon^2 & Y_{23}^d \\ Y_{31}^d\epsilon^4 & Y_{32}^d\epsilon^2 & Y_{33}^d \end{pmatrix}, \quad \text{(B9)}$$

$$Y^e = \epsilon^p \begin{pmatrix} Y^e_{11}\epsilon^5 & Y^e_{12}\epsilon^4 & Y^e_{13}\epsilon^4 \\ Y^e_{12}\epsilon^3 & Y^e_{22}\epsilon^2 & Y^e_{23}\epsilon^2 \\ Y^e_{13}\epsilon & Y^e_{23} & Y^e_{33} \end{pmatrix}, \tag{B10}$$

with $\epsilon \ll 1$, $p = 0, 1, 2$ corresponding to large, medium, and small values of $\tan\beta$, and all $Y^{u,d,e}_{ij}$ being of order one. By keeping only the leading ϵ^0 terms we obtain

$$\Delta\beta_{Y^u_{i3}}(M_{\mathrm{mess}}) = \frac{Y^u_{i3}}{16\pi^2}\lambda'^2_b, \tag{B11}$$

$$\Delta\beta_{Y^e_{ij}}(M_{\mathrm{mess}}) = \frac{Y^e_{ij}}{16\pi^2}(\lambda'^2_{\tau^c} + 3\lambda'^2_b), \quad i \neq 3 \tag{B12}$$

$$\Delta\beta_{Y^e_{3i}}(M_{\mathrm{mess}}) = 3\frac{Y^e_{3i}}{16\pi^2}(\lambda'^2_{\tau^c} + \lambda'^2_b), \tag{B13}$$

with all other contributions suppressed. The beta-functions for λ'_b and λ'_{τ^c} above M_{mess} are given by

$$\beta_{\lambda'_b>}(M_{\mathrm{mess}}) = \frac{\lambda'_b}{16\pi^2}\Big(6\lambda'^2_b + \lambda'^2_e + (Y^u_{33})^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big), \tag{B14}$$

$$\beta_{\lambda'_{\tau^c}>}(M_{\mathrm{mess}}) = \frac{\lambda'_{\tau^c}}{16\pi^2}\Big(4\lambda'^2_{\tau^c} + 3\lambda'^2_b - 3g_2^2 - \frac{9}{5}g_1^2\Big). \tag{B15}$$

Note that $[\gamma_>, \gamma_<] = [\Delta\gamma, \gamma_<]$. Plugging Eqs. (B3)–(B15) into Eqs. (B1) and (B2) and keeping the leading power of ϵ we obtain

$$\delta\tilde{m}^2_{e^c} \sim \delta\tilde{m}^2_{e^c_3}\begin{pmatrix} \epsilon^{8+2p} & \epsilon^{6+2p} & \epsilon^{4+2p} \\ \epsilon^{6+2p} & \epsilon^{4+2p} & \epsilon^{2+2p} \\ \epsilon^{4+2p} & \epsilon^{2+2p} & 1 \end{pmatrix}, \tag{B16}$$

$$\delta A_e \sim \frac{\Lambda\epsilon^p}{(16\pi^2)}\begin{pmatrix} \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^1 & 3(\lambda'^2_b + \lambda^2_{\tau^c}) & 3(\lambda'^2_b + \lambda^2_{\tau^c}) \end{pmatrix}, \tag{B17}$$

$$\delta A_d \sim \delta A_b \epsilon^p \begin{pmatrix} \epsilon^5 & \epsilon^3 & \epsilon \\ \epsilon^4 & \epsilon^2 & 1 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{B18}$$

$$\delta\tilde{m}^2_Q \sim \delta\tilde{m}^2_{Q_3}\begin{pmatrix} 0 & 0 & \epsilon^4 \\ 0 & 0 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{B19}$$

$$\delta\tilde{A}_t = -\frac{\Lambda}{16\pi^2}Y^u_{33}\lambda'^2_b. \tag{B20}$$

Here $\delta\tilde{m}^2_{e^c_3}$, $\delta\tilde{m}^2_{Q_3}$ and δA_b are given respectively by Eqs. (11), (10), and (14).

2. SOFT MASS PARAMETERS IN THE 10 + $\overline{10}$ MODEL

From the superpotential $W_{10+\overline{10}}$ of Eq. (17), we can write $\Delta\gamma_Q$, $\Delta\gamma_{u^c}$ and $\Delta\gamma_{H_u}$ as

$$\Delta\gamma_Q(M_{\mathrm{mess}}) = -\frac{\lambda'^2_{t^c}}{8\pi^2}, \tag{B21}$$

$$\Delta\gamma_{u^c}(M_{\mathrm{mess}}) = -\frac{2\lambda'^2_t}{8\pi^2}, \tag{B22}$$

$$\Delta\gamma_{H_u}(M_{\mathrm{mess}}) = -\frac{3}{8\pi^2}(\lambda'^2_t + \lambda'^2_{t^c} + \lambda'^2_m). \tag{B23}$$

The beta-functions for the mixed Yukawa couplings appearing in these matrices for momenta above M_{mess} are

$$\beta_{\lambda'_{t^c}>}(M_{\mathrm{mess}}) = \frac{\lambda'_{t^c}}{16\pi^2}\Big(5\lambda'^2_m + 6\lambda'^2_{t^c} + 3\lambda'^2_t + 4(Y^u_{33})^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big), \tag{B24}$$

$$\beta_{\lambda'_t>}(M_{\mathrm{mess}}) = \frac{\lambda'_t}{16\pi^2}\Big(4\lambda'^2_m + 6\lambda'^2_t + 3\lambda'^2_{t^c} + 5(Y^u_{33})^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big). \tag{B25}$$

The anomalous dimension matrix $\gamma_{Q<}$ for the Q fields are the same as the ones given in Eq. (B6) for this model. For the u^c fields, it is given by

$$\gamma_{u^c_{ij}<}(M_{\mathrm{mess}}) = -\frac{1}{8\pi^2}\Big(2Y^u_{ik}Y^{*u}_{jk} - \frac{16}{6}g_3^2 - \frac{8}{15}g_1^2\Big). \tag{B26}$$

We also have

$$\Delta\beta_{Y^u_{i3}}(M_{\mathrm{mess}}) = \frac{Y^u_{i3}}{16\pi^2}(3\lambda'^2_m + 4\lambda'^2_{t^c} + 3\lambda'^2_t), \quad i \neq 3 \tag{B27}$$

$$\Delta\beta_{Y^u_{3i}}(M_{\mathrm{mess}}) = \frac{Y^u_{3i}}{16\pi^2}(3\lambda'^2_m + 3\lambda'^2_{t^c} + 5\lambda'^2_t), \quad i \neq 3 \tag{B28}$$

$$\Delta\beta_{Y^u_{33}}(M_{\mathrm{mess}}) = \frac{Y^u_{33}}{16\pi^2}(3\lambda'^2_m + 4\lambda'^2_{t^c} + 5\lambda'^2_t). \tag{B29}$$

Using Eqs. (B1) and (B2) we obtain

$$\delta\tilde{m}^2_Q \sim \delta\tilde{m}^2_{Q_3}\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{B30}$$

$$\delta\tilde{m}^2_{u^c} \sim \delta\tilde{m}^2_{u^c_3}\begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{B31}$$

$$\delta\tilde{A}_u \sim \delta A_t \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \qquad \text{(B32)}$$

$$\delta\tilde{A}_d \sim \delta A_b \begin{pmatrix} 0 & 0 & \epsilon^4 \\ 0 & 0 & \epsilon^2 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \text{(B33)}$$

where $\delta\tilde{m}^2_{Q_3}$, $\delta\tilde{m}^2_{u_3^c}$, and δA_t, and δA_b are given respectively by Eqs. (18), (19), (21), and (22). Here order one coefficients multiplying each term are to be understood.

The coupling $\lambda\epsilon^p\bar{5}10_m\bar{5}_d$ induces flavor changing mass terms and trilinear A–terms in the $\tilde{d}^c$ and the $\tilde{e}$ sectors. These terms are obtained following the same steps as in the $5+\bar{5}$ model as

$$\delta\tilde{m}^2_L \sim \delta\tilde{m}^2_{d^c} \sim \frac{\Lambda^2}{2(16\pi^2)^2} \begin{pmatrix} 0 & 0 & \epsilon^{1+4p} \\ 0 & 0 & \epsilon^{4p} \\ \epsilon^{1+4p} & \epsilon^{4p} & \epsilon^{2p} \end{pmatrix}, \qquad \text{(B34)}$$

$$\delta A_e \sim \frac{\Lambda}{2(16\pi^2)} \begin{pmatrix} \epsilon^{5+2p} & \epsilon^{4+2p} & \epsilon^{4+2p} \\ \epsilon^{3+2p} & \epsilon^{2+2p} & \epsilon^{2+2p} \\ \epsilon^{1+2p} & \epsilon^{2p} & \epsilon^{2p} \end{pmatrix}, \qquad \text{(B35)}$$

$$\delta A_d \sim \frac{\Lambda}{2(16\pi^2)} \begin{pmatrix} \epsilon^{5+2p} & \epsilon^{3+2p} & \epsilon^{1+2p} \\ \epsilon^{4+2p} & \epsilon^{2+2p} & \epsilon^{2p} \\ \epsilon^{4+2p} & \epsilon^{2+2p} & \epsilon^{2p} \end{pmatrix}. \qquad \text{(B36)}$$

Here order one couplings multiplying each term are not shown, but should be understood.

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