

Dissociation of quarkonium in an anisotropic hot QCD mediumLata Thakur,¹ Najmul Haque,² Uttam Kakade,¹ and Binoy Krishna Patra^{1,*}¹*Department of Physics, Indian Institute of Technology Roorkee, Roorkee 247667, India*²*Theory Division, Saha Institute of Nuclear Physics, 1/AF Bidhanagar, Kolkata 700064, India*

(Received 14 December 2012; published 23 September 2013)

We have studied the properties of quarkonium states in the presence of momentum anisotropy by correcting the full Cornell potential through the hard-loop resummed gluon propagator. The in-medium modification to the potential causes less screening, so quarkonium states become more tightly bound than in isotropic medium. In addition, the anisotropy in the momentum space introduces a characteristic angular dependence in the potential and as a result the quark pairs aligned in the direction of anisotropy are bound stronger than those of perpendicular alignment. Since the weak anisotropy represents a perturbation to the (isotropic) spherical potential, we use the quantum mechanical perturbation theory to obtain the first-order correction due to the small anisotropic contribution to the energy eigenvalues of spherically symmetric potential.

DOI: [10.1103/PhysRevD.88.054022](https://doi.org/10.1103/PhysRevD.88.054022)

PACS numbers: 12.39.-x, 11.10.St, 12.38.Mh, 12.39.Pn

I. INTRODUCTION

Ultrarelativistic heavy-ion experiments have shown very rich physics which cannot be interpreted by mere extrapolation from elementary nucleon-nucleon collisions to nucleus-nucleus collisions. This is evidenced from the suppression of the high transverse momentum region of hadron spectra up to a factor of 5 relative to nucleon-nucleon collisions, which is an indication for strong absorption of high-energy partons traversing the medium [1]. Also the inclusive production of charm quark bound states are suppressed by a factor of 3–5 at both Super Proton Synchrotron and Relativistic Heavy Ion Collider (RHIC) experiments which hints their dissolution in the medium [2,3]. These accumulated evidences indicate that a new form of matter has been produced in the heavy-ion collision experiments, called quark-gluon plasma (QGP). Among different experimental observations which may be served as the signals for the QGP formation, quarkonium suppression has been proposed a long time ago as a clear probe of the QGP formation in the collider experiments [4,5]. First, in a pioneering work by Matsui and Satz [6] and later in a follow-up quantitative calculation [7], it was shown that the suppression of J/ψ yields could be explained by the (color) screening of the potential between a heavy quark and antiquark by the surrounding deconfined light quarks and gluons. Theoretically, one can study the quarkonium states by using the effective field theories. Since the mass of heavy quark, m_Q , is much larger than the intrinsic scale in the theory of quantum chromodynamics (QCD), Λ_{QCD} , the heavy quark and antiquark are expected to move slowly with a relative velocity $v \ll 1$ and results in the nonrelativistic version of QCD [8,9]. However, nonrelativistic version of QCD does not fully exploit the smallness of v which further gives rise to

another effective theory, known as potential nonrelativistic QCD (pNRQCD) [10,11] by integrating out the momentum scale.

The heavy quark pairs formed in relativistic nuclear collisions develop into the physical resonances and traverse the plasma and then hot hadronic matter before decaying into dilepton pairs. Even before the resonance is formed it may be absorbed by the nucleons streaming past it [12] and by the time the resonance is formed, the screening of the color force in the plasma may inhibit the formation of bound states. The resonance(s) could also be dissociated either by hard gluons [13–17] or by comoving hot hadrons [18]. In order to disentangle these sequential effects [19], we must know how the properties of quarkonium states change in medium. The basic tools of the phenomenological approach to study the properties of quarkonium states are potential models where the possible relativistic effects for excited states of charmonium may also be incorporated there. At zero temperature, the potential model has made great success. At finite temperature, the essence of the potential model in the context of deconfinement is to use a finite temperature extension of the potential. Quantitative understanding of the bound state properties needs the exact potential at finite temperature which, in principle, should be derived directly from QCD, like the Cornell potential at zero temperature has been derived from pNRQCD from the zeroth-order matching coefficient. Such derivations at finite temperature for weakly coupled plasma have recently come up in the literature [20,21] but they are, however, plagued by the existence of temperature-driven hard as well as soft scales, T , gT , g^2T , respectively. Due to these difficulties in finite temperature extension in effective field theories, the lattice-based potentials become popular. However, neither the free energy nor the internal energy can be directly used as the potential. In fact, what kind of screened potentials should be used in the Schrödinger equation which

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describes well the bound states at finite temperature are still an open question. However, recently more involved calculations of quarkonium spectral functions and meson current correlators obtained from potential models have been performed and compared to first-principle QCD calculations performed numerically on lattices [22–26]. In addition to the uncertainties of the correct form of the finite-temperature potential, there is also arbitrariness of the criteria of dissociation. Besides the binding energy for a particular potential as an criterion of dissociation, the decay width is another important quantity to determine the dissociation of the bound states. The calculation based on a real-valued potential model does not include the true width of a state. By performing an analytical continuation of the Euclidean Wilson loop to Minkowski space, the potential has an imaginary part due to Landau damping which clearly broadens the peak [21,27] and facilitates the early dissociation of the bound states.

In the RHIC or LHC era (small μ_B), recent lattice studies have confirmed that the transition from nuclear matter to QGP is not a phase transition, rather a crossover [28]. The large distance property of the heavy quark interaction is important for our understanding of the bulk properties of the QCD plasma phase, e.g., the screening property of the quark gluon plasma [29], the equation of state [30,31], etc. In these studies, deviations from perturbative calculations and the ideal gas behavior are found beyond the deconfinement temperature. It is then reasonable to assume that the string tension does not vanish abruptly at the deconfinement point [32–34], so one should study its effects on heavy quark potential even above T_c . This issue, usually overlooked in the literature where only a screened Coulomb potential was assumed above T_c and the linear/string term was assumed zero, was certainly worth investigation. Recently a heavy quark potential at finite temperature was derived by correcting the full Cornell potential, not its Coulomb part alone, with a dielectric function encoding the effects of the deconfined medium [35]. This was found to have an additional long-range Coulomb term, in addition to the conventional Yukawa term. In the short distance limit, the potential is reduced to vacuum potential, i.e., the $Q\bar{Q}$ pair does not see the medium, giving rise to the duality between $V(r, T = 0)$ and $V(0, T)$. On the other hand, in the large distance limit (where the screening occurs), potential is reduced to a long-range Coulomb potential with a dynamically screened-color charge. Thereafter the binding energies and dissociation temperatures of the ground and the lowest-lying states of charmonium and bottomonium spectra have been determined [35,36] which matches with the finding of recent works based on potential models [37–39] with the Debye mass extracted from the lattice free energy. However, when the Debye mass in leading order was used, the results [35] match with the lattice correlator studies or with the stronger (lattice) binding potential, i.e., internal

energy [40,41]. In a way, their findings address the reason of arbitrariness of the results on dissociation temperatures [42] and ensues a basic question about the nature of dissociation of quarkonium in a hot QCD medium.

However, all the works described above were limited to an isotropic medium but the partonic system generated in an ultrarelativistic heavy-ion collision cannot be homogeneous and isotropic because at the very early time of collision, asymptotic weak coupling enhances the longitudinal expansion substantially more than the radial expansion so the system becomes colder in the longitudinal direction than in the transverse direction. As a result, an anisotropy in the momentum space sets in and causes the parton system produced unstable with respect to the chromomagnetic plasma modes [43] which facilitate to isotropize the system [44,45]. In our work, we restricted ourselves to a *weakly anisotropic medium* because by the time ($t_F = \gamma\tau_F$, τ_F is the quarkonium formation time in its rest frame) quarkonia are formed in the plasma, the plasma becomes almost equilibrated. Motivated with this preamble on the anisotropy generated in the very stage of collision, we wish to investigate the effect of weak anisotropy on the heavy-quark potential and subsequently on the dissociation of quarkonia states in an anisotropic medium.

Our work is organized as follows. In Sec. II, we will discuss the medium modifications to a heavy quark potential both in isotropic and anisotropic medium. In Sec. II A, we start with the in-medium modification to the heavy quark potential in an isotropic medium and then extend it to a medium which exhibits a local anisotropy in momentum space in Sec. II B. To do that, we first obtain the self-energy tensor for an anisotropic medium to obtain the hard-loop resummed gluon propagator. Thereafter the dielectric permittivity obtained in terms of retarded gluon propagator and its Fourier transform at vanishing frequency gives the desired nonrelativistic potential at finite temperature. The potential thus obtained depends not only on the relative separation of the $Q\bar{Q}$ pair but also on their relative orientation with respect to the direction of anisotropy and is found always deeper than in an isotropic medium. It is found that in the weak-anisotropy limit, the correction arising due to anisotropy to the isotropic part of the potential is small and thus has been treated as a perturbation. So, using the first-order perturbation theory, we estimate the shift in energy eigenvalues due to the small anisotropic correction to the energy eigenvalues from the spherically symmetric part in the isotropic medium and determine their dissociation temperatures in Sec. III. Finally, we conclude in Sec. IV.

II. HEAVY-QUARK EFFECTIVE POTENTIAL

A. For isotropic medium ($\xi = 0$)

Potential models are based on the assumption that the interaction between a heavy quark and its antiquark can be described by a potential. At $T = 0$, the hierarchy of well

separated energy scales, $m_Q \gg m_Q v \gg m_Q v^2$, allows one to systematically integrate out the different scales and obtain the nonrelativistic potential QCD (pNRQCD) where the Cornell potential indeed shows up as the zeroth-order matching coefficient [10,11]. Inspired by its success at zero temperature, the potential model has been applied at finite temperature, with the main assumption that medium effects can be accounted by a temperature-dependent potential.

Recent lattice results [28] indicate the phase transition in full QCD appears to be a crossover rather than a “true” phase transition with the related singularities in thermodynamic observables. In light of the above findings, one cannot simply ignore the effects of string tension between the quark-antiquark pairs beyond T_c . This is indeed a very important effect which needs to be incorporated while setting up the criterion for the dissociation. Recently this issue has successfully been addressed for the dissociation of quarkonium in QGP by one of us [35,36] and we closely follow their work in brief. Let us now start with a heavy quark potential (Cornell potential) at $T = 0$:

$$V(r) = -\frac{\alpha}{r} + \sigma r, \quad (1)$$

where α and σ are the phenomenological parameters. The former accounts for the effective coupling between a heavy quark and its antiquark and the latter gives the string coupling. The medium modification enters through the Fourier transform of heavy quark potential as

$$\tilde{V}(k) = \frac{V(k)}{\epsilon(k)}, \quad (2)$$

where $V(k)$ is the Fourier transform (FT) of the Cornell potential which requires a regularization. We regulate both terms in the potential by multiplying with an exponential damping factor and it is switched off after the FT is evaluated. This can be done by assuming r - as distribution ($r \rightarrow r \exp(-\gamma r)$). The FT of the linear part $\sigma r \exp(-\gamma r)$ is

$$= -\frac{i}{k\sqrt{2\pi}} \left(\frac{2}{(\gamma - ik)^3} - \frac{2}{(\gamma + ik)^3} \right). \quad (3)$$

After putting $\gamma = 0$, we obtain the FT of the linear term σr as

$$(\tilde{\sigma}r) = -\frac{4\sigma}{k^4\sqrt{2\pi}} \quad (4)$$

and for the full Cornell potential, the FT is

$$V(k) = -\sqrt{(2/\pi)} \frac{\alpha}{k^2} - \frac{4\sigma}{\sqrt{2\pi}k^4}. \quad (5)$$

The dielectric permittivity $\epsilon(k)$ is given in terms of the static limit of the longitudinal part of the gluon self-energy [46]. In the isotropic case, it can be decomposed into longitudinal (Π_L) and transverse (Π_T) components which, in the static limit, are associated with the screening of electric and magnetic fields, respectively. However, in the

static limit, the transverse part $\Pi_T(0, k \rightarrow 0, T)$ vanishes, i.e., static magnetic fields are not screened. In perturbation theory, the quantity that enters in the Fourier transform of the potential at finite temperature is the static limit of the longitudinal gauge boson self-energy which was calculated long ago [47] at the one-loop level,

$$\lim_{k \rightarrow 0} \Pi_L(0, k, T) = g^2 T^2 \left(\frac{N_c}{3} + \frac{N_f}{6} \right) \equiv m_D^2, \quad (6)$$

where m_D is defined as the screening mass. Since the static limit of the self-energy is momentum independent, the pole of the inverse of dielectric permittivity is simply the gauge invariant Debye mass m_D , so it leads to an exponential damping of the potential $V(r) \sim \exp(-m_D r)/r$. In particular, this form of Π_L has the consequence that gluons screen the strong interaction, in contrast to the zero temperature case, over long-distance scale. If one assumes nonperturbative effects such as the string tension which survives even above the deconfinement point [48] then the dependence of the dielectric function on the Debye mass may get modified. However, we assume the same screening mass scale m_D which emerges in the Debye screened Coulomb potential also appears in the nonperturbative long-distance contribution due to string. In the following section, we take over this assumption to anisotropic medium too. However, different scales for the Coulomb and linear pieces of the $T = 0$ potential, rather than a single one, were already employed in Refs. [49,50]. Moreover, they developed a theoretical model to include nonperturbative effects beyond the deconfinement temperature through dimension-two gluon condensates to calculate the heavy quark-free energy. Interestingly, their model predicts a duality between the zero temperature $Q\bar{Q}$ potential and the quark self-energy and explains the lattice data well.

Finally, one can define a dielectric permittivity in one loop by

$$\epsilon(k) = \left(1 + \frac{\Pi_L(0, k, T)}{k^2} \right) \equiv \left(1 + \frac{m_D^2}{k^2} \right). \quad (7)$$

After substituting the dielectric permittivity in the Fourier transform of Cornell potential (2) and then evaluating its inverse FT, one obtains the medium modified potential in the coordinate space [35,36]

$$\begin{aligned} V(r, T) &= \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{r}} \tilde{V}(k) \\ &= \left(\frac{2\sigma}{m_D} - \alpha m_D \right) \frac{\exp(-\hat{r})}{\hat{r}} - \frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D \end{aligned} \quad (8)$$

with the dimensionless variable $\hat{r} = m_D r$. The constant terms are introduced to yield the correct limit of $V(r, T)$ as $T \rightarrow 0$. Such terms could arise naturally from the basic computations of real time static potential in hot QCD [51] and also from the real and imaginary time correlators in a

thermal QCD medium [52]. The medium modified potential thus obtained has an additional long-range Coulomb term with an (reduced) effective charge, in addition to the conventional Yukawa term.

In the small distance limit, $r \ll 1/m_D$, the above potential reduces to the Cornell potential, i.e., $Q\bar{Q}$ does not see the medium. On the other hand, in the screening region $r \gg 1/m_D$, the potential (8) reduces to

$$V(r, T) \sim -\frac{2\sigma}{m_D^2 r} - \alpha m_D, \quad (9)$$

which, apart from a constant term, looks like a Coulomb potential encountered in the hydrogen-atom problem after identifying the fine structure constant e^2 with the effective charge $2\sigma/m_D^2$. The binding energies and the dissociation temperatures for quarkonium states can thus be determined by solving the Schrödinger equation numerically either with the full potential (8) or analytically with the approximated form (9). In addition, one can also exploit the advantage to demonstrate the flavor dependence of the dissociation process where the dissociation temperatures for 2-flavor are found to be higher than the 3-flavor case [35,36].

B. For anisotropic medium ($\xi \neq 0$)

1. Dielectric permittivity tensor

To study the perturbative potential with an anisotropic parton distribution, consider a hot QCD plasma which, due to expansion and finite (momentum) relaxation time, manifests a local anisotropy in momentum space through the distribution function,

$$f_{\text{aniso}}(\mathbf{k}) = f_{\text{iso}}\left(\sqrt{\mathbf{k}^2 + \xi(\mathbf{k}\cdot\mathbf{n})^2}\right), \quad (10)$$

i.e., $f_{\text{aniso}}(\mathbf{k})$ is obtained from an isotropic distribution $f_{\text{iso}}(|\mathbf{k}|)$ by removing particles with a large momentum component along the direction of anisotropy, \mathbf{n} [43]. We shall restrict ourselves to a plasma close to equilibrium and so that $f_{\text{aniso}}(\mathbf{k})$ is either a Bose-Einstein $n_B(k)$ or a Fermi-Dirac $n_F(k)$ distribution function. This may be true because by the time quarkonia have been formed in the plasma medium from the $Q\bar{Q}$ pairs produced at very early stages of the collision, the system may not be then highly anisotropic rather closer to isotropic distribution. In the limit of small anisotropy, anisotropy parameter ξ is related to the shear viscosity-to-entropy density (η/s) through the one-dimensional Navier-Stokes formula by

$$\xi = \frac{10}{T\tau} \frac{\eta}{s}, \quad (11)$$

where $1/\tau$ denotes the expansion rate of the fluid element. However, the degree of momentum-space anisotropy is generically defined by the parameter

$$\xi = \frac{\langle \mathbf{k}_T^2 \rangle}{2\langle k_L^2 \rangle} - 1, \quad (12)$$

where $k_L = \mathbf{k} \cdot \mathbf{n}$ and $\mathbf{k}_T = \mathbf{k} - \mathbf{n}(\mathbf{k} \cdot \mathbf{n})$ are the components of momentum parallel and perpendicular to the direction of anisotropy, \mathbf{n} , respectively. The positive and negative values of ξ correspond to the squeezing and the stretching of the distribution function in the direction of anisotropy, respectively. In the relativistic nucleus-nucleus collisions, ξ is however, found to be positive. The calculation of the real part of the potential at finite anisotropy was first obtained in Refs. [43,51,53] and was later extended to calculate the imaginary part [21,52,54,55] which is seen as a generic feature of the medium. To study the effect of anisotropy on the in-medium potential, one need to calculate first the self-energy in an anisotropic medium. With the specified anisotropic distribution function, we can compute the gluon self-energy analytically [56]. We will restrict our consideration to the spatial part of the self-energy, $\Pi^{\mu\nu}$, for simplicity and the timelike components can be easily obtained by using the symmetry and transversality of the gluon self-energy tensor. The spatial components of the retarded self-energy tensor reads [53]

$$\Pi^{ij}(P) = -g^2 \int d^3\mathbf{k} v^i \frac{\partial f(\mathbf{k})}{\partial k^l} \left(\delta^{jl} + \frac{v^j p^l}{P \cdot V + i\epsilon} \right), \quad (13)$$

where $P^\mu \equiv (p_0, p)$ is the four momentum of the external gluon, $p = |\mathbf{p}|$ is the amplitude of the spatial momentum. The four-velocity, V^μ ($1, \mathbf{v} = \mathbf{k}/|\mathbf{k}|$), is a lightlike four vector with $v = |\mathbf{v}|$ and the partial derivative, $\partial f/\partial k^l$, in terms of new variable \tilde{k} , is given by

$$\frac{\partial f(\mathbf{k})}{\partial k^l} = \frac{v^l + \xi(\mathbf{v} \cdot \mathbf{n})n^l}{\sqrt{1 + \xi(\mathbf{v} \cdot \mathbf{n})^2}} \frac{\partial f(\tilde{k}^2)}{\partial \tilde{k}}, \quad (14)$$

where $\tilde{k}^2 = k^2(1 + \xi(\mathbf{v} \cdot \mathbf{n})^2)$. After integrating out over the modified momentum \tilde{k} , the self-energy, Π_{ij} is simplified into [53]

$$\Pi^{ij}(P) = m_D^2 \int \frac{d\Omega}{4\pi} v^i \frac{v^l + \xi(\mathbf{v} \cdot \mathbf{n})n^l}{(1 + \xi(\mathbf{v} \cdot \mathbf{n})^2)^2} \left(\delta^{jl} + \frac{v^j p^l}{P \cdot V + i\epsilon} \right), \quad (15)$$

where the square of the Debye mass is defined by

$$m_D^2 = -\frac{g^2}{2\pi^2} \int_0^\infty dk k^2 \frac{df_{\text{iso}}(k^2)}{dk}. \quad (16)$$

Unlike in the isotropic medium, the self-energy Π^{ij} shows an extra dependence on the preferred anisotropic direction (\mathbf{n}), therefore, it can no longer be decomposed into transverse and longitudinal parts; rather it becomes a tensor [43,57,58] with more basis vectors. So the self-energy tensor is decomposed into four structure functions as [43]

$$\Pi^{ij} = \alpha A^{ij} + \beta B^{ij} + \gamma C^{ij} + \delta D^{ij}, \quad (17)$$

where the coefficients α , β , γ , and δ can be determined for any value of ξ [43]. Using these structure functions, one can find the gluon propagator in the temporal axial gauge as [43]

$$\begin{aligned} \Delta_{ij}(\omega, k) &= \frac{(A_{ij} - C_{ij})}{k^2 - \omega^2 + \alpha} \\ &+ \frac{(k^2 - \omega^2 + \alpha + \gamma)B_{ij}}{(k^2 - \omega^2 + \alpha + \gamma)(\beta - \omega^2) - k^2 \tilde{n}^2 \delta^2} \\ &+ \frac{(\beta - \omega^2)C_{ij} - \delta D_{ij}}{(k^2 - \omega^2 + \alpha + \gamma)(\beta - \omega^2) - k^2 \tilde{n}^2 \delta^2}. \end{aligned} \quad (18)$$

In order to see how the anisotropy affects the response to static electric field, we examine the propagator in the static limit ($\omega \rightarrow 0$). Defining the masses in the static limit [43]

$$\begin{aligned} m_\alpha^2 &= \lim_{\omega \rightarrow 0} \alpha, & m_\beta^2 &= \lim_{\omega \rightarrow 0} -\frac{k^2}{\omega^2} \beta, \\ m_\gamma^2 &= \lim_{\omega \rightarrow 0} \gamma, & m_\delta^2 &= \lim_{\omega \rightarrow 0} \frac{\tilde{n} k^2}{\omega} \text{Im} \delta, \end{aligned} \quad (19)$$

the gluon propagator becomes

$$\lim_{\omega \rightarrow 0} \Delta_{ij}(\omega, k) = -\frac{(k^2 + m_\alpha^2 + m_\gamma^2)k_i k_j}{\omega^2 [(k^2 + m_\alpha^2 + m_\gamma^2)(k^2 + m_\beta^2) - m_\delta^4]}. \quad (20)$$

We can now factorize the denominator of the gluon propagator as

$$\begin{aligned} (k^2 + m_\alpha^2 + m_\gamma^2)(k^2 + m_\beta^2) - m_\delta^4 \\ = (k^2 + m_+^2)(k^2 + m_-^2), \end{aligned} \quad (21)$$

where

$$\begin{aligned} 2m_\pm^2 &= M^2 \pm \sqrt{M^4 - 4(m_\beta^2(m_\alpha^2 + m_\gamma^2) - m_\delta^4)}, \\ M^2 &= m_\alpha^2 + m_\beta^2 + m_\gamma^2. \end{aligned} \quad (22)$$

Thus the dielectric permittivity for an anisotropic medium in the temporal axial gauge can be obtained from the definition [59]

$$\begin{aligned} \epsilon^{-1}(k) &= -\lim_{\omega \rightarrow 0} \omega^2 \frac{k_i k_j}{k^2} \Delta_{ij}(\omega, k) \\ &= \frac{k^2(k^2 + m_\alpha^2 + m_\gamma^2)}{(k^2 + m_+^2)(k^2 + m_-^2)}. \end{aligned} \quad (23)$$

If the anisotropy is small, the pole masses (19) can be simplified, by retaining only the linear term in ξ , into [43]

$$\begin{aligned} m_\alpha^2 &= -\frac{\xi}{6}(1 + \cos 2\beta_n)m_D^2, \\ m_\beta^2 &= \left(1 + \frac{\xi}{6}(3 \cos 2\beta_n - 1)\right)m_D^2, \\ m_\gamma^2 &= \frac{\xi}{3} \sin^2 \beta_n m_D^2, \\ m_\delta^2 &= -\xi \frac{\pi}{4} \sin \beta_n \cos \beta_n m_D^2, \end{aligned} \quad (24)$$

where β_n is the angle between \mathbf{k} and \mathbf{n} . So the explicit dependencies of m_\pm on the anisotropy, in the small ξ limit, are given by

$$\begin{aligned} m_+^2 &= \left(1 + \frac{\xi}{6}(3 \cos 2\beta_n - 1)\right)m_D^2, \\ m_-^2 &= -\frac{\xi}{3} \cos 2\beta_n m_D^2. \end{aligned} \quad (25)$$

In the isotropic limit, all masses become zero except one ($m_\alpha^2 = m_\gamma^2 = m_\delta^2 = m_-^2 = 0$, $m_+^2 = m_D^2$), which is the only pole in the isotropic medium (7).

2. Medium modification to heavy quark potential

Once we have obtained the dielectric permittivity in anisotropic medium (23), we substitute it in the Fourier transform (2) and then evaluate its inverse Fourier transform to obtain the medium modified potential in an anisotropic medium:

$$\begin{aligned} V(\mathbf{r}, \xi, T) &= \frac{1}{(2\pi)^{3/2}} \int d^3 \mathbf{k} \tilde{V}(k) e^{i\mathbf{k} \cdot \mathbf{r}} \\ &= -\frac{\alpha}{2\pi^2} \int d^3 \mathbf{k} \frac{(k^2 + m_\alpha^2 + m_\gamma^2)}{(k^2 + m_+^2)(k^2 + m_-^2)} e^{i\mathbf{k} \cdot \mathbf{r}} \\ &\quad - \frac{4\sigma}{(2\pi)^2} \int d^3 \mathbf{k} \frac{(k^2 + m_\alpha^2 + m_\gamma^2)}{k^2(k^2 + m_+^2)(k^2 + m_-^2)} e^{i\mathbf{k} \cdot \mathbf{r}}. \end{aligned} \quad (26)$$

After substituting the pole masses, m_+ and m_- (in the small ξ limit) from (25), the potential becomes

$$\begin{aligned} V(\mathbf{r}, \xi, T) &= -\frac{\alpha}{2\pi^2} \int \frac{d^3 \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}}}{k^2 + m_D^2(1 + \frac{\xi}{6}(3 \cos 2\beta_n - 1))} \\ &\quad - \frac{4\sigma}{(2\pi)^2} \int \frac{d^3 \mathbf{k} e^{i\mathbf{k} \cdot \mathbf{r}}}{k^2 [k^2 + m_D^2(1 + \frac{\xi}{6}(3 \cos 2\beta_n - 1))]} \\ &\equiv V_1(\mathbf{r}, \xi, T) + V_2(\mathbf{r}, \xi, T), \end{aligned} \quad (27)$$

where $V_1(\mathbf{r}, \xi, T)$ and $V_2(\mathbf{r}, \xi, T)$ are the medium-modified potential corresponding to short-distance Coulombic and long-distance string term, respectively, which can be rewritten as

$$V_1(\mathbf{r}, \xi, T) = -\frac{\alpha}{2\pi^2} \int d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{1}{(k^2 + m_D^2)} \times \left(1 + \frac{\xi}{6} \frac{m_D^2}{(k^2 + m_D^2)} (3 \cos 2\beta_n - 1)\right)^{-1}. \quad (28)$$

Expanding the integrand in terms of the anisotropy parameter (ξ) and retaining the term linear in ξ (weak-anisotropy limit, $\xi < 1$), $V_1(\mathbf{r}, \xi, T)$ can be written as

$$V_1(\mathbf{r}, \xi, T) = -\frac{\alpha}{2\pi^2} \int d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}} \left[\frac{1}{(k^2 + m_D^2)} - \frac{\xi}{6} \frac{m_D^2}{(k^2 + m_D^2)^2} (3 \cos 2\beta_n - 1) \right] \equiv V_1^{(1)}(r, \xi = 0, T) + V_1^{(2)}(\mathbf{r}, \xi, T), \quad (29)$$

where $V_1^{(1)}(r, \xi = 0, T)$ and $V_1^{(2)}(\mathbf{r}, \xi, T)$ are the isotropic and anisotropic contributions due to the medium modification of the Coulomb term, respectively. Similarly $V_2(\mathbf{r}, \xi, T)$ can be decomposed into isotropic and anisotropic parts:

$$V_2(\mathbf{r}, \xi, T) = V_2^{(1)}(r, \xi = 0, T) + V_2^{(2)}(\mathbf{r}, \xi, T), \quad (30)$$

where $V_2^{(1)}(r, \xi = 0, T)$ and $V_2^{(2)}(\mathbf{r}, \xi, T)$ are the isotropic and anisotropic contributions due to the medium modification of the linear term, respectively. Let us now calculate them one by one.

The isotropic part $V_1^{(1)}(r, \xi = 0, T)$ of the Coulomb term [already calculated in (8)] is given by ($\hat{r} = rm_D$)

$$V_1^{(1)}(r, \xi = 0, T) = -\frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} - \alpha m_D, \quad (31)$$

and the anisotropic part, $V_1^{(2)}(\mathbf{r}, \xi, T)$ is given by

$$V_1^{(2)}(\mathbf{r}, \xi, T) = -\xi \frac{\alpha m_D^2}{2\pi^2} \int \frac{d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}}{(k^2 + m_D^2)^2} \left(\frac{2}{3} - \cos^2\beta_n\right). \quad (32)$$

One immediately observes that, unlike the isotropic part $V_1^{(1)}(r, \xi = 0, T)$, momentum anisotropy ($\xi \neq 0$) causes the potential to depend on angle, in addition to interparticle distance (r). Before deriving a general angular dependence we first illustrate the two cases which are especially interesting to grasp the effect of anisotropy on the heavy-quark potential. However, they will be used further to derive the general angular dependence. First, we consider that \mathbf{r} is parallel to the direction of anisotropy, \mathbf{n} , where we have taken the direction of anisotropy \mathbf{n} along the z axis and the angle between the r and \mathbf{k} is θ . So the anisotropic part $V_1^{(2)}(\mathbf{r}, \xi, T)$ for the medium modification to the Coulomb term becomes

$$V_1^{(2)}(\mathbf{r} \parallel \mathbf{n}, \xi, T) = -\xi \frac{\alpha m_D^2}{\pi} \int \frac{d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}}{(k^2 + m_D^2)^2} \left(\frac{2}{3} - \cos^2\theta\right). \quad (33)$$

$$V_1^{(2)}(\mathbf{r} \parallel \mathbf{n}, \xi, T) = \xi V_1^{(1)}(r, \xi = 0, T) \left(2 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{2}{\hat{r}} - \frac{\hat{r}}{6} - 1\right). \quad (34)$$

$$V_1(\mathbf{r} \parallel \mathbf{n}, \xi, T) = V_1^{(1)}(r, \xi = 0, T) + V_1^{(2)}(\mathbf{r} \parallel \mathbf{n}, \xi, T) = -\frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} - \alpha m_D - \xi \frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} \left(2 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{2}{\hat{r}} - \frac{\hat{r}}{6} - 1\right). \quad (35)$$

Next we consider the other scenario, i.e., when \mathbf{r} is transverse to the direction of anisotropy \mathbf{n} , where we take \mathbf{r} along the z axis and \mathbf{n} lying in the $x - y$ plane, so ϕ is the azimuthal angle and ϕ_n is the angle between \mathbf{n} with the x axis. Then $V_1^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T)$ becomes

$$V_1^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T) = -\xi \frac{\alpha m_D^2}{2\pi^2} \int \frac{d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}}{(k^2 + m_D^2)^2} \times \left(\frac{2}{3} - \cos^2(\phi - \phi_n) \sin^2\theta\right). \quad (36)$$

After the angular integration (exploiting the cylindrical symmetry), it is simplified into

$$V_1^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T) = \xi V_1^{(1)}(r, \xi = 0, T) \left(\frac{1 - e^{\hat{r}}}{\hat{r}^2} + \frac{1}{\hat{r}} + \frac{\hat{r}}{3} + \frac{1}{2}\right). \quad (37)$$

Thus, the complete in-medium modification to the Coulomb term [Eqs. (31) and (37)], for the transverse alignment becomes

$$V_1(\mathbf{r} \perp \mathbf{n}, \xi, T) = V_1^{(1)}(r, \xi = 0, T) + V_1^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T) = -\frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} - \alpha m_D + \xi \frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} \left(\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{1}{\hat{r}} - \frac{\hat{r}}{3} - \frac{1}{2}\right). \quad (38)$$

Next we calculate the in-medium modification to the linear term $V_2(\mathbf{r}, \xi, T)$, where the isotropic part [from (8)] is given by

$$V_2^{(1)}(r, \xi = 0, T) = \frac{2\sigma}{m_D \hat{r}} e^{-\hat{r}} - \frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D} \quad (39)$$

and the anisotropic contribution, when \mathbf{r} is parallel to the direction of anisotropy \mathbf{n} , is given by

$$V_2^{(2)}(\mathbf{r} \parallel \mathbf{n}, \xi, T) = -\xi \frac{4\sigma}{(2\pi)^2} \int \frac{d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2(k^2 + m_D^2)^2} \times \left(\frac{2}{3} - \cos^2\beta_n \right). \quad (40)$$

After the angular integration, it becomes

$$V_2^{(2)}(\mathbf{r} \parallel \mathbf{n}, \xi, T) = -\xi \frac{4\sigma}{m_D \hat{r}} e^{-\hat{r}} \left(2 \frac{(1-e^{\hat{r}})}{\hat{r}^2} + \frac{e^{\hat{r}}+2}{3} + \frac{2}{\hat{r}} + \frac{\hat{r}}{12} \right). \quad (41)$$

Thus, the complete in-medium modification to the linear term [Eqs. (39) and (41)], for $\mathbf{r} \parallel \mathbf{n}$ becomes

$$V_2(\mathbf{r} \parallel \mathbf{n}, \xi, T) = V_2^{(1)}(r, \xi = 0, T) + V_2^{(2)}(\mathbf{r} \parallel \mathbf{n}, \xi, T) = -\frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D \hat{r}} e^{-\hat{r}} + \frac{2\sigma}{m_D} + \xi \frac{4\sigma}{m_D \hat{r}} e^{-\hat{r}} \left(2 \frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{e^{\hat{r}}+2}{3} - \frac{2}{\hat{r}} - \frac{\hat{r}}{12} \right). \quad (42)$$

On the other hand, when \mathbf{r} is transverse to the direction of anisotropy, \mathbf{n} , the anisotropic contribution to the linear term becomes

$$V_2^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T) = -\xi \frac{4\sigma}{(2\pi)^2} \int \frac{d^3\mathbf{k} e^{i\mathbf{k}\cdot\mathbf{r}}}{k^2(k^2 + m_D^2)^2} \times \left(\frac{2}{3} - \cos^2(\phi - \phi_n) \sin^2\theta \right), \quad (43)$$

which is simplified into

$$V_2^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T) = -\xi \frac{4\sigma}{m_D \hat{r}} e^{-\hat{r}} \left(\frac{(e^{\hat{r}}-1)}{\hat{r}^2} + \frac{(e^{\hat{r}}-7)}{12} - \frac{1}{\hat{r}} - \frac{\hat{r}}{6} \right), \quad (44)$$

after the angular integration. Thus, the complete in-medium modification to the linear term [Eqs. (39) and (44)] for $\mathbf{r} \perp \mathbf{n}$ becomes

$$V_2(\mathbf{r} \perp \mathbf{n}, \xi, T) = V_2^{(1)}(r, \xi = 0, T) + V_2^{(2)}(\mathbf{r} \perp \mathbf{n}, \xi, T) = -\frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D \hat{r}} e^{-\hat{r}} + \frac{2\sigma}{m_D} - \xi \frac{4\sigma}{m_D^2 \hat{r}} e^{-m_D r} \left(\frac{(e^{\hat{r}}-1)}{\hat{r}^2} + \frac{(e^{\hat{r}}-7)}{12} - \frac{1}{\hat{r}} - \frac{\hat{r}}{6} \right). \quad (45)$$

Therefore, the full medium-modified potential, consisting of Coulomb and linear term [Eqs. (35) and (42), respectively], for the quark pairs aligned to the direction of anisotropy, yields as

$$V(\mathbf{r} \perp \mathbf{n}, \xi, T) = \left(\frac{2\sigma}{m_D} - \alpha m_D \right) \frac{e^{-\hat{r}}}{\hat{r}} - \frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D + \xi \left[\frac{4\sigma}{m_D \hat{r}} e^{-\hat{r}} \left(2 \frac{e^{\hat{r}}-1}{\hat{r}^2} - \frac{e^{\hat{r}}+2}{3} - \frac{2}{\hat{r}} - \frac{\hat{r}}{12} \right) - \frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} \left(2 \frac{(e^{\hat{r}}-1)}{\hat{r}^2} - \frac{2}{\hat{r}} - \frac{\hat{r}}{6} - 1 \right) \right] \equiv V_{\text{iso}}(r, T) + V_{\text{aniso}}^{\parallel}(r, \xi, T). \quad (46)$$

In the short distance limit ($r \ll 1/m_D$), the potential reduces to the vacuum potential (Cornell) for $\xi = 0$, i.e., $Q\bar{Q}$ pairs are not affected by the medium. On the other hand, in the long-distance limit ($r \gg 1/m_D$) (where the screening occurs), we can neglect the Yukawa term and for large values of temperatures, the product αm_D will be much greater than $2\sigma/m_D$. Thus, the potential is simplified into the following form:

$$V(\mathbf{r} \parallel \mathbf{n}, \xi, T) \stackrel{\hat{r} \gg 1}{\simeq} -\frac{2\sigma}{m_D^2 r} - \alpha m_D - \frac{4\xi}{6} \left(\frac{2\sigma}{m_D^2 r} \right) \stackrel{\hat{r} \gg 1}{\equiv} V_{\text{iso}}(\hat{r} \gg 1, T) + V_{\text{aniso}}^{\parallel}(\hat{r} \gg 1, \xi, T), \quad (47)$$

which clearly shows that the potential for $Q\bar{Q}$ pairs aligned in the direction of anisotropy gets screened less, i.e., becomes stronger compared to isotropic medium (9).

On the other hand, when the quark pairs are aligned transverse to the direction of anisotropy ($\mathbf{r} \perp \mathbf{n}$), the medium modification to the Coulombic and linear terms together [Eqs. (38) and (45), respectively] gives rise to the following form:

$$V(\mathbf{r} \perp \mathbf{n}, \xi, T) = \left(\frac{2\sigma}{m_D} - \alpha m_D \right) \frac{e^{-\hat{r}}}{\hat{r}} - \frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D - \xi \left[\frac{4\sigma}{m_D \hat{r}} e^{-\hat{r}} \left(\frac{(e^{\hat{r}}-1)}{\hat{r}^2} + \frac{(e^{\hat{r}}-7)}{12} - \frac{1}{\hat{r}} - \frac{\hat{r}}{6} \right) - \frac{\alpha m_D}{\hat{r}} e^{-\hat{r}} \left(\frac{(e^{\hat{r}}-1)}{\hat{r}^2} - \frac{1}{\hat{r}} - \frac{1}{2} - \frac{\hat{r}}{3} \right) \right] \equiv V_{\text{iso}}(r, T) + V_{\text{aniso}}^{\perp}(r, \xi, T). \quad (48)$$

Similarly, the $Q\bar{Q}$ pair does not see the medium in the short-distance limit whereas in the long-distance limit, the potential is simplified into a Coulombic form with a dynamically screened color charge ($2\sigma/m_D^2$):

$$V(\mathbf{r} \perp \mathbf{n}, \xi, T) \stackrel{\hat{r} \gg 1}{\simeq} -\frac{2\sigma}{m_D^2 r} - \alpha m_D - \frac{\xi}{6} \left(\frac{2\sigma}{m_D^2 r} \right) \stackrel{\hat{r} \gg 1}{\equiv} V_{\text{iso}}(\hat{r} \gg 1, T) + V_{\text{aniso}}^{\perp}(\hat{r} \gg 1, \xi, T), \quad (49)$$

which again shows that the potential for the transverse alignment is still stronger than in isotropic medium but less strong than the former.

Until now we have demonstrated the effects of momentum anisotropy on the heavy-quark interaction for the special cases viz. the interparticle separation (r) may be parallel or perpendicular to the direction of anisotropy (n). We would now like to derive a potential which depends on both the interparticle separation (r) and the angle (θ_n) between \mathbf{r} and n , explicitly.

Let us assume that \mathbf{r} is parallel to the z component of \mathbf{k} and the direction of anisotropy \mathbf{n} lies in the $y-z$ plane (cylindrical symmetry) in the momentum space. We may further assume that, given the weak anisotropy, the potential in the anisotropic medium represents a perturbation to the central potential in the isotropic medium as

$$V(r, \theta_n, T) = V(r, T) + V_{\text{tensor}}(r, \theta_n, T) \quad (50)$$

$$= V_{\text{iso}}(r, T) + \xi F(r, \theta_n, T), \quad (51)$$

where the tensorial part $V_{\text{tensor}}(r, \theta_n, T)$ represents a small perturbation to the central one $V(r, T)$ and ξ is the

strength of noncentral component of the potential. The function $F(r, \theta_n, T)$ can be expanded as

$$F(r, \theta_n, T) = f_0(r, T) + f_1(r, T) \cos 2\theta_n. \quad (52)$$

The potentials for the angles $\theta_n = 0$ and $\theta_n = \pi/2$ help us to determine the functions $f_0(r, T)$ and $f_1(r, T)$ in terms¹ of $\hat{r}(=rm_D)$ as

$$f_0(\hat{r}, T) = \frac{2\sigma}{m_D} \frac{e^{-\hat{r}}}{\hat{r}} \left(\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{5e^{\hat{r}}}{12} - \frac{1}{\hat{r}} + \frac{\hat{r}}{12} - \frac{1}{12} \right) - \frac{\alpha m_D}{2} \frac{e^{-\hat{r}}}{\hat{r}} \left(\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{1}{\hat{r}} + \frac{\hat{r}}{6} - \frac{1}{2} \right) \quad (53)$$

and

$$f_1(\hat{r}, T) = \frac{2\sigma}{m_D} \frac{e^{-\hat{r}}}{\hat{r}} \left(3 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{e^{\hat{r}}}{4} - \frac{3}{\hat{r}} - \frac{\hat{r}}{4} - \frac{5}{4} \right) - \frac{\alpha m_D}{2} \frac{e^{-\hat{r}}}{\hat{r}} \left(3 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{3}{\hat{r}} - \frac{\hat{r}}{2} - \frac{3}{2} \right). \quad (54)$$

So after substituting f_0 and f_1 into Eq. (51), we obtain the complete angular dependence of the potential in the limit of weak anisotropy ($\xi \ll 1$):

$$\begin{aligned} V(r, \theta_n, T) &= \left(\frac{2\sigma}{m_D} - \alpha m_D \right) \frac{e^{-\hat{r}}}{\hat{r}} - \frac{2\sigma}{m_D \hat{r}} + \frac{2\sigma}{m_D} - \alpha m_D \\ &+ \xi \left(\frac{2\sigma}{m_D} \frac{e^{-\hat{r}}}{\hat{r}} \left[\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{5e^{\hat{r}}}{12} - \frac{1}{\hat{r}} + \frac{\hat{r}}{12} - \frac{1}{12} \right] - \frac{\alpha m_D}{2} \frac{e^{-\hat{r}}}{\hat{r}} \left[\frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{1}{\hat{r}} + \frac{\hat{r}}{6} - \frac{1}{2} \right] \right. \\ &+ \left. \left(\frac{2\sigma}{m_D} \frac{e^{-\hat{r}}}{\hat{r}} \left[3 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{e^{\hat{r}}}{4} - \frac{3}{\hat{r}} - \frac{\hat{r}}{4} - \frac{5}{4} \right] - \frac{\alpha m_D}{2} \frac{e^{-\hat{r}}}{\hat{r}} \left[3 \frac{e^{\hat{r}} - 1}{\hat{r}^2} - \frac{3}{\hat{r}} - \frac{\hat{r}}{2} - \frac{3}{2} \right] \right) \cos 2\theta_n \right) \\ &= V(r, T) + V_{\text{tensor}}(r, \theta_n, T). \end{aligned} \quad (55)$$

Thus, the anisotropy in the momentum space introduces an angular (θ_n) dependence, in addition to the interparticle separation (r), to the potential in the coordinate space which was earlier only r dependent in the isotropic medium. We can now identify the ξ -independent term with $V(r, T)$ in (50) which depends only on the separation (r) distance and the ξ -dependent term with the tensorial component $V_{\text{tensor}}(r, \theta_n, T)$ in (50) which depends on both r and θ_n . Thus, the full potential in an anisotropic medium, $V(r, \theta_n, T)$, needs to be solved by the three-dimensional Schrödinger equation. Since we are restricted in the small ξ limit, the tensorial component $V_{\text{tensor}}(r, \theta_n, T)$ is much smaller than the (isotropic) central component $V(r, T)$ and hence may be treated as the perturbation by a first-order perturbation theory in quantum mechanics. However, the isotropic component may be solved numerically by the one-dimensional (radial) Schrödinger equation.

¹The variable \hat{r} should not be confused with the usual notation of unit vector in the coordinate system.

The heavy quark interaction at short and intermediate distances ($rm_D \leq 1$) is important for the understanding of in-medium modification of heavy quark bound states and the large distance property ($rm_D > 1$) helps to understand the bulk properties of the QGP phase which also affects the in-medium properties of the quarkonium states. So we wish to see how the potential in anisotropic medium behaves in these (short, intermediate, and long) limiting cases. In the short-distance limit, the vacuum contribution dominates over the medium contribution and this is exactly what happens here:

$$V(r, \theta_n, T) \stackrel{\hat{r} \ll 1}{\approx} \sigma r - \frac{\alpha}{r} \quad (56)$$

for $\xi = 0$. On the other hand, in the long-distance limit ($\hat{r} \gg 1$), the potential is reduced to a long-range Coulombic interaction after identifying the factor $2\sigma/m_D^2$ with the coupling (g_s^2) of the interaction

$$\begin{aligned} V(r, \theta_n, T) &\stackrel{\hat{r} \gg 1}{\approx} -\frac{2\sigma}{m_D^2 r} - \alpha m_D - \frac{5\xi}{12} \frac{2\sigma}{m_D^2 r} \left(1 + \frac{3}{5} \cos 2\theta_n \right) \\ &\equiv V_{\text{iso}}(\hat{r} \gg 1, T) + V_{\text{tensor}}(\hat{r} \gg 1, \theta_n, T). \end{aligned} \quad (57)$$

Since the resulting potential is Coulombic plus a subleading anisotropic contribution, it then has to satisfy the condition $a_0 m_D \gg 1$, where a_0 is the Bohr radius and m_D is the Debye mass. Since the Bohr radius a_0 is proportional to $m_D^2/(m_Q \sigma)$, the above condition for the long-distance limit implies that $m_D^3/(m_Q \sigma)$ should be greater than 1. Thus, this inequality results in a condition on the Debye mass and hence on the temperature. It is seen that the above condition is satisfied for the temperatures above the critical temperature ($>T_c$) for the charmonium states and above $1.6T_c$ for the bottomonium states. The temperature ranges (T_c and $1.6T_c$) for $c\bar{c}$ and $b\bar{b}$ states above which the effective potential looks Coulombic are smaller than their respective dissociation temperatures and thus seems justified to approximate the potential in the long-distance limit. In the intermediate distance ($rm_D \simeq 1$) scale, the interaction becomes complicated and thus the potential does not look simpler in contrast to the asymptotic limits, so this limit needs to be dealt numerically with the full potential in a Schrödinger equation.

We have thus noticed overall that, in the short distance limit, the potential has not been affected in the isotropic limit. On the contrary, in the long-distance limit, the momentum anisotropy transpires an angular dependence in the potential and gives rise to a characteristic angular (θ_n) dependence between the relative separation (\mathbf{r}) and the direction of anisotropy (\mathbf{n}). As a corollary, the quark pairs aligned along the direction of anisotropy feel more attraction than the transverse alignment because the interquark potential along the direction of anisotropy is screened less than the transverse alignment. However, the potential in the anisotropic medium is always stronger than in the isotropic medium.

To see the effects of anisotropy, we have shown the potentials for $Q\bar{Q}$ pairs in an anisotropic medium in Figs. 1 and 2, for $\theta_n = 0$ (parallel) and $\theta_n = \pi/2$ (perpendicular), respectively. The immediate observation common to all figures is that the interquark potential in anisotropic

medium is always more attractive than in isotropic medium. This can be understood physically: In the small anisotropic limit, the anisotropic distribution function may be obtained from an isotropic distribution $f_{\text{iso}}(|\mathbf{k}|)$ by removing particles with a large momentum component along \mathbf{n} , i.e., $f_{\text{iso}}(\sqrt{\mathbf{k}^2 + \xi(\mathbf{k} \cdot \mathbf{n})^2})$. This transpires in the reduction of the number of partons (around a static test heavy quark) more than in isotropic medium, i.e., $n_{\text{aniso}}(\xi) = n_{\text{iso}}/\sqrt{1 + \xi}$. Therefore, the (effective) Debye mass always becomes smaller and results in less screening of the potential than in isotropic medium.

The second observation is that the quark pairs aligned along ($\theta_n = 0$) the direction of anisotropy are stronger than aligned perpendicular ($\theta_n = \pi/2$) to the direction of anisotropy because, for the parallel alignment, the component of momentum to be removed is higher than the transverse alignment so the distribution function for the parallel alignment case contributing to the Debye mass is smaller than the transverse alignment. Hence, the potential for the parallel case will be screened less compared to the transverse case. However, the difference between the two scenarios will be not much different because the contributions to the Debye mass from the partons having higher momenta are very small.

To understand the effect of the linear term on the medium modified potential quantitatively, in addition to the Coulomb term, we have plotted separately the medium modifications to the linear term, the Coulomb term, and their sum in the right panels of Figs. 1 and 2, for parallel and transverse case, respectively. Medium modification to the Cornell potential contains two parts: one is due to the medium modifications of the linear term (σr) and the other one is due to the medium modifications of the Coulomb term. As usually done in the literature, medium modification to the linear term does not arise because the string tension was assumed to be zero [42,60–62] at or beyond deconfinement temperature [63]. Since string tension is

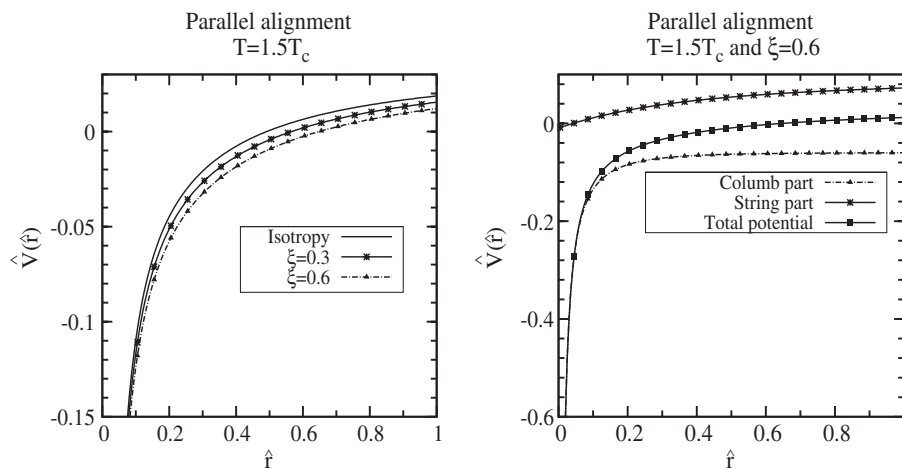


FIG. 1. The left panel represents the potential divided by $(g^2 C_F m_D)$ and the right panel represents the contribution of Coulomb, string, and both together as a function of $\hat{r}(= rm_D)$ for quark pairs parallel to the direction of anisotropy, \mathbf{n} .

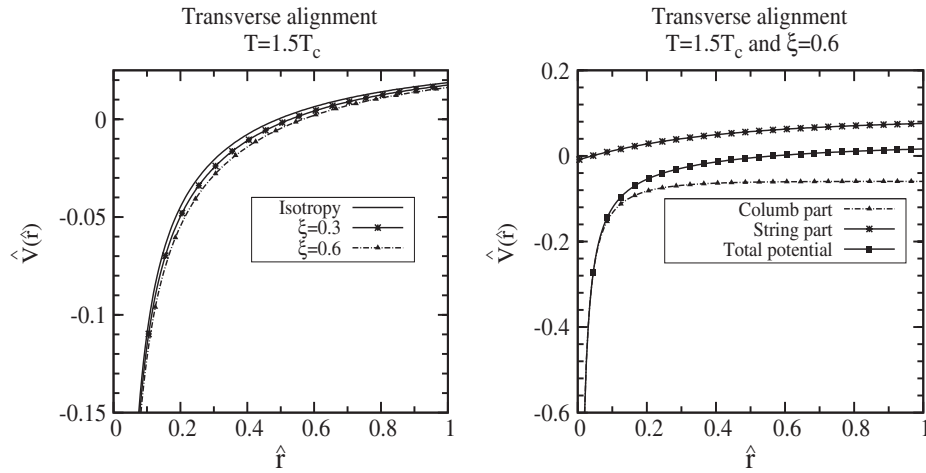


FIG. 2. The notations are the same as in Fig. 1 but for quark pairs perpendicular to the direction of anisotropy, \mathbf{n} .

found to be nonzero at T_c rather it approaches zero much beyond T_c [32–34] and hence the medium modification to the linear term may be a nonzero contribution to the potential even at temperatures beyond T_c , although it is very small. In the isotropic medium, medium modification to the linear term remains positive up to 2–3 T_c , making the potential less attractive compared to $T = 0$. On contrast, in the anisotropic case medium modification to the linear term becomes negative and the overall full potential becomes more attractive.

As mentioned earlier we used the same screening scale for both the linear and Coulombic terms in our calculation which does not look plausible. It would thus be interesting to see the effects of different scales for the Coulomb and linear pieces of the $T = 0$ potential [49,50]. To illustrate it graphically, we have compared our results with their results (in Fig. 3) for the isotropic case. The difference in

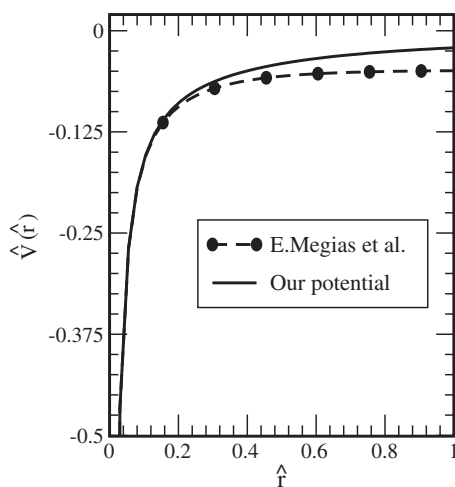


FIG. 3. The dotted circle represents the results from Megias *et al.* [50] where different scales were used for linear and Coulomb terms separately whereas the solid line represents our work.

the large distance limit arises due to the difference in the potential at infinity ($-\sigma/m_D$) so the potential in Ref. [50] is more attractive than our potential.

III. PROPERTIES OF QUARKONIUM IN AN ANISOTROPIC MEDIUM

A. Binding energy

To understand the in-medium properties of the quarkonium states, we need to model the heavy quark potential as a function of temperature and solve the resulting Schrödinger equation. The potential thus obtained in anisotropic medium (55), in contrast to the (spherically symmetric) potential in isotropic medium, is nonspherical and so one cannot simply obtain the energy eigenvalues by solving the radial part of the Schrödinger equation only because the radial part is no longer sufficient due to the angular dependence in the potential. Another way to understand is that, because of the anisotropic screening scale, the wave functions are no longer radially symmetric for $\xi \neq 0$. So one has to solve the potential in anisotropic medium through the Schrödinger equation in three dimensions. However, we have seen in the potential (55) that in the small ξ limit, the spherically nonsymmetric component $V_{\text{tensor}}(r, \theta_n, T)$ is much smaller in comparison to spherically symmetric (isotropic) component $V(r, T)$ and thus can be treated as perturbation. This can be understood physically: The tensorial (nonsphericity) nature of the potential in the coordinate space is arisen due to anisotropy in the momentum space. However, we are restricted to a plasma which is very much close to equilibrium because, by the time quarkonium states are formed in the plasma around $(1-2)T_c$, the plasma becomes almost isotropized. Thus, this weak (momentum) anisotropy ($\xi \ll 1$) transpires feeble angular dependence in the potential so the potential will be spherically abundant with a tiny nonspherical component. So we could treat the anisotropic component through the perturbation theory in quantum

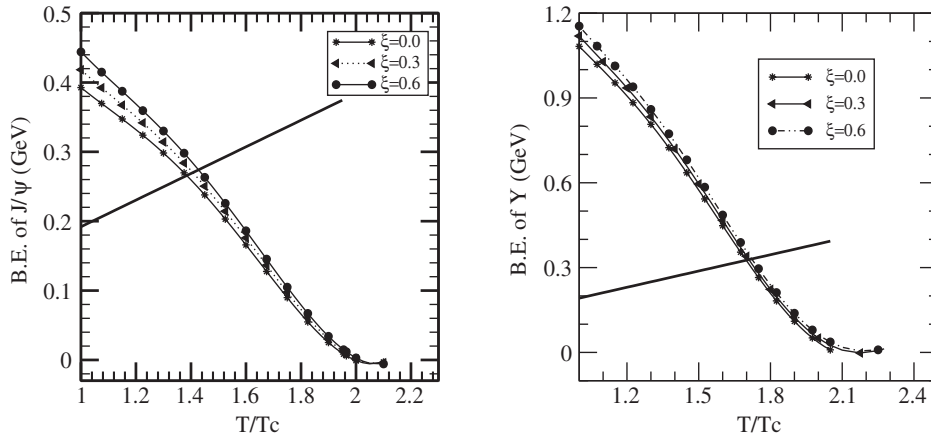


FIG. 4. Variation of J/ψ and Y binding energy (in GeV) with the temperature in an anisotropic hot QCD medium.

mechanics and the isotropic part should be handled numerically by the one-dimensional radial Schrödinger equation.

There are some numerical methods to solve the Schrödinger equation either in partial differential form (time-dependent) or eigenvalue form (time-independent/stationary) by the finite difference time domain method or the matrix method [64], respectively. However, we choose the matrix method to solve the stationary Schrödinger equation with the isotropic part of the potential (55) in anisotropic medium. In this method, the Schrödinger equation can be cast in a matrix form through a discrete basis, instead of the continuous real-space position basis spanned by the states $|\vec{x}\rangle$. Here the confining potential V is subdivided into N discrete wells with potentials V_1, V_2, \dots, V_{N+2} such that for i th boundary potential, $V = V_i$ for $x_{i-1} < x < x_i$; $i = 2, 3, \dots, (N + 1)$. Therefore for the existence of a bound state, there must be an exponentially decaying wave function in the region $x > x_{N+1}$ as $x \rightarrow \infty$ and has the form

$$\Psi_{N+2}(x) = P_E \exp[-\gamma_{N+2}(x - x_{N+1})] + Q_E \exp[\gamma_{N+2}(x - x_{N+1})], \quad (58)$$

where, $P_E = \frac{1}{2}(A_{N+2} - B_{N+2})$, $Q_E = \frac{1}{2}(A_{N+2} + B_{N+2})$, and, $\gamma_{N+2} = \sqrt{2\mu(V_{N+2} - E)}$. The eigenvalues can be obtained by identifying the zeros of Q_E .

Therefore, the corrected energy eigenvalue comes from the solution of the Schrödinger equation of the isotropic component $V_{\text{iso}}(r, T)$, using the above-mentioned matrix method plus the first-order perturbation due to the anisotropic component $V_{\text{aniso}}(r, \theta; \xi, T)$ (55) through the quantum mechanical perturbation theory. The variations of the binding energies with the temperature are shown in Fig. 4 for J/ψ and Y for different values of anisotropy parameter ξ , to see the effect of anisotropy on the binding energies compared to the isotropic case.

There are mainly two observations: First, as the anisotropy increases, the binding of $Q\bar{Q}$ pairs get stronger with

respect to their isotropic counterpart because the potential becomes deeper with the increase of anisotropy due to weaker screening. It seems that the (effective) Debye mass $m_D(\xi, T)$ in an anisotropic medium is always smaller than in an isotropic medium. As a result the screening of the Coulomb and string contribution are less accentuated and hence the quarkonium states become more stronger than in an isotropic medium. However, the effects of anisotropy on the excited states are not so pronounced compared to the ground states because they are generically weakly bound. Secondly, there is a strong decreasing trend with the temperature. This is due to the fact that the screening becomes always stronger with the increase of temperature, so the potential becomes weaker compared to $T = 0$ and results in early dissolution of quarkonia in the medium. Our results on the temperature dependence of the binding energies show an agreement with the similar variations in other calculations [55].

In our calculation, we use the Debye mass ($m_D^L = 1.4m_D^{LO}$) obtained by fitting the (color-singlet) free energy in lattice QCD [38] where both one- and two-loop expressions [61,65,66] for coupling have been used to explore the effects of running coupling on the dissociation process.

Thus, the study of the temperature dependence of the binding energies is poised to provide a wealth of information about the dissociation pattern of quarkonium states in an anisotropic thermal medium that can be used to determine the dissociation temperatures of different states in the next section.

B. Dissociation temperatures for heavy quarkonia

Dissociation of a two-body bound state in an thermal medium can be understood qualitatively: When the binding energy of a resonance state drops below the mean thermal energy of a parton, the state becomes feebly bound. The thermal fluctuation then can easily dissociate by exciting them into the continuum. The spectral function technique in potential models defines the dissociation temperature as the temperature above which the quarkonium spectral

TABLE I. Dissociation temperatures (T_D) for the quarkonium states with one-loop QCD coupling.

State	$\xi = 0.0$	$\xi = 0.3$	$\xi = 0.6$
J/ψ	1.38	1.41	1.43
Υ	1.70	1.71	1.72

function shows no resonancelike structures but the widths shown in spectral functions from current potential model calculations are not physical. The broadening of states with the increase in temperature is not included in any of these models. The authors has argued in Ref. [38] that one need not to reach the binding energy (E_{bin}) to be zero for the dissociation rather a weaker condition $E_{\text{bin}} < T$ causes a state weakly bound. In fact, when $E_{\text{bin}} \simeq T$, the resonances have been broadened due to direct thermal activation, so the dissociation of the bound states may be expected to occur roughly around $E_{\text{bin}} \simeq T$.

Using the binding energies calculated earlier in Sec. III A, the dissociation temperatures (T_D) (shown in Table I) are found minimum for the isotropic case and increase with the increase of anisotropy ($\xi > 0$) viz. J/ψ is dissociated at $1.38T_c$ in an isotropic medium while in an anisotropic medium with the anisotropies $\xi = 0.3$ and 0.6 , they will survive higher temperatures, $1.41T_c$ and $1.43T_c$, respectively. Similarly the dissociation temperatures of Υ for $\xi = 0.3$ and 0.6 are $1.71T_c$ and $1.72T_c$, respectively, corresponding to the value ($1.70T_c$) in an isotropic medium.

Finally, we wish to explore the effects of perturbative as well as nonperturbative contributions on the dissociation of quarkonia states qualitatively in terms of the Debye mass. Instead of lattice Debye mass (m_D^L), if we use the leading-order Debye mass ($m_D^{LO} < m_D^L$), the screening of the potential will be much smaller and hence the binding energies ($1/m_D^4$) will be enhanced substantially and result in the increase of the dissociation temperatures. On the other hand, if we include the nonperturbative corrections of order $O(g^2T)$ and $O(g^3T)$ to the leading-order Debye mass [67], the dissociation temperatures become unrealistically small which looks unfeasible. Thus, this study provides us a handle to decipher the extent up to which and how much nonperturbative effects should be incorporated into the Debye mass.

IV. CONCLUSIONS AND OUTLOOK

In conclusion, we have studied the dissociation of quarkonia by correcting the full Cornell potential with a dielectric function embodying the effects of an weakly anisotropic medium where the in-medium modification causes less screening of the interaction and hence the potential gets stronger than in an isotropic medium. Anisotropy further introduces a characteristic angular (θ) dependence to the potential in the coordinate space, in addition to the interparticle separation r , making it spherically

nonsymmetric and needs to be solved numerically by the three-dimensional Schrödinger equation. However, in the small ξ limit, the spherically nonsymmetric component is much smaller in comparison to the spherically symmetric component and can be treated in a perturbation theory and the symmetric (isotropic) component is solved numerically by the one-dimensional radial Schrödinger equation. So the corrected binding energy is obtained by the direction-independent shift due to the spherically nonsymmetric component to the eigenvalues of spherically symmetric potential.

We have observed that the quarkonia states are always more bound and as a consequence, they survive higher temperature compared to the isotropic medium. Our results are found relatively higher compared to similar calculation [53], which may be due to the absence of three-dimensional medium modification of the linear term in their calculation. In fact, the one-dimensional Fourier transform of the Cornell potential yields the similar form used in the lattice QCD in which one-dimensional color flux tube structure was assumed [68]. However, at finite temperature that may not be the case since the flux tube structure may expand in more dimensions [42]. Therefore, it would be better to consider the three-dimensional form of the medium modified Cornell potential which has been done exactly in the present work.

In brief, J/ψ is found to be dissociated at $1.38T_c$ and $1.43T_c$ for $\xi = 0$ and 0.6 , respectively, whereas the corresponding temperatures for the Υ state are $1.70T_c$ and $1.72T_c$. Moreover, we explore the effects of perturbative as well as nonperturbative effects on the dissociation process qualitatively. For example, the perturbative result of Debye mass gives much higher values of dissociation temperatures whereas the inclusion of nonperturbative corrections to it gives unrealistically smaller values. It may be important to note that in the weakly coupled regime, the effects of (nonperturbative) terms viz. g^2T , g^3T etc. may be checked separately but in the strong-coupling regime, this may not be possible because they are no longer uncoupled. These findings envisage a basic question about the nature of dissociation of quarkonium in an anisotropic hot QCD medium.

Apart from the uncertainty of the correct form of the potential, there is an arbitrariness in the definition of dissociation temperature. So, in the future, we wish to investigate the dissociation through the decay width of quarkonium bound states calculated from the imaginary part of the potential because it is now well understood that potential in thermal medium always has an imaginary component [51,69].

ACKNOWLEDGMENTS

We are thankful for some financial assistance from CSIR Project No. CSR-656-PHY, Government of India. We also thank M. G. Mustafa for some suggestions.

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