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Multiple scattering effects on inclusive particle production in the large-x regime

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We study the multiple scattering effects on inclusive particle production in p + A and γ + A collisions. Specifically, we concentrate on the region where the parton momentum fraction in the nucleus $x \sim \mathcal{O}(1)$ and incoherent multiple interactions are relevant. By taking into account both initial-state and final-state double scattering, we derive the nuclear size-enhanced power corrections to the differential cross section for single inclusive hadron production in p + A and γ + A reactions, and for prompt photon production in p + A reactions. We find that the final result can be written in a simple compact form in terms of four-parton correlation functions, in which the second-derivative, first-derivative and nonderivative terms of the correlation distributions share the same hard-scattering functions. We expect our result to be especially relevant for understanding the nuclear modification of particle production in the backward rapidity regions in p + A and e + A collisions.

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I. INTRODUCTION

Medium-induced modification of moderate and high transverse momentum particle production in both proton-nucleus (p + A) and nucleus-nucleus (A + A) collisions relative to the naive binary collision-scaled proton-proton (p + p) baseline expectation is an excellent tool to diagnose the properties of dense QCD matter [1,2]. Multiple parton scattering has played an important role in understanding novel effects that contribute to the observed nuclear dependence, such as dynamical shadowing, Cronin effect, parton energy loss and jet quenching [3]. To better extract the QCD matter properties from experimental measurements, it is critical to elucidate the differences between the elastic, inelastic, coherent, and incoherent scattering regimes.

Different theoretical approaches are possible in studying the same physics effect in high energy nuclear collisions. An illustrative example is the calculation of medium-induced parton splitting and radiative energy loss that leads to the "jet quenching" phenomena in A + A reactions [4–10]. In p + A collisions, most attention has been devoted to the nontrivial QCD dynamics in the small-x regime and the existence of very dense gluonic systems. In this regime, the probe can cover several nucleons inside the nucleus and interact with all of them *coherently*. Two approaches on the market treat this coherent kinematic limit. One of them is the color glass condensate approach [11–15], which focuses on the nonlinear corrections to QCD evolution equations. It is only applicable at very small x and for transverse momenta $\lesssim Q_s$, the saturation scale, where all the multiple scatterings are equally important. The other approach is the high-twist expansion approach, which treats the multiple scatterings as power-suppressed corrections to the cross sections. It follows a generalized QCD factorization formalism [16–19] and computes the corrections order by order in a power series, where any additional correlated scattering is suppressed by an extra power of the momentum transfer. Within this approach, in the forward rapidity limit, i.e. the proton direction, all nuclear size-enhanced power corrections [20,21] to the differential cross sections for both single hadron and dihadron production in p + A collisions have been resummed. When combined with cold nuclear matter energy loss [22], successful description of the single hadron suppression and dihadron correlation in the forward rapidity region has been demonstrated [23].

In this paper, we will focus on a different regime, the region where the parton momentum fraction in the nucleus $x \sim \mathcal{O}(1)$ (outside small x). Incoherent multiple interactions, relevant to the Cronin effect, have been resummed before for uniform nuclear matter described by mean squared momentum transfer and parton scattering length [24]. Here, we follow the same generalized factorization theorem, discussed above, and attribute the first nontrivial multiple scattering (double scattering) contributions to the twist-4 power-suppressed corrections to the differential cross section. We demonstrate explicitly that in the $x \sim \mathcal{O}(1)$ regime only the incoherent multiple interactions are relevant. We take into account both initial-state and final-state double scattering effects and find that the final result can be written in terms of the well-defined twist-4 four-parton correlation functions. It depends on a universal combination of second-derivative, first-derivative and nonderivative terms of these correlation functions that shares the same hard-scattering function.

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The rest of our paper is organized as follows. In Sec. II, we introduce our notation and study the double scattering contribution to the single inclusive hadron production in p + A collisions. Take one particular partonic subprocess $qq' \to qq'$ as an example to explain in detail how we derive the result and what are the approximations we have used. In Sec. III, we extend our method to the physical processes involving a photon. In particular, we study the double scattering contribution to the prompt photon production in p + A collisions, and to single hadron production in γ + A collisions. Both results depend on our findings from Sec. II. We summarize our paper in Sec. IV. We defer the phenomenological study, based on our result, to future publications.

II. MULTIPLE SCATTERING EFFECTS FOR SINGLE INCLUSIVE HADRON PRODUCTION IN p + A COLLISIONS

A. Single scattering contribution

In this section we study single inclusive hadron production in p + A collisions,

$$p(P') + A(P) \to h(P_h) + X, \tag{1}$$

where P' is the momentum for the incoming proton, and P is defined as the average momentum per nucleon in the nucleus. In general, the differential cross section for the above process can be expressed as a sum of contributions from single scattering, double scattering, and even higher multiple scattering [25]:

$$d\sigma_{pA\to hX} = d\sigma^{(S)}_{pA\to hX} + d\sigma^{(D)}_{pA\to hX} + \cdots, \tag{2}$$

where the superscript "(S)" indicates the single scattering contribution, and "(D)" represents the double scattering contribution. As illustrated in Fig. 1, in the single scattering contribution one parton a from the proton interacts with one single parton b inside the nucleus to produce a parton c, which will then fragment into the final observed hadron h. On the other hand, in the double scattering contribution the same parton a from the proton will interact with two partons b, b' from the nucleus simultaneously to produce the final parton c.

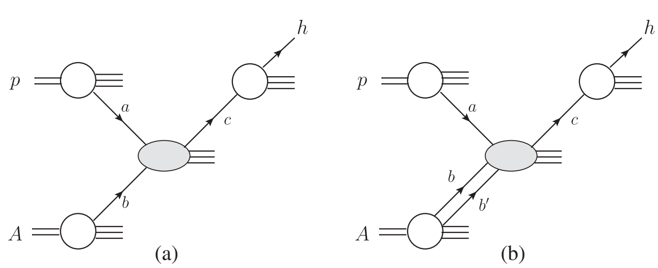

FIG. 1. Generic diagrams for (a) single scattering amplitude, where the parton a from the proton interacts with one single parton b inside the nucleus. (b) Double scattering amplitude, where the parton a from the proton interacts with two partons b, b' from the nucleus simultaneously. Eventually, the hard scattering produces a parton c, which then fragments into the final observed hadron h.

The single scattering contribution follows the usual leading-twist perturbative QCD factorization [26], and the differential cross section per nucleon at leading order in the strong coupling α_s is given by

$$E_h\frac{d\sigma^{(S)}}{d^3P_h} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}f_{a/p}(x')$$
$$\times\int\frac{dx}{x}f_{b/A}(x)H^U_{ab\to cd}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \tag{3}$$

where $\sum_{a,b,c}$ runs over all parton flavors, $S = (P' + P)^2$, $f_{a/p}(x')$ and $f_{b/A}(x)$ are the parton distribution functions inside the proton and the nucleus, respectively, and $D_{c\to h}(z)$ is the fragmentation function for parton c transforming into hadron h. The scale dependencies are suppressed for brevity. The hard-scattering functions $H^U_{ab\to cd}(\hat{s},\hat{t},\hat{u})$ are the squared averaged matrix elements for the subprocess $a(p_a) + b(p_b) \to c(p_c) + d(p_d)$ with $p_a = x'P'$, $p_b = xP$, $p_c = P_h/z$ and the usual partonic Mandelstam variables:

$$\hat{s} = (x'P' + xP)^2, \quad \hat{t} = (x'P' - p_c)^2, \quad \hat{u} = (xP - p_c)^2. \tag{4}$$

These hard-scattering functions $H^U_{ab\to cd}(\hat{s},\hat{t},\hat{u})$ are well known [23,27,28] and we reproduce them here for later convenience:

$$H^U_{qq'\to qq'} = \frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right], \tag{5}$$

$$H^U_{qq\to qq} = \frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right] - \frac{N_c^2-1}{N_c^3}\left[\frac{\hat{s}^2}{\hat{t}\hat{u}}\right], \tag{6}$$

$$H^U_{q\bar{q}\to q'\bar{q}'} = \frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right], \tag{7}$$

$$H^U_{q\bar{q}\to q\bar{q}} = \frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}+\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right] - \frac{N_c^2-1}{N_c^3}\left[\frac{\hat{u}^2}{\hat{s}\hat{t}}\right], \tag{8}$$

$$H^U_{qg\to qg} = H^U_{gq\to gq} = -\frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{s}}{\hat{u}}+\frac{\hat{u}}{\hat{s}}\right] + \left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right], \tag{9}$$

$$H^U_{gq\to qg} = H^U_{qg\to gq} = -\frac{N_c^2-1}{2N_c^2}\left[\frac{\hat{s}}{\hat{t}}+\frac{\hat{t}}{\hat{s}}\right] + \left[\frac{\hat{s}^2+\hat{t}^2}{\hat{s}^2}\right], \tag{10}$$

$$H^U_{q\bar{q}\to gg} = \frac{(N_c^2-1)^2}{2N_c^3}\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right] - \frac{N_c^2-1}{N_c}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right], \tag{11}$$

$$H^U_{gg\to q\bar{q}} = \frac{1}{2N_c}\left[\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right] - \frac{N_c}{N_c^2-1}\left[\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right], \quad (12)$$

$$H^U_{gg\to gg} = \frac{4N_c^2}{N_c^2-1}\left[3 - \frac{\hat{t}\,\hat{u}}{\hat{s}^2} - \frac{\hat{s}\,\hat{u}}{\hat{t}^2} - \frac{\hat{s}\,\hat{t}}{\hat{u}^2}\right]. \quad (13)$$

B. Double scattering contribution: $qq' \to qq'$ as an example

Let us now concentrate on the double scattering contribution to the differential cross section. Since there are many partonic channels which contribute to the single inclusive hadron production, including $qq' \to qq'$, $qq \to qq$, $q\bar{q} \to q'\bar{q}'$, $q\bar{q} \to q\bar{q}$, $qg \to qg$, $gq \to gq$, $gq \to qg$, $qg \to gq$, $q\bar{q} \to gg$, $gg \to q\bar{q}$, and $gg \to gg$, we will take a simple partonic channel $qq' \to qq'$ as an example to demonstrate our method. The derivation for all other channels is similar.

The double scattering diagrams can have both initial-state contributions, as shown in Fig. 2, and final-state contributions, as shown in Fig. 3. The chosen process $qq' \to qq'$ is rather simple as only one Feynman diagram (the t-channel diagram which has one gluon exchange between the different quark flavors q and q') is relevant for the single scattering contribution. The Feynman diagrams shown in Figs. 2 and 3 are the complete set for double scattering contributions. However, other partonic processes typically involve many more diagrams, which are all taken into account carefully in our calculation. Let us start with initial-state double scattering, for which there are three Feynman diagrams: in the first diagram Fig. 2(L), both gluons are on the left side of the cut line; in the second diagram Fig. 2(R), both gluons are on the right side of the cut line; in the third diagram Fig. 2(M), the cut line is in the middle of the two gluons. Physically, Fig. 2(M) is the real diagram representing the classical double scattering picture, while both Figs. 2(L) and 2(R) are the interference diagrams.

These double scatterings manifest themselves as twist-4 contributions to the differential cross section. To derive these contributions, a generalized factorization theorem, the so-called high-twist power expansion approach, was developed in [16–19] some time ago. Since then, this approach has been used to study cold nuclear matter effects in either e + A or p + A collisions, such as energy loss, broadening effects, and dynamical nuclear shadowing. For examples beyond the ones mentioned in the Introduction, see [29–33].

Following this generalized factorization theorem, the double scattering contribution in Fig. 2 can be expressed in terms of a twist-4 four-parton correlation function as follows:

$$E_h\frac{d\sigma^{(D)}}{d^3P_h} \propto \int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}f_{a/p}(x') \times\int dx_1dx_2dx_3T(x_1,x_2,x_3)\left(-\frac{1}{2}g^{\rho\sigma}\right) \times\left[\frac{1}{2}\frac{\partial^2}{\partial k_\perp^\rho\partial k_\perp^\sigma}H(x_1,x_2,x_3,k_\perp)\right]_{k_\perp\to 0}, \quad (14)$$

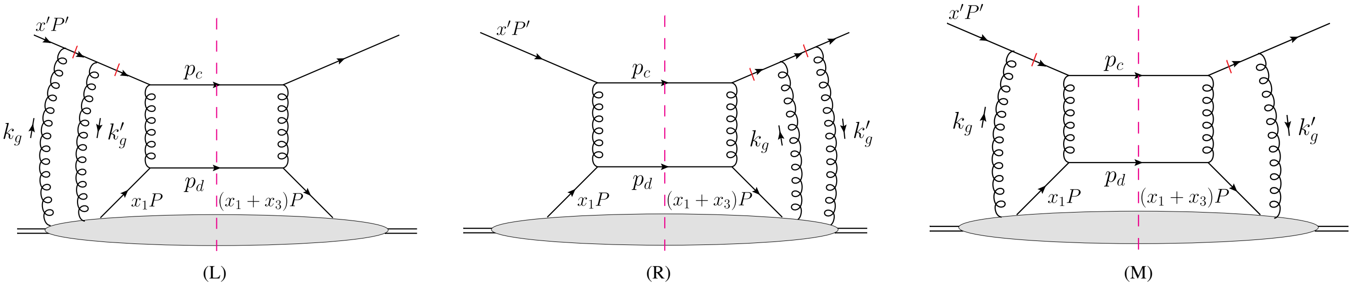

FIG. 2 (color online). Initial-state double scattering contributions to the partonic subprocess $qq' \to qq'$. Here gluon momenta $k_g = x_2P + k_\perp$ and $k'_g = (x_2 - x_3)P + k_\perp$.

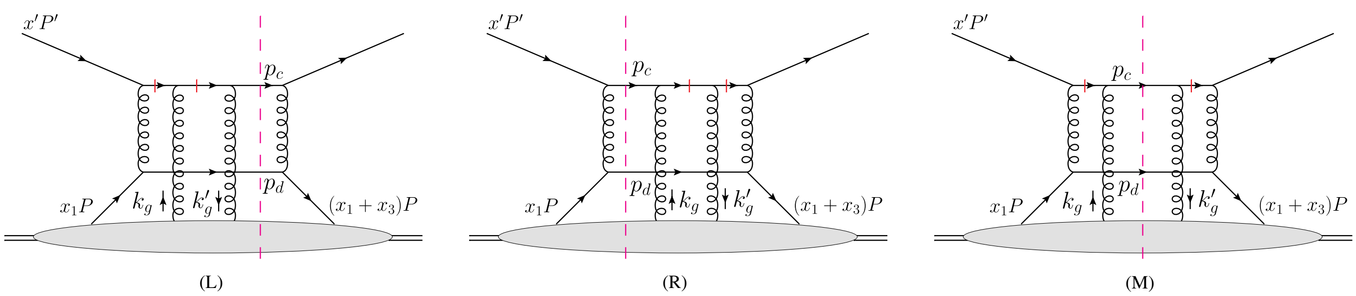

FIG. 3 (color online). Final-state double scattering contributions to the partonic subprocess $qq' \to qq'$. Here, the gluon momenta $k_g = x_2P + k_\perp$ and $k'_g = (x_2 - x_3)P + k_\perp$.

where $k_\perp$ is a small transverse momentum kick due to the multiple scattering and $T(x_1, x_2, x_3)$ is a twist-4 two-quark-two-gluon correlation function defined by

$$T(x_1, x_2, x_3) = \int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}\frac{dy_2^-}{2\pi} e^{ix_1P^+y^-} e^{ix_2P^+(y_1^- - y_2^-)} e^{ix_3P^+y_2^-} \times \frac{1}{2}\langle P|F_\alpha{}^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|P\rangle. \quad (15)$$

$H(x_1, x_2, x_3, k_\perp)$ are the corresponding partonic hard-scattering functions, and the x_1, x_2, x_3 are the independent collinear momentum fractions carried by the partons from the nucleus, as shown in Fig. 2.

Here, the expansion around $k_\perp = 0$ to second order will extract the twist-4 contributions. This so-called collinear expansion is usually rather complicated and/or tedious in practice. In this paper, we will use a slightly improved technique for performing the collinear expansion and, thus, will be able to use it for single inclusive hadron production, which contains many partonic subprocesses. Such an improved technique was first developed for twist-3 expansion in studying transverse spin-dependent observables [34,35]. It involves first integrating out the parton momentum fractions x_1, x_2, and x_3 with the help of either a kinematic δ function or contour integrals, then performing the $k_\perp$ expansion directly. Though a small improvement, it enables us to perform the $k_\perp$ expansion with the help of the Mathematica package, instead of doing it by hand, as in the past.

We will now explain in detail how this works for our chosen example process, $qq' \to qq'$. We start with the first diagram Fig. 2(L). In this diagram, we have an on-shell condition for the unobserved quark d:

$$\delta(p_d^2) = \delta[((x_1 + x_3)P + x'P' - p_c)^2] = -\frac{x}{\hat{t}}\delta(x_1 + x_3 - x), \quad (16)$$

which can be used to integrate out x_1 in Eq. (14), fixing $x_1 = x - x_3$. At the same time, there are propagators marked by a short bar in the diagram. These propagators are the so-called "pole" propagators, which will be used to perform contour integrals to eliminate the remaining two momentum fractions. They are given by the following expressions:

$$\frac{1}{(x'P' + x_2P + k_\perp)^2 + i\epsilon} = \frac{x}{\hat{s}}\frac{1}{x_2 + x\frac{k_\perp^2}{\hat{s}} + i\epsilon}, \quad (17)$$

$$\frac{1}{(x'P' + x_3P)^2 + i\epsilon} = \frac{x}{\hat{s}}\frac{1}{x_3 + i\epsilon}. \quad (18)$$

Now, the first propagator can be used to integrate out x_2,

$$\int dx_2 e^{ix_2P^+(y_1^- - y_2^-)}\frac{1}{x_2 + x\frac{k_\perp^2}{\hat{s}} + i\epsilon} = -2\pi i\theta(y_2^- - y_1^-)e^{-i\frac{k_\perp^2}{\hat{s}}xP^+(y_1^- - y_2^-)}, \quad (19)$$

which fixes $x_2 = -x\frac{k_\perp^2}{\hat{s}}$. On the other hand, the second propagator will be used to integrate out x_3,

$$\int dx_3 e^{ix_3P^+(y_2^- - y^-)}\frac{1}{x_3 + i\epsilon} = -2\pi i\theta(y^- - y_2^-), \quad (20)$$

thus fixing $x_3 = 0$. Eventually, for both gluons on the left side of the cut line, we have the contribution proportional to

$$T_L(x_1, x_2, x_3)H_L(x_1, x_2, x_3, k_\perp), \quad (21)$$

with all the momentum fractions given by

$$x_1 = x, \qquad x_2 = -x\frac{k_\perp^2}{\hat{s}}, \qquad x_3 = 0, \quad (22)$$

and the relevant twist-4 correlation function

$$T_L\left(x_1 = x, x_2 = -x\frac{k_\perp^2}{\hat{s}}, x_3 = 0\right) = \int \frac{dy^-}{2\pi}\frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+y^-} e^{-i\frac{k_\perp^2}{\hat{s}}xP^+(y_1^- - y_2^-)}\theta(y_2^- - y_1^-)\theta(y^- - y_2^-)\frac{1}{2}\langle P|F_\alpha^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|P\rangle. \quad (23)$$

For the double scattering diagram shown in Fig. 2(R), performing similar analysis, we have the contribution proportional to

$$T_R(x_1, x_2, x_3)H_R(x_1, x_2, x_3, k_\perp), \quad (24)$$

where the parton momentum fractions x_1, x_2, x_3 are the same as those in Fig. 2(L) and are given by Eq. (22). The relevant twist-4 correlation function is slightly different:

$$T_R\left(x_1 = x, x_2 = -x\frac{k_\perp^2}{\hat{s}}, x_3 = 0\right) = \int\frac{dy^- dy_1^- dy_2^-}{2\pi\ \ 2\pi} e^{ixP^+y^-} e^{-i\frac{k_\perp^2}{\hat{s}}xP^+(y_1^- - y_2^-)}\theta(y_1^- - y_2^-)\theta(-y_1^-) \times\frac{1}{2}\langle P|F_\alpha^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|P\rangle. \quad (25)$$

Finally let us study the double scattering contribution in Fig. 2(M). In this case, the on-shell condition for the unobserved parton d gives

$$\delta(p_d^2) = \delta[((x_1 + x_2)P + x'P' + k_\perp - p_c)^2]$$
$$= -\frac{x}{\hat{t}}\delta\left[x_1 + x_2 - x - x\frac{k_\perp^2 - 2p_c\cdot k_\perp}{\hat{t}}\right], \tag{26}$$

which fixes $x_1 = x + x\frac{k_\perp^2 - 2p_c\cdot k_\perp}{\hat{t}} - x_2$. The two pole propagators marked with short bars are given by the following expressions:

$$\frac{1}{(x'P' + x_2P + k_\perp)^2 + i\epsilon} = \frac{x}{\hat{s}}\frac{1}{x_2 + x\frac{k_\perp^2}{\hat{s}} + i\epsilon}, \tag{27}$$

$$\frac{1}{(x'P' + (x_2 - x_3)P + k_\perp)^2 - i\epsilon} = -\frac{x}{\hat{s}}\frac{1}{x_3 - x_2 - x\frac{k_\perp^2}{\hat{s}} + i\epsilon}, \tag{28}$$

which can be used to integrate over x_2 and x_3. Finally, we have

$$x_1 = x + x\frac{(k_\perp^2 - 2k_\perp\cdot p_c)}{\hat{t}} + x\frac{k_\perp^2}{\hat{s}}, \quad x_2 = -x\frac{k_\perp^2}{\hat{s}},$$
$$x_3 = 0. \tag{29}$$

Thus for this diagram, the contribution can be written as

$$-T_M(x_1, x_2, x_3)H_M(x_1, x_2, x_3, k_\perp), \tag{30}$$

with momentum fractions x_1, x_2, x_3 given by Eq. (29). The minus sign in front of the expression emphasizes the relative sign difference between Fig. 2(M) and Figs. 2 (L),(R) in the contour integration process. The relevant correlation function $T_M(x_1, x_2, x_3)$ has different θ functions given by the following expression:

$$T_M\left(x_1 = x + x\frac{(k_\perp^2 - 2k_\perp\cdot p_c)}{\hat{t}} + x\frac{k_\perp^2}{\hat{s}}, x_2 = -x\frac{k_\perp^2}{\hat{s}}, x_3 = 0\right)$$
$$= \int\frac{dy^-}{2\pi}\frac{dy_1^- dy_2^-}{2\pi}e^{i(1+\frac{(k_\perp^2-2k_\perp\cdot p_c)}{\hat{t}}+\frac{k_\perp^2}{\hat{s}})xP^+y^-}e^{-i\frac{k_\perp^2}{\hat{s}}xP^+(y_1^- - y_2^-)}\theta(y^- - y_1^-)\theta(-y_2^-)\frac{1}{2}\langle P|F_\alpha^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|P\rangle. \tag{31}$$

The next critical step is to combine these three contributions, Eqs. (21), (24), and (30), and to perform the $k_\perp$ expansion:

$$\frac{\partial^2}{\partial k_\perp^\rho \partial k_\perp^\sigma}[T_L(x_1, x_2, x_3)H_L(x_1, x_2, x_3, k_\perp) + T_R(x_1, x_2, x_3)H_R(x_1, x_2, x_3, k_\perp) - T_M(x_1, x_2, x_3)H_M(x_1, x_2, x_3, k_\perp)]. \tag{32}$$

Here, we use the following useful identity:

$$\frac{\partial^2}{\partial k_\perp^\rho \partial k_\perp^\sigma}[T(x_1, x_2, x_3)H(x_1, x_2, x_3, k_\perp)] = \frac{\partial^2 T}{\partial x_i \partial x_j}\left[\frac{\partial x_i}{\partial k_\perp^\rho}\frac{\partial x_j}{\partial k_\perp^\sigma}H\right] + \frac{\partial T}{\partial x_i}\left[\frac{\partial^2 x_i}{\partial k_\perp^\rho \partial k_\perp^\sigma}H + \frac{\partial x_i}{\partial k_\perp^\rho}\frac{\partial H}{\partial k_\perp^\sigma} + \frac{\partial x_i}{\partial k_\perp^\sigma}\frac{\partial H}{\partial k_\perp^\rho}\right] + T\frac{\partial^2 H}{\partial k_\perp^\rho \partial k_\perp^\sigma}, \tag{33}$$

where repeated indices imply summation. With this identity in hand, we substitute the relevant parton momentum fractions x_1, x_2, x_3, given in Eqs. (22) and (29), and then use Mathematica to perform the $k_\perp$ expansion automatically. For example, for the double-derivative term $\frac{\partial^2 T(x_1,x_2,0)}{\partial x_2^2}$, we obtain an expression proportional to

$$\propto \int\frac{dy^-}{2\pi}e^{ixP^+y^-}\int\frac{dy_1^- dy_2^-}{2\pi}(y_1^- - y_2^-)^2\langle P|F_\alpha^+(y_2^-)\bar{\psi}_q(0)\gamma^+\psi_q(y^-)F^{+\alpha}(y_1^-)|P\rangle H(x, 0, 0, 0)[\theta(y_2^- - y_1^-)\theta(y^- - y_2^-)$$
$$+ \theta(y_1^- - y_2^-)\theta(-y_1^-) - \theta(y^- - y_1^-)\theta(-y_2^-)], \tag{34}$$

where we have used the fact

$$H_L(x_1, x_2, x_3, k_\perp) = H_R(x_1, x_2, x_3, k_\perp) = H_M(x_1, x_2, x_3, k_\perp)$$
$$\equiv H(x, 0, 0, 0) \tag{35}$$

for $x_1 = x$, $x_2 = 0$, $x_3 = 0$, and $k_\perp = 0$. It is important to observe that in Eq. (34) the factor

$$[\theta(y_2^- - y_1^-)\theta(y^- - y_2^-) + \theta(y_1^- - y_2^-)\theta(-y_1^-)$$
$$- \theta(y^- - y_1^-)\theta(-y_2^-)] \tag{36}$$

is equivalent to the restrictions [17–19]

$$|y^-| > |y_1^-| > |y_2^-|, \tag{37}$$

i.e., the integration $\int dy_1^- dy_2^-$ becomes an ordered integral limited by the value of y^-. In the region where the parton momentum fraction in the nucleus $x \sim \mathcal{O}(1)$, the rapidly oscillating exponential phase $e^{ixP^+y^-}$ will effectively restrict $y^- \sim 1/(xP^+) \to 0$, and thus also restricts $y_{1,2}^- \to 0$ through Eq. (37). Physically, this means that all the y^- integrations in such a term are localized, and therefore, will not have nuclear size enhancement *for the double scattering contribution*. For this reason, such a term that is proportional to Eq. (36) is sometimes referred to as "contact" term [8].

It is instructive and important to point out that in the small-x region, where $x \to 0$, the above argument does not hold any more: in this case, even though one still has y^- restricted to $y^- \sim 1/(xP^+)$, y^- integration is not localized at $y^- \sim 0$ when $x \to 0$. Physically, this means that in the small-x case, the probe (incoming parton from the proton) can cover the whole nucleus. Thus, it will interact *coherently* with the partons from different nucleons at the same impact parameter inside the nucleus. In the small-x region one cannot drop such contact term contributions as in Eq. (36). In this regime the multiple scattering contributions to single hadron and dihadron production in p + A collisions were studied in the forward rapidity limit in [20,21]. It has been shown that they lead to the so-called "dynamical shadowing" effect, which has been used to successfully describe both single hadron suppression and dihadron correlation in the forward rapidity region at the relativistic heavy ion collider (RHIC) energies [21,23].

In this paper, we will concentrate on the region where the parton momentum fraction in the nucleus $x \sim \mathcal{O}(1)$, i.e. outside the small-x regime. We, thus, follow the original study [18,19] and neglect all these contact terms that are proportional to Eq. (36). Finally, from Eqs. (32) and (33), we have the initial-state double scattering contribution to the $qq' \to qq'$ process as

$$\propto \left[x^2 \frac{\partial^2 T^{(I)}_{q'/A}(x)}{\partial x^2} - x \frac{\partial T^{(I)}_{q'/A}(x)}{\partial x} + T^{(I)}_{q'/A}(x)\right] c^I H^I_{qq'\to qq'}(\hat{s}, \hat{t}, \hat{u}), \tag{38}$$

where the prefactor $c^I = -\frac{1}{\hat{t}} - \frac{1}{\hat{s}}$, the associated hard-scattering function $H^I_{qq'\to qq'} = C_F H^U_{qq'\to qq'}$, and the relevant two-quark-two-gluon correlation function $T^{(I)}_{q/A}(x)$ is given by [23,30,31]

$$T^{(I)}_{q/A}(x) = \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \int \frac{dy_1^- dy_2^-}{2\pi} \theta(y^- - y_1^-)\theta(-y_2^-) \times \frac{1}{2} \langle P | F_\alpha^+(y_2^-) \bar{\psi}_q(0) \gamma^+ \psi_q(y^-) F^{+\alpha}(y_1^-) | P \rangle. \tag{39}$$

The integration over $dy_1^- dy_2^-$ leads to the nuclear size enhancement $\propto A^{1/3}$, as demonstrated in [19]. There are several comments in order. First, in the intermediate steps we have three independent two-quark-two-gluon correlation functions, which can be seen clearly from the different θ functions in Eqs. (23), (25), and (31). They are associated with the three different cuts, as shown in Fig. 2. However, only one correlation function $T^{(I)}_{q/A}(x)$ remains in the final result and it is associated with the diagram Fig. 2(M). Those associated with Figs. 2(L) and 2(R) eventually lead to the contact combination, as in Eq. (36). In other words, in the $x \sim \mathcal{O}(1)$ (outside the small-x) region, where all the contact terms are suppressed, the final result only depends on the real diagram shown in Fig. 2(M). The classical double scattering picture is preserved. It is because of this reason that the *coherent* nature in the multiple scattering disappears. Thus, we should not expect to see the dynamical shadowing effect shown in [20,21].

Second, the simple and compact form in Eq. (38) is quite remarkable, i.e. the final results for the combined second-derivative, first-derivative and nonderivative terms have a *common* hard-scattering function for this process, even though there could have been three separate hard-scattering functions multiplying $\frac{\partial^2 T^{(I)}_{q/A}(x)}{\partial x^2}$, $\frac{\partial T^{(I)}_{q/A}(x)}{\partial x}$, and $T^{(I)}_{q/A}(x)$, as in Eq. (33). Similar simple structure was first observed in studying the transverse spin asymmetry at the twist-3 level [34,35], where one has only first-derivative and nonderivative terms and they share a single hard-scattering function. A more fundamental reason why this is the case deserves further investigation [36].

Let us now turn to the final-state double scattering contributions to $qq' \to qq'$. The relevant Feynman diagrams are shown in Fig. 3, where the *observed* outgoing parton c undergoes double scattering (absorb soft gluons) directly. Following the same approach as above, the final result can again be written in a compact form:

$$\propto \left[x^2 \frac{\partial^2 T^{(F)}_{q'/A}(x)}{\partial x^2} - x \frac{\partial T^{(F)}_{q'/A}(x)}{\partial x} + T^{(F)}_{q'/A}(x)\right] \times c^F H^F_{qq'\to qq'}(\hat{s}, \hat{t}, \hat{u}), \tag{40}$$

where we have a different prefactor $c^F = -\frac{1}{\hat{t}} - \frac{1}{\hat{u}}$, and the final-state hard-scattering function is the same as the initial-state hard-scattering function as above $H^F_{qq'\to qq'} = C_F H^U_{qq'\to qq'}$. The relevant final-state two-quark-two-gluon correlation function $T^{(F)}_{q/A}(x)$ is the same as $T^{(I)}_{q/A}(x)$, except for the θ functions that are replaced as follows [20,23,30,31]:

$$\theta(y^- - y_1^-)\theta(-y_2^-) \to \theta(y_1^- - y^-)\theta(y_2^-). \tag{41}$$

Here, the final result again depends only on the real diagram Fig. 3(M), which preserves the classical double scattering picture. In principle, there are also Feynman diagrams in which the *unobserved* outgoing parton d undergoes multiple scattering. The sum over different cuts for these diagrams will always lead to the contact term as in Eq. (36), as demonstrated already in semi-inclusive deep inelastic scattering [19,20]. Thus, in the kinematic region $x \sim \mathcal{O}(1)$, we neglect such contributions.

C. Final result: A compact form

Likewise, we have computed both initial-state and final-state double scattering contributions to all other partonic channels: $qq \to qq$, $q\bar{q} \to q'\bar{q}'$, $q\bar{q} \to q\bar{q}$, $qg \to qg$, $gq \to gq$, $gq \to qg$, $qg \to gq$, $q\bar{q} \to gg$, $gg \to q\bar{q}$, and $gg \to gg$. For these processes, besides the two-quark-two-gluon correlation functions $T_{q/A}^{(I,F)}(x)$ defined above, four-gluon correlation functions $T_{g/A}^{(I,F)}(x)$ also contribute and they have the following definitions [23,30,31]:

$$T_{g/A}^{(I)}(x) = \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \int \frac{dy_1^- dy_2^-}{2\pi} \theta(y^- - y_1^-)\theta(-y_2^-) \times \frac{1}{xP^+} \langle P| F_\alpha^+(y_2^-) F^{\sigma+}(0) F_\sigma^+(y^-) F^{+\alpha}(y_1^-)|P\rangle. \tag{42}$$

$T_{g/A}^{(F)}(x)$ is given by the same expression with the θ-functions replacement specified in Eq. (41). We find that the double scattering contributions for all these partonic processes follow the same compact form as in Eqs. (38) and (40), with the following expression to the single hadron differential cross section:

$$E_h \frac{d\sigma^{(D)}}{d^3P_h} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right) \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to h}(z) \int \frac{dx'}{x'} f_{a/p}(x') \times \int \frac{dx}{x} \delta(\hat{s}+\hat{t}+\hat{u}) \sum_{i=I,F} \left[x^2 \frac{\partial^2 T_{b/A}^{(i)}(x)}{\partial x^2} - x \frac{\partial T_{b/A}^{(i)}(x)}{\partial x} + T_{b/A}^{(i)}(x) \right] c^i H_{ab\to cd}^i(\hat{s},\hat{t},\hat{u}), \tag{43}$$

where $\sum_{i=I,F}$ denotes the summation over initial-state and final-state double scattering, c^i are given by

$$c^I = -\frac{1}{\hat{t}} - \frac{1}{\hat{s}}, \tag{44}$$

$$c^F = -\frac{1}{\hat{t}} - \frac{1}{\hat{u}}. \tag{45}$$

The hard-scattering functions $H_{ab\to cd}^i(\hat{s},\hat{t},\hat{u})$ receive contributions from both initial-state and final-state double scattering and are always proportional to the unpolarized hard-part functions $H_{ab\to cd}^U(\hat{s},\hat{t},\hat{u})$ as follows:

$$H_{ab\to cd}^I = \begin{cases} C_F H_{ab\to cd}^U & a = \text{quark} \\ C_A H_{ab\to cd}^U & a = \text{gluon}, \end{cases} \tag{46}$$

$$H_{ab\to cd}^F = \begin{cases} C_F H_{ab\to cd}^U & c = \text{quark} \\ C_A H_{ab\to cd}^U & c = \text{gluon}. \end{cases} \tag{47}$$

In other words, they only depend on the color of the incoming and outgoing partons which undergo the multiple scattering. It is important to emphasize again that the final result depends only on the diagrams in which the two gluons are on different sides of the $t = \infty$ cut and preserve the classical double scattering picture. All the interference diagrams drop out in the final result because they all show up in the contact terms and thus don't lead to the nuclear size enhancement in the $x \sim \mathcal{O}(1)$ (outside small-x) region, which we are interested in. Finally, if one replaces the hadron fragmentation function $D_{c\to h}(z)$ by $\delta(1-z)$, we immediately obtain the double scattering contribution to the single jet production in p + A collisions (to lowest order in the jet structure).

III. MULTIPLE SCATTERING EFFECTS IN PHYSICAL PROCESSES INVOLVING A PHOTON

In this section we study the multiple scattering contributions to the physical processes which involve a photon in either the initial or the final state. In particular, we study the prompt photon production in p + A collisions, and single inclusive hadron production in γ + A collisions. For both processes, our results derived in last section will be directly relevant, as we will show below.

A. Multiple scattering in prompt photon production in p + A collisions

The prompt photon production can receive two contributions: the so-called "direct" photons and "fragmentation" photons [37–39]. Thus, the single scattering contribution to the prompt photon production can be written as

$$E_\gamma \frac{d\sigma^{(S)}}{d^3P_\gamma} = E_\gamma \frac{d\sigma^{(S)}}{d^3P_\gamma}\bigg|_{\text{direct}} + E_\gamma \frac{d\sigma^{(S)}}{d^3P_\gamma}\bigg|_{\text{frag}}. \tag{48}$$

The direct photon contribution at the leading order has the following form:

$$E_\gamma \frac{d\sigma^{(S)}}{d^3P_\gamma}\bigg|_{\text{direct}} = \frac{\alpha_{em}\alpha_s}{S} \sum_{a,b} \int \frac{dx'}{x'} f_{a/p}(x') \times \int \frac{dx}{x} f_{b/A}(x) H_{ab\to\gamma d}^U(\hat{s},\hat{t},\hat{u}) \delta(\hat{s}+\hat{t}+\hat{u}), \tag{49}$$

with the hard-scattering functions given by [27,40]

$$H_{qg\to\gamma q}^U = e_q^2 \frac{1}{N_c}\left[-\frac{\hat{s}}{\hat{t}} - \frac{\hat{t}}{\hat{s}}\right], \tag{50}$$

$$H_{gq\to\gamma q}^U = e_q^2 \frac{1}{N_c}\left[-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}}\right], \tag{51}$$

$$H_{q\bar{q}\to\gamma g}^U = e_q^2 \frac{N_c^2-1}{N_c^2}\left[\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right]. \tag{52}$$

On the other hand, the single scattering contribution to the fragmentation photon production can be written as

$$E_\gamma \frac{d\sigma^{(S)}}{d^3P_\gamma}\bigg|_{\text{frag}} = \frac{\alpha_s^2}{S} \sum_{a,b,c} \int \frac{dz}{z^2} D_{c\to\gamma}(z) \int \frac{dx'}{x'} f_{a/p}(x') \times \int \frac{dx}{x} f_{b/A}(x) H_{ab\to cd}^U(\hat{s},\hat{t},\hat{u}) \delta(\hat{s}+\hat{t}+\hat{u}), \tag{53}$$

i.e., one replaces parton-to-hadron fragmentation function $D_{c\to h}(z)$ in Eq. (3) by parton-to-photon fragmentation function $D_{c\to\gamma}(z)$.

Let us now study the double scattering contributions to the prompt photon production. For the direct photon component, in which the photon is produced in the hard scattering, we only have initial-state double scattering. The calculation follows the same method as the last section and we have the result:

$$E_\gamma \frac{d\sigma^{(D)}}{d^3P_\gamma}\bigg|_{\rm direct} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\alpha_{em}\alpha_s}{S}\sum_{a,b}\int\frac{dx'}{x'}f_{a/p}(x')\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u}) \times\left[x^2\frac{\partial^2 T_{b/A}^{(I)}(x)}{\partial x^2} - x\frac{\partial T_{b/A}^{(I)}(x)}{\partial x} + T_{b/A}^{(I)}(x)\right]c^I H^I_{ab\to\gamma d}(\hat{s},\hat{t},\hat{u}), \quad (54)$$

where c^I is given in Eq. (44), and the associated hard-scattering functions $H^I_{ab\to\gamma d}$ are given by

$$H^I_{qg\to\gamma q} = C_F H^U_{qg\to\gamma q}, \quad (55)$$

$$H^I_{gq\to\gamma q} = C_A H^U_{gq\to\gamma q}, \quad (56)$$

$$H^I_{q\bar{q}\to\gamma g} = C_F H^U_{q\bar{q}\to\gamma g}. \quad (57)$$

This initial-state double scattering contribution to direct photon production was first derived in [41]. Our approach allows for the result to be written in a compact form, as in our Eq. (54).

In the fragmentation photon contribution Eq. (53) one first produces a parton c through the hard partonic process $ab\to cd$. This parton then fragments into the final observed photon. In this case, both the incoming parton a and the outgoing parton c can interact with the partons in the nucleus, resulting in both initial-state and final-state multiple scattering effects. These interactions are exactly the same as the ones in single inclusive hadron production, as shown in the previous section. We, thus, obtain the double scattering contribution to fragmentation photons as the following:

$$E_\gamma \frac{d\sigma^{(D)}}{d^3P_\gamma}\bigg|_{\rm frag} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to\gamma}(z) \times\int\frac{dx'}{x'}f_{a/p}(x')\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u}) \times\sum_{i=I,F}\left[x^2\frac{\partial^2 T_{b/A}^{(i)}(x)}{\partial x^2} - x\frac{\partial T_{b/A}^{(i)}(x)}{\partial x} + T_{b/A}^{(i)}(x)\right]c^i H^i_{ab\to cd}(\hat{s},\hat{t},\hat{u}), \quad (58)$$

i.e., the final result is the same as Eq. (43) with the replacement of the parton-to-hadron fragmentation function with the parton-to-photon fragmentation function.

B. Single inclusive hadron production in photon + nucleus collisions

Let us now study the single hadron photo-production $\gamma+A\to h+X$, which can also receive two contributions: the so-called "direct" and "resolved" components [42–47]. The "direct" component corresponds to the case where the photon interacts directly with a parton in the nucleus. On the other hand, in the resolved contribution, the photon acts as a source of partons which collide with the partons in the nucleus. One can write the single scattering contribution to hadron photo-production as follows:

$$E_h\frac{d\sigma^{(S)}}{d^3P_h} = E_h\frac{d\sigma^{(S)}}{d^3P_h}\bigg|_{\rm direct} + E_h\frac{d\sigma^{(S)}}{d^3P_h}\bigg|_{\rm resolved}. \quad (59)$$

The direct component can be written as

$$E_h\frac{d\sigma^{(S)}}{d^3P_h}\bigg|_{\rm direct} = \frac{\alpha_{em}\alpha_s}{S}\sum_{b,c}\int\frac{dz}{z^2}D_{c\to h}(z) \times\int\frac{dx}{x}f_{b/A}(x)H^U_{\gamma b\to cd}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \quad (60)$$

where $S=(P_\gamma+P)^2$ is the center-of-mass energy squared, and the hard-scattering functions are given by [31]

$$H^U_{\gamma q\to qg} = e_q^2\frac{N_c^2-1}{N_c}\left[-\frac{\hat{s}}{\hat{t}}-\frac{\hat{t}}{\hat{s}}\right], \quad (61)$$

$$H^U_{\gamma q\to gq} = e_q^2\frac{N_c^2-1}{N_c}\left[-\frac{\hat{s}}{\hat{u}}-\frac{\hat{u}}{\hat{s}}\right], \quad (62)$$

$$H^U_{\gamma g\to q\bar{q}} = e_q^2\left[\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right]. \quad (63)$$

On the other hand, the resolved component can be written as

$$E_h\frac{d\sigma^{(S)}}{d^3P_h}\bigg|_{\rm resolved} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}f_{a/\gamma}(x') \times\int\frac{dx}{x}f_{b/A}(x)H^U_{ab\to cd}(\hat{s},\hat{t},\hat{u})\delta(\hat{s}+\hat{t}+\hat{u}), \quad (64)$$

where $f_{a/\gamma}(x')$ is the parton distribution function in a photon, e.g. see Ref. [43] for a parametrized functional form. All the partonic subprocesses $ab\to cd$ are exactly the same as those in the single hadron production in p + A collisions, with the same hard-scattering functions $H^U_{ab\to cd}(\hat{s}, \hat{t}, \hat{u})$ as given in the last section.

Now, we turn our attention to the double scattering contribution to single hadron production in $\gamma + A$ collisions, which can be studied experimentally in a future electron ion collider [48]. For the direct component, we have only a final-state double scattering contributions. The final result is

$$E_h \frac{d\sigma^{(D)}}{d^3P_h}\bigg|_{\text{direct}} = \left(\frac{8\pi^2\alpha_s}{N_c^2-1}\right)\frac{\alpha_{em}\alpha_s}{S}\sum_{b,c}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx}{x}\delta(\hat{s}+\hat{t}+\hat{u}) \times\left[x^2\frac{\partial^2 T^{(F)}_{b/A}(x)}{\partial x^2} - x\frac{\partial T^{(F)}_{b/A}(x)}{\partial x} + T^{(F)}_{b/A}(x)\right] \times c^F H^F_{\gamma b\to cd}(\hat{s}, \hat{t}, \hat{u}), \tag{65}$$

where c^F is given in Eq. (45), and the hard-scattering function $H^F_{\gamma b\to cd}$ is related to the unpolarized hard-part function $H^U_{\gamma b\to cd}$ just like in Eq. (47):

$$H^F_{\gamma q\to qg} = C_F H^U_{\gamma q\to qg}, \tag{66}$$

$$H^F_{\gamma q\to gq} = C_A H^U_{\gamma q\to gq}, \tag{67}$$

$$H^F_{\gamma g\to q\bar{q}} = C_F H^U_{\gamma g\to q\bar{q}}. \tag{68}$$

On the other hand, in the resolved component, the incoming particle is a parton resolved inside the photon and it can interact with the nucleus via strong interaction. In this case, we have both initial-state and final-state multiple scattering effects. Again, the double scattering contributions will have the same form as in Eq. (43), except that $f_{a/p}(x')$ is replaced by $f_{a/\gamma}(x')$. Finally, if one replaces the fragmentation function $D_{c\to h}(z)$ by $\delta(1-z)$, one immediately obtains the double scattering contributions to single jet production in $\gamma + A$ collisions at lowest order in the jet substructure. The double scattering contribution to the direct component for jet photo-production was first derived in [17]. Our approach allows us to write it in a simple compact form, Eq. (65).

IV. SUMMARY AND DISCUSSION

In this paper we studied the double scattering contribution to the differential cross section for single inclusive hadron production in p + A collisions within the high-twist factorization approach to parton interactions in cold nuclear matter. Unlike most recent studies, we concentrated on the region where the parton momentum fraction in the nucleus $x \sim \mathcal{O}(1)$. This regime, outside small x, represents the *incoherent* double scattering contribution that is most relevant at backward rapidities, i.e. in the direction of the nucleus. Including both initial-state and final-state double scattering contributions, we found that the final result is proportional to a simple combination of second-derivative, first-derivative, and nonderivative terms of well-defined four-parton correlation functions that share the same hard-scattering functions. We further extended our method to study the double scattering contribution to prompt photon production in p + A collisions and to the single inclusive hadron production in $\gamma + A$ collisions. All final results follow the same simple compact form, which is the main finding of this work.

We leave phenomenological studies for the future, since they require detailed modeling of the four-parton correlation functions. Nevertheless, by direct inspection of our analytic results, we see that in the incoherent regime the double scattering gives a positive contribution to the differential cross section for all processes considered here. Qualitatively, such Cronin-like enhancement [49] is directly comparable to the findings of alternative approaches to independent multiple parton scattering [24]. Owing to its power-suppressed nature, the nuclear effect is expected to disappear at large transverse momenta. At the same time, since even higher-order multiple scattering will result in additional power suppression in this approach, we expect that double scattering contributions will be the most relevant ones for phenomenology [50]. At backward rapidities (i.e. in the direction of the nucleus) and transverse momenta up to a few GeV, p + A reaction always show enhancement of particle production relative to the naive binary collision-scaled p + p result [51–53]. At midrapidity, the sign and magnitude of the nuclear enhancement depends on the center-of-mass energy. In the center-of-mass energy range up to 5 TeV [54], where measurements in p + A reactions exist, Cronin effect is present but its magnitude is significantly reduced in going from the fixed target experiments to RHIC and, finally, to the LHC. We expect that our work will shed light on the origin of cold nuclear matter effects in the unexplored backward rapidity region in both p + A and e + A reactions. It will also help understand the transition from incoherent to coherent multiple scattering effects [3] at forward rapidity.

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