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Top-quark decay into Higgs boson and a light quark at next-to-leading order in QCD

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Neutral flavor-changing transitions are hugely suppressed in the Standard Model and therefore they are very sensitive to new physics. We consider the decay rate of $t \to u_i h$ where $u_i = u, c$ using an effective field theory approach. We perform the calculation at next-to-leading order (NLO) in QCD including the relevant dimension-six operators. We find that at NLO the contribution from the flavor-changing chromomagnetic operator is as important as the standard QCD correction to the flavor-changing Yukawa coupling. In addition to improving the accuracy of the theoretical predictions, the NLO calculation provides information on the operator mixing under the renormalization group.

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I. INTRODUCTION

The discovery of a particle of about 125 GeV mass [1,2] that resembles the Higgs boson of the Standard Model (SM) [3–5] has opened a new era in particle physics. A new realm of possibilities for exploring the electroweak breaking sector and new exciting opportunities to search for new physics in general have appeared. From the existence of new symmetries to new space-time dimensions, from new matter to a richer scalar sector, many are still viable options. For instance, extra scalar states could exist mostly (or exclusively) coupling to the Higgs boson or new particles could decay into final states involving the SM scalar boson. In addition to direct searches, the accurate measurement of the coupling strengths and structures to the SM particles could point to the scale of new physics.

In this perspective, the study of neutral flavor-changing (NFC) couplings involving the top quark and the Higgs boson is of special interest. In the Standard Model, NFC interactions are absent at tree level and hugely suppressed by the Glashow-Iliopoulos-Maiani mechanism at one loop. Finding evidence for such processes taking place at measurable rates would basically always imply new physics not too far from the TeV scales. The recently observed excess of $B \to D^{(*)}\tau\nu$ [6] could hint to NFC mediated by the Higgs boson [7].

In this work we consider the decay of a top quark into a light u_i (up or charm) quark and the Higgs boson, assuming new physics residing at a scale $\Lambda > m_t$. The SM contribution to the branching ratio is extremely small, at order 10^{-13}–10^{-15} [8–10]. Indirect bounds on $\mathrm{BR}(t \to ch)$ have been set, for example, in Refs. [11,12], and are found to be at $\sim 10^{-3}$ level. Collider searches for these interactions have been discussed in [10,13–16]. The first limit at LHC, $\mathrm{BR}(t \to ch) < 2.7\%$, was given in [17].

In this Letter we present the calculation at next-to-leading order (NLO) in QCD of the inclusive top-quark decay into a Higgs boson via NFC interactions in an effective field theory (EFT) [18] approach. We consider all lowest dimensional operators O_i compatible with the symmetries of the SM,

$$\mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i O_i}{\Lambda^2} + \mathrm{H.c.}, \tag{1}$$

where Λ represents the scale of new physics. Our calculation completes the set of NLO results available for two-body top decays, such as $t \to bW$ and $t \to cV$ with $V = \gamma, g, Z$ [19–22], which we have independently checked.

II. SETUP

A complete and minimal list of dimension-six operators that can be written with SM fields and are compatible with the symmetries of the SM can be found in Ref. [23]. We use the same operator basis, employing the following notation for quark fields:

Q: third-generation left-handed quark doublet,

q: first- or second-generation left-handed quark doublet,

t: right-handed top quark,

u, c: right-handed up and charm quark,

φ: Higgs boson doublet,

and $\tilde{\varphi} = i\sigma^2\varphi$. There are two main Lorentz structures that contribute to the $t \to u_i h$ decay, the dimension-six Yukawa interaction $O_{u\varphi}$, and the chromomagnetic operator O_{uG}. The latter contributes only at NLO. Considering the possible flavor assignments, they read

$$O_{uG}^{(1,3)} = y_t g_s (\bar{q}\sigma^{\mu\nu}T^A t)\tilde{\varphi}G^A_{\mu\nu}, \tag{2}$$

$$O_{u\varphi}^{(1,3)} = -y_t^3(\varphi^\dagger\varphi)(\bar{q}t)\tilde{\varphi}, \tag{3}$$

$$O_{uG}^{(3,1)} = y_t g_s (\bar{Q}\sigma^{\mu\nu}T^A u)\tilde{\varphi}G^A_{\mu\nu}, \tag{4}$$

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$$O_{u\varphi}^{(3,1)} = -y_t^3(\varphi^\dagger\varphi)(\bar{Q}u)\tilde{\varphi}, \qquad (5)$$

where superscript (1, 3) and (3, 1) denote the flavor structure. The Hermitian conjugates of the (3, 1) operators contribute to $t \to u_i h$ with the opposite chirality of the corresponding (1, 3) operators. In addition, replacing the up-quark field with the charm-quark field gives the same set of operators with (2, 3) and (3, 2) flavor structures.

Note that the operators have been normalized by attaching appropriate factors of the top-quark Yukawa coupling y_t and the strong coupling g_s. The powers of these factors are determined by requiring that, whenever these operators give rise to a SM-like vertex, the coupling strength relative to the SM coupling is always one of the following factor:

$$C_i\frac{m_t^2}{\Lambda^2}, \qquad C_i\frac{m_tE}{\Lambda^2}, \qquad C_i\frac{E^2}{\Lambda^2}, \qquad (6)$$

where E is the typical energy of the particles entering the vertex. This helps to determine the order of the mixing between these operators. With this convention, the operator mixing induced by a gluon exchange is always of order α_s, even if the gluon vertex comes from an effective operator. In fact the normalization coefficient of these operators depends on the details of the full theory beyond Λ. In short our convention states that for any bilinear quark operators, we attach a y_t to each Higgs field, and a g_s for each gluon field. We remark that in this work we choose y_t to be defined in terms of the on-shell top-quark mass m_t

$$y_t = \frac{\sqrt{2}m_t}{v}. \qquad (7)$$

This is just for simplicity. As a result, it does not contribute to the anomalous dimension of the operators at order α_s.

In the following we focus on operators with (1, 3) and (3, 1) flavor structure. The extension of the results from $t \to uh$ to $t \to ch$ is trivial. In addition, since there is no mixing between operators of type (1, 3) and (3, 1) we can omit the superscripts (1, 3) and (3, 1), and only consider the (1, 3) case. Results for (3, 1) can be obtained by flipping the chirality of the quarks.

At the tree level, only $O_{u\varphi}$ contributes. The effective Lagrangian describing the interaction of a top quark, a light quark, and the Higgs boson h reads

$$\mathcal{L}_{tuh} = -C_{u\varphi}\frac{m_t^2}{\Lambda^2}\frac{3y_t}{\sqrt{2}}(\bar{u}P_Rt)h + \text{H.c.} \qquad (8)$$

In addition, the terms in $O_{u\varphi}$ involving the vacuum expectation value v of the Higgs field give rise to $u_L - t_R$ mixing. The standard way to deal with this effect is to perform a set of transformations that diagonalize the mass matrix:

$$u_L \to u_L + C_{u\varphi}\frac{m_t^2}{\Lambda^2}t_L, \qquad (9)$$

$$t_L \to t_L - C_{u\varphi}^*\frac{m_t^2}{\Lambda^2}u_L, \qquad (10)$$

FIG. 1. Feynman diagrams for $t \to u + h$ at tree level. Squares represent an insertion of $O_{u\varphi}$.

and similarly for $O_{u\varphi}^{(3,1)}$. As a result Eq. (8) is modified and the tuh interaction reads

$$\mathcal{L}'_{tuh} = -C_{u\varphi}\frac{m_t^2}{\Lambda^2}\frac{2y_t}{\sqrt{2}}(\bar{u}P_Rt)h + \text{H.c.} \qquad (11)$$

Equivalently, one can add a dimension-four counterterm such that the operator $O_{u\varphi}$ becomes

$$O_{u\varphi} \to O_{u\varphi} + m_t^2y_t(\bar{q}t)\tilde{\varphi} = -y_t^3\left(\varphi^\dagger\varphi - \frac{v^2}{2}\right)(\bar{q}t)\tilde{\varphi}, \qquad (12)$$

and the mixing term disappears.

Another possibility is to keep the quark fields not diagonal, and simply include the external leg corrections to the diagrams, as shown in Fig. 1. This gives the same result for $t \to uh$. At the one-loop level, we choose this point of view to take into account the loop-induced tu mixing.

The decay rate up to next-to-leading corrections in the strong coupling can be written as

$$\Gamma(t \to u_ih) = \Gamma^{(0)} + \alpha_s\Gamma^{(1)}, \qquad (13)$$

with LO result [24]

$$\Gamma^{(0)} = \frac{|C_{u\varphi}|^2}{\Lambda^4}\frac{\sqrt{2}G_Fm_t^7}{8\pi}\left(1 - \frac{m_h^2}{m_t^2}\right)^2, \qquad (14)$$

where the light quark mass is neglected.

III. NLO CALCULATION STRATEGY

We briefly describe our strategy for the computation of NLO corrections in QCD to the decay rate.

First, we aim at NLO accuracy in QCD but only LO in C/Λ^2 EFT expansion. Calculation of higher orders of C/Λ^2 requires complete knowledge of dimension-eight operators, and it is beyond the scope of this paper. As there are no SM FCNC decays $t \to u_ih$ at LO, the first nonzero contribution to the decay width from new physics is order $(C/\Lambda^2)^2$.

To regulate both ultraviolet (UV) and infrared (IR) divergences, we employ dimensional regularization [25] and work in $D = 4 - 2\epsilon$ dimensions. Whenever γ^5 is present in our computation, we use the following prescription based on the 't Hooft-Veltman scheme [26,27]:

$$\gamma^5 \to (1 - 8a_s)\frac{i}{4!}\epsilon_{\nu_1\nu_2\nu_3\nu_4}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4}, \qquad (15)$$

$$\gamma_\mu\gamma^5 \to (1 - 4a_s)\frac{i}{3!}\epsilon_{\mu\nu_1\nu_2\nu_3}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}, \qquad (16)$$

$$\sigma_{\mu\nu}\gamma^5 \to -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}, \qquad (17)$$

where $a_s = C_F\alpha_s/(4\pi)$.

IR divergences cancel between virtual and real diagrams when sufficiently inclusive observables are considered. The rest of the calculation involves the following:

(1) a UV-divergent part, which gives rise to operator mixing and renormalization group equations,
(2) UV-finite part, which gives the actual corrections to the matrix elements.

In the first step, we calculate the UV-divergent part arising from the loop diagrams, and identify the UV counterterms by applying the $\overline{\rm MS}$ scheme and requiring that the UV-divergent terms cancel. The outcome of this procedure is a set of counterterms for dimension-six operators. We then proceed to work out the anomalous dimension and the renormalization group equations of these operators. These equations can be used to evolve the coefficients of these operators from a higher scale down to the scale of top-quark mass.

In the second step, we calculate the UV-finite part. The final result is given in terms of the coefficients of these operators defined at the scale of top-quark mass.

Throughout this paper we ignore the light quark masses, and assume $V_{tb} = 1$.

IV. OPERATOR RENORMALIZATION

The following counterterms for the SM part are used. For the external fields:

$$\delta Z_2^{(t)} = -\frac{\alpha_s}{3\pi} D_\epsilon\left(\frac{1}{\epsilon_{\rm UV}} + \frac{2}{\epsilon_{\rm IR}} + 4\right), \tag{18}$$

$$\delta Z_2^{(q)} = -\frac{\alpha_s}{3\pi} D_\epsilon\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right), \tag{19}$$

$$\delta Z_2^{(\varphi)} = 0, \tag{20}$$

$$\delta m_t/m_t = -\frac{\alpha_s}{3\pi} D_\epsilon\left(\frac{3}{\epsilon_{\rm UV}} + 4\right), \tag{21}$$

while for the couplings:

$$\delta Z_{g_s} = \frac{\alpha_s}{4\pi}\Gamma(1+\epsilon)(4\pi)^\epsilon\left(\frac{N_f}{3} - \frac{11}{2}\right)\frac{1}{\epsilon_{\rm UV}} + \frac{\alpha_s}{12\pi} D_\epsilon \frac{1}{\epsilon_{\rm UV}}, \tag{22}$$

$$\delta Z_{y_t} = -\frac{\alpha_s}{3\pi} D_\epsilon\left(\frac{3}{\epsilon_{\rm UV}} + 4\right), \tag{23}$$

where $D_\epsilon \equiv \Gamma(1+\epsilon)(\frac{4\pi\mu^2}{m_t^2})^\epsilon$. This set of counterterms corresponds to renormalizing the external fields and the top Yukawa coupling on shell, and the strong coupling in the $\overline{\rm MS}$ scheme. We consider five light flavors in the running of α_s. We then apply the $\overline{\rm MS}$ scheme to the dimension-six operators and require that dimension-six operators only mix with dimension-six operators. The counterterms are given by

$$C_i^0 \to Z_{i,j} C_j = (\mathbf{1} + \delta Z)_{i,j} C_j. \tag{24}$$

We first consider $O_{u\varphi}$. Including counterterms, the Lagrangian can be written as

$$\mathcal{L}_{\rm Eff} = \mathcal{L}_{tu} + \mathcal{L}_{tuh}, \tag{25}$$

$$\mathcal{L}_{tu} = -\frac{C_{u\varphi} m_t^3}{\Lambda^2}(\bar{u}_L t_R)(1 + \delta Z_{u\varphi}), \tag{26}$$

$$\mathcal{L}_{tuh} = -\frac{C_{u\varphi} m_t^2}{\Lambda^2}\frac{3y_t}{\sqrt{2}}(\bar{u}_L t_R h)(1 + \delta Z_{u\varphi}), \tag{27}$$

where the counterterm $\delta Z_{u\varphi}$ is

$$\begin{aligned}\delta Z_{u\varphi} &= \delta Z_{u\varphi,u\varphi} + \frac{1}{2} Z_2^{(t)} + \frac{1}{2}\delta Z_2^{(q)} \\ &= \delta Z_{u\varphi,u\varphi} - \frac{\alpha_s}{3\pi}\frac{1}{\epsilon} + \cdots,\end{aligned} \tag{28}$$

where the dots stand for additional finite terms. The renormalization mirrors that of the SM Yukawa terms. $\delta Z_{u\varphi,u\varphi}$ is determined by the $1/\epsilon$ part of the two-point (ut) and three-point (uth) functions. We find

$$\delta Z_{u\varphi,u\varphi} = -\frac{\alpha_s}{\pi}\frac{1}{\epsilon}. \tag{29}$$

Now we consider O_{uG}. The utg vertex is given by

$$\mathcal{L}_{\rm Eff} = -\frac{C_{uG}}{\Lambda^2} 2 m_t g_s (\bar{u}_L \sigma^{\mu\nu} T^A t_R)\partial_\nu G_\mu^A. \tag{30}$$

This gives rise to a $u - t$ mixing at one loop. We find

$$\begin{aligned}\Pi_{ut}(p^2) = -\frac{C_{uG} m_t}{\Lambda^2}\bigg[& m_t^2\left(\delta Z_{u\varphi,uG} + \frac{2\alpha_s}{\pi}\frac{1}{\epsilon}\right)P_R \\ & - P_R(2p^2 - m_t \not{p})\frac{a_s}{\pi}\frac{1}{\epsilon}\bigg] + \cdots.\end{aligned} \tag{31}$$

The first term in the bracket implies

$$\delta Z_{u\varphi,uG} = -\frac{2\alpha_s}{\pi}\frac{1}{\epsilon}. \tag{32}$$

One could also determine this counterterm by calculating the three-point (uth) function. The pole in the second term can be dealt with through the following dimension-six counterterms:

$$O_{(1)} = -y_t(\bar{q}\overleftarrow{\not{D}}\not{D}t)\tilde{\varphi}, \tag{33}$$

$$O_{(2)} = -\frac{i}{2} y_t^2(\bar{Q}\tilde{\varphi})(\tilde{\varphi}^\dagger \not{D} q). \tag{34}$$

However, these operators vanish when the equations of motions are considered (on-shell or off-shell quark does not matter). Therefore one can simply ignore these operators as the $1/\epsilon$ poles always cancel out when combining with the vertex contributions in a physical amplitude.

Finally, the renormalization of O_{uG} requires computation of utg at one loop. We find

$$\delta Z_{uG,uG} = \frac{\alpha_s}{6\pi}\frac{1}{\epsilon}, \tag{35}$$

while $Z_{uG,u\varphi}$ is zero because there is no contribution from $O_{u\varphi}$ to utg at one loop at order α_s.

In summary, we have the following anomalous dimension matrix for O_{uG} and $O_{u\varphi}$:

$$\gamma = \frac{2\alpha_s}{\pi}\begin{pmatrix} \frac{1}{6} & 0 \\ -2 & -1 \end{pmatrix}. \tag{36}$$

The operator O_{uG} can also renormalize other operators. The complete operator mixing can be extracted from the calculation of $t \to u_i V$. However, at order α_s no other operator can renormalize $O_{u\varphi}$ and O_{uG}, so for the process $t \to u_i h$ the anomalous dimensions given in Eq. (36) are sufficient.

V. FINITE CORRECTIONS

We proceed to carry out the UV-finite part of the calculation. At NLO, the loop corrections and real corrections are shown in Fig.. 2. To simplify the calculation, we rotate the u and t quark fields to remove the mixing due to $O_{u\varphi}$. More specifically, given the following effective Lagrangian,

$$\mathcal{L}_{\rm Eff} = \frac{C_{u\varphi}}{\Lambda^2}(-y_t^3)(\varphi^\dagger\varphi)(\bar{q}t\tilde{\varphi})\left[1 + \delta Z_{u\varphi,u\varphi} + \frac{1}{2}\delta Z_2^t + \frac{1}{2}\delta Z_2^q\right] + \frac{C_{uG}}{\Lambda^2}[y_t g_s(\bar{q}\sigma^{\mu\nu}T^A t)G^A_{\mu\nu} + \delta Z_{u\varphi,uG}(-y_t^3)(\varphi^\dagger\varphi)(\bar{q}t\tilde{\varphi})], \tag{37}$$

we perform the following rotation:

$$u_L \to u_L + C_{u\varphi}\frac{m_t^2}{\Lambda^2}\left[1 + \delta Z_{u\varphi,u\varphi} + \frac{1}{2}\delta Z_2^t + \frac{1}{2}\delta Z_2^q\right]t_L, \tag{38}$$

$$t_L \to t_L - C^*_{u\varphi}\frac{m_t^2}{\Lambda^2}\left[1 + \delta Z_{u\varphi,u\varphi} + \frac{1}{2}\delta Z_2^t + \frac{1}{2}\delta Z_2^q\right]u_L. \tag{39}$$

As a result the $u - t$ mixing due to the first term in Eq. (37) is removed; therefore, only the vertex correction needs to be considered for $O_{u\varphi}$. On the other hand, for O_{uG}, the second terms in Eq. (37) contains a $u - t$ mixing counterterm that is not rotated away. This term cancels the UV-divergent terms from O_{uG} at one loop in $u - t$ mixing. The remaining finite part is included in the leg correction diagrams in Fig. 2. Because the rotation of fields is of order C/Λ^2, and the LO contribution to $t \to uh$ is already of order C/Λ^2, the rotation does not affect our results. As a check, we have computed the loop corrections without redefining the fields. In this case there are four more diagrams, as shown in Fig. 3.

Including both virtual and real corrections, the total NLO correction to the decay rate is ($x = m_h/m_t$):

$$\begin{aligned}\frac{8\pi\Gamma^{(1)}}{\sqrt{2}G_F m_t^7} = {} & \frac{|C_{uG}|^2}{\Lambda^4}\frac{1}{36\pi}\Big[x^8 - 8x^6 - 342x^4 + 620x^2 - 271 \\ & + 6x\sqrt{4-x^2}(26-5x^2)\Big(\pi - 6\sin^{-1}\frac{x}{2}\Big) + 12(9x^4 + 76x^2 - 8)\log x\Big] \\ & - \frac{\mathrm{Re}(C_{uG}C^*_{u\varphi})}{\Lambda^4}\frac{2}{9\pi}\Bigg\{6\Bigg[6(1-x^2)^2\log\frac{m_t}{\mu} + (5x^4 + 2x^2 + 4\log(1-x^2) - 2\log x)\log x \\ & + \Bigg(\sqrt{\frac{4}{x^2}-1}(x^4 - 6x^2 + 8) + 2\pi\Bigg)\arcsin\frac{x}{2} + 6\arcsin^2\frac{x}{2}\Bigg] \\ & + 12\Bigg[\mathrm{Li}_2(x^2) - 2\,\mathrm{ReLi}_2\Bigg(\Bigg(x - \frac{1}{x}\Bigg)\Bigg(\frac{x}{2} - i\sqrt{1-\frac{x^2}{4}}\Bigg)\Bigg)\Bigg] \\ & + \Big[3\pi\sqrt{4-x^2}(x^2-2)x - 3(x^4 + 8x^2 - 9)x^2 - 5\pi^2\Big]\Bigg\} \\ & - \frac{|C_{u\varphi}|^2}{\Lambda^4}\frac{(1-x^2)^2}{9\pi}\Bigg[\Bigg(36\log\frac{m_t}{\mu} + 4\pi^2 - 51\Bigg) + 24\mathrm{Li}_2 x^2 + 24\log x\log(1-x^2) \\ & + 24\frac{x^2}{1-x^2}\log x + 6\Bigg(5 - \frac{2}{x^2}\Bigg)\log(1-x^2)\Bigg].\end{aligned} \tag{40}$$

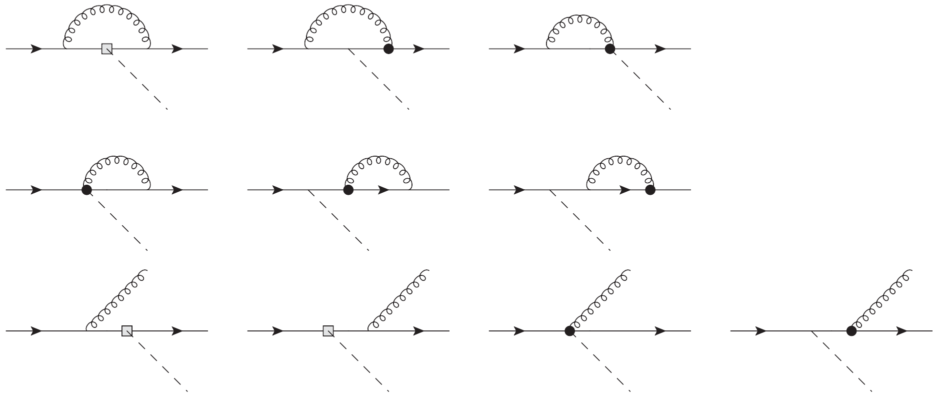

FIG. 2. Virtual and real corrections for $t \to uh$. The squares represent the contribution from $O_{u\varphi}$, while the black dots represent the contribution from O_{uG}.

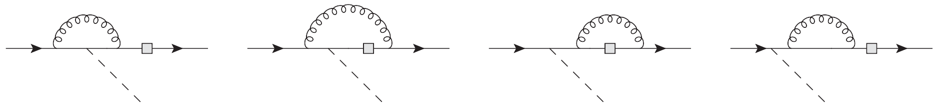

FIG. 3. Additional corrections for $t \to uh$.

The $|C_{uG}|^2$ term does not have a $\log \frac{m_t}{\mu}$ dependence. This is because the tree-level amplitude does not have a contribution from O_{uG}. As a result, the $|C_{uG}|^2$ term entirely comes from real corrections (virtual corrections are interferences between tree- and one-loop level amplitudes), and it is independent of μ.

In addition, the C_{uG}^2 term contains $\log x$ which is divergent in the limit $m_h \to 0$. This corresponds to a soft Higgs emission which in the $m_h \to 0$ limit is divergent when $E_h = 0$. In this limit, we have

$$\alpha_s \Gamma^{(1)} = -\frac{\alpha_s G_F}{144\sqrt{2}\pi^2} \frac{|C_{uG}|^2}{\Lambda^4} m_t^7 (96 \log x + 271) + \mathcal{O}(x). \tag{41}$$

As this term is purely from the real corrections, it can be thought of as the real Higgs emission correction to the decay mode $t \to u + g$. The soft divergence is expected to be canceled by the wave function renormalization of the top quark in the process $t \to u + g$, coming from a virtual Higgs bubble diagram. As a check, we have computed this diagram and find

$$\delta Z_{2,h}^{(t)} = -\frac{m_t^2 G_F}{16\sqrt{2}\pi^2} D_\epsilon \left(\frac{1}{\epsilon_{\rm UV}} - 8 \log x - 3 \right) + \mathcal{O}(x). \tag{42}$$

The corresponding contribution to the virtual correction to the decay width of $t \to u + g$ is

$$\Gamma_{t \to u+g}^{(\rm virt)} = \Gamma_{t \to u+g}^{(0)} \times \delta Z_{2,h}^{(t)} = \left(\frac{4\alpha_s}{3} \frac{|C_{uG}|^2}{\Lambda^4} m_t^5 \right) \times \delta Z_{2,h}^{(t)} = -\frac{\alpha_s G_F}{12\sqrt{2}\pi^2} m_t^7 D_\epsilon \left(\frac{1}{\epsilon_{\rm UV}} - 8 \log x - 3 \right) + \mathcal{O}(x), \tag{43}$$

which exactly cancels the $\log x$ term in Eq. (41).

The other two terms in $\Gamma^{(1)}$, $|C_{u\varphi}^2|$ and $\mathrm{Re}(C_{uG} C_{u\varphi}^*)$ do not have this divergence. In particular, the interference term [which is proportional to $\mathrm{Re}(C_{uG} C_{u\varphi}^*)$] is finite in the $x \to 0$ limit, even though it contains $\log x$ terms and Li_2 functions. This is because the two real correction diagrams from $O_{u\varphi}$ cancel each other when $p_h = 0$.

VI. NUMERICAL ANALYSIS

For the numerical analysis we assume $\Lambda = 1$ TeV. For the input parameters, we use [28]

$$m_t = 173.5 \text{ GeV}, \tag{44}$$

$$m_h = 125.3 \text{ GeV}, \tag{45}$$

$$G_F = 1.1664 \times 10^{-5} \text{ GeV}^{-2}. \tag{46}$$

With these parameters we find

$$\Gamma^{(0)} = 7.11 |C_{u\varphi}(\mu)|^2 \times 10^{-4} \text{ GeV}, \tag{47}$$

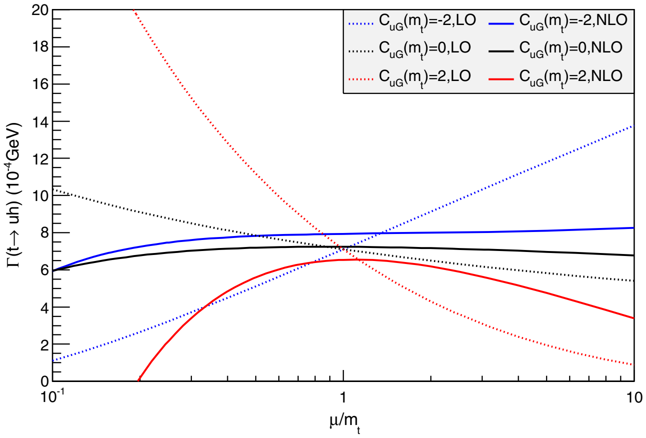

FIG. 4 (color online). Renormalization scale dependence of the width $\Gamma(t \to uh)$, assuming $C_{u\varphi}(m_t) = 1$ and $\Lambda = 1$ TeV.

$$\Gamma^{(1)} = \left\{\left[1.19 - 9.05 \log\left(\frac{m_t}{\mu}\right)\right]|C_{u\varphi}(\mu)|^2 - \left[3.26 + 18.1 \log\left(\frac{m_t}{\mu}\right)\right]\mathrm{Re}C_{uG}(\mu)C^*_{u\varphi}(\mu) + 9.33 \times 10^{-5}|C_{uG}(\mu)|^2\right\} \times 10^{-4}\ \mathrm{GeV}. \quad (48)$$

The C^2_{uG} term is 4 orders of magnitude smaller than the other two terms, and thus it is interesting to understand such a suppression. As we have mentioned, this term only receives contributions from real emission. We find that, due to the $\sigma^{\mu\nu}p_{g\nu}$ structure of the coupling from O_{uG}, the squared amplitude for $t \to u + h + g$ depends on $p_g \cdot p_u$. We find

$$|M|^2 = 128\pi^2\alpha\alpha_s \frac{|C_{uG}|^2}{\Lambda^4} \frac{m_t^6}{m_W^2 s_W^2} \frac{\hat{t}^2}{(1-\hat{t})^2} \times (\hat{t}^2 + \hat{t}\,\hat{s} + (2 - x^2)\hat{t} - \hat{s} + 1), \quad (49)$$

where $\hat{s} = (p_t - p_u)^2/m_t^2$ and $\hat{t} = (p_g \cdot p_u)/m_t^2$.

As a result, this term is dominated by the phase space region where $\hat{t}$ is large. However, the maximum value of $\hat{t}$ is $(1-x)^2/2$ and is therefore suppressed for a large Higgs mass. In fact, for $m_h = 125$ GeV this suppression factor for $|M|^2$ already reaches the 10^{-3} level. The phase space itself accounts for one additional order of magnitude, so the total decay width from $|O_{uG}|^2$ is small for $m_h = 125$ GeV. On the other hand, the other two terms $[|C^2_{u\varphi}|$ and $\mathrm{Re}(C_{uG}C^*_{u\varphi})]$ are not affected by this factor as their main contribution comes from virtual $1 \to 2$ topologies.

We now consider the impact of the NLO corrections to phenomenological applications. In the following we assume both $C_{u\varphi}$ and C_{uG} to be real. At order α_s the contribution from C_{uG} is even more important than that from $C_{u\varphi}$. Neglecting the $|C_{uG}|^2$ term, the ratio between the NLO and LO result is

$$\frac{\alpha_s\Gamma^{(1)}}{\Gamma_{(0)}} = 0.018 - 0.049\frac{C_{uG}}{C_{u\varphi}} \quad (50)$$

at $\mu = m_t$. Here we have used $\alpha_s(m_t) = 0.1079$, which we obtain with the program RunDec [29] from the value $\alpha_s(m_Z) = 0.1184$ [28]. Without O_{uG} the QCD correction is about 2%, while if O_{uG} and $O_{u\varphi}$ are similar in size, the QCD correction can reach the 10% level.

The residual theoretical uncertainties can be estimated by checking the scale dependence of the decay width. Using the anomalous dimension matrix given in Eq. (36), we solve the scale dependence of the coefficients $C_{u\varphi}$ and C_{uG}:

$$C_{u\varphi}(\mu) = C_{u\varphi}(m_t)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{\frac{4}{\beta_0}} + \frac{12}{7}C_{uG}(m_t)\left[\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{\frac{4}{\beta_0}} - \left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{2}{3\beta_0}}\right], \quad (51)$$

$$C_{uG}(\mu) = C_{uG}(m_t)\left(\frac{\alpha_s(\mu)}{\alpha_s(m_t)}\right)^{-\frac{2}{3\beta_0}}, \quad (52)$$

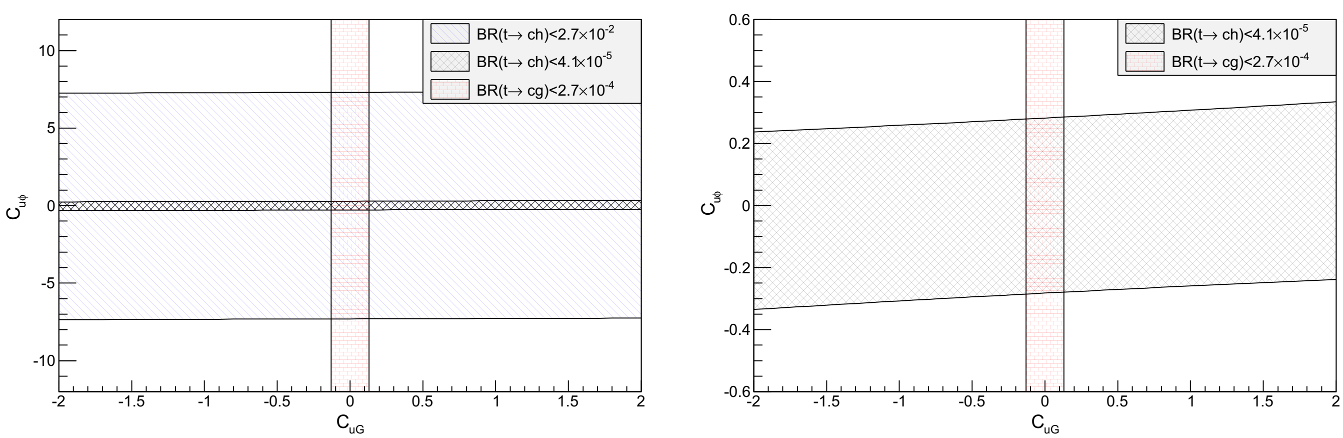

FIG. 5 (color online). Limits on C_{uG} and $C_{u\varphi}$ plane. Left: the blue region corresponds to current bound on branching ratio $\mathrm{BR}(t \to ch)$ from the LHC, the black region is the projected sensitivity for $t \to ch$, and the red region comes from bounds on $t \to cg$. Right: y axis is zoomed in to show the effects of C_{uG}.

where $\beta_0 = 11 - 2N_f/3$. The running of $C_{u\varphi}$ is affected by the operator O_{uG}. In Fig. 4, we show the μ dependence of both LO and NLO results for the width, with different values of C_{uG}. We can see that the renormalization scale dependence at LO can be quite large depending on the value of C_{uG}, and that it is greatly reduced at NLO in QCD.

Finally, for the sake of illustration, in Fig. 5 we plot the limits on the C_{uG} and $C_{u\varphi}$ plane. The region in the parameter space corresponding to the current bound $\mathrm{BR}(t \to ch) < 2.7\%$ from CMS [17] is shown, as well as the 95% upper limit for $t \to ch$ as estimated in Ref. [10], i.e. $\mathrm{BR}(t \to qh) < 4.1 \times 10^{-5}$ for an integrated luminosity of 100 fb^{-1}. In our results, the $|C_{uG}|^2$ term has been neglected and the next-to-next-to-leading-order top quark width result $\Gamma(t \to bW) = 1.39$ GeV [30] is used. The constraints on C_{uG} coming from $\mathrm{BR}(t \to cg) < 2.7 \times 10^{-4}$ [31] are also shown.

VII. CONCLUSION

We have presented a calculation for the decay width of $t \to u_i h$ in the EFT approach at NLO in QCD. Two operators contribute at LO, while at NLO two additional operators (and their mixing) need to be included. We find that QCD correction can reach the 10% level, depending on the relative size of these operators. The possibly large scale dependence of the LO results is tamed at NLO.

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