QCD: Restoration of chiral symmetry and deconfinement for large N_f

A. Bashir,[1] A. Raya,[1] and J. Rodríguez-Quintero[2]

[1] *Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Ciudad Universitaria, Morelia, Michoacán 58040, México*

[2] *Departamento de Física Aplicada, Facultad de Ciencias Experimentales, Universidad de Huelva, Huelva 21071, Spain*

(Received 23 February 2013; published 4 September 2013)

Exploiting the recent lattice results for the infrared gluon propagator with light dynamical quarks, we solve the gap equation for the quark propagator. We thus model the chiral symmetry-breaking mechanism with an increasing number of flavors and study confinement (intimately tied with the analytic properties of QCD Schwinger functions) order parameters. We obtain, with this approach, clear signals of chiral symmetry restoration and deconfinement when the number of light quark flavors exceeds a critical value of $N_f^c \approx 8 \pm 1$, in agreement with the state-of-the-art direct lattice analysis of chiral symmetry restoration in QCD.

DOI: 10.1103/PhysRevD.88.054003 PACS numbers: 12.38.−t, 11.15.Tk, 11.30.Rd

I. INTRODUCTION

QCD with a large number of massless fermion flavors has seen a resurgence of interest due to its connection with technicolor models, originally proposed by Weinberg and Susskind [1], which fall into the category of *beyond the Standard Model theories*. They possess intrinsically attractive features. They do not resort to fundamental scalars to reconcile local gauge symmetries with massive mediators of interactions and have a close resemblance to well-studied fundamental strong interactions, i.e., QCD. However, their simple versions do not live up to the experimental electroweak precision constraints, in particular the ones related to flavor-changing neutral currents. Walking models containing a conformal window and an infrared fixed point can possibly cure this defect and become phenomenologically viable [2]. This scenario motivates the investigation of QCD for similar characteristics. One looks for such behavior of QCD for a large number of light flavors, albeit less than the critical value where asymptotic freedom sets in, i.e., $N_f^{c_1} = 16.5$, a Nobel prize-winning result known since the advent of QCD, [3]. Just as N_f dictates the peculiar behavior of QCD in the ultraviolet, we expect it to determine the onslaught of its emerging phenomena in the infrared, i.e., chiral symmetry breaking and confinement.

Whereas the self-interaction of gluons provides antiscreening, the production of virtual quark-antiquark pairs screens and debilitates the strength of this interaction of non-Abelian origin. For real QCD, light flavors are small in number and hence yield to the gluonic influence which triggers confinement and chiral symmetry breaking. One needs to establish if there is another critical value $N_f^{c_2} < N_f^{c_1}$ which can sufficiently dilute the gluon-gluon interactions to restore chiral symmetry and deconfine color degrees of freedom. Such a phase transition lies at the nonperturbative boundary of the interactions under scrutiny and hence we cannot expect to extract sufficiently reliable information from multiloop calculations of the QCD β function. Purely nonperturbative techniques are required to tackle the problem. Lattice studies in the infrared indicate that just below $N_f^{c_1}$, chiral symmetry remains unbroken and color degrees of freedom are unconfined [4]. Below this conformal window, for $8 < N_f^{c_2} < 12$, the evolution of the beta function in the infrared is such that QCD enters the phase of dynamical mass generation as well as confinement.

Modern lattice analyses appear to argue in favor of a restoration of the chiral-symmetric phase taking place somewhere between $N_f \sim 8$ and $N_f \sim 10$ [5,6]. In particular, the authors of Ref. [6], with their study of the meson spectrum in lattice QCD with eight light flavors using the highly improved staggered quark action, gathered some striking evidences that $N_f = 8$ QCD still lies in the broken-chiral-symmetry phase but, at the same time, suffers the effects from a remnant of the infrared conformality (a large anomalous dimension for the quark-mass renormalization constant), indicating that the unbroken phase is recovered near above $N_f \sim 8$. In the present work, we intend to combine the Schwinger-Dyson machinery—which is well adjusted to account for QCD phenomenology in the pion sector—with the latest lattice data including twisted-mass dynamical light flavors in order to provide a model for the chiral restoration mechanism, in quantitative agreement with the above-mentioned lattice studies.

II. CHIRAL-PHASE-TRANSITION PICTURE FROM SCHWINGER-DYSON AND LATTICE GLUON PROPAGATORS

In the continuum, Schwinger-Dyson equations (SDEs) of QCD provide an ideal framework to study its infrared properties [7]. These are the fundamental equations of any quantum field theory, linking all its defining Green functions to one another through intricately coupled nonlinear integral equations. As their formal derivation through the variational principle makes no appeal to the weakness of the interaction strength, they naturally connect the perturbative ultraviolet physics with its emerging nonperturbative properties in the infrared sector within the same framework. The simplest two-point quark propagator is a basic object to analyze dynamical chiral symmetry breaking and confinement. Within the formalism of the SDEs, the inverse quark propagator can be expressed as $S^{-1}(p) = Z_2(i\gamma \cdot p + m) + \Sigma(p)$, where $\Sigma(p)$ is the quark self-energy,

$$\Sigma(p) = Z_1 \int \frac{d^4q}{(2\pi)^4} g^2 \Delta_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p), \tag{1}$$

where $Z_1 = Z_1(\mu^2, \Lambda^2)$ and $Z_2 = Z_2(\mu^2, \Lambda^2)$ are the renormalization constants associated, respectively, with the quark-gluon vertex and the quark propagator. Λ is the ultraviolet regulator and μ is the renormalization point. The solution to this equation is

$$S^{-1}(p) = \frac{i\gamma \cdot p + M(p^2)}{Z(p^2, \mu^2)}, \tag{2}$$

where $Z(p^2, \mu^2)$ is the quark wave-function renormalization and the quark-mass function $M(p^2)$ is renormalization-group invariant. This equation involves the quark-gluon vertex $\Gamma^a_\nu(q,p)$ and the gluon propagator $\Delta_{\mu\nu}(p-q)$.

A. Modeling the flavor behavior for the gluon propagator

As a consequence of a patient effort spanning several decades to unravel the gluon propagator $\Delta_{\mu\nu}$ in the infrared, lattice as well as SDE studies have finally converged on its massive or so-called decoupling solution; see for example Ref. [8]. After the gluon-propagator solution in the quenched approximation has been chiselled, we now have the first quantitatively reliable glimpses of its quark-flavor dependence by incorporating $N_f = 0, 2$ light dynamical quark flavors [9] and $2+1+1$ (two light degenerate quarks, with masses ranging from 20 to 50 MeV, and two nondegenerate flavors for the strange and the charm quarks, with their respective masses set to 95 MeV and 1.51 GeV) [16]. As we will demonstrate shortly, in this last $2+1+1$ case the number of light quarks corresponds effectively to 3. This is exactly the result derived from the recently developed "*partially unquenched*" approach to incorporate flavor effects in the gluon SDE [17]. Their work is in agreement with that of Ref. [16] when the charm flavor is assumed to decouple from gluons. In any case, this two-point function serves as a crucial input to study the quark propagator. The only other ingredient is the three-point quark-gluon vertex $\Gamma^a_\nu(q,p)$. Significant advances have been made in pinning it down through its key attributes in the ultraviolet and infrared domains [18]. More recently, the seeds of the most general ansatz for the fermion-boson vertex appeared in Ref. [19] and its full-blown extension was presented in Ref. [20]. Significantly, this ansatz contains nontrivial factors associated with those tensors whose appearance is expressly driven by dynamical chiral symmetry breaking in a perturbatively massless theory. This novel feature enables a direct and positive comparison with the best available symmetry-preserving solutions of the inhomogeneous Bethe-Salpeter equation for the vector vertex. This encouraging outcome indicates that this model is likely to provide a much-needed tool for use in Poincaré-covariant symmetry-preserving studies of hadron electromagnetic form factors. Furthermore, given the general nature of constraints and the simplicity of the construction, a straightforward extension of this approach is expected to yield an ansatz adequate for the task of representing the dressed-quark-gluon vertex. Before this is achieved, we restrict ourselves to an efficacious approach. Following the lead of Maris *et. al.* [21], we employ the following suitable ansatz which has sufficient integrated strength in the infrared to achieve dynamical mass generation,

$$Z_1 g^2 \Delta_{\mu\nu}(p-q)\Gamma_\nu(p,q) \to g^2_{\rm eff}(q^2)\Delta^N_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\nu, \tag{3}$$

where

$$\Delta^N_{\mu\nu}(q) = \frac{D(q^2)}{q^2}\left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]. \tag{4}$$

The effective coupling $g_{\rm eff}$ is chosen to correctly reproduce the static as well as dynamic properties of mesons below 1 GeV and reproduce perturbation theory in the ultraviolet; see, for example, the review in Ref. [22] and references therein. Moreover, our modern understanding of the flavor dependence of the gluon propagator provides us with the solid basis to use the following nonperturbative model [23],

$$D(q^2) = \frac{z(\mu^2)q^2(q^2+M^2)}{q^4 + q^2(M^2 - 13g^2\langle A^2\rangle/24) + M^2 m_0^2}, \tag{5}$$

to describe the gluon dressing renormalized in the momentum-subtraction scheme at $q^2 = \mu^2$. This model is based on the tree-level gluon propagator obtained with the renormalized Gribov-Zwanziger (RGZ) action [24] which has been shown to properly describe the lattice data in the infrared sector (see Refs. [23,25]). The overall factor $z(\mu^2)$ is introduced to guarantee the multiplicative renormalization prescription, namely, $D(\mu^2) = 1$, and implies no physical consequence as the effective coupling $g_{\rm eff}(q^2)$ is further adjusted to properly reproduce the meson phenomenology. We obtain the mass parameters of Eq. (5) by fitting it to the gluon-propagator lattice data analyzed in Ref. [16]. M^2 is related to the condensate of auxiliary fields, emerging merely to preserve locality for the RGZ action. A free fit of the lattice data suggests that it does not depend on the number of fermion flavors (we find $M^2 = 4.85\ \mathrm{GeV}^2$). The dimension-two gluon condensate $\langle A^2\rangle$ [26] and $m_0^2 = z(\mu^2)\lim_{q^2\to 0} q^2/D(q^2)$ are flavor dependent and we look for their best fits. In order to cover a wide range of possibilities within reason, we assume their evolution with the flavor number driven either by a simple linear scaling law,

$$m_0^{-1}(N_f) = m_0^{-1}(0)(1 - AN_f),\qquad g^2\langle A^2\rangle(N_f) = g^2\langle A^2\rangle(0)(1 - BN_f), \tag{6}$$

as data appear to suggest, or by an exponential law,

$$m_0^{-1}(N_f) = m_0^{-1}(0)e^{-AN_f},\qquad g^2\langle A^2\rangle(N_f) = g^2\langle A^2\rangle(0)e^{-BN_f}, \tag{7}$$

which allows for the possibility that the gluon propagator becomes infinitely massive only when the number of light quark flavors tends to infinity. The best fit of the m_0 and $g^2\langle A^2\rangle$ from lattice data will require $m_0(0) = 0.333$ GeV and $g^2\langle A^2\rangle(0) = 7.856$ in both cases; $A = 0.083$ and $B = 0.080$ for the linear case and $A = 0.095$ and $B = 0.091$ for the exponential one. Equation (5) now provides a prediction for the gluon propagator for arbitrarily large N_f, as can be seen in Fig. 1, while Fig. 2 shows the corresponding gluon propagator along with the lattice data superimposed [16]. We also include some very recent gluon-propagator data obtained from lattice simulations with four degenerate light twisted-mass flavors [27]. These new data are rather well described by Eq. (5) evaluated for the mass parameters extrapolated to $N_f = 4$ with Eq. (6) (see the zoomed plot in Fig. 3). This observation strongly supports that $N_f = 2 + 1 + 1$ gluon data indeed correspond to three light flavors.

Thus, we can efficaciously model the dilution of the gluon-gluon interactions with increasing flavor number in order to study the chiral restoration mechanism. We can

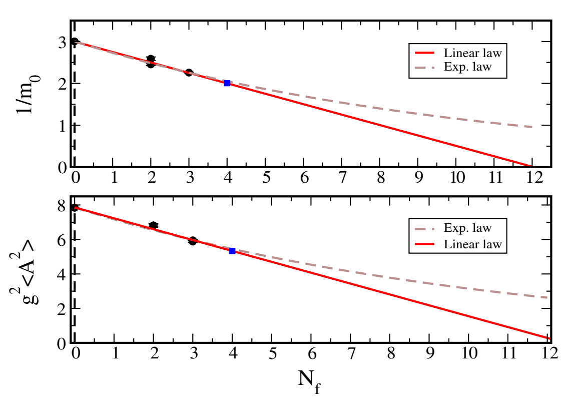

FIG. 1 (color online). Parameters $g^2\langle A^2\rangle$ and $1/m_0^2$ in terms of the numbers of flavors and the fits with Eqs. (6) and (7). The blue squares stand for the extrapolated results at $N_f = 4$ we used for Fig. 3.

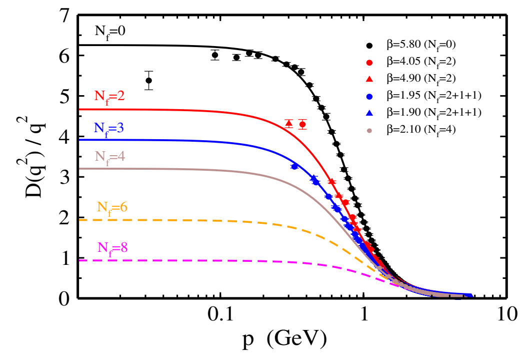

FIG. 2 (color online). Lattice gluon-propagator data in terms of momenta for different number of fermion flavors and fits with Eq. (5) and the parameters of Eq. (6).

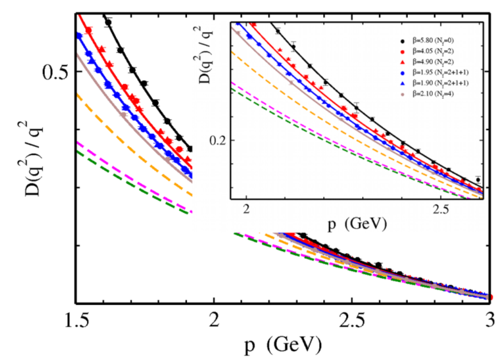

FIG. 3 (color online). The same as Fig. 2 but with the parameters of set 2 and incorporating new small-volume lattice data for four degenerate fermion flavors.

now employ the gap equation to provide quantitative details of chiral symmetry breaking in terms of the quark-mass function for an increasing number of light quarks.

B. Results

In the following, we mostly discuss the results obtained by employing the linear law and state the effect of exponential extrapolation afterwards. We take the effective coupling $g_{\mathrm{eff}}(q^2)$ to be independent of N_f, which is justified by the results of Ref. [16] [see Eq. (5.2)] which suggest that an effective coupling can be constructed such that there is an absence of any flavor dependence in the infrared region, more precisely starting from $q^2 \lesssim 1\ \mathrm{GeV}^2$. Note that we have not considered the flavor dependence that would arise from the quark-gluon vertex. No explicit handle on this dependence is available at the moment. Within the Abelian theory of QED, restrictions imposed by the all-order multiplicative renormalizability of the photon propagator may provide a handle on the transverse part of the electron-photon vertex; see the last reference in Ref. [18]. A consequent nonperturbative construction of such a vertex with imprints of the massless charged fermion flavors and its subsequent extension to QCD is still not available. Once the quark mass function is available for varying light quark flavors (see Fig. 4 for the linear case), one can investigate any of the interrelated order parameters, namely, the Euclidean pole mass defined as $m_{\rm dyn}^2 + M^2(p^2 = m_{\rm dyn}^2) = 0$, the quark-antiquark condensate which is obtained from the trace of the quark propagator or the pion leptonic decay constant f_π defined through the Pagel-Stokar equation [29], or through considering the residue at the pion pole of the meson propagator. Each of these quantities involves the quark wave-function renormalization, the mass function and/or its derivatives and is hence calculable from the solution for the full quark propagator. Moreover, these order parameters can help locate the critical number of flavors above which chiral symmetry is restored.

We investigate these three order parameters and choose to present here the Euclidean pole mass of the quark in Fig. 5 for the linear (exponential) case and show that, at a critical value of about $N_f^c \approx 7.1$ ($N_f^c \approx 9.4$), chiral symmetry appears restored. The phase transition appears to be second order, described by the following mean-field behavior (solid lines in Fig. 5):

$$m_{\rm dyn} \sim \sqrt{N_f^{c_2} - N_f}. \tag{8}$$

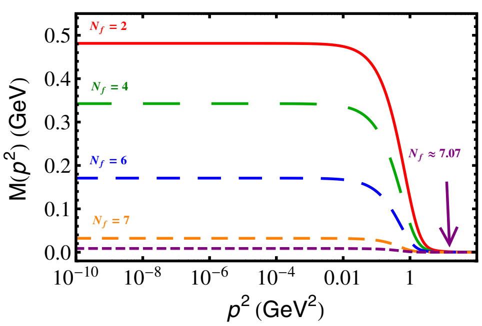

FIG. 4 (color online). The quark-mass function diminishes in height for increasing light quark flavors [here with Eq. (6)]. Above $N_f \approx 7.07$, only the chirally symmetric solution exists.

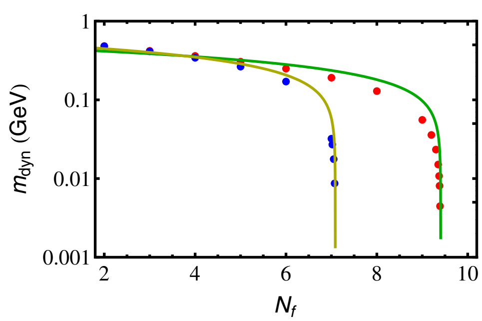

FIG. 5 (color online). The quark pole mass in the Euclidean space clearly demonstrates that chiral symmetry is restored above a critical number of quark flavors. Blue (red) points with smaller N_f^c (larger N_f^c)correspond to the linear (exponential) case. The gold (green) solid line is the mean-field scaling, Eq. (8).

This behavior of QCD resembles that of the toy version of QED with large electromagnetic coupling with or without the inclusion of four-fermion operators to render the theory closed [30].

It has been established that confinement is related to the analytic properties of QCD Schwinger functions which are the Euclidean-space Green functions, namely, propagators and vertices. One deduces from the reconstruction theorem [31] that the only Schwinger functions which can be associated with expectation values in the Hilbert space of observables—namely, the set of measurable expectation values—are those that satisfy the axiom of reflection positivity. When that happens, the real-axis mass pole splits, moving into pairs of complex conjugate singularities. No mass-shell can be associated with a particle whose propagator exhibits such a singularity structure. We define the Schwinger function

$$\Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4}\, e^{i(p_4 t + \mathbf{p}\cdot\mathbf{x})} \sigma_s(p^2) \tag{9}$$

to study the analytic properties of the quark propagator, where $\sigma_s(p^2)$ is the scalar term for the quark propagator in Eq. (2), which can be written in terms of the quark wave-function renormalization and mass function as $Z(p^2, \mu^2)M(p^2)/(p^2 + M(p^2))$. One can show that if there is a stable asymptotic state associated with this propagator, with a mass m, then $\Delta(t) \sim e^{-mt}$, whereas two complex conjugate mass-like singularities with complex masses $\mu = a \pm ib$ lead to an oscillating behavior of the sort $\Delta(t) \sim e^{-at}\cos(bt + \delta)$ for large t [32]. Figure 6 analyzes this function for varying N_f in the linear extrapolation case. The existence of oscillations clearly demonstrates that the quarks correspond to a confined excitation for small N_f. With increasing N_f, the onslaught of oscillations moves towards higher values of t and eventually never takes place above a critical N_f when quarks deconfine and correspond to a stable asymptotic state. As an order

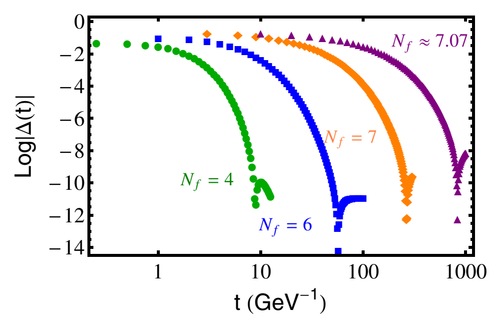

FIG. 6 (color online). The spatially averaged Euclidean-space two-point Schwinger function $\Delta(t)$ develops oscillations for large times which corresponds to the nonexistence of asymptotically stable free quark states. For sufficiently large values of N_f, the first minimum of these oscillations is pushed all the way to infinity, thus ensuring the existence of a pole on the time-like axis, a property of free-particle propagators.

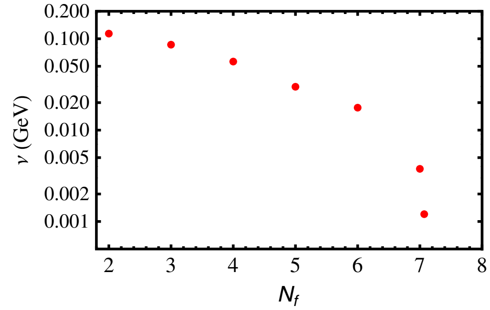

FIG. 7 (color online). The order parameter for confinement $\nu(N_f) = 1/\tau_1(N_f)$, where $\tau_1(N_f)$ is the location of the first zero of Eq. (9). A comparison with Fig. 5 suggests that quarks get deconfined when chiral symmetry is restored.

parameter of confinement, we therefore employ $\nu(N_f) = 1/\tau_1(N_f)$, where $\tau_1(N_f)$ is the location of the first singularity [33]. The first oscillation is pushed to infinity when confinement is lost. It is notable that when the dynamically generated mass approaches zero, $\nu(N_f)$ diminishes rapidly (see Fig. 7). This highlights the intimate connection between chiral symmetry restoration and deconfinement. In fact, within our numerical accuracy, N_f^c is found to be the same for both the transitions.

The results with the exponential and linear flavor extrapolations are qualitatively the same, leading to identical conclusions. They only quantitatively differ by the critical flavor numbers, although both are pretty much in the same ballpark: $N_f^c \simeq 7.1$ and $N_f^c \simeq 9.4$. Note that both the parametrizations—so far apart as to have an infinitely massive gluon at $N_f \approx 12$ or $N_f \Rightarrow \infty$—restore chiral symmetry and trigger deconfinement at so similar a value of light quark flavors.

III. CONCLUSIONS

We have performed a Poincare-covariant SDE analysis of the latest lattice results for the quark-flavor dependence of the gluon propagator in the infrared, provided a model for the dilution of the gluon-gluon interaction with an increasing number of light quarks and finally provided a picture for the chiral restoration mechanism. The quantitative analysis, following this approach, hints at the restoration of chiral symmetry and deconfinement in QCD when the number of light quark flavors exceeds a critical value of $N_f^{c_2} \approx 8.2 \pm 1.2$. This is in perfect agreement with the state-of-the-art lattice investigations of chiral symmetry restoration in QCD [5,6] and shows that the model presented here for the chiral restoration mechanism is properly capturing the relevant physics for the problem. That being said, it will surely be illuminating to incorporate and study the effect of the flavor-dependent quark-gluon vertex and, moreover, solve the coupled system of the Green functions involved simultaneously. All of this is for future work.

ACKNOWLEDGMENTS

We acknowledge D. Schaich for a fruitful communication. This work was supported by the grants: CIC, UMICH, Mexico, 4.10 and 4.22, CONACyT (Mexico) No. 82230 and No. 128534, and MINECO (Spain) research Project No. FPA2011-23781.

[1] S. Weinberg, Phys. Rev. D **13**, 974 (1976); S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).
[2] B. Holdom, Phys. Lett. **150B**, 301 (1985); K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. W. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).
[3] D. J. Gross and F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973); H. D. Politzer, Phy. Rev. Lett. **30**, 1346 (1973).
[4] T. Appelquist *et al.*, Phys. Rev. Lett. **104**, 071601 (2010); T. Appelquist, G. T. Fleming, and E. T. Neil, Phys. Rev. D **79**, 076010 (2009); Z. Fodor, K. Holland, J. Kuti, D. Nógrádi, and C. Schroeder, Phys. Lett. B **681**, 353 (2009); K.-I. Nagai, M. Carrillo-Ruiz, G. Koleva, and R. Lewis, Phys. Rev. D **80**, 074508 (2009); L. Del Debbio, B. Lucini, A. Patella, C. Pica, and A. Rago, Phys. Rev. D **82**, 014510 (2010); A. Hasenfratz, Phys. Rev. D **82**, 014506 (2010).
[5] M. Hayakawa, K.-I. Ishikawa, Y. Osaki, S. Takeda, S. Uno, and N. Yamada, Phys. Rev. D **83**, 074509 (2011); T. Appelquist, R. C. Brower, M. I. Buchoff, M. Cheng, S. D. Cohen, G. T. Fleming, J. Kiskis, M. Lin, H. Na, E. T. Neil, J. C. Osborn, C. Rebbi, D. Schaich, C. Schroeder, G. Voronov, and P. Vranas, arXiv:1204.6000; Y. Iwasaki, arXiv:1212.4343; A. Cheng, A. Hasenfratz, G. Petropoulos, and D. Schaich, J. High Energy Phys. 07 (2013) 061.
[6] Y. Aoki, T. Aoyama, M. Kurachi, T. Maskawa, K-I Nagai, H. Ohki, A. Shibata, K. Yamawaki, and T. Yamazaki, Phys. Rev. D **87**, 094511 (2013).
[7] F. Dyson, Phys. Rev. **75**, 1736 (1949); J. S. Schwinger, Proc. Nat. Acad. Sci. **37**, 452 (1951); J. S. Schwinger, Proc. Nat. Acad. Sci. **37**, 455 (1951).
[8] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, Phys. Lett. B **676**, 69 (2009); A. C. Aguilar, D. Binosi, and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008); P. Boucaud, J. Leroy, A. L. Yaouanc, J. Micheli, O. Péne, and J. Rodríguez-Quintero, J. High Energy Phys. 06 (2008) 099; D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. D **78**, 065047 (2008).

[9] The dynamical flavors have been generated, within the framework of the ETM collaboration [10–13], with the mass-twisted lattice action [14], while $N_f = 0$ data have been borrowed from Ref. [15].
[10] B. Blossier, Ph. Boucaud, F. De soto, V. Morenas, M. Gravina, O. Pène, and J. Rodríguez-Quintero (ETM Collaboration), Phys. Rev. D **82**, 034510 (2010).
[11] B. Blossier, Ph. Boucaud, M. Brinet, F. De Soto, X. Du, M. Gravina, V. Morenas, O. Pène, K. Petrov, and J. Rodríguez-Quintero, Phys. Rev. D **85**, 034503 (2012),
[12] R. Baron *et al.*, J. High Energy Phys. 06 (2010) 111.
[13] R. Baron *et al.* (ETM Collaboration), Proc. Sci., LATTICE (**2010**) 123.
[14] R. Frezzotti *et al.* (Alpha Collaboration), J. High Energy Phys. 08 (2001) 058.
[15] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Proc. Sci., LAT (**2007**) 290.
[16] A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti and J. Rodriguez-Quintero, Phys. Rev. D **86**, 074512 (2012).
[17] A. C. Aguilar, D. Binosi, and J. Papavassiliou, Phys. Rev. D **86**, 014032 (2012).
[18] J. S. Ball and T-W. Chiu, Phys. Rev. D **22**, 2542 (1980); D. C. Curtis and M. R. Pennington, Phys. Rev. D **42**, 4165 (1990); Z. Dong, H. J. Munczek, and C. D. Roberts, Phys. Lett. B **333**, 536 (1994); A. Bashir and M. R. Pennington, Phys. Rev. D **50**, 7679 (1994); A. Bashir, A. Kizilersu, and M. R. Pennington, Phys. Rev. D **62**, 085002 (2000); A. Kizilersu and M. R. Pennington, Phys. Rev. D **79**, 125020 (2009).
[19] A. Bashir, A. Raya, and S. Sánchez-Madrigal, Phys. Rev. D **84**, 036013 (2011).
[20] A. Bashir, R. Bermudez, L. Chang, and C. D. Roberts, Phys. Rev. C **85**, 045205 (2012).
[21] P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B **420**, 267 (1998); P. Maris and C. D. Roberts, Phys. Rev. C **58**, 3659 (1998).
[22] A. Bashir, L. Chang, I. C. Cloët, B. El-Bennich, Y-X. Liu, C. D. Roberts, and P. C. Tandy, Commun. Theor. Phys. **58**, 79 (2012).
[23] D. Dudal, O. Oliveira and J. Rodriguez-Quintero, Phys. Rev. D **86**, 105005 (2012).
[24] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. D **78**, 065047 (2008).
[25] D. Dudal, O. Oliveira, and N. Vandersickel, Phys. Rev. D **81**, 074505 (2010).
[26] P. Boucaud, A. Le Yaouanc, J. P. Leroy, J. Micheli, O. Pene, and J. Rodríguez-Quintero, Phys. Lett. B **493**, 315 (2000); F. V. Gubarev and V. I. Zakharov, Phys. Lett. B **501**, 28 (2001); K.-I. Kondo, Phys. Lett. B **514**, 335 (2001); H. Verschelde, K. Knecht, K. Van Acoleyen, and M. Vanderkelen, Phys. Lett. B **516**, 307 (2001); D. Dudal, H. Verschelde, and S. P. Sorella, Phys. Lett. B **555**, 126 (2003); E. Ruiz Arriola, P. O. Bowman and W. Broniowski, Phys. Rev. D **70**, 097505 (2004).
[27] The gluon propagator lattice data for four light flavors have been taken from ETMC [28]. Simulated at small volumes, they are only available for momenta above 1.25 GeV and hardly allow for a fit with Eq. (5). Nevertheless, they can be used to check our modeling of the flavor evolution.
[28] B. Blossier *et al.* (ETM Collaboration) (to be published).
[29] H. Pagels and S. Stokar, Phys. Rev. D **20**, 2947 (1979).
[30] A. Bashir, C. Calcaneo-Roldan, L. X. Gutierrez-Guerrero, and M. E. Tejeda-Yeomans, Phys. Rev. D **83**, 033003 (2011); F. Akram, A. Bashir, L. X. Gutierrez-Guerrero, B. Masud, J. Rodriguez-Quintero, C. Calcaneo-Roldan, and M. E. Tejeda-Yeomans, Phys. Rev. D **87**, 013011 (2013).
[31] R. F. Streater and A. S. Wightman, *PCT, spin and statistics, and all that* (Princeton University Press, Princeton, 1989); J. Glimm and A. Jaffee, *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York, 1981).
[32] P. Maris, Phys. Rev. D **52**, 6087 (1995).
[33] A. Bashir, A. Raya, S. Sánchez-Madrigal, and C. D. Roberts, Few Body Syst. **46**, 229 (2009).