PHYSICAL REVIEW D **88,** 046001 (2013)

Higher-spin realization of a de Sitter static patch/cut-off CFT correspondence

Andreas Karch[1,*] and Christoph F. Uhlemann[2,†]

[1]*Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA*

[2]*Institut für Theoretische Physik und Astrophysik, Universität Würzburg, 97074 Würzburg, Germany*

(Received 12 June 2013; published 5 August 2013)

We derive a holographic relation for the de Sitter (dS) static patch with the dual field theory defined on the observer horizon. The starting point is the duality of higher-spin theory on AdS_4 and the $O(N)$ vector model. We build on a similar analytic continuation as used recently to obtain a realization of dS/CFT and adapt it to the static patch. The resulting duality relates higher-spin theory on the dS_4 static patch to a cutoff conformal field theory on the cylinder $\mathbb{R} \times S^2$. The construction permits a derivation of the finite thermodynamic entropy associated to the horizon of the static patch from the dual field theory. As a further brick, we recover the spectrum of quasinormal frequencies from the correlation functions of the boundary theory. In the last part, we incorporate the dS/dS correspondence as an independent proposal for holography on dS and show that a concrete realization can be obtained by similar reasoning.

DOI: 10.1103/PhysRevD.88.046001 PACS numbers: 11.25.Tq, 04.50.−h, 04.62.+v

I. INTRODUCTION

The AdS/CFT correspondence [1] has stimulated remarkable progress in the understanding of gauge theories. Moreover, it provides a means to study quantum aspects of gravity with asymptotically anti–de Sitter (AdS) boundary conditions in terms of the dual conformal field theory (CFT). However, our Universe is likely not asymptotically AdS. It would therefore be desirable to have a holographic definition of (quantum) gravity in terms of a dual boundary theory also on the physically more directly relevant de Sitter (dS) space. There has indeed been a proposal for a dS/CFT correspondence [2], which exploits the conformal properties of dS to establish a dual CFT description on the spacelike conformal boundary at future/past infinity $\mathcal{I}^{\pm}$. With the explicit realization obtained in Ref. [3], this proposal has recently been lifted to a very concrete level. However, with the dual CFT defined at $\mathcal{I}^{\pm}$, these dS/CFT correspondences are formulated in terms of dS meta-observables, accessible only to an unphysical meta-observer [4].

Restricting to only the region accessible to a physical observer crucially complicates things. There is no notion of a conformal boundary, and we instead only have the horizon and the observer worldline as distinguished places. Moreover, this region is symmetric only under a subgroup of the dS isometries. Nevertheless, understanding quantum gravity on the region accessible to a single observer arguably is the most interesting question to pose. We will therefore aim to explicitly realize holography in that setting. Motivation for the existence of a holographic description comes in the first place from the Bekenstein bound, which certainly suggests that there is a holographic description also for gravity on the dS static patch. The screen may, in principle, be anywhere. For the flat slicing of dS, the conformal boundary is a preferred place since the full dS isometry group nicely acts on it. With that option unavailable for the static patch, the possibility of a dual quantum mechanics description on the observer worldline has been investigated in Ref. [5]. However, the covariant construction of holographic screens in generic spacetimes [6] suggests that the screen is at the horizon.

In this paper we aim to make this discussion more precise. Similarly to Ref. [3], we shall start from AdS/CFT dualities involving higher-spin theories [7,8] in the bulk, which may be seen as tensionless limits of string theory. More precisely, the bulk theory is the parity-invariant minimal bosonic version of Vasiliev gravity, with massless symmetric tensor fields of all even spins. We will exploit that there is a nice analytic continuation from AdS to dS for that theory [9]. This will allow us to derive from the well-understood Giombi-Klebanov-Polyakov-Yin (GKPY) AdS/CFT duality [10,11] by a double Wick rotation a dual description for higher-spin gravity on the static patch of dS_4. The dual theory will be defined on the observer horizon and will be a cutoff version of the $Sp(N)$ CFT_3 of anticommuting scalars [12], which was obtained as dual theory at $\mathcal{I}^{+}$ in Ref. [3]. Without the geometric bells and whistles which are at the heart of the more conventional (A)dS/CFT dualities, establishing an analog of the bulk-boundary dictionary is a bit more subtle. Building on the discussion of horizon holography in Ref. [13], we will work out in detail how such a dictionary can be realized for the static patch of dS and present some first applications. We then turn to the dS/dS correspondence proposed in Refs. [14,15], where the dual theories are similarly defined at a horizon. It provides an independent approach to dS holography, and we will adapt our construction to also obtain a concrete realization.

The outline is as follows. In Sec. II we discuss an analytic continuation relating the dS static patch to an inner

*akarch@u.washington.edu
†uhlemann@physik.uni-wuerzburg.de

shell of Euclidean AdS. In Sec. III we build on the role of the AdS radial coordinate as an energy scale in the dual CFT to analytically continue a cutoff version of AdS/CFT to a holographic relation for the static patch. This will first be restricted to an inner region before we recover the entire static patch in Sec. III B. The static-patch entropy will be derived from the dual theory in Sec. III C. We then study the duality more explicitly for bulk scalar fields in Sec. IV. The realization of dS/dS correspondences will be derived in Sec. V, and we conclude in Sec. VI.

II. DS STATIC PATCH AS PART OF ADS

After reviewing the analytic continuation from dS to AdS as used in Ref. [3], we will in this section derive a similar relation of the dS static patch to an inner shell of AdS. We start off from the well-known fact that dS_{d+1} of signature $(-,+\cdots+)$ and Euclidean AdS can both be defined as hyperboloids in $(d+2)$-dimensional flat space with metric $\eta=\mathrm{diag}(-1,1,\ldots,1)$ by

$$\begin{aligned}\mathrm{dS}_{d+1}&:\ -X_0^2+\sum_{i=1}^{d+1}X_i^2=H^2,\\ \mathrm{AdS}_{d+1}&:\ -X_0^2+\sum_{i=1}^{d+1}X_i^2=-L^2.\end{aligned}\tag{1}$$

Correspondingly, their symmetry groups $\mathrm{SO}(1,d+1)$ coincide. The defining equations are related by $H=iL$, and this can be exploited to relate their coordinatizations as follows. The usual Poincaré coordinates can be introduced on AdS_{d+1} by solving Eq. (1) in terms of

$$\begin{aligned}X_{0/1}&=\frac{u}{2}\left(1+\frac{1}{u^2}(\vec{x}^2\pm L^2)\right),\\ X_i&=\frac{Lx^{i-1}}{u},\quad \forall\ i=2,\ldots,d+1,\end{aligned}\tag{2}$$

which results in the line element $ds^2=L^2u^{-2}(du^2+d\vec{x}^2)$. The coordinates cover all of Euclidean AdS, and $u=0$ corresponds to almost all of the conformal boundary. The flat slicing of Lorentzian dS space, which covers half of the hyperboloid, can now be obtained by the analytic continuation used in Ref. [3],

$$L=iH,\qquad u=i\eta.\tag{3}$$

That results in $X_{0/1}\to iX_{0/1}$, such that their roles are exchanged and we are dealing with a double Wick rotation in the ambient space. The resulting line element $ds^2=H^2\eta^{-2}(-d\eta^2+d\vec{x}^2)$ corresponds to the flat slicing of dS_{d+1}, where u has become the time coordinate. Building on that simple geometric identification along with the analytic continuation from AdS to dS for Vasiliev's higher-spin theory via Eq. (3), a concrete realization of dS/CFT has been derived in Ref. [3].

We will now discuss a similar relation of the dS static patch to an inner shell of Euclidean AdS. To this end we turn to a different global coordinatization of Euclidean AdS. This is obtained by solving Eq. (1) in terms of

$$\begin{aligned}X_0&=\sqrt{L^2+r^2}\cosh\tau,\qquad X_1=\sqrt{L^2+r^2}\sinh\tau,\\ X_i&=rz_i\quad\forall\ i=2,\ldots,d+1,\end{aligned}\tag{4}$$

where the z_i parametrize the sphere S^{d-1}, i.e. $\sum_i z_i^2=1$. Euclidean AdS_{d+1} as the interior of a unit ball B^{d+1} is covered by these coordinates as follows. Sections of fixed time correspond to fixed latitude. The north/south poles correspond to $t=\pm\infty$. The axis through them is $r=0$, and the surface of the ball with the two poles removed corresponds to $r=\infty$. Intermediate r interpolates between these two extremes. The boundary at $r=\infty$ therefore is a cylinder $\mathbb{R}\times S^{d-1}$. Adding the two points corresponding to $t=\pm\infty$ completes the boundary to S^{d-1}. The resulting line element takes the form

$$ds^2=(L^2+r^2)d\tau^2+\frac{1}{1+r^2/L^2}dr^2+r^2d\Omega_{d-1}^2.\tag{5}$$

The rescaling of the time coordinate as compared to the standard form of that metric is just for technical convenience. The parametrizations (2) and (4) can be combined to derive the coordinate transformation connecting them. We then straightforwardly find that the transformation corresponding to Eq. (3) with $X_{0/1}\to iX_{0/1}=:\tilde{X}_{1/0}$ is in the coordinates (4) realized by simply setting

$$H=iL.\tag{6}$$

With that analytic continuation, the coordinatization of Euclidean AdS (4) becomes the dS parametrization

$$\begin{aligned}\tilde{X}_0&=\sqrt{H^2-r^2}\sinh\tau,\qquad \tilde{X}_1=\sqrt{H^2-r^2}\cosh\tau,\\ X_i&=rz_i\quad\forall\ i=2,\ldots,d+1.\end{aligned}\tag{7}$$

The resulting line element is—up to a rescaling of the time coordinate—that of the usual static-patch metric and reads

$$ds^2=-(H^2-r^2)d\tau^2+\frac{1}{1-r^2/H^2}dr^2+r^2d\Omega_{d-1}^2.\tag{8}$$

For $r\in[0,H)$ Eq. (7) parametrizes the static patch of dS which is thus related to the inner shell $r\in[0,L)$ of Euclidean AdS, as illustrated in Fig. 1. The transformation (6) then directly realizes the transformation discussed above in Eq. (3) and used in Ref. [3]. It is obtained by simply transforming coordinates on both sides of that identification. The relation of the dS static patch to only the inner shell of AdS reflects the fact that of the $\mathrm{SO}(1,d+1)$ isometries of dS_{d+1} only an $\mathrm{SO}(d)\times\mathbb{R}$ subgroup, corresponding to the symmetries of S^{d-1} and the timelike Killing field, preserves the horizon. Likewise, restricting to the inner shell of AdS also breaks the radial isometries. The parametrization (7) can also be continued to $r>H$, where r becomes timelike and t spacelike. The roles of X_0 and X_1 are simply switched then, again

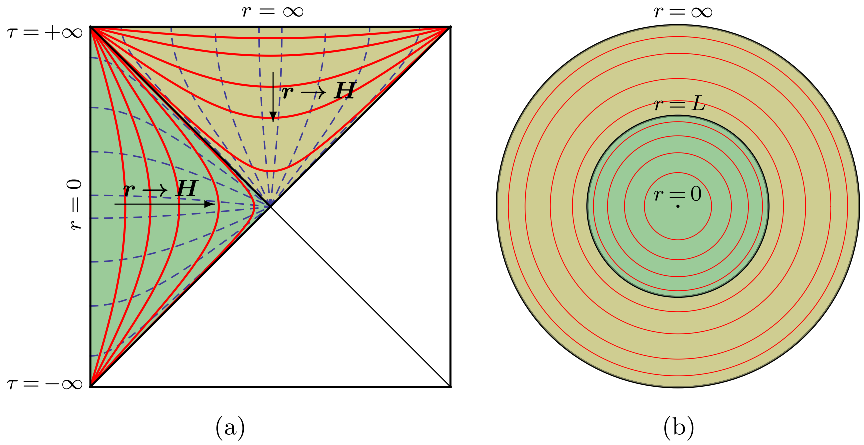

FIG. 1 (color online). The static patch of global dS and the extension to $r > H$ covering the expanding Poincaré patch are shown on the left-hand side. The solid red and dashed blue curves correspond to constant r and τ, respectively. Note the exchange of timelike and spacelike characters for these curves when extending to $r > H$. On the right-hand side is a section of constant τ, i.e. of constant latitude, through Euclidean AdS realized as the interior of a unit ball with coordinates (5). Thanks to the Killing vector field ∂_τ, the sections are all equivalent. The analytic continuation (6) identifies the inner green and outer yellow regions with the left and upper triangles of dS as shown on the left-hand side, respectively.

by a double Wick rotation in the ambient space. Only for $r = H$, corresponding to the horizon of the static patch, the transformation becomes singular. The extension including $r > H$ relates almost all of the dS Poincaré patch to global AdS. For definiteness we choose the expanding patch, such that the conformal boundary of AdS is mapped to the spacelike conformal boundary of dS at $\mathcal{I}^+$. The boundary arising if a finite cutoff is imposed on r is timelike/spacelike so long as the cutoff is below/above the radius of curvature H.

III. STATIC PATCH HOLOGRAPHY FROM ADS/CFT

Building on the higher-spin realization of dS/CFT via analytic continuation of AdS/CFT in Ref. [3] and the identification of the static patch as part of AdS, we will now attempt to realize static patch holography. This will strongly build on the role of the AdS radial coordinate as an energy scale in the dual CFT. Since many of the arguments do not depend on the spacetime dimension, we will mostly keep it general and only specialize to (A)dS$_4$ for certain points.

A. From cutoff AdS/CFT to cutoff dS/CFT

In AdS/CFT cutting off the infrared part of the bulk geometry corresponds to a UV modification of the dual CFT and realizes a high-energy cutoff [16,17]. In the semiclassical limit, the values of on-shell bulk fields at fixed radial position can then be interpreted as running couplings in the dual CFT, and the bulk field equations were related to renormalization group equations of the boundary theory in Ref. [18]. Recently, more systematic approaches to a holographic realization of the Wilsonian renormalization group have been discussed in Refs. [19,20]. These discussions employed AdS in Poincaré coordinates. To fix notation and set the stage, we now discuss the analog in the coordinates (4) and (5).

We recall that AdS is conformally compact and choose a function $f = 1/r$, defining the boundary via $\partial\mathcal{M} := \{p \in \mathcal{M} | f(p) = 0\}$. The rescaled metric $\bar{g} := f^2 g$ then induces a representative of the boundary conformal structure on the conformal boundary. For explicitness we consider a bulk Klein-Gordon field ϕ of mass $m^2 = -\Delta_+\Delta_- L^{-2}$ with $\Delta_\pm = d/2 \pm \nu$ in the following, but we expect similar results for fields of higher spin. The asymptotic expansion of solutions is $\phi = f^{\Delta_-}\varphi_- + f^{\Delta_+}\varphi_+$, and we choose the standard quantization where the boundary-dominant part φ_- is interpreted as a source for the dual operator. The AdS/CFT prescription then reads

$$\mathcal{Z}[\varphi_-] := \int \mathcal{D}\phi|_{\phi\to f^{\Delta_-}\varphi_-} e^S = \left\langle \exp\left\{\int d^d x \sqrt{g_\infty}\varphi_-(x)\mathcal{O}(x)\right\}\right\rangle_{\mathrm{CFT}}. \quad (9)$$

On the right-hand side, g_∞ denotes the representative of the boundary conformal structure as explained above. We then introduce a fixed value $r_\kappa := \kappa$ for the radial coordinate and split the path integral into the parts corresponding to $r \le r_\kappa$ and $r \ge r_\kappa$

$$\Psi_{\mathrm{IR}}[\phi_\kappa] := \int^{\phi(r_\kappa)=f^{\Delta_-}\phi_\kappa} \mathcal{D}\phi|_{r\le r_\kappa} e^{S+S_\kappa},$$
$$\Psi_{\mathrm{UV}}[\varphi_-, \phi_\kappa] := \int_{\phi(r_\kappa)=f^{\Delta_-}\phi_\kappa}^{\phi\to f^{\Delta_-}\varphi_-} \mathcal{D}\phi|_{r\ge r_\kappa} e^{S-S_\kappa}. \quad (10)$$

$S_\kappa = S_\kappa[\phi_\kappa]$ is an arbitrary boundary action at $r = r_\kappa$, which just introduces a multiplicative renormalization of both objects since it is fixed by the boundary condition at r_κ and can be pulled out of the path integral. We have normalized the boundary condition at r_κ such that we can conveniently take the limit $\kappa \to \infty$. The full path integral becomes

$$\mathcal{Z}[\varphi_-] = \int \mathcal{D}\phi_\kappa \Psi_{\rm IR}[\phi_\kappa]\Psi_{\rm UV}[\varphi_-, \phi_\kappa]. \quad (11)$$

Following Refs. [19,20], the generating function for correlators in the CFT with a cutoff at an energy scale Λ_κ is then identified with $\Psi_{\rm IR}$ by

$$\Psi_{\rm IR}[\phi_\kappa] = \left\langle \exp\left\{\int d^d x\sqrt{g_\kappa}\phi_\kappa \mathcal{O}\right\}\right\rangle_{{\rm CFT},\Lambda_\kappa}. \quad (12)$$

Since Eq. (12) restricts the bulk theory to a part of AdS, the $\mathsf{SO}(1, d+1)$ bulk isometries are broken to those preserving the radial cutoff, $\mathbb{R} \times \mathsf{SO}(d)$. In the dual theory, that corresponds to the breaking of conformal invariance by the UV cutoff. The analog in Poincaré coordinates preserves the boundary Euclidean symmetries $\mathsf{SO}(d)\ltimes\mathbb{R}^d$, and we recover the fact that while the conformal symmetries of the cylinder and the plane agree, their isometries do not. The metric on the right-hand side would naturally be that induced by $\bar{g}$ at $r = r_\kappa$. However, since the boost symmetries mixing the time and spatial directions are broken anyway, we can also use $g_{\kappa,\tau\tau} = (L^2 + f^{-2})^{-1} g_{\tau\tau}$ and $g_{\kappa,ij} = f^2 g_{ij}$ to extract the CFT metric. Asymptotically that becomes equivalent to $\bar{g}$, and we recover the usual induced conformal structure. For finite κ this keeps the time component of the boundary metric normalized and ensures that, similarly to the prescription in Poincaré coordinates, changes in κ have a purely field-theoretic interpretation.[1]

1. Analytic continuation to dS

Combining the above discussions with the arguments used in Ref. [3], we can now derive a holographic description for the static patch with radial cutoff as follows. We start in the same way from the GKPY duality relating the $\mathsf{O}(N)$ CFT_3 to the minimal version of Vasiliev's higher-spin theory on AdS_4. However, for the bulk AdS, we employ the coordinates (4) and (5), such that the dual CFT at $r \to \infty$ is defined on the cylinder. We then use the same analytic continuation, which in our coordinates is realized by Eq. (6), but apply it to the cutoff AdS/CFT prescription (12) with $\kappa < L$. That transforms the cutoff AdS bulk geometry to the corresponding part of the dS static patch with $r < \kappa$, while the Euclidean CFT_3 on the cylinder is Wick rotated to Lorentzian signature. For the scalar discussed above, it also switches the sign of the mass, in agreement with Ref. [9]. Following Ref. [3] we note that, since $N \propto (\Lambda G_{\rm N})^{-1}$ in the GKPY duality, this should on the CFT side be combined with $N \to -N$. A little more formally, we obtain

$$\Psi^{\rm dS}_{\rm IR}[\phi_\kappa] := \Psi_{\rm IR}[\phi_\kappa]|_{L=iH} = \left\langle \exp\left\{\int d^d x\sqrt{g_\kappa}\phi_\kappa \mathcal{O}\right\}\right\rangle_{{\rm CFT},\Lambda_\kappa,\tau\to i\tau,N\to -N}. \quad (13)$$

This results in a duality of higher-spin gravity on a part of the dS static patch and a cutoff version of the $\mathsf{Sp}(N)$ CFT_3 of Ref. [3] on the Lorentzian-signature cylinder. Since we have restricted to κ smaller than the radius of curvature, the bulk coordinates are regular on both the dS and AdS sides of the analytic continuation. The cylinder as boundary geometry arises straightforwardly from the bulk, and we have a dual description for the static patch with a radial cutoff. Note that this derivation is valid for the cutoff arbitrarily close to the horizon. While the proposal is rather straightforward to implement from the bulk perspective, its interpretation on the CFT side poses some nontrivial questions. In the AdS/CFT picture, the radial direction is understood to be encoded in the CFT as renormalization group flow and thus has a clear interpretation. In the usual dS/CFT setting on the other hand, the time direction itself has to emerge in a nontrivial way from the CFT. It is not quite clear therefore how cutting off, e.g., the upper yellow-shaded triangle of the dS geometry in Fig. 1 to arrive at the static patch is reflected in the CFT at $\mathcal{I}^+$. A related issue is that the $\mathsf{Sp}(N)$ CFT becomes nonunitary when Wick rotated to Lorentzian signature. While such a continuation is not desired in Ref. [3], this is what happens with the cutoff version of the theory in Eq. (13). How a UV cutoff can restore unitarity has been investigated in Ref. [22], and one could hope for a similar mechanism to be realized here.

B. Holography for the static patch

We now want to obtain a duality defined on the entire dS static patch. To this end we have to consider the analytic continuation of the cutoff AdS/CFT duality (12) to a cutoff static-patch holography (13) in the limit where κ approaches the radius of curvature. There are no particular complications arising for the AdS bulk theory with cutoff at $r = L$, and it simply corresponds to the $\mathsf{O}(N)$ CFT_3 on the cylinder with a particular value for the UV cutoff. We could thus perform calculations on both sides of the AdS/CFT correspondence and then define the dS static patch/cutoff CFT picture by analytic continuation. As discussed above, for $\kappa < L$ there is also no problem in the analytic continuation of the bulk $\leftrightarrow$ boundary dictionary itself from Eqs. (12) to (13) to obtain a duality which is intrinsically defined on the dS static patch.

However, for the holographic dictionary intrinsically on the static patch, the limit $\kappa \to H$ is nontrivial due to the infinite redshift factor in the bulk metric at the horizon. As a result, only the S^{d-1} part of the boundary cylinder

[1] Alternatively, we can take as induced geometrical data on the boundary that appropriate for a nonrelativistic theory, i.e. a spatial metric along with an orthogonal timelike one-form, as discussed in Ref. [21].

$\mathbb{R}\times S^{d-1}$ arises naturally from the bulk geometry in position space: sending r to H with the other coordinates fixed reduces the bulk geometry by two dimensions, as can be seen in Fig. 1(a) from the fact that the constant-τ surfaces meet at a point on the horizon. That challenges the interpretation of $\phi|_{r=H}$ as a source in the dual CFT and obscures the boundary geometry. It is therefore convenient for the formulation of a holographic dictionary for the entire static patch to exploit the existence of a timelike Killing field and employ the Fourier transform, following Ref. [13]. For notational convenience we change the radial coordinate to $r = H\operatorname{sech}\frac{z}{H}$ such that the horizon $r\to H$ corresponds to $z\to 0$. As we shall verify in Sec. IV, the asymptotic form of the bulk field as $z\to 0$ is given by

$$\phi(\tau, z, x) = \int \frac{d\omega}{\sqrt{2\pi}} e^{-i\omega\tau}(\varphi^+_\omega(z,x)z^{i\omega} + \varphi^-_\omega(z,x)z^{-i\omega}), \tag{14}$$

where $\varphi^\pm_\omega$ have regular power series expansions in z. These correspond to the presence of left- and right-moving modes near the horizon,[2] and we have to adapt the Dirichlet boundary condition accordingly. The situation is actually similar to a scalar field on AdS with mass below the Breitenlohner-Freedman stability bound, see Appendix A of Ref. [23], and we derive admissible boundary conditions from the demand that we have to find a well-defined symplectic structure. We focus on the standard Klein-Gordon product defined from the canonical symplectic current $j_\mu = i\phi_1 \overleftrightarrow{\partial}_\mu \phi_2$, noting that other choices may be of interest as well [24]. The flux through a surface of constant r approaching the horizon is given by

$$\begin{aligned}\mathcal{F} &= \int_{r\to H^-} d^d x\sqrt{-g_{\text{ind}}}\, n^\mu j_\mu \\ &= \int_{S^{d-1}}\int d\omega 2\omega H^{d-1}(\varphi^+_{1,\omega}\varphi^+_{2,-\omega} - \varphi^-_{1,\omega}\varphi^-_{2,-\omega}),\end{aligned} \tag{15}$$

where $n = \sqrt{g^{rr}}\partial_r$ is the unit normal vector field and the volume form in the integration over S^{d-1} is implicit. Note that the vanishing volume form is compensated by the radial derivative combined with the oscillatory behavior of ϕ, yielding a finite result. Conservation then demands $\mathcal{F} = 0$, and we thus find that the natural way to impose boundary conditions is on the oscillatory parts of the Fourier modes at the horizon. Admissible boundary conditions for quantum fluctuations are, for example, given by demanding for all $\omega > 0$,

$$(i)\ \delta\varphi^+_\omega = \delta\varphi^-_{-\omega} = 0 \quad \text{or} \quad (ii)\ \delta\varphi^-_\omega = \delta\varphi^+_{-\omega} = 0. \tag{16}$$

[2]The Fourier modes in Eq. (14) become rapidly oscillating as the horizon is approached and the Fourier transform becomes singular, since the timelike Killing field degenerates.

How the entire bulk field can be reconstructed once the boundary values are fixed has been studied in Ref. [13]. We note that, with the horizon at $z = 0$, (i)/(ii) correspond to outgoing/ingoing boundary conditions, respectively. The situation is similar to AdS fields close to the Breitenlohner-Freedman bound, where two quantization prescriptions are available and either the boundary-dominant or subdominant component is identified as source for the dual operator [25].[3] More general mixed boundary conditions are possible in both cases. As a book-keeping device we may shift $\omega\to\omega(1\pm i\epsilon)$ with $\epsilon\to 0^+$ understood. The boundary condition (16) then fixes the non-normalizable modes, and it is natural to identify the corresponding boundary values as sources for gauge-invariant operators $\mathcal{O}_{\pm\omega}$ of the dual theory. The dictionary (13) thus becomes

$$\begin{aligned}&\Psi^{\text{dS}}_{\text{IR}}[\varphi^\pm_\omega, \varphi^\mp_{-\omega}] \\ &\quad = \left\langle \exp\left\{ i\int_{S^{d-1}}\int_{\omega\ge 0} d\omega(\varphi^\pm_\omega \mathcal{O}_{-\omega} + \varphi^\mp_{-\omega}\mathcal{O}_\omega)\right\}\right\rangle_{\text{CFT},\Lambda_\kappa},\end{aligned} \tag{17}$$

where the upper choice of signs in $\varphi^\pm/\varphi^\mp$ corresponds to imposing the boundary condition (i) and the lower choice to imposing (ii). We have just split $\{\mathcal{O}_\omega, \omega\in\mathbb{R}\}$ into $\{(\mathcal{O}_{-\omega}, \mathcal{O}_\omega), \omega\ge 0\}$ and re-assigned the sources. Since we have identified the Fourier modes individually with dual operators building on the fact that the Fourier transform becomes singular on the horizon, transforming back to position space in the dual theory could be delicate. The most conservative picture would be to understand the dual theory on S^{d-1}, with a family of operators labeled by ω that encodes the bulk time evolution. However, the discussion of Sec. III A, which was valid for the cutoff arbitrarily close to the horizon, strongly suggests that this data organizes into a cutoff CFT on the cylinder. The boundary geometry just arises differently from the bulk: the S^{d-1} directly arises as a holographic screen, while the $\mathbb{R}$ factor naturally arises in Fourier space. The identification of the Dirichlet boundary condition to be imposed in Eq. (13) with those resulting from Eq. (15) seems nontrivial and deserves further investigation. We will leave that for the future and for the time being note that Eq. (17) naturally realizes a concise holographic dictionary intrinsically on the dS static patch.

C. Static patch entropy

Associated to the cosmological horizon for the static-patch observer on dS is a finite thermodynamic entropy [26]. Much like for the general case of black holes, the microscopic origin of that entropy has remained elusive, in particular, whether it is related to a counting of microstates in a quantum-gravitational description. The dual description

[3]In fact, with a radial cutoff on AdS, Neumann and Dirichlet modes are normalizable independently of the mass.

of higher-spin gravity on the static patch in terms of a cutoff CFT on the horizon provides a handle to gain some insight. More concretely, we can derive the number of bulk degrees of freedom by counting those of the dual theory.

The identifications (13) and (17) relate the static patch of dS with an optional radial cutoff to a dual quantum field theory on the boundary. Being defined on S^2, this boundary theory naturally has an IR cutoff. Moreover, it is the analytic continuation of a boundary theory on AdS where the bulk has a radial cutoff. The boundary theory therefore also has a cutoff in the UV, and we expect a finite number of degrees of freedom. To make this more precise, we start with the cutoff AdS/CFT picture (12) and repeat the analysis of Ref. [16] for the higher-spin theory in the bulk AdS_4 and the $\mathsf{O}(N)$ CFT_3 on the $\mathbb{R}\times S^2$ boundary. A convenient way to implement the UV cutoff corresponding to the bulk IR cutoff $r \le r_\kappa$ in the boundary theory is by introducing a minimal length and discretizing the boundary geometry to obtain a lattice. Defining a dimensionless parameter by $r_\kappa =: L\delta^{-1}$, such that $\delta \to 0$ corresponds to full AdS, the S^2 is then naturally composed of $\mathcal{O}(\delta^{-2})$ cells, with each boundary field having 1 degree of freedom per cell. The overall coefficient depends on the specific realization of the UV cutoff and shall not bother us. The vectorlike boundary theories we are dealing with only have $\mathcal{O}(N)$ degrees of freedom, as compared to $\mathcal{O}(N^2)$ in the usual Yang-Mills theories. We thus find the total number n of degrees of freedom in the boundary theory $n \propto N\delta^{-2}$. A more geometric meaning can be given to the bulk radial cutoff by noting that the surface area of the cutoff AdS is $A_\kappa \propto r_\kappa^2 = L^2\delta^{-2}$. We thus find $n \propto NA_\kappa L^{-2}$. As noted in Sec. III A, we have $N \propto (\Lambda G_{\mathrm{N}})^{-1}$ in the GKPY duality, where G_{N} and Λ are Newton's and the cosmological constant in the bulk, respectively. Combining that with $L \propto \Lambda^{-1/2}$ in the four-dimensional bulk theory, we arrive at

$$n \propto \frac{A_\kappa}{G_{\mathrm{N}}}. \tag{18}$$

This is the number of degrees of freedom for the cutoff CFT in the AdS/CFT picture (12). With the GKPY duality, we have thus obtained that the higher-spin bulk theory on AdS respects a holographic bound of the form discussed in Ref. [27].

The analytic continuation in Eq. (13) does not change the number of degrees of freedom of the boundary theories, and we can thus transfer this result to our static-patch/cutoff-CFT duality: the dual description of the static patch also has $n \propto A_\kappa/G_{\mathrm{N}}$ degrees of freedom. Deriving the corresponding entropy is particularly simple for the boundary theory with anticommuting scalars. In a Fock-space representation, the occupancy of each degree of freedom is at most one, such that the dimension of the Hilbert space $\mathcal{H}$ is 2^n. For the entropy we thus find

$$\mathcal{S} = \log\ \dim \mathcal{H} \propto \frac{A_\kappa}{G_{\mathrm{N}}}. \tag{19}$$

For the specific case that we holographically describe the entire static patch, the cutoff is at $r_\kappa = H$ and $A_\kappa = 4\pi H^2$. Up to the undetermined overall coefficient, Eq. (19) then reproduces the horizon entropy. Note that as $H \to \infty$, where flat space is recovered, also $n \to \infty$, and we correctly find an infinite entropy.

IV. SCALAR FIELDS EXPLICITLY

In this part we specialize to a free bulk scalar field and explicitly verify the transformation from cutoff AdS/CFT to (cutoff) static-patch holography for the two-point functions of the dual operators. We will be particularly interested in the entire static patch as bulk geometry, for which we recover the quasinormal frequencies from the dual theory on the horizon. Although the scalar of Vasiliev's minimal higher-spin theory has $m^2L^2 = -2$ and $d = 3$, we will keep the mass and spacetime dimension general. We also find it convenient to work with an action reproducing the scalar field equations, although that may not be available for the full higher-spin theory. For Euclidean AdS with the metric (5), we start from

$$S_{\mathrm{AdS}} = -\frac{1}{2}\int d^{d+1}x\sqrt{g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \mu^2L^{-2}\phi^2 + V[\phi]), \tag{20}$$

such that the usual parametrization of the mass translates to $\mu^2 = \Delta_+\Delta_-$. Performing the analytic continuation to dS via Eq. (6), we note that the mass term switches the sign. That realizes the analytic continuation in the higher-spin theory [9] and leaves $\Delta_\pm$ unchanged. This is in fact also necessary to make sense of the boundary conditions in Eq. (13) for all values of the radial cutoff. The resulting dS action reads

$$S_{\mathrm{dS}} = -\frac{1}{2}i\int d^{d+1}x\sqrt{-g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \mu^2H^{-2}\phi^2 + V[\phi]), \tag{21}$$

where the metric is that of Eq. (8). To have a positive mass on dS, admissible ν are thus restricted to $0 \le \nu \le d/2$, and we end up in the complementary series on dS.[4] For holographic applications the bulk actions (20) and (21) have to be renormalized in the usual way by adding counterterms at the conformal boundaries to render the combination finite on shell. In the following we will simply drop contact terms in the correlators without explicitly constructing the counterterms. For notational convenience we introduce λ, which is defined by $\lambda = L$ on AdS and $\lambda = iH$ on dS. The Klein-Gordon equation on (A)dS resulting from Eqs. (20) and (21) then reads

[4] A snapshot review and references can be found in Ref. [23]. The fundamental series corresponds to imaginary ν, and such fields on AdS were also discussed briefly in Ref. [23].

$$(\Box + \mu^2\lambda^{-2})\phi = 0,$$
$$\Box = r^{1-d}\partial_r(1 + r^2/\lambda^2)r^{d-1}\partial_r + \frac{\partial_\tau^2}{\lambda^2 + r^2} + r^{-2}\triangle_{S^{d-1}}. \tag{22}$$

To exploit the symmetries for its solution, we employ in both cases the Fourier ansatz

$$\phi = e^{-i\omega\tau}Y_{\vec{\ell}}(\Omega_{d-1})\chi_\ell(r), \tag{23}$$

with the spherical harmonics satisfying $\Delta_{S^{d-1}}Y_{\vec{\ell}} = -\ell(\ell + d - 2)Y_{\vec{\ell}}$. With $\chi_\ell = (1-u)^{\Delta_+/2}u^{\ell/2}h(u)$, where $u = 1/(1 + \lambda^2/r^2)$, the Klein-Gordon equation then translates to a hypergeometric differential equation for h. The origin of AdS corresponds to $u = 0$ approached from above and the position of the static-patch observer on dS to $u = 0$ approached from below. The solution which is normalizable at $r = 0$ is thus in both cases given by

$$\chi_\ell(r,\omega) = C_\kappa u^{\ell/2}(1-u)^{\Delta_+/2}{}_2F_1\left(\delta_+, \delta_-; \frac{d}{2} + \ell; u\right),$$
$$\delta_\pm = \frac{1}{2}(\Delta_+ + \ell \pm i\omega). \tag{24}$$

Note that χ_ℓ is real for appropriately chosen C_κ and $\chi_\ell(r,\omega) = \chi_\ell(r,-\omega)$. Furthermore, since the definitions of u on dS and AdS are related by Eq. (6), so are the solutions.

A. Two-point functions in cutoff (A)dS/CFT

As discussed in Ref. [28], it is convenient to employ for the split path integral (11) a subtraction scheme where the boundary action S_κ in Eq. (10) coincides with the UV counterterms at the conformal boundary in the limit $\kappa \to \infty$, i.e. $S_\kappa = -\frac{1}{2}\int_{r=\kappa}\lambda^{-1}\Delta_-\phi^2 + \cdots$. This ensures that $\Psi_{\rm IR}$ becomes the partition function of the full theory as $\kappa \to \infty$. In the semiclassical limit, the inner part of the bulk path integral reduces to $\Psi_{\rm IR} = e^{S_{\rm IR}}$, where

$$S_{\rm IR} = -\frac{1}{2}\int_{r=\kappa} d^dx\sqrt{g_{\rm ind}}\phi\sqrt{g^{rr}}\partial_r\phi + S_\kappa. \tag{25}$$

The volume form becomes imaginary for dS and reproduces the overall factor i in Eq. (21). The constant C_κ in Eq. (24) has to be chosen appropriately to satisfy the boundary condition at r_κ; e.g. for the Dirichlet boundary condition normalized as in Eq. (10), it follows from $\chi_\ell|_{r=\kappa} = f^{\Delta_-}$. The bulk solution with the boundary condition $\phi|_{r=\kappa} = f^{\Delta_-}\phi_\kappa$ then reads

$$\phi = \sum_{\vec{\ell}}\int\frac{d\omega}{\sqrt{2\pi}}e^{-i\omega\tau}\tilde{\phi}_{\kappa,\vec{\ell}}(\omega)Y_{\vec{\ell}}(\Omega)\chi_\ell(r,\omega), \tag{26}$$

where $\tilde{\phi}_{\kappa,\vec{\ell}}(\omega)$ are the Fourier modes of ϕ_κ on $\mathbb{R}\times S^{d-1}$. Inserting Eq. (26) into Eq. (25), we arrive at

$$S_{\rm IR} = \frac{1}{2}\sum_{\vec{\ell}}\int_{r=\kappa} d\omega\tilde{\phi}_{\kappa,\vec{\ell}}(\omega)G_{\kappa,\ell}(\omega)\tilde{\phi}_{\kappa,\vec{\ell}}(-\omega), \tag{27}$$

where $G_{\kappa,\ell}(\omega) = r^{d-1}\sqrt{1 + r^2/\lambda^2}\chi_\ell(r,\omega)[\sqrt{\lambda^2 + r^2}\partial_r\chi_\ell(r,\omega) + \Delta_-\chi_\ell(r,\omega)]$. The cutoff CFT two-point functions are then obtained from the Fourier transforms of Eqs. (12) and (13). Similarly to the transformation from Eq. (20) to (21) via Eq. (6), the dS version picks up a factor i, which we absorb in $c_\lambda = e^{i\arg\lambda}$. We then arrive at

$$\Psi_{\rm IR}^{\rm (A)dS}[\tilde{\phi}_{\kappa,\vec{\ell}}] = \left\langle\exp\left\{-c_\lambda\sum_{\vec{\ell}}\int d\omega\tilde{\phi}_{\kappa,\vec{\ell}}(\omega)\tilde{\mathcal{O}}_{\vec{\ell}}(-\omega)\right\}\right\rangle_{{\rm CFT},\Lambda_\kappa}, \tag{28}$$

which yields $\langle\tilde{\mathcal{O}}_{\vec{\ell}_1}(\omega_1)\tilde{\mathcal{O}}_{\vec{\ell}_2}(\omega_2)\rangle_{{\rm CFT},\Lambda_\kappa} = \delta_{\vec{\ell}_1\vec{\ell}_2}\delta(\omega_1 + \omega_2)c_\lambda^{-2}G_{\kappa,\ell_1}(\omega_1)$. Evaluating $G_{\kappa,\ell}(\omega)$ at $r = \kappa$ using standard identities for hypergeometric functions as found, e.g., in Ref. [29] and dropping contact terms results in

$$G_{\kappa,\ell}(\omega) = 2C_\kappa^2\lambda^{d-1}u_\kappa^{\ell+d/2}(1-u_\kappa)^\nu X\left(\delta_+, \delta_-, \frac{d}{2} + \ell, u_\kappa\right), \tag{29}$$

where $u_\kappa = 1/(1 + \lambda^2/\kappa^2)$ and $X(a,b,c,x) = c^{-1}(c-a)(c-b){}_2F_1(a,b,c+1,x){}_2F_1(a,b,c,x)$. We have thus obtained the two-point functions of the dual cutoff CFTs on dS and AdS, with the choice of the constant C_κ encoding the choice of boundary condition on the cutoff surface. The parallel calculations leading to Eq. (29) establish their relation by the analytic continuation (6). As compared to Ref. [3], the continuation does not just affect the overall normalization, reflecting the fact that the bulk radial cutoffs have different but apparently still related interpretations in the boundary theories on AdS and dS. The two-point function (29) obtained from AdS with $\lambda = L$ has no poles for real ω, as expected for the Euclidean boundary theory. The same applies for dS with $\kappa > H$, where the cutoff surface is spacelike and the boundary theory Euclidean. For smaller κ and $\lambda = iH$, the poles appear for real ω, as expected for the Lorentzian boundary theory on the dS static patch. The analytic continuation $L \to iH$ then yields the $i\epsilon$ prescription corresponding to the Wick rotation on the right-hand side of Eq. (13).

To complete the discussion, we now consider the limit $\kappa \to \infty$. The cutoff surface then approaches $\mathcal{I}^+$ on dS and correspondingly the conformal boundary of AdS, such that $\Psi_{\rm IR}^{\rm (A)dS}$ becomes the full path integral. On AdS as well as on dS, $\kappa \to \infty$ corresponds to $u_\kappa \to 1$, see Fig. 2, and the asymptotic expansions coincide. With $\lim_{\kappa\to\infty}C_\kappa = \Gamma(\delta_+)\Gamma(\delta_-)/(\lambda^{\Delta_-}c_\lambda^{2\nu}\Gamma(\nu)\Gamma(d/2 + \ell))$, we find

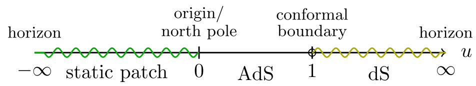

FIG. 2 (color online). The ranges of u corresponding to the different parts of (A)dS. The colors for the dS parts correspond to those in Fig. 1(a). To avoid the branch points of the hypergeometric function, the analytic continuation from $\lambda = L$ to $\lambda = iH$ should proceed through finite nonzero values, e.g. via $\lambda = (1-t)L + itH$.

$$\lim_{\kappa\to\infty} G_{\kappa,\ell}(\omega) = 2\lambda^{2\nu-1} c_\lambda^{-2\nu} \frac{\Gamma(1-\nu)\Gamma(\delta_+)\Gamma(\delta_-)}{\Gamma(\nu)\Gamma(\delta_+-\nu)\Gamma(\delta_--\nu)}. \tag{30}$$

The dependence on κ has dropped out, and the effect of the analytic continuation (6) is now restricted to the overall normalization, in accordance with the discussion of Ref. [3]. As a CFT two-point function, Eq. (30) should be conformally covariant, restricting its form to a power of the invariant distance of the two points on the cylinder. Transforming back to position space for (A)dS$_2$, we indeed find $\langle \mathcal{O}(0)\mathcal{O}(t,x)\rangle \propto (\sinh\frac{\tau}{2})^{-2\Delta_+}$. The poles and zeros in Eq. (30) also encode the (anti)quasinormal frequencies of dS$_{d+1}$, $\pm i\omega = 2n + \ell + \Delta_\pm$, which have been calculated in Ref. [30]. That they can be recovered from the dual CFT at $\mathcal{I}^+$ in the dS/CFT proposal of Ref. [2] has been emphasized already in Ref. [31]. A quantization prescription based on the quasinormal modes can be found in Ref. [24].

B. Static-patch holography and quasinormal frequencies

We now specialize to the entire static patch, which is recovered for $\kappa \to H^-$. The boundary data corresponding to sources for dual operators were naturally identified in Fourier space, and we calculate the two-point functions using the dictionary (17). Note first of all that the asymptotic expansion of the radial mode (24) around the horizon at $u \to -\infty$ confirms Eq. (14), which we used to derive the dictionary. To actually calculate the left-hand side of Eq. (17) in the saddle point approximation, we have to evaluate $\Psi_{\rm IR}^{\rm dS}[\varphi_\omega^\pm, \varphi_{-\omega}^\mp] = e^{S_{\rm IR}^{\rm dS}}$. Evaluating the action (21) on shell with the expansion (14) yields

$$S^{\rm dS}_{{\rm IR},\kappa\to H^-} = -H^{d-1}\int_{S^{d-1}}\int_{\omega\geq 0} d\omega \underbrace{\frac{1}{2} i[\tilde\phi(\omega) z\partial_z\tilde\phi(-\omega) - \tilde\phi(-\omega)z\partial_z\tilde\phi(\omega)]_{z=0}}_{=\omega(\varphi_\omega^+\varphi_{-\omega}^+ - \varphi_\omega^-\varphi_{-\omega}^-)}. \tag{31}$$

For fixed $\omega > 0$ and the boundary condition (i) in Eq. (16), the functions φ_ω^+ and $\varphi_{-\omega}^+$ evaluated at $z=0$ thus constitute a pair of source and expectation value for the dual operator $\mathcal{O}_{-\omega}$, while $\varphi_{-\omega}^-$ and φ_ω^- are source and expectation value for $\mathcal{O}_\omega$, respectively. For the alternative quantization with the boundary condition (ii), the analogous statement applies with φ^+ and φ^- exchanged. For the two-point functions, we then find $\langle \mathcal{O}_{\omega_1,\vec\ell_1}\mathcal{O}_{\omega_2,\vec\ell_2}\rangle = -\delta_{\vec\ell_1\vec\ell_2}\delta(\omega_1+\omega_2)G_{\omega_1,\ell_1}$, where

$$G^{(i)}_{\omega,\ell} = \frac{\delta}{\delta\varphi_{-\omega}^-}\frac{\delta}{\delta\varphi_\omega^+} e^{S^{\rm dS}_{{\rm IR},\kappa\to H^-}} = 2\omega H^{d-1}\frac{\delta\varphi_\omega^-}{\delta\varphi_\omega^+},$$
$$G^{(ii)}_{\omega,\ell} = -2\omega H^{d-1}\frac{\delta\varphi_\omega^+}{\delta\varphi_\omega^-}. \tag{32}$$

They are therefore almost reciprocal to each other. The precise forms are most conveniently derived from the asymptotic expansion of the full bulk solution around $z=0$. To implement the boundary condition on the horizon, we have to fix $C_\kappa^{(i)} = C_\kappa^{(ii)}|_{\omega\to-\omega} = \frac{H^{i\omega}\Gamma(\delta_-)\Gamma(\delta_--\nu)}{i^\ell\Gamma(-i\omega)\Gamma(d/2+\ell)}$ in Eq. (24), and the bulk field is then again given by Eq. (26). From the expansion of Eq. (24) with $u = -{\rm csch}^2(z/H)$, we then find

$$G^{(i)}_{\omega,\ell} = G^{(ii)}_{-\omega,\ell} = -2iH^{d+2i\omega-1}\frac{\Gamma(1+i\omega)}{\Gamma(-i\omega)}\frac{\Gamma(\delta_-)\Gamma(\delta_--\nu)}{\Gamma(\delta_+)\Gamma(\delta_+-\nu)}. \tag{33}$$

The boundary condition on AdS corresponding to the choice of C_κ is not directly Dirichlet, but since the bulk solution is fixed uniquely, the two can be related. With the choices naturally appearing at the horizon, $C_\kappa = C_\kappa^{(i)/(ii)}$, we could have obtained Eq. (33) up to contact terms also from the near-horizon limit of Eq. (29). The frequency dependence of the right-hand side of Eq. (33) in particular encodes the dS$_{d+1}$ quasinormal frequencies: we find poles and zeros for

$$i\omega = \Delta_\pm + \ell + 2n \quad \text{and} \quad -i\omega = \Delta_\pm + \ell + 2n, \tag{34}$$

respectively, where $n \in \mathbb{N}$. These are precisely the (anti) quasinormal frequencies arising for scalar perturbations as found in Ref. [30]. Since they are naturally associated to the static patch of dS$_{d+1}$, it is desirable to recover them from a dual description, which is intrinsically defined on the static patch. Previously they have been recovered from a putative quantum mechanics on the observer worldline in Ref. [3], and here we find them encoded directly on the horizon of the static patch as the natural place to define the dual theory.

V. INCORPORATING THE DS/DS CORRESPONDENCE

In this section we come back to a seemingly disconnected proposal for a holographic description of dS: building on the complementary holographic interpretation of

Randall-Sundrum setups [32] and warped geometries where dS_{d+1} is sliced by dS_d, a dual description for dS_{d+1} gravity in terms of cutoff CFTs on dS_d was proposed in Ref. [14,15]. The discussion of static-patch holography above can nicely be adapted to this dS/dS correspondence and then similarly allows us to lift it to a concrete level. We can thus incorporate the dS/dS correspondence into a coherent picture of dS holography via cutoff AdS/CFT. In the first part of the section, we derive a relation of the dS/dS geometry to a part of AdS, similarly to the discussion of Sec. II. Building on that identification, we can then realize analytic continuations from AdS/CFT to dS/dS correspondences.

The geometries for the dS/dS correspondence are obtained from the fact that, with a parametrization x_i of dS_d as hyperboloid with radius of curvature h in $\mathbb{R}^{1,d}$, one obtains a parametrization of dS_{d+1} via

$$X_k = \frac{H}{h}\cos\frac{R}{H}x_k, \qquad \forall\ k = 0,\ldots,d, \qquad X_{d+1} = H\sin\frac{R}{H}. \tag{35}$$

From $-x_0^2 + \sum_{i=1}^d x_i^2 = h^2$ we find that these coordinates cover some part of dS_{d+1} with radius of curvature H as in Eq. (1). The part which is covered comprises all of the spatial S^d factor for $X_0 = 0$ and shrinks to an S^{d-1} subspace at $\mathcal{I}^{\pm}$, see Fig. 3(a). To actually fix the geometry, we have yet to specify which part of dS^d the slices cover. We may choose global dS_d slices, for which the above discussion applies, or restrict, e.g., to the expanding Poincaré patch as illustrated for dS_{d+1} in Fig. 1(a). That choice covers the $\mathbb{Z}_2$ quotient of dS_d relevant for the "elliptic interpretation" going back to Ref. [33]. A third option is to choose only the static patches of the dS_d slices, and we will refer to the corresponding dS/dS geometries as global, elliptic, and static dS/dS. Our focus for deriving dS/dS correspondences will be on the elliptic and static dS/dS geometries, for which a correspondingly smaller part of dS_{d+1} is covered. More precisely, the Poincaré coordinates cover only the part $x_0 > x_1$ of the slices. Via Eq. (35) this implies also $X_0 > X_1$, and the elliptic dS/dS geometry thus is a part of the Poincaré patch of dS_{d+1} illustrated in Fig. 1(a). We can therefore employ the analytic continuations used in Ref. [3] and similarly to the construction in Sec. III restrict to the region appropriate for the dS/dS geometry. The same applies for static dS/dS. In that case the bulk geometry is manifestly static, and the coordinates indeed cover the dS_{d+1} static patch. The resulting line element in any case reads

$$ds^2_{d+1} = dR^2 + \frac{H^2}{h^2}\cos^2\frac{R}{H}\,ds^2_{dS_d}. \tag{36}$$

To obtain an analytic continuation to an AdS geometry, we employ the Wick rotations discussed in Sec. II and apply it to the slices, e.g. the analog of Eq. (3) for the slices of elliptic dS/dS, or similarly Eq. (6) for static-patch slices. We then perform in Eq. (35) the analytic continuation

$$H \to iL, \qquad h \to il, \qquad \ldots, \tag{37}$$

where the dots denote the possible further transformations to complete $h \to il$ to an analytic continuation of the dS_d slices to AdS_d. The parametrization (35) becomes

$$X_k = \frac{L}{l}\cosh\frac{R}{L}x_k, \qquad \forall\ k = 0,\ldots,d, \qquad X_{d+1} = L\sinh\frac{R}{L}, \tag{38}$$

and since $-X_0^2 + \sum_i X_i^2 = -L^2$ we find a parametrization of a part of AdS_{d+1}. The line element becomes

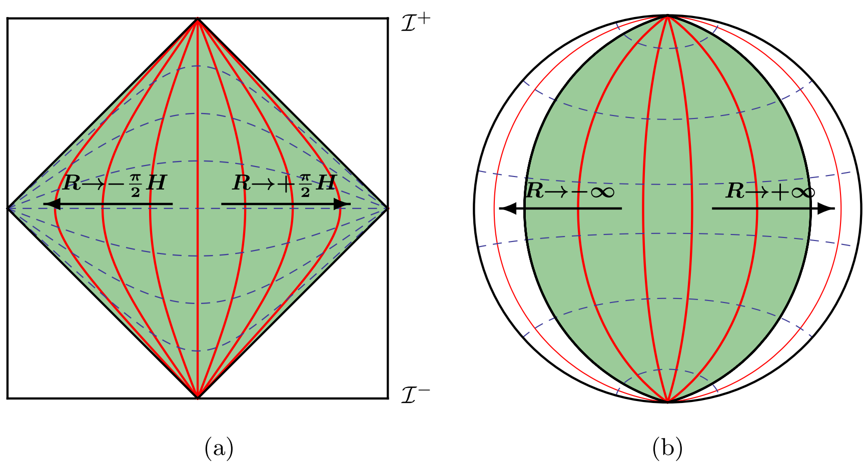

FIG. 3 (color online). The left-hand side shows the slicing of dS_{d+1} by dS_d hypersurfaces. On the horizontal axis is the polar angle on S^d, and the boundaries correspond to the north and south pole. The red solid and blue dashed curves correspond to constant R and global dS_d time, respectively. The right-hand side similarly shows the slicing of AdS_{d+1} by AdS_d hypersurfaces. The green shaded regions are identified by the analytic continuation (37). Note that, with a cutoff on R, also on the AdS_{d+1} side only an S^{d-1} of the S^d conformal boundary is covered.

$$ds^2_{d+1} = dR^2 + \frac{L^2}{l^2}\cosh^2\frac{R}{L}ds^2_{\mathrm{AdS}_d}, \tag{39}$$

and that geometry is illustrated in Fig. 3(b). The complete AdS_{d+1} corresponds to $R \in \mathbb{R}$. The analytic continuation (37) relates the dS_d slicing of dS_{d+1} to an inner part of the AdS_d slicing of AdS_{d+1}, where the radial coordinate is restricted to $|R| \leq \pi L/2$. For elliptic dS/dS the analytic continuation yields complete AdS_d slices, while for static dS/dS it introduces a cutoff not only on the AdS_{d+1} radial coordinate R but also on the radial coordinate of the AdS_d slices.

We have thus obtained for the elliptic and static dS/dS geometries a geometric identification with a part of AdS and can proceed to implement a similar analytic continuation of the AdS/CFT dictionary as we have done for the static patch in Eq. (13). For static dS/dS the spatial cutoff on the AdS_d slices of the corresponding AdS_{d+1} geometry calls for a separate treatment, and we therefore start with a discussion of elliptic dS/dS. In the AdS/CFT picture, the conformal boundary of AdS_{d+1} comprises two copies of AdS_d joined at their conformal boundaries, as illustrated in Fig. 3(b). The bulk theory is thus dual to a pair of CFTs on AdS. One may study them with transparent boundary conditions or identify them by considering $\mathrm{AdS}_{d+1}/\mathbb{Z}_2$, with the slices at $\pm R$ identified, as in Ref. [23,34]. Each choice leads to a distinct AdS/CFT duality, and we keep that general in the following. Introducing a radial cutoff $|R| \leq \pi\kappa/2 =: R_\kappa$ on AdS would again be interpreted as a UV cutoff in the dual CFTs, and the analog of the cutoff AdS/CFT duality (12) then reads

$$\Psi_{\mathrm{IR}}[\phi_\kappa^+, \phi_\kappa^-] = \left\langle \exp\left\{\int_{\mathrm{AdS}_d}(\phi_\kappa^+\mathcal{O}_+ + \phi_\kappa^-\mathcal{O}_-)\right\}\right\rangle_{\mathrm{CFT},\Lambda_\kappa}. \tag{40}$$

The inner part of the bulk path integral Ψ_{IR} is defined analogously to Eq. (10), with the boundary conditions $\phi(\pm R_\kappa) = f^{\Delta_-}\phi_\kappa^\pm$ on the two components of the boundary and $f = l/L\,\mathrm{sech}(R/L)$. On the right-hand side, $\mathcal{O}_\pm$ denotes the corresponding dual operators of the CFTs at $\pm R_\kappa$, where the rescaled bulk metric $\bar{g} := f^2 g$ naturally induces AdS_d metrics. The starting point to obtain concrete dS/dS correspondences is Vasiliev's minimal higher-spin gravity on that AdS_{d+1} bulk geometry with $d = 3$, which is then dual to the $\mathsf{O}(N)$ vector model on the two copies of AdS_d constituting the conformal boundary. With $\kappa \leq L$ the cutoff version of that duality (40), which corresponds to introducing a symmetric UV cutoff in both CFTs, provides a holographic relation defined on a part of the green shaded region in Fig. 3(b). We can thus apply the analytic continuation (37), which realizes the transformation from AdS to dS used in Ref. [3] just adapted to our choice of coordinates, resulting in

$$\begin{aligned}\Psi_{\mathrm{IR}}^{\mathrm{dS/dS}}[\phi_\kappa^+, \phi_\kappa^-] &:= \Psi_{\mathrm{IR}}[\phi_\kappa^+, \phi_\kappa^-]|_{(37)}\\ &= \left\langle \exp\left\{i\int_{\mathrm{dS}_d}(\phi_k^+\mathcal{O}_+ + \phi_k^-\mathcal{O}_-)\right\}\right\rangle_{\mathrm{CFT},\Lambda_k,N\to -N}.\end{aligned} \tag{41}$$

The bulk geometry is transformed to the dS_d slicing of dS_{d+1} and the CFT metrics accordingly to dS_d. We have thus obtained a duality of higher-spin theory on the dS/dS bulk geometry to a pair of dual cutoff CFTs on dS_d, again with the continuation from $\mathsf{O}(N)$ to $\mathsf{Sp}(N)$. The dS/dS duality (41) is similar to the static-patch duality (13): both involve higher-spin theory in the bulk and cutoff versions of the $\mathsf{Sp}(N)$ CFT_3 on the boundary. The different bulk geometries are reflected in the fact that the boundary theories are defined on different spaces as well—on a cylinder in Eq. (13) as compared to two copies of dS_d in Eq. (41)—and with different cutoff implementations.

In the AdS/CFT duality (40), we had intentionally left the choice of boundary conditions for the CFTs on AdS_d unspecified. For each choice of CFT boundary conditions or $\mathbb{Z}_2$ identification on the AdS side, the analytic continuation yields a corresponding dS/dS duality (41). The limit where the entire bulk geometry is recovered calls for a special treatment, analogously to the discussion for the static patch in Sec. III B. In that context the boundary conditions (16) appearing naturally at the horizon may be of interest. We expect something similar to appear for dS/dS, in particular, also for the bulk metric. As the horizon boundary conditions are not pure Dirichlet, the boundary cutoff CFTs would naturally be coupled to dynamical gravity, as anticipated in Refs. [14,15]. We leave more detailed investigations for the future and note that the picture nicely agrees with the general discussions of dS/dS so far. It substantiates the discussion by providing a concrete example and also a new perspective on the expected features.

We have seen that one and the same field theory, formulated on different background geometries and with different cutoffs, can be dual to different regions of dS. Detailed discussions were given for the static-patch holography and for elliptic dS/dS, and related dualities for other patches of dS can be obtained by similar analytic continuation. It would be particularly interesting to study in more detail the static dS/dS slicing, where the pair of dual cutoff CFTs in the corresponding AdS/CFT picture is defined with a spatial cutoff on AdS_d in addition to the UV cutoff. Since the bulk geometry is static in that case, this setting allows for a direct comparison to the static-patch holography of Sec. III.

VI. DISCUSSION AND OUTLOOK

We have argued that the dynamics of minimal higher-spin gravity on the dS_4 static patch, optionally with a radial cutoff, is encoded in a cutoff version of the $\mathsf{Sp}(N)$ CFT_3 of anticommuting scalars on the Lorentzian-signature cylinder. The discussion was based on a relation of the dS static patch to an inner shell of AdS via double Wick rotation and a corresponding analytic continuation in the dynamics of the bulk and boundary theories. As discussed in Sec. III B, the limit where the dS bulk geometry becomes the entire static patch has to be taken carefully. The spatial part of the boundary cylinder straightforwardly arises as a holographic screen, while the time direction only arises naturally in Fourier space. The proposed duality allowed us to transfer lessons learned from AdS/CFT to the description of (quantum) gravity on the static patch. With the concrete dual description in terms of a cutoff CFT on the cylinder, we have derived the number of degrees of freedom on the dS static patch from the dual theory. It is finite and respects a holographic bound, and the corresponding entropy reproduces the functional form of the thermodynamic horizon entropy. To make the discussion more explicit, we have then studied the two-point functions of the boundary theories on cutoff dS and AdS. We found them related by the expected analytic continuation, which, reflecting the different but related cutoff interpretations in the boundary theories, did not solely affect their normalization. For the entire static patch as bulk geometry, we have recovered the spectrum of quasinormal frequencies from the correlators of the boundary theory on the horizon, and as a limiting case, we have also recovered the proposal of Ref. [3]. Although the explicit discussion was limited to free bulk scalars, we expect the established analytic continuation from AdS to dS to extend to perturbatively interacting fields. We have then derived a similar relation of the geometries underlying the dS/dS correspondence to an inner part of AdS, which allowed us to similarly provide an explicit realization. It also results in a coherent picture of dS holography identifying the various incarnations as different forms of cutoff AdS/CFT.

There are also open questions which we think would be interesting to study in the future. As discussed in Sec. IV B, the boundary conditions arising naturally on the horizon of the static patch can be translated to AdS, and the two settings are then connected by analytic continuation. However, it would be of interest to find an independent interpretation of these boundary conditions intrinsically on AdS. In that context alternative interpretations of the AdS Dirichlet problem may be relevant: since a boundary condition at fixed r_κ uniquely determines the bulk solution, one may identify a localized source for the cutoff CFT at r_κ with a nonlocal source for the dual operator in the full CFT, or vice versa. A possibly related issue discussed briefly at the end of Sec. III A is the unitarity of the boundary theory and the question whether it is indeed restored by the mandatory cutoff. As an extension of the discussion here, it would certainly be of interest to include fields of higher spin, in particular, a dynamical bulk metric. To this end it would be desirable to have a characterization of the spacetimes where the above construction can be carried out, as available, e.g., for AdS/CFT in the form of asymptotically AdS spaces. It may also be possible to obtain further concrete examples by applying the discussion to other AdS/CFT dualities. We note in that context that on the group-theoretic level, a similar analytic continuation as from $\mathsf{SO}(N)$ to $\mathsf{Sp}(N)$ is also available for $\mathsf{SU}(N)$ [35]. The crucial point will certainly be the extension of the analytic continuation from AdS to dS to the actual bulk dynamics, apart from which the discussion was pretty general already. That may be possible for other variants of higher-spin theory, as relevant, e.g., for the minimal-model holography [36].

ACKNOWLEDGMENTS

We thank Thorsten Ohl for useful discussions. The work of A. K. is supported in part by the U.S. Department of Energy under Grant No. DE-FG02-96ER40956. C. F. U. is supported by Deutsche Forschungsgemeinschaft through the Research Training Group GRK 1147 *Theoretical Astrophysics and Particle Physics*.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).
[2] A. Strominger, J. High Energy Phys. 10 (2001) 034.
[3] D. Anninos, T. Hartman, and A. Strominger, arXiv:1108.5735.
[4] E. Witten, arXiv:hep-th/0106109.
[5] D. Anninos, S. A. Hartnoll, and D. M. Hofman, Classical Quantum Gravity **29**, 075002 (2012).
[6] R. Bousso, J. High Energy Phys. 06 (1999) 028.
[7] M. A. Vasiliev, Phys. Lett. B **243**, 378 (1990).
[8] M. A. Vasiliev, arXiv:hep-th/9910096.
[9] C. Iazeolla, E. Sezgin, and P. Sundell, Nucl. Phys. **B791**, 231 (2008).
[10] I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **550**, 213 (2002).
[11] S. Giombi and X. Yin, J. High Energy Phys. 09 (2010) 115.
[12] D. J. Robinson, E. Kapit, and A. LeClair, J. Math. Phys. (N.Y.) **50**, 112301 (2009).
[13] I. Sachs and S. N. Solodukhin, Phys. Rev. D **64**, 124023 (2001).
[14] M. Alishahiha, A. Karch, E. Silverstein, and D. Tong, AIP Conf. Proc. **743**, 393 (2004).
[15] M. Alishahiha, A. Karch, and E. Silverstein, J. High Energy Phys. 06 (2005) 028.
[16] L. Susskind and E. Witten, arXiv:hep-th/9805114.

[17] A. W. Peet and J. Polchinski, Phys. Rev. D **59**, 065011 (1999).
[18] J. de Boer, E. P. Verlinde, and H. L. Verlinde, J. High Energy Phys. 08 (2000) 003; V. Balasubramanian and P. Kraus, Phys. Rev. Lett. **83**, 3605 (1999).
[19] T. Faulkner, H. Liu, and M. Rangamani, J. High Energy Phys. 08 (2011) 051.
[20] I. Heemskerk and J. Polchinski, J. High Energy Phys. 06 (2011) 031.
[21] S. F. Ross, Classical Quantum Gravity **28**, 215019 (2011).
[22] T. Andrade, T. Faulkner, and D. Marolf, J. High Energy Phys. 05 (2012) 011.
[23] T. Andrade and C. F. Uhlemann, J. High Energy Phys. 01 (2012) 123.
[24] D. L. Jafferis, A. Lupsasca, V. Lysov, G. Seng Ng, and A. Strominger, arXiv:1305.5523.
[25] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B **115**, 197 (1982); P. Breitenlohner and D. Z. Freedman, Ann. Phys. (N.Y.) **144**, 249 (1982); E. Witten, arXiv:hep-th/0112258.
[26] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).
[27] L. Susskind, J. Math. Phys. (N.Y.) **36**, 6377 (1995).
[28] V. Balasubramanian, M. Guica, and A. Lawrence, J. High Energy Phys. 01 (2013) 115.
[29] *NIST Handbook of Mathematical Functions*, edited by F. W. J. Olver, D. W. Lozier, R. F. Boisvert, and C. W. Clark (Cambridge University Press, Cambridge, England, 2010).
[30] A. Lopez-Ortega, Gen. Relativ. Gravit. **38**, 1565 (2006).
[31] E. Abdalla, K. H. C. Castello-Branco, and A. Lima-Santos, Phys. Rev. D **66**, 104018 (2002).
[32] M. J. Duff and J. T. Liu, Phys. Rev. Lett. **85**, 2052 (2000); S. B. Giddings, E. Katz, and L. Randall, J. High Energy Phys. 03 (2000) 023.
[33] E. Schrödinger, *Expanding Universes* (Cambridge University Press, Cambridge, England, 1956).
[34] O. Aharony, D. Marolf, and M. Rangamani, J. High Energy Phys. 02 (2011) 041.
[35] G. V. Dunne, J. Phys. A **22**, 1719 (1989).
[36] M. R. Gaberdiel and R. Gopakumar, Phys. Rev. D **83**, 066007 (2011).