Supergauge theories in aether superspace

A. C. Lehum,[1,2,*] J. R. Nascimento,[3,†] A. Yu. Petrov,[3,‡] and A. J. da Silva[1,§]

[1]*Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, São Paulo, Brazil*

[2]*Escola de Ciências e Tecnologia, Universidade Federal do Rio Grande do Norte, Caixa Postal 1524, 59072-970, Natal, Rio Grande do Norte, Brazil*

[3]*Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970, João Pessoa, Paraíba, Brazil*

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Within the superfield method, we extend the formulation of the Lorentz-breaking aether superspace for supergauge theories, in both three- and four-dimensional cases.

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I. INTRODUCTION

The idea of the Lorentz symmetry breaking has been intensively discussed during the last years (for a review, see for example [1]). Interest in this line of study is motivated by the fact that the presence of the Lorentz-breaking additive modifications of field theory models essentially enriches their structure [2]. At the same time, since supersymmetry is treated as a fundamental physical symmetry, a natural question is whether the Lorentz-breaking field theory can be supersymmetric. A systematic methodology used to address this problem is based on the Kostelecky-Berger construction [3] involving the deformation of the supersymmetry algebra, which, in principle, can be applied to different kinds of superfield theories, formulated in different space-time dimensions, and allows for the arising of the CPT-even Lorentz-breaking terms on the component level. Other possible solutions for this problem involve one or more extra superfields whose components depend on the Lorentz-breaking parameters [4] (which can allow for the arising of the CPT-odd Lorentz-breaking terms on the component level) or a straightforward addition of Lorentz-breaking, superfield-dependent terms like $k^{ab}\partial_a\Phi\partial_b\bar{\Phi}$ (where Φ, $\bar{\Phi}$ are the superfields, and the k^{ab} is a constant tensor). To the best of our knowledge, however, the last method has not been systematically used yet, and it is clear that it involves higher derivatives.

In this paper we develop a method based on the Kostelecky-Berger (KB) construction to introduce a Lorentz-breaking deformation of the supersymmetry (SUSY) algebra for supergauge field theories. Earlier, this method was successfully applied to supersymmetric scalar field theories [5], where it was shown that the application of the KB construction allows the generation of aetherlike terms [6] in the action of the theories at the component level, while the effective action can be calculated on the basis of the superfield approach, in a way that is as simple as the usual, Lorentz-invariant case. We develop this methodology, in both three- and four-dimensional cases, and one of the key results of our consideration consists in a natural arising of a new form of gauge symmetry, involving the Lorentz-breaking parameter, for the vector component of the superfield.

The paper is organized as follows. In Sec. II we discuss the generalization of the three-dimensional aether superspace to gauge theories, including some perturbative calculations on three-dimensional supersymmetric quantum electrodynamics and Chern-Simons-matter model. In Sec. III we deal with the four-dimensional case, and we apply the aether superspace methodology in the computation of the effective potential to the supersymmetric quantum electrodynamics. In Sec. IV we discuss the possibility of equivalence between our modification of the supersymmetry generators and some coordinate transformations. Finally, in Sec. V we present our final remarks.

II. THREE-DIMENSIONAL AETHER SUPERSPACE

Just as we have discussed in our previous paper [5], the extension of the usual superspace to a three-dimensional deformed superspace is stated through the deformed SUSY generators

$$Q_\alpha = i[\partial_\alpha - i\theta^\beta\gamma^m_{\beta\alpha}(\partial_m + k_{mn}\partial^n)] = i[\partial_\alpha - i\theta^\beta\gamma^m_{\beta\alpha}\nabla_m], \tag{1}$$

satisfying the anticommutation relation

$$\{Q_\alpha, Q_\alpha\} = 2i\gamma^m_{\alpha\beta}\nabla_m, \tag{2}$$

where ∂_α is the derivative with respect to the Grassmannian coordinates θ^α and $\nabla_m = \partial_m + k_{mn}\partial^n$, with ∂_m the derivative with respect to x^m. Latin indices assume values of three-dimensional space-time coordinates (0, 1, 2) and k_{mn} is a constant tensor that can be chosen to assume an aetherlike form $k_{mn} = \alpha u_m u_n$, with α a small parameter (cf. [6]) and u^m a constant vector with $u^m u_m$ equal either to 1, -1 or 0. In general, we use the conventions and notations as well as normalization factors as in Ref. [7], but the symbols for the (supergauge

*andrelehum@ect.ufrn.br
†jroberto@fisica.ufpb.br
‡petrov@fisica.ufpb.br
§ajsilva@fma.if.usp.br

covariant) derivatives appearing in the present paper are slightly different.

It is important to remark that the new supercovariant derivative that anticommutes with Q_α is given by

$$D_\alpha = \partial_\alpha + i\theta^\beta \gamma^m_{\beta\alpha}\nabla_m, \tag{3}$$

where the operator ∇_m commutes with D_α, as well as with the SUSY generators.

For superscalar field theories constructed in this deformed superspace, we can define an action such as

$$S = -\frac{1}{2}\int d^5z[(\overline{D^\alpha\Phi})(D_\alpha\Phi) - f(\bar{\Phi}\Phi)], \tag{4}$$

where $f(\bar{\Phi}\Phi)$ is some function of the bilinear $\bar{\Phi}\Phi$, which is invariant under $U(1)$ global transformations ($\Phi' = \mathrm{e}^{iK}\Phi$). Our aim in this paper is to extend such action to theories that are invariant under local (gauge) transformations ($\Phi' = \mathrm{e}^{iK(x,\theta)}\Phi$, with $K(x,\theta)$ being a *real* scalar superfield).

To do this, let us introduce a supergauge covariant derivative $\mathcal{D}_\alpha = (D_\alpha - i\Gamma_\alpha)$ such tht $\mathcal{D}_\alpha\Phi$ transforms covariantly under $U(1)$ gauge transformations ($\mathcal{D}'_\alpha\Phi' = \mathrm{e}^{iK(x,\theta)}\mathcal{D}_\alpha\Phi$), allowing us to write a gauge invariant action,

$$\begin{aligned} S &= -\frac{1}{2}\int d^5z[(\overline{\mathcal{D}^\alpha\Phi})(\mathcal{D}_\alpha\Phi) - f(\bar{\Phi}\Phi)] \\ &= -\frac{1}{2}\int d^5z[(\overline{D^\alpha\Phi})(D_\alpha\Phi) - i\overline{D^\alpha\Phi}\Gamma_\alpha\Phi \\ &\qquad + i\Gamma^\alpha\bar{\Phi}D_\alpha\Phi + \Gamma^\alpha\Gamma_\alpha\bar{\Phi}\Phi - f(\bar{\Phi}\Phi)], \end{aligned} \tag{5}$$

where the spinor gauge connection transforms as $\Gamma'_\alpha = \Gamma_\alpha + D_\alpha K$. Note that the gauge transformations themselves are defined as deformed ones.

The components of the spinor superfield connection can be defined as

$$\begin{aligned} \chi_\alpha &= \Gamma_\alpha|_{\theta=0}, \qquad B = \frac{1}{2}D^\alpha\Gamma_\alpha|_{\theta=0}, \\ V_{\alpha\beta} &= -\frac{i}{2}D_{(\alpha}\Gamma_{\beta)}|_{\theta=0}, \qquad \lambda_\alpha = \frac{1}{2}D^\beta D_\alpha\Gamma_\beta|_{\theta=0}, \end{aligned} \tag{6}$$

where $V_{\alpha\beta} = (\gamma^m)_{\alpha\beta}A_m$. In analogy with the usual three-dimensional superspace, the components of the scalar superfield are conveniently defined as

$$\varphi = \Phi|_{\theta=0}, \qquad \psi = D^\alpha\Phi|_{\theta=0}, \qquad F = D^2\Phi|_{\theta=0}, \tag{7}$$

with similar definitions for the components of $\bar{\Phi}$.

Therefore, in terms of the components of the superfields, the action Eq. (5) can be cast as

$$\begin{aligned} S = \int d^3x\Big\{&\bar{F}F + \bar{\psi}^\alpha(\gamma^m)_\alpha{}^\beta[i\nabla_m - A_m]\psi_\beta \\ &+ (i\bar{\psi}^\alpha\lambda_\alpha\varphi + \text{H.c.}) + (\nabla^m - iA^m)\bar{\varphi}(\nabla_m + iA_m)\varphi \\ &+ \frac{1}{2}f'(\bar{\varphi}\varphi)[\bar{F}\varphi + \bar{\varphi}F + 2\bar{\psi}^\beta\psi_\beta] \\ &+ \frac{1}{2}f''(\bar{\varphi}\varphi)[2\bar{\varphi}\varphi\bar{\psi}^\beta\psi_\beta + \varphi^2\bar{\psi}^\beta\bar{\psi}_\beta + \bar{\varphi}^2\psi^\beta\psi_\beta]\Big\}, \end{aligned} \tag{8}$$

where $f'(\bar{\varphi}\varphi) = \frac{\partial f(\bar{\Phi}\Phi)}{\partial(\bar{\Phi}\Phi)}|_{\bar{\Phi}\Phi=\bar{\varphi}\varphi}$.

Gauge covariant superfield strength can be defined just as in the usual case, $W_\alpha = \frac{1}{2}D^\beta D_\alpha\Gamma_\beta$. A SUSY Maxwell Lorentz-breaking action can be constructed as

$$\begin{aligned} S &= \int d^5z\frac{1}{2}W^\alpha W_\alpha \\ &= \int d^3x\Big[\lambda^\alpha i(\gamma^m)_\alpha{}^\beta\nabla_m\lambda_\beta - \frac{1}{2}f^{\alpha\beta}f_{\alpha\beta}\Big], \end{aligned} \tag{9}$$

where $\lambda_\alpha \equiv W_\alpha|_{\theta=0}$ and $f_{\alpha\beta} = D_\alpha W_\beta|_{\theta=0} = D_\beta W_\alpha|_{\theta=0}$. In terms of the gauge field A_m, $f_{\alpha\beta}$ can be written as $f_{\alpha\beta} = \frac{1}{2}\epsilon_r^{mn}(\gamma^r)_{\alpha\beta}\nabla_m A_n$. Therefore, the physical content of the SUSY Maxwell-like action is given by

$$S = \int d^3x\Big[\lambda^\alpha i(\gamma^m)_\alpha{}^\beta\nabla_m\lambda_\beta - \frac{1}{4}(\nabla_m A_n - \nabla_n A_m)^2\Big], \tag{10}$$

where $\nabla_m = \partial_m + k_{mn}\partial^n$.

A. Three-dimensional quantum electrodynamics in aether superspace

As a first example of the power of the superspace techniques even in Lorentz-breaking scenarios, let us evaluate the one-loop correction to the self-energy of the gauge superfield in the super quantum electrodynamics in three dimensions. To do this it is necessary to compute the superpropagators of the model. Considering the theory defined by the action Eqs. (5)–(9) plus the gauge-fixing and the corresponding Fadeev-Popov term, and proceeding as usual, we can write the following propagators in the aether superspace,

$$\begin{aligned} &\langle\Phi(p,\theta_1)\Phi(-p,\theta_2)\rangle \\ &\quad = \frac{(D^2 - m)}{\tilde{p}^2 + m^2}\delta^2(\theta_1 - \theta_2), \\ &\langle\Gamma_\alpha(p,\theta_1)\Gamma_\beta(-p,\theta_2)\rangle \\ &\quad = \frac{1}{\tilde{p}^2}\Big[\frac{(1+\xi)}{2}C_{\beta\alpha} - \frac{(1-\xi)}{2}\frac{(\gamma^m)_{\beta\alpha}\tilde{p}_m D^2}{\tilde{p}^2}\Big]\delta^2(\theta_1 - \theta_2), \end{aligned} \tag{11}$$

where $\tilde{p}_m = p_m + k_{mn}p^n$, $\tilde{p}^2 = p^2 + 2k_{mn}p^m p^n + k^{mn}k_{ml}p_n p^l$, $D^2 = \partial^2 - \theta^\beta(\gamma^m)_{\beta\alpha}\tilde{p}_m\partial^\alpha + \theta^2\tilde{p}^2$. As we commented in our previous work [5], this dispersion relation has a structure common for the propagators in the CPT-even Lorentz-breaking theories (see e.g. [8]).

We are able to compute the radiative corrections to the super quantum electrodynamics Lorentz-breaking theory, Eq. (9), and choosing $f(\bar{\Phi}\Phi) = M\bar{\Phi}\Phi$ (i.e., a mass term to the scalar superfield), the diagrams that contribute to the effective action are depicted in Fig. 1. The corresponding expression can be cast as

$$S_{2l} = \int \frac{d^3p}{(2\pi)^3} d^2\theta [W^\alpha W_\alpha - M\Gamma^\alpha W_\alpha] \times \int \frac{d^3q}{(2\pi)^3} \frac{1}{(\tilde{q}^2 + M^2)[(\tilde{q} - \tilde{p})^2 + M^2]}. \quad (12)$$

This last integral can be evaluated by changing the variable of integration q to $\tilde{q}$. In the case of $p^2 \approx 0$, we can write $\int d^3q = \Delta \int d^3\tilde{q}$, where $\Delta = \det(\frac{\partial q^m}{\partial \tilde{q}^n}) = \det^{-1}(\delta^m_n + k^m_n)$ is the Jacobian of the changing of variables. For $k_{mn} = \alpha u_m u_n$ with a small α, $\Delta \approx (1 - \alpha u^2)$. So, the final result is

$$S_{2l} = \frac{\Delta}{8\pi|M|} \int \frac{d^3p}{(2\pi)^3} d^2\theta [W^\alpha W_\alpha - M\Gamma^\alpha W_\alpha]. \quad (13)$$

We can observe that the one-loop quantum correction is finite. This model is known to be finite to all loop orders in perturbation theory in the usual superspace [9,10], and it is natural to expect that this issue persists in the aether superspace, since the power counting of the model is not affected by the presence of the Lorentz-breaking terms introduced through the aether superspace. We also observe the generation of a super Chern-Simons Lorentz-breaking term, whose corresponding bosonic local part has the form

$$\int d^3x M\Delta\epsilon^{lmn}A_l\nabla_m A_n = \int d^3x M\Delta\epsilon^{lmn}[A_l\partial_m A_n + k_{ms}A_l\partial^s A_n]. \quad (14)$$

We note that the Chern-Simons action, instead of the usual gauge transformations, is invariant under the new ones

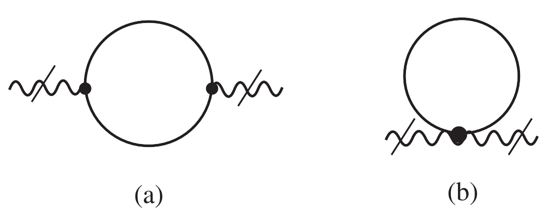

FIG. 1. One-loop contributions to the gauge superfield effective action. Continuous lines represent the scalar superfield propagator, and wave crossed lines represent the external gauge superfield.

$\delta A_n = \nabla_n\xi$, with ξ being a parameter of the gauge transformation.

Let us now discuss the Maxwell action. It is easy to see that after doing the Fourier transformation and reducing to the component fields, the Maxwell-like contribution from (13) looks like

$$S_M = -\frac{1}{4}\frac{\Delta}{8\pi|M|}\int d^3x\tilde{F}_{mn}\tilde{F}^{mn}, \quad (15)$$

where

$$\tilde{F}_{mn} = \nabla_m A_n - \nabla_n A_m \quad (16)$$

is a new stress tensor. We note that the derivatives ∇_m emerge from the supercovariant spinor derivatives. So, as can be seen, a new action for the vector field A_m is generated, which is (as the Chern-Simons one) invariant under the new gauge transformations $\delta A_n = \nabla_n\xi$. We note that the action (15) essentially differs from the usual aetherlike action for the gauge field [6], which is invariant under the usual gauge transformations $\delta A_n = \partial_n\xi$ and cannot be reduced to it.

B. Chern-Simons-matter model in aether superspace

Quantum field theories defined in a three-dimensional space-time are widely discussed in the literature because they offer a very rich structure, working as excellent theoretical laboratories. They can also be applied to some almost planar condensed matter systems, such as the quantum Hall effect [11]. More recently, supersymmetric gauge field theories in three dimensions could be related to M2-branes [12–14]. Of special interest is the computation of the effective superpotential of the supersymmetric Chern-Simons-matter model [15–20], which can be used to evaluate the possibility of spontaneous (super)symmetry breaking via the Coleman-Weinberg mechanism [21]. The presence of Lorentz-symmetry violating terms in the Lagrangian could be a source of spontaneous SUSY breaking [22] .

Let us start by defining the classical action of the model

$$S = \int d^5z\left\{\Gamma^\alpha W_\alpha - \frac{1}{2}\overline{\mathcal{D}^\alpha\Phi}\mathcal{D}_\alpha\Phi + \lambda(\bar{\Phi}\Phi)^2\right\}, \quad (17)$$

where $W^\alpha = (1/2)D^\beta D^\alpha\Gamma_\beta$ is the gauge superfield strength as defined before and $\mathcal{D}^\alpha = (D^\alpha - ie\Gamma^\alpha)$ is the supercovariant derivative.

The action Eq. (17) possesses manifest $\mathcal{N} = 1$ SUSY, and it can be lifted to $\mathcal{N} = 2$ by the elimination of the fermion-number violating terms [23], from which we identify the coupling constants as $\lambda = -e^2/8$. In the usual superspace, supersymmetric Chern-Simons–matter theory is superconformal invariant at the classical level, but in the aether superspace the presence of the constant vector k_{mn} explicitly breaks this invariance; even so, we should expect that an analog (or extended) symmetry could emerge from the action Eq. (17), but we will not extend such analysis in this paper.

The presence of a mass term like $\int d^5 z M\bar{\Phi}\Phi$, with a wrong sign, in the action Eq. (17) would generate a spontaneously (gauge) symmetry-broken phase at the classical level [24], but here we are interested in spontaneous symmetry breaking induced by radiative corrections (Coleman-Weinberg mechanism), and so we will keep the model massless at this level. To this end, let us shift the superfields $\bar{\Phi}$ and Φ by the classical background superfield φ as

$$\bar{\Phi} = \frac{1}{\sqrt{2}}(\varphi + \Phi_1 - i\Phi_2)\Phi = \frac{1}{\sqrt{2}}(\varphi + \Phi_1 + i\Phi_2), \quad (18)$$

where $\varphi = \varphi_1 - \theta^2\varphi_2$, with φ_1 and φ_2 being real constant fields.

Assuming the vanishing of the vacuum expectation values of the quantum superfields, i.e., $\langle\Phi_1\rangle = \langle\Phi_2\rangle = 0$ at any order of perturbation theory, the gauge-invariant action Eq. (17) results in

$$\begin{aligned} S = \int d^5 z\Big\{&\Gamma^\alpha W_\alpha - \frac{e^2\varphi^2}{4}\Gamma^\alpha\Gamma_\alpha - \frac{e\varphi}{2}D^\alpha\Gamma_\alpha\Phi_2 + \frac{1}{2}\Phi_1(D^2 + 3\lambda\varphi^2)\Phi_1 + \frac{1}{2}\Phi_2(D^2 + \lambda\varphi^2)\Phi_2 + \frac{1}{2}\varphi D^2\varphi + \frac{\lambda}{4}\varphi^4 \\ &+ \frac{e}{2}D^\alpha\Phi_2\Gamma_\alpha\Phi_1 - \frac{e}{2}D^\alpha\Phi_1\Gamma_\alpha\Phi_2 - \frac{e^2}{2}(\Phi_1^2 + \Phi_2^2)\Gamma^2 - e^2\varphi\Phi_1\Gamma^2 + \frac{\lambda}{4}(\Phi_1^4 + \Phi_2^4) + \frac{\lambda}{2}\Phi_1^2\Phi_2^2 + \lambda\varphi\Phi_1(\Phi_1^2 + \Phi_2^2) \\ &- eD^\alpha\varphi\Phi_2\Gamma_\alpha + (\lambda\varphi^3 + D^2\varphi)\Phi_1 + \frac{1}{2\alpha}\Big(D^\alpha\Gamma_\alpha + \alpha\frac{e\varphi}{2}\Phi_2\Big)^2 + \bar{c}D^2c + \frac{\alpha}{4}e^2\varphi^2\bar{c}c + \frac{\alpha}{4}e^2\varphi\bar{c}\Phi_1 c\Big\}. \end{aligned} \quad (19)$$

In the last line we added a gauge fixing and the corresponding Faddeev-Popov terms. We have used an R_ξ gauge condition to eliminate the mixing between Γ and Φ_2 superfields, but this procedure is not enough to completely eliminate this mixing. Even so, the remaining term, $-eD^\alpha\varphi\Phi_2\Gamma_\alpha$, can be disregarded in the Kählerian approximation of the effective superpotential, because it contains a supercovariant derivative applied to the background superfield φ.

The knowledge of the Kählerian effective superpotential is enough to determine the possibility of spontaneous SUSY and gauge symmetry breaking [18,25]. We will evaluate it at two-loop order, where such effects are expected to show up [18,26,27].

The Feynman rules derived from Eq. (19) are given, in the Kählerian approximation (that is, by preserving the dependence in φ and dropping the dependences on $D_\alpha\varphi$ and $D^2\varphi$), by

$$\begin{aligned} &\langle T\Phi_1(k,\theta)\Phi_1(-k,\theta')\rangle \\ &\quad = -i\frac{D^2 - M_{\Phi_1}}{k^2 + M^2_{\Phi_1}}\delta^{(2)}(\theta - \theta'), \\ &\langle T\Phi_2(k,\theta)\Phi_2(-k,\theta')\rangle \\ &\quad = -i\frac{D^2 - M_{\Phi_2}}{k^2 + M^2_{\Phi_2}}\delta^{(2)}(\theta - \theta'), \\ &\langle T\Gamma_\alpha(k,\theta)\Gamma_\beta(-k,\theta')\rangle \\ &\quad = \frac{i}{4}\Big[\frac{(D^2 + M_A)D^2 D_\beta D_\alpha}{k^2(k^2 + M^2_\Gamma)} + \alpha\frac{(D^2 - \alpha M_\Gamma)D^2 D_\alpha D_\beta}{k^2(k^2 + \alpha^2 M^2_\Gamma)}\Big] \\ &\qquad \times \delta^{(2)}(\theta - \theta'). \end{aligned} \quad (20)$$

For simplicity, let us choose the SUSY Landau gauge $\alpha = 0$ (we have to remark that the effective superpotential is a gauge-dependent quantity [28]). With this choice, the ghost superfields are decoupled from the model, and we can identify the poles of the propagators of the interacting superfields as

$$M_{\Phi_1} = 3\lambda\varphi^2, \qquad M_{\Phi_2} = \lambda\varphi^2, \qquad M_\Gamma = \frac{e^2\varphi^2}{4}. \quad (21)$$

Proceeding as described in [29], considering the two-loop corrections depicted in Fig 2, and performing the integrals using the regularization by dimensional reduction [30], the two-loop Kählerian effective superpotential can be cast as

$$K(\varphi) = -\frac{b_2}{4}\varphi^4\Big(\frac{b_1}{b_2} - \frac{1}{2}\Delta^2 + \Delta^2\ln\frac{\varphi^2}{\mu}\Big) - \frac{B}{4}\varphi^4, \quad (22)$$

where B is a counterterm, μ is a mass scale introduced by the regularization, b_1 is a function of the coupling constants of Δ and of $1/\epsilon \equiv \frac{1}{3-D}$ (D is the dimension of the space-time). The quantity b_2 is explicitly given by

$$\begin{aligned} b_2 = -(&116e^6 + 543e^4\lambda + 432e^2\lambda^2 \\ &- 71552\lambda^3)/(12288\pi^2). \end{aligned} \quad (23)$$

The counterterm B is fixed through the following renormalization condition,

$$\frac{\lambda}{4} \equiv \frac{1}{4!}\frac{\partial^4 K(\varphi)}{\partial\varphi^4}\Big|_{\varphi=v}, \quad (24)$$

where v is the renormalization point. By substituting B in Eq. (22), the Kählerian effective superpotential results in

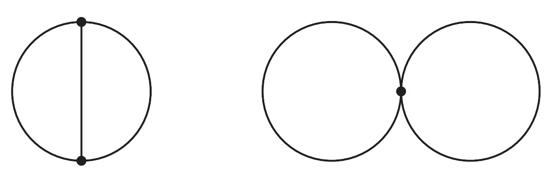

FIG. 2. Topologies of two-loop diagrams that contribute to the Kählerian effective superpotential.

$$K(\varphi) = -\frac{b_2\Delta^2}{4}\varphi^4 \ln\left[\frac{\varphi^2}{v^2}\exp\left(-\frac{\lambda}{b_2\Delta^2} - \frac{25}{6}\right)\right]. \quad (25)$$

Now we are able to study the spontaneous generation of mass to the physical superfields induced by the radiative corrections. First, let us impose the condition to extremize the Kählerian effective superpotential. It reads

$$\frac{\partial K(\varphi)}{\partial\varphi} = \frac{\varphi^3}{3}\left[3\lambda + 3b_2\Delta^2\left(\frac{11}{3} - \ln\frac{\varphi^2}{v^2}\right)\right] = 0. \quad (26)$$

The nontrivial solutions are given by

$$\varphi = \pm v \exp\left(\frac{11}{6} + \frac{\lambda}{2b_2\Delta^2}\right). \quad (27)$$

Since we have computed the effective superpotential for constant configurations of the background superfields, we expect that our approximation is valid for small fluctuations around the mass scale v, used as the renormalization point. This expectation constrains the exponential of the above equation to approximately 1. Therefore, the coupling constants satisfy $\frac{11}{6} + \frac{\lambda}{2b_2\Delta^2} \approx 0$, which results in the following condition:

$$\begin{aligned}\lambda &\approx \frac{11}{3}b_2\Delta^2 \\ &\approx [-(4\times 10^{-3})e^6 - (16\times 10^{-3})e^4\lambda \\ &\quad - (13\times 10^{-3})e^2\lambda^2 + 2\lambda^3]\Delta^2. \end{aligned} \quad (28)$$

For the Coleman-Weinberg mechanism, this last equation is directly related to the compatibility of the effective superpotential calculations with the assumptions of perturbation theory. We can see from Eq. (28) that λ should be of the order of $(4\times 10^{-3})e^6\Delta^2 + \mathcal{O}(e^{10})$, so that for small e we are in the regime of validity of the perturbative expansion.

The second derivative of the Kählerian effective superpotential with respect to the background field φ evaluated in the minimum of the superpotential, i.e. $\varphi \simeq \pm v$, is interpreted as the mass of the matter (background) superfield φ. If positive, this condition guarantees that Eq. (27) is a minimum of $K(\varphi)$. In fact, using Eqs. (27) and (28), we obtain

$$M_\varphi = \frac{d^2K(\varphi)}{d\varphi^2}\bigg|_{\varphi=v} \approx (2\times 10^{-3})e^6 v^2\Delta^2, \quad (29)$$

and the mass of the gauge superfield induced by the radiative corrections is given by

$$\frac{M_\Gamma}{M_\varphi} \sim \frac{e^2}{12\lambda} \sim -21\frac{e^{-4}}{\Delta^2}, \quad (30)$$

where we can notice that the mass of the gauge superfield is much larger than the mass of the matter superfield, since for a small violation of the Lorentz symmetry, Δ should be approximately 1.

One interesting remark is that all the information on the presence of Lorentz-violating terms in the original action is manifested in the presence of the Δ factor in the effective superpotential and, consequently, in the induced masses. Since the Kählerian effective superpotential has a consistent minimum, we can affirm that SUSY cannot be spontaneously broken via the Coleman-Weinberg mechanism. To search for SUSY breaking induced by the Lorentz-violating terms, we should probably compute the whole effective action, using some more sophisticated technology.

III. FOUR-DIMENSIONAL AETHER SUPERSPACE

Now, let us consider gauge theories within the four-dimensional aether superspace. In this case, the spinor supercovariant derivatives look like

$$D_\alpha = \partial_\alpha + i\bar{\theta}^{\dot\beta}\sigma^m_{\dot\beta\alpha}\nabla_m; \quad \bar{D}_{\dot\alpha} = \partial_{\dot\alpha} + i\theta^\beta\bar{\sigma}^m_{\beta\dot\alpha}\nabla_m, \quad (31)$$

where again $\nabla_m = \partial_m + k_{mn}\partial^n$. It is clear that these spinor derivatives satisfy the usual properties,

$$D_\alpha D_\beta D_\gamma = 0; \qquad \bar{D}_{\dot\alpha}\bar{D}_{\dot\beta}\bar{D}_{\dot\gamma} = 0. \quad (32)$$

Now, let us define the following Abelian gauge theory,

$$S = \int d^6z W^\alpha W_\alpha, \quad (33)$$

where

$$W_\alpha = \frac{1}{8}\bar{D}^2(e^{-v}D_\alpha e^v) = \frac{1}{8}\bar{D}^2 D_\alpha v. \quad (34)$$

In principle, the non-Abelian generalization of this theory can be constructed along the same lines.

This action can be rewritten as

$$S_W = -\frac{1}{16}\int d^8z v D^\alpha\bar{D}^2D_\alpha v. \quad (35)$$

As can be seen, its form does not differ from the usual action of gauge theories (see for example [7]); the only difference, from the usual case, consists in the replacement of the common spinor supercovariant derivative with a new one given by (31). It is clear that this action is invariant under the gauge transformations $\delta v = \Lambda + \bar{\Lambda}$, where Λ is a chiral superfield and $\bar{\Lambda}$ is an antichiral one.

Following the general principles, we suggest that the component expansion of the real scalar superfield v is the same as in the usual case, i.e., it depends on the relevant vector (gauge) field A_m as

$$v = -\frac{i}{2}(\bar{\theta}\sigma^m\theta)A_m(x) + \cdots. \quad (36)$$

By reducing the action (35) to the component fields, its bosonic part can be shown to have the form (15) up to the numerical factor, with the only difference being that the integral is now performed over the four-dimensional space-time.

Then, we must add the following gauge-fixing action,

$$S_{gf} = \frac{1}{16\alpha}\int d^8 z v D^2 \bar{D}^2 v, \quad (37)$$

where α is the gauge-fixing parameter.

The corresponding propagator looks like

$$\langle v(z_1)v(z_2)\rangle = -\frac{1}{\tilde{\Box}}\left(-\frac{D^\alpha \bar{D}^2 D_\alpha}{8\tilde{\Box}} + \alpha\frac{\{\bar{D}^2, D^2\}}{16\tilde{\Box}}\right)\delta^8(z_1 - z_2), \quad (38)$$

involving the new projection operators,

$$\Pi_0 = \frac{\{\bar{D}^2, D^2\}}{16\tilde{\Box}}, \qquad \Pi_{1/2} = -\frac{D^\alpha \bar{D}^2 D_\alpha}{8\tilde{\Box}}.$$

We then couple the gauge field to the chiral matter field ϕ by introducing the following action:

$$S_\Phi = \int d^8 z \bar{\phi} e^{gv}\phi. \quad (39)$$

The propagators of the chiral field look like (cf. [5])

$$\begin{aligned}\langle\phi(z_1)\bar{\phi}(z_2)\rangle &= \frac{\bar{D}^2 D^2}{16\tilde{\Box}}\delta^8(z_1 - z_2)\\ \langle\bar{\phi}(z_1)\phi(z_2)\rangle &= \frac{D^2 \bar{D}^2}{16\tilde{\Box}}\delta^8(z_1 - z_2).\end{aligned} \quad (40)$$

To calculate the one-loop Kählerian effective potential, we can use the well-developed methodology of calculating the superfield effective potential elaborated in [31–33]. As usual, one can begin with constructing the one-loop Feynman diagrams contributing to the superfield effective potential. The structure of the supergraphs does not essentially differ from the usual case [31]. The first set depicted in Fig. 3 involves only gauge propagators. Their sum is given by

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty}\frac{(-1)^n}{2n}\left(g^2\Phi\bar{\Phi}\frac{1}{\Box}(\Pi_{1/2} + \alpha\Pi_0)\right)^n \times \delta_{12}|_{\theta_1=\theta_2}, \quad (41)$$

where $\frac{1}{n}$ is a symmetry factor.

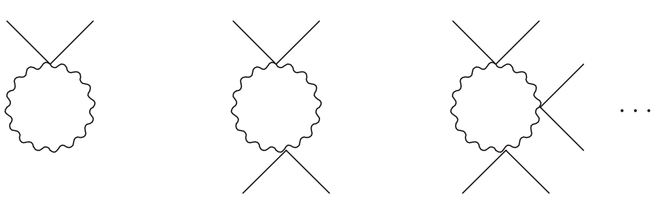

FIG. 3. Supergraphs composed by gauge propagators only.

Proceeding just as in [31], we find that

$$K_a^{(1)} = \int d^8 z \int\frac{d^4 p}{(2\pi)^4}\frac{1}{\tilde{p}^2}\left[\ln\left(1 + \frac{g^2\Phi\bar{\Phi}}{\tilde{p}^2}\right) - \ln\left(1 + \frac{\alpha g^2\Phi\bar{\Phi}}{\tilde{p}^2}\right)\right], \quad (42)$$

where $\tilde{p}^2 = (p_m + k_{mn}p^n)^2$ is a Fourier transform for $\tilde{\Box}$. Notice that at $\alpha = 0$ (Landau gauge), the second term in (42) vanishes. Using the notations adopted in [34], one can introduce a "dressed" propagator involving a sum over quartic vertices (see Fig. 4),

$$\langle vv\rangle_D = -\left(\frac{1}{\tilde{\Box} + g^2\Phi\bar{\Phi}}\Pi_{1/2} + \frac{\alpha}{\tilde{\Box} + \alpha g^2\Phi\bar{\Phi}}\Pi_0\right)\delta^8(z_1 - z_2). \quad (43)$$

The triple vertices will enter the Feynman diagrams only through the links depicted at Fig. 5, and the contribution from this sector is given by the sum of the supergraphs depicted at Fig. 6. It is equal to

$$K_b^{(1)} = \int d^8 z \int\frac{d^4 p}{(2\pi)^4}\frac{1}{\tilde{p}^2}\left[\ln\left(1 + \frac{\alpha g^2\Phi\bar{\Phi}}{\tilde{p}^2}\right)\right]. \quad (44)$$

The total result which is the sum of $K_a^{(1)}$ and $K_b^{(1)}$, is gauge invariant and equal to

$$K^{(1)} = \int d^8 z \int\frac{d^4 p}{(2\pi)^4}\frac{1}{\tilde{p}^2}\ln\left(1 + \frac{g^2\Phi\bar{\Phi}}{\tilde{p}^2}\right). \quad (45)$$

To calculate these integrals, we can change the variables as in [5]. After integration and subtracting the divergences we arrive at

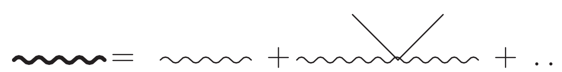

FIG. 4. Dressed propagator.

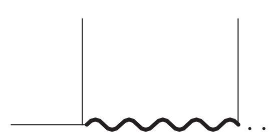

FIG. 5. A link involving gauge and matter propagators.

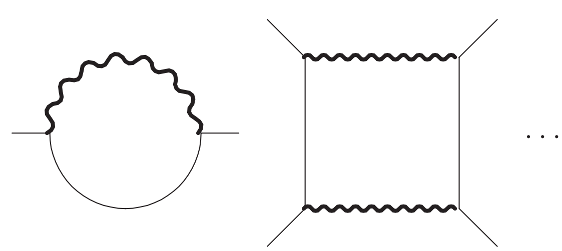

FIG. 6. Supergraphs composed by gauge and matter propagators.

$$K^{(1)} = -\frac{1}{32\pi^2}\Delta g^2\Phi\bar{\Phi}\ln\frac{g^2\Phi\bar{\Phi}}{\mu^2}, \qquad (46)$$

where Δ is again a Jacobian of the change of variables $k^m \to \tilde{k}^m$. We see that the result only differs from the usual case [32] by the multiplicative factor Δ.

IV. LORENTZ-BREAKING MODIFICATION OF THE SUPERSYMMETRY GENERATORS AND COORDINATE TRANSFORMATIONS

To close the paper, let us discuss the possible impact of the Lorentz-breaking modification of the supersymmetry generators for the generic quantum contributions to an effective action of arbitrary superfield theory. It follows from the the definitions of modified supersymmetry generators and covariant derivatives (1) and (31) that the methodology of Lorentz symmetry breaking adopted by us implies the change of all momenta that emerge through the D-algebra transformations by the rule $p_m \to p_m + k_{mn}p^n$ or, in the coordinate space, $\partial_m \to \partial_m + k_{mn}\partial^n$ (which corresponds to the linear coordinate change $x_m \to (\delta_m^n + k_m^n)^{-1}x_n$). Therefore, one can elaborate the following geometric interpretation of this Lorentz-breaking modification of SUSY algebra.

It was shown in our previous paper [5] that the one-loop contribution to the two-point function in a $3D$ self-coupled scalar superfield model looks like

$$\Gamma_2^{(1)} = \frac{\Delta}{8\pi|m|}\int d^3x d^2\theta \Phi(D^2 - 2m)\Phi. \qquad (47)$$

By projecting this action to components, we arrive at

$$\Gamma_2^{(1)} = \frac{\Delta}{8\pi|m|}\int d^3x(-\eta^{mn}\nabla_m\phi\nabla_n\phi + \psi^\alpha i(\gamma^m)_\alpha^\beta\nabla_m\psi_\beta + F^2 - 2m(\psi^2 + \phi F)). \qquad (48)$$

Let us perform, for this action, the analysis carried out in [35]. It is clear that one can formally introduce the upper-index metric

$$g^{ab} = \eta^{mn}(\delta_m^a + k_m^a)(\delta_n^b + k_n^b), \qquad (49)$$

with g_{ab} introduced as usual to be the inverse of g^{ab}. The Jacobian $\Delta = \det^{-1}(\delta_a^m + k_a^m)$ can naturally be treated as a contribution to the integral measure, since $\Delta = \sqrt{|\det g_{ab}|} = \sqrt{|g|}$. One can also introduce the modified Dirac matrices $\tilde{\gamma}^m = \gamma^m + k_n^m\gamma^n$, which satisfy the modified anticommutation relation

$$\{\tilde{\gamma}^a, \tilde{\gamma}^b\} = 2g^{ab}, \qquad (50)$$

where g^{ab} is given by (49). Therefore, the action (48) can be rewritten as

$$\Gamma_2^{(1)} = \frac{1}{8\pi|m|}\int d^3x\sqrt{|g|}(-g^{mn}\partial_m\phi\partial_n\phi + \psi^\alpha i(\tilde{\gamma}^m)_\alpha^\beta\partial_m\psi_\beta + F^2 - 2m(\psi^2 + \phi F)). \qquad (51)$$

Therefore, we can say that for the scalar superfield, our Lorentz-breaking modification of the supersymmetry generators is equivalent to the introduction of a new metric and, therefore, a new geometry (this is an affine geometry since the new metric is related to the Minkowski one through a constant matrix). It is easy to check that an analogous situation occurs also in the four-dimensional chiral superfield theory, that is, in our extension of the Wess-Zumino model.

However, the situation differs for the contributions involving external gauge legs. We have shown above that in the $3D$ gauge theory, the quantum correction is given by the expression (13). If we project it into components, the result in the purely gauge sector will be

$$S_M = \frac{\Delta}{8\pi|M|}\int d^3x\left(-\frac{1}{4}\eta^{ma}\eta^{nb}\tilde{F}_{mn}\tilde{F}_{ab} - M\epsilon^{abc}A_a\nabla_bA_c\right). \qquad (52)$$

Let us consider the Maxwell term, and, more precisely, one contribution to it, for example,

$$S_{M1} = \frac{\Delta}{8\pi|M|}\int d^3x\left(-\frac{1}{4}\right)\eta^{ma}\eta^{nb}\nabla_mA_n\nabla_aA_b. \qquad (53)$$

Repeating identically the arguments above, we can rewrite this expression as

$$S_{M1} = \frac{1}{8\pi|M|}\int d^3x\sqrt{|g|}\left(-\frac{1}{4}\right)g^{ma}\eta^{nb}\partial_mA_n\partial_aA_b. \qquad (54)$$

We see that while we succeeded in replacing the Minkowski metric with a new metric g^{ab} in a sector involving only the space-time derivatives, there is no way to form a new "curved" metric g^{ab} in a sector involving vector fields. This is related to the fact that within our methodology only the geometry (that is, coordinates, derivatives, metric and Dirac matrices) suffers transformations due to the introduction of the Lorentz-breaking parameters k_{ab}, but not the vector fields. Thus, there is no way to reabsorb the Lorentz breaking completely within a corresponding coordinate transformation in the gauge sector. This conclusion is similar to the one performed in [35].

V. SUMMARY

In this work we developed a gauge superfield method to construct Lorentz-breaking supersymmetric field theories based on the Kostelecky-Berger construction [3]. The methodology of superfields is a powerful tool for studying, among other subjects, the perturbative aspects of supersymmetric theories. Even though SUSY and its algebra is closely related to Lorentz symmetry, we could extend the superfield formalism to include the Lorentz violating terms, therefore allowing us to use the most attractive properties of superspace formalism. The aether superspace is a natural way to deal with Lorentz-violating supersymmetric models.

In this context, we presented some applications of the aether superspace techniques in three- and four-dimensional space-time, discussing perturbative aspects of supersymmetric quantum electrodynamics and the super Chern-Simons-matter model. We showed that, from the methodological viewpoint, the calculations do not essentially differ from the usual Lorentz-invariant case. However, as we have noted, the new theory involving the Lorentz symmetry breaking can be reduced, through simple rules, to the usual Lorentz-invariant theory, only if it is being considered in the purely scalar sector. If we deal with vector or spinor fields whose actions involve metrics contracted to fields, the redefinition of coordinates will not allow us to redefine completely all of the action, since it will imply variations of the fields that are not suggested by the initial structure of our modification of the supersymmetry generators (cf. [35]).

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