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Effective superpotential in the generic higher-derivative superfield supersymmetric three-dimensional gauge theory

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We formulate a generic three-dimensional superfield higher-derivative gauge theory coupled to matter, which in certain cases reduces to the three-dimensional scalar super-QED, supersymmetric Maxwell-Chern-Simons, or Chern-Simons theories with matter. For this theory, we explicitly calculate the one-loop effective potential.

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I. INTRODUCTION

The effective potential is a central object of quantum field theory, efficiently describing its low-energy effective dynamics [1]. In the supersymmetric theories, the most adequate method for its calculation is based on the superfield formalism, allowing one to maintain a manifest explicit supersymmetry at all steps of the calculation. The superfield methodology for calculating the effective potential was originally developed in Ref. [2] for a four-dimensional spacetime and, further, was successfully applied to the Wess-Zumino model in Ref. [3], to a more general model involving chiral superfields only in Ref. [4], to the supergauge theories in Ref. [5], and to the higher-derivative superfield theories in Refs. [6,7].

However, preliminary discussions of the superfield approach to the study of three-dimensional supersymmetric field theories themselves—especially the Chern-Simons theory [8]—and of the effective potential in three-dimensional superfield theories [9] began in the 1980s; in a more or less systematic way the superfield methodology for studying the effective potential has been formulated only recently [10,11]. However, the interest in three-dimensional field theories has grown recently, especially due to the study of the $N = 6$ and $N = 8$ Chern-Simons theories which display finiteness and conformal invariance [12]. Other important studies of the extended supersymmetric three-dimensional theories were presented in Ref. [13], where, in particular, the $N = 2$ and $N = 3$ superfield descriptions of these theories were given explicitly. Different issues related to the superfield Chern-Simons theories have also been considered in Refs. [14,15].

However, up to now no studies of the higher-derivative three-dimensional superfield theories have been carried out, whereas such a study could certainly be interesting (for example, it is natural to study the famous problem of ghosts in higher-derivative theories [16], especially when taking into account that in the three-dimensional superspace the convergence is better and the formulation is simpler). The only consideration of the higher-derivative supersymmetric theories in three dimensions has been carried out within the component approach in Ref. [17], where a one-loop effective potential for a model involving only scalar fields and their superpartners was calculated. In this work we suggest to fill this lack. We formulate the generic three-dimensional superfield higher-derivative gauge theory coupled to matter. For this theory, we present the generic methodology for calculating the effective potential, and calculate it in an explicit way. Throughout the paper, we follow the notations and conventions adopted in Ref. [18]. Our calculations will be carried out in Euclidean space.

II. HIGHER-DERIVATIVE SUPERSYMMETRIC GAUGE THEORY

We start with the following three-dimensional free generic Abelian gauge theory:

$$S = \frac{1}{2e^2}\int d^5z A^\beta \hat{R} D^\gamma D_\beta A_\gamma. \tag{1}$$

Here, $\hat{R}$ is some scalar operator commuting with $D^\gamma D_\beta$, and hence it is a function of D^2, spacetime derivatives, and some constants. This theory is evidently invariant under usual gauge transformations $\delta A_\alpha = D_\alpha K$, with K an arbitrary scalar superfield parameter. It is clear that if (up to the multiplicative constants) $\hat{R} = 1$ we have a Chern-Simons theory, if $\hat{R} = D^2$ we have a three-dimensional QED, and if $\hat{R} = D^2 + m$ we have a Maxwell-Chern-Simons theory. If $\hat{R}$ involves higher degrees of D^2—in particular, the d'Alembertian operator $\Box$ and its functions—we have the higher-derivative supersymmetric gauge theory. Earlier, the one-loop effective potential for this theory was calculated for only the supersymmetric Chern-Simons theory, $\hat{R} = 1$ [19], and the supersymmetric scalar QED $\hat{R} = D^2$ was discussed in Ref. [20]. We note that while the non-Abelian extension of this theory would be rather sophisticated—involving the vertices of self-couplings of the gauge superfield—the one-loop effective potential will be the same as in the Abelian case, up to the

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constant depending on the algebraic factor, since, at the one-loop level, only the vertices involving the external scalar legs give nontrivial contributions to the effective potential.

We add to this action the following gauge-fixing term:

$$S_{\rm GF} = \frac{1}{2e^2\alpha}\int d^5z A^\beta \hat{R} D_\beta D^\gamma A_\gamma, \tag{2}$$

which is a natural higher-derivative generalization of the usual gauge-fixing term. We note that this theory is Abelian, and therefore the ghosts completely decouple.

Now, let us couple the gauge superfield to the scalar matter. It is clear that the gauge covariant derivative is $D_\alpha - iA_\alpha$, so the object $(D_\alpha - iA_\alpha)\Phi$ [and, similarly, $(D_\alpha + iA_\alpha)\bar\Phi$] is transformed covariantly, i.e., if one transforms $\Phi \to e^{iK}\Phi$ and takes the gauge transformation of the field $A_\alpha \to A_\alpha + D_\alpha K$, one will arrive at $(D_\alpha - iA_\alpha)\Phi \to e^{iK}(D_\alpha - iA_\alpha)\Phi$. Hence, in principle, if we introduce $\nabla_\alpha \equiv D_\alpha - iA_\alpha$, we can introduce a higher-derivative kinetic term,

$$S^K_\Phi = -\frac{1}{2}\int d^5z \nabla^\alpha\nabla^\beta\ldots\nabla^\gamma\Phi(\nabla_\alpha\nabla_\beta\ldots\nabla_\gamma\Phi)^*. \tag{3}$$

We can also introduce the mass for the scalar field and the self-coupling for the scalar field of the form $\frac{\lambda}{2}\int d^5z(\Phi\bar\Phi)^n$. More generally, we will consider an arbitrary potential $V(\bar\Phi,\Phi)$. So, the complete action of the theory would look like

$$S_t = \int d^5z\Big[\frac{1}{2}\Big(A^\beta\frac{1}{e^2}\hat{R}\Big(D^\gamma D_\beta + \frac{1}{\alpha}D_\beta D^\gamma\Big)A_\gamma - \nabla^\alpha\nabla^\beta\ldots\nabla^\gamma\Phi(\nabla_\alpha\nabla_\beta\ldots\nabla_\gamma\Phi)^*\Big) + V(\bar\Phi,\Phi)\Big]. \tag{4}$$

However, for the first attempt we suggest that the higher derivatives be present only in the gauge sector, as it occurs in Ref. [7]. Therefore, the equation above reduces to

$$S_t = \int d^5z\Big[\frac{1}{2}\Big(A^\beta\frac{1}{e^2}\hat{R}\Big(D^\gamma D_\beta + \frac{1}{\alpha}D_\beta D^\gamma\Big)A_\gamma - \nabla^\alpha\Phi(\nabla_\alpha\Phi)^*\Big) + V(\bar\Phi,\Phi)\Big]. \tag{5}$$

The standard method of calculating the effective action is based on the methodology of the loop expansion [21]. To do this, we make a shift $\Phi \to \Phi + \phi$ in the superfield Φ (together with the analogous shift for the $\bar\Phi$), where now Φ is a background (super)field and ϕ is a quantum one. We suppose that the gauge field A_α is taken to be a purely quantum one. In order to calculate the effective action at the one-loop level, we have to keep only the quadratic terms in the quantum fluctuations ϕ, $\bar\phi$, and A_α. By using this prescription, we get

$$\begin{aligned}S_2[\Phi,\bar\Phi;\phi,\bar\phi,A_\alpha] &= \frac{1}{2}\int d^5z\Big[A^\beta\frac{1}{e^2}\hat{R}\Big(D^\gamma D_\beta + \frac{1}{\alpha}D_\beta D^\gamma\Big)A_\gamma \\ &\quad + 2\bar\phi D^2\phi + 2V_{\bar\Phi\Phi}\bar\phi\phi + i\Phi A^\alpha D_\alpha\bar\phi - i\bar\Phi A^\alpha D_\alpha\phi \\ &\quad + V_{\Phi\Phi}\phi^2 + V_{\bar\Phi\bar\Phi}\bar\phi^2 - \bar\Phi\Phi A^\alpha A_\alpha\Big],\end{aligned} \tag{6}$$

where the irrelevant terms were omitted, including those involving covariant derivatives of the background scalar superfields. Moreover, we use a shorthand notation: $V_{\bar\Phi\Phi} = \frac{\partial^2 V(\bar\Phi,\Phi)}{\partial\bar\Phi\partial\Phi}$, $V_{\Phi\Phi} = \frac{\partial^2 V(\bar\Phi,\Phi)}{\partial\Phi^2}$, $V_{\bar\Phi\bar\Phi} = \frac{\partial^2 V(\bar\Phi,\Phi)}{\partial\bar\Phi^2}$.

From Eq. (6), it follows that the propagators are given by

$$\begin{aligned}\langle A_\gamma(1)A^\alpha(2)\rangle &= \frac{e^2}{4k^2\hat{R}_1}(D^\alpha_1 D_{1\gamma} + \alpha D_{1\gamma}D^\alpha_1)\delta_{12}, \\ \langle\bar\phi(1)\phi(2)\rangle &= \frac{D^2_1}{k^2}\delta_{12},\end{aligned} \tag{7}$$

where $\delta_{12} \equiv \delta^2(\theta_1 - \theta_2)$ is the usual Grassmann delta function.

Now, let us study the Kähler potential. At one-loop order, the basic supergraphs contributing to the effective action in the theory under consideration are of three types: first, those with internal lines composed of only scalar propagators; second, those composed of only gauge propagators; third, those involving alternating gauge and matter propagators. However, as was argued in Ref. [19], if we consider the Landau gauge ($\alpha = 0$) then the last case need not be considered in our calculations, since the gauge superfield propagator $\langle A^\alpha A^\beta\rangle$ in this gauge is proportional to $D^\beta D_\alpha$, while the vertex to which this propagator is associated looks like $(\Phi A^\alpha D_\alpha\bar\phi - \bar\Phi A^\alpha D_\alpha\phi)$, so—after integration by parts—the D_α acts on the propagator $\langle A^\alpha A^\beta\rangle$, annihilating it due to the identity $D_\alpha D^\beta D^\alpha = 0$. From now on, all the calculations presented in this work will be performed in the Landau gauge for simplicity.

Since the vertices $(\Phi A^\alpha D_\alpha\bar\phi - \bar\Phi A^\alpha D_\alpha\phi)$ are irrelevant in the Landau gauge, we can discard them and rewrite the functional (6) as

$$\begin{aligned}S_2[\Phi,\bar\Phi;\phi,\bar\phi,A_\alpha] &= \frac{1}{2}\int d^5z\Big[A^\beta\frac{1}{e^2}\hat{R}\Big(D^\gamma D_\beta + \frac{1}{\alpha}D_\beta D^\gamma\Big)A_\gamma - \bar\Phi\Phi A^\alpha A_\alpha \\ &\quad + \phi^i P_i^j D^2\phi_j + \phi^i M_i^j\phi_j\Big],\end{aligned} \tag{8}$$

where

$$\begin{aligned}\phi_i &= \begin{pmatrix}\phi \\ \bar\phi\end{pmatrix}, \qquad \phi^i = (\phi \quad \bar\phi), \\ P_i^j &= \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}, \qquad M_i^j = \begin{pmatrix}V_{\Phi\Phi} & V_{\Phi\bar\Phi} \\ V_{\Phi\bar\Phi} & V_{\bar\Phi\bar\Phi}\end{pmatrix}.\end{aligned} \tag{9}$$

Therefore, the new propagators are ($\alpha = 0$)

$$\langle A_\gamma(1)A^\alpha(2)\rangle = \frac{e^2}{4k^2\hat{R}_1}D_1^\alpha D_{1\gamma}\delta_{12},$$
$$\langle \phi_i(1)\phi^j(2)\rangle = \frac{P_i^j D_1^2}{k^2}\delta_{12}. \tag{10}$$

These propagators will be used for the one-loop calculations.

III. ONE-LOOP CALCULATIONS

Let us start the calculations of the one-loop supergraphs contributing to the purely scalar sector, that is, those involving the scalar superfield propagators (10) connecting the vertices $\phi^i M_i{}^j \phi_j$. Such supergraphs exhibit the structures shown in Fig. 1.

We can compute all the contributions by noting that each supergraph above is formed by n "subgraphs," like those shown in Fig. 2.

Hence, the contribution of this subgraph is given by

$$(Q_{12})_i{}^j = (M_1)_i{}^k P_k{}^j \frac{D_1^2}{k^2}\delta_{12} = (\tilde{M}_1)_i{}^j \frac{D_1^2}{k^2}\delta_{12}, \tag{11}$$

$$\tilde{M} = \begin{pmatrix} V_{\Phi\bar{\Phi}} & V_{\Phi\Phi} \\ V_{\bar{\Phi}\bar{\Phi}} & V_{\Phi\bar{\Phi}} \end{pmatrix}. \tag{12}$$

It follows from the result above that the contribution of a supergraph formed by n subgraphs is given by

$$I_n = \int d^3x \frac{1}{2n}\int d^2\theta_1 d^2\theta_2 \ldots d^2\theta_n \int \frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\{(Q_{12})_i{}^j (Q_{23})_j{}^k \ldots (Q_{n-1,n})_l{}^m (Q_{n,1})_m{}^p\}$$
$$= \int d^3x \frac{1}{2n}\int d^2\theta_1 d^2\theta_2 \ldots d^2\theta_n \int \frac{d^3k}{(2\pi)^3}\,\mathrm{Tr}\Big\{\Big[(\tilde{M}_1)_i{}^j \frac{D_1^2}{k^2}\delta_{12}\Big]\Big[(\tilde{M}_2)_j{}^k \frac{D_2^2}{k^2}\delta_{23}\Big]\ldots\Big[(\tilde{M}_n)_m{}^p \frac{D_n^2}{k^2}\delta_{n,1}\Big]\Big\}, \tag{13}$$

where Tr denotes the trace over the matrix indices and $2n$ is a symmetry factor. Such a factor takes into account the Taylor series expansion coefficients of the effective action, the usual symmetry factor of each supergraph, and the number of topologically distinct supergraphs [22]. The external momenta have to be taken to be zero in the calculation of the effective potential.

We can integrate the expression I_n by parts to get

$$I_n = \int d^5z \int \frac{d^3k}{(2\pi)^3}\frac{1}{2n}\,\mathrm{Tr}[\tilde{M}^n]\Big(\frac{D^2}{k^2}\Big)^n \delta_{\theta\theta'}|_{\theta=\theta'}. \tag{14}$$

The effective action is given by the sum of all supergraphs I_n,

$$\Gamma_1^{(1)} = \sum_{n=1}^{\infty} I_n$$
$$= \int d^5z \int \frac{d^3k}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{2n}\,\mathrm{Tr}[\tilde{M}^n]\Big(\frac{D^2}{k^2}\Big)^n \delta_{\theta\theta'}|_{\theta=\theta'}. \tag{15}$$

It is not difficult to prove that $(D^2)^m\delta_{\theta\theta'}|_{\theta=\theta'} = 0$ for $m = 2l$, and that $(D^2)^m\delta_{\theta\theta'}|_{\theta=\theta'} = (\sqrt{-k^2})^{m-1}$ for $m = 2l+1$, where l is a non-negative integer. It follows that

$$\Gamma_1^{(1)} = \int d^5z \int \frac{d^3k}{(2\pi)^3}\sum_{l=0}^{\infty}\frac{(-1)^l}{2(2l+1)}\mathrm{Tr}[\tilde{M}^{2l+1}]\frac{1}{(k^2)^{l+1}}$$
$$= \int d^5z \int \frac{d^3k}{(2\pi)^3}\sum_{l=0}^{\infty}\frac{(-1)^l}{2(2l+1)}[\lambda_1^{2l+1} + \lambda_2^{2l+1}]\frac{1}{(k^2)^{l+1}}, \tag{16}$$

where the λ's are the eigenvalues of the matrix $\tilde{M}$, namely $\lambda_{1,2} = V_{\bar{\Phi}\Phi} \pm (V_{\Phi\Phi}V_{\bar{\Phi}\bar{\Phi}})^{1/2}$. Hence, by substituting these eigenvalues into Eq. (16) and summing over all l we get

$$\Gamma_1^{(1)} = \frac{1}{2}\int d^5z \int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}$$
$$\times\Big[\arctan\Big(\frac{V_{\bar{\Phi}\Phi} + (V_{\Phi\Phi}V_{\bar{\Phi}\bar{\Phi}})^{1/2}}{|k|}\Big) + \arctan\Big(\frac{V_{\bar{\Phi}\Phi} - (V_{\Phi\Phi}V_{\bar{\Phi}\bar{\Phi}})^{1/2}}{|k|}\Big)\Big]. \tag{17}$$

Finally, we can compute these integrals to get

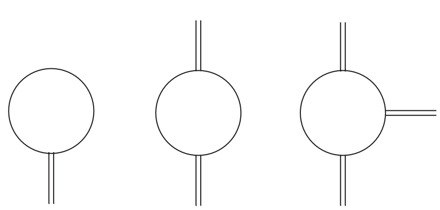

FIG. 1. One-loop supergraphs in a purely scalar sector.

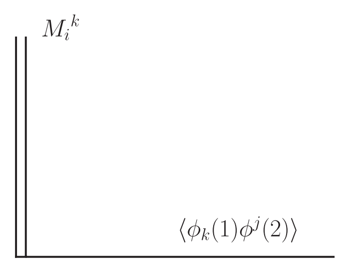

FIG. 2. A typical vertex in one-loop supergraphs in the matter sector.

$$\Gamma_1^{(1)} = -\frac{1}{8\pi}\int d^5z(V^2_{\bar{\Phi}\Phi} + V_{\Phi\Phi}V_{\bar{\Phi}\bar{\Phi}}). \tag{18}$$

We conclude here that this contribution to the one-loop effective action does not display any divergences, independently of the form of the potential $V(\bar{\Phi}, \Phi)$. We note that, unlike Ref. [19], here we used only the supergraph summation instead of the functional trace calculations.

Let us move on to the calculation of the one-loop supergraphs involving the gauge superfield propagator connecting the vertices, $-\bar{\Phi}\Phi A^\alpha A_\alpha$. Such supergraphs exhibit the structure shown in Fig. 3.

As before, we can compute all the contributions by noting that each supergraph above is formed by n subgraphs, like those depicted in Fig. 4.

This subgraph provides the contribution

$$(P_{12})_{\alpha_1}{}^{\alpha_2} = -\frac{e^2(\bar{\Phi}\Phi)_1}{4k^2}\frac{1}{\hat{R}_1}D_1^{\alpha_2}D_{1,\alpha_1}\delta_{12}. \tag{19}$$

It follows from the result above that the contribution of a supergraph formed by n subgraphs is given by

$$\begin{aligned} J_n &= (2\pi)^3\delta^3(0)\frac{1}{2n}\int d^2\theta_1 d^2\theta_2\ldots d^2\theta_n\int\frac{d^3k}{(2\pi)^3}(P_{12})_{\alpha_1}{}^{\alpha_2}(P_{23})_{\alpha_2}{}^{\alpha_3}\ldots(P_{n-1,n})_{\alpha_{n-1}}{}^{\alpha_n}(P_{n,1})_{\alpha_n}{}^{\alpha_1} \\ &= (2\pi)^3\delta^3(0)\frac{1}{2n}\int d^2\theta_1 d^2\theta_2\ldots d^2\theta_n\int\frac{d^3k}{(2\pi)^3}\left[-\frac{e^2(\bar{\Phi}\Phi)_1}{4k^2}\frac{1}{\hat{R}_1}D_1^{\alpha_2}D_{1,\alpha_1}\delta_{12}\right] \\ &\quad\times\left[-\frac{e^2(\bar{\Phi}\Phi)_2}{4k^2}\frac{1}{\hat{R}_2}D_2^{\alpha_3}D_{2,\alpha_2}\delta_{23}\right]\cdots\left[-\frac{e^2(\bar{\Phi}\Phi)_n}{4k^2}\frac{1}{\hat{R}_n}D_n^{\alpha_1}D_{n,\alpha_n}\delta_{n,1}\right]. \end{aligned} \tag{20}$$

After successive integrations by parts and summing all supergraphs J_n, we get the effective action

$$\begin{aligned} \Gamma_2^{(1)} = \int d^5z\int\frac{d^3k}{(2\pi)^3}\sum_{n=1}^{\infty}\frac{1}{2n}\left(-\frac{e^2\bar{\Phi}\Phi}{4k^2}\right)^n \\ \times\frac{1}{\hat{R}^n}D^{\alpha_2}D_{\alpha_1}D^{\alpha_3}D_{\alpha_2}\ldots D^{\alpha_n}D_{\alpha_{n-1}} \\ \times D^{\alpha_1}D_{\alpha_n}\delta_{\theta\theta'}|_{\theta=\theta'}. \end{aligned} \tag{21}$$

At this stage of the calculation, we have to specify the operator $\hat{R}$ in order to proceed with the calculation of $\Gamma_2^{(1)}$. The most general choice is $\hat{R} = f(\Box) + g(\Box)D^2$ (recall that this operator is a scalar). This expression is rather generic. The result of the complete evaluation of the D-algebra essentially depends on the explicit form of the operator $\hat{R}$. So, let us consider two characteristic examples where the final result is expressed in closed form and in terms of elementary functions.

The first example is $f = 0$ and $g \neq 0$, so we have

$$\hat{R} = g(\Box)D^2 \Rightarrow \frac{1}{\hat{R}^n} = \left(\frac{-1}{g(k^2)k^2}\right)^n(D^2)^n. \tag{22}$$

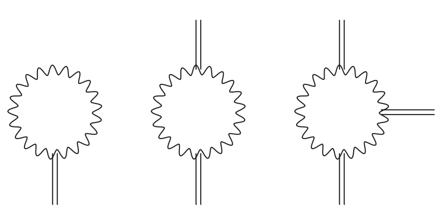

FIG. 3. One-loop supergraphs in a gauge sector.

It follows from the covariant derivative algebra that $(D^2)^nD^{\alpha_2}D_{\alpha_1}D^{\alpha_3}D_{\alpha_2}\ldots D^{\alpha_1}D_{\alpha_n}\delta_{\theta\theta'}|_{\theta=\theta'} = 0$ for all n. Therefore, from the Eqs. (21) and (22), we have

$$\Gamma_2^{(1)} = 0. \tag{23}$$

In conclusion, the complete one-loop Kähler effective potential is completely given by the expression (18),

$$\begin{aligned} K^{(1)}(\Phi, \bar{\Phi}) = -\frac{1}{8\pi}(V^2_{\bar{\Phi}\Phi} + V_{\Phi\Phi}V_{\bar{\Phi}\bar{\Phi}}), \\ \text{for } f(\Box) = 0 \quad \text{and} \quad g(\Box) \neq 0. \end{aligned} \tag{24}$$

This result is consistent with the claim made in Ref. [20] that, in the case where the self-coupling of the scalar field is absent, the one-loop Kähler effective potential for the three-dimensional QED (that is, $g = 1$) identically vanishes. We have shown that the same situation occurs for all classes of theories in which $g \neq 1$ but $f = 0$.

Our second example is $f = \xi(-\Box)^m$ and $g = 0$, where ξ is a parameter with a nontrivial mass dimension

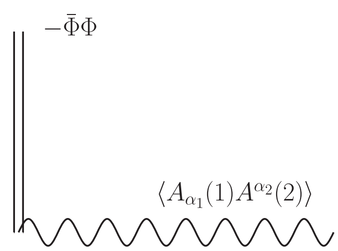

FIG. 4. A typical vertex in one-loop supergraphs in the matter sector.

$[\xi] = [M]^{-2m}$, $\xi > 0$, and m is a non-negative integer. Consequently, we have trivially

$$\hat{R} = \xi(-\Box)^m \Rightarrow \frac{1}{\hat{R}^n} = \left(\frac{1}{\xi(k^2)^m}\right)^n. \tag{25}$$

It can be shown that $D^{\alpha_2}D_{\alpha_1}D^{\alpha_3}D_{\alpha_2}\ldots D^{\alpha_1}D_{\alpha_n}\delta_{\theta\theta'}|_{\theta=\theta'} = 0$ for $n = 2l$, and $D^{\alpha_2}D_{\alpha_1}D^{\alpha_3}D_{\alpha_2}\ldots D^{\alpha_1}D_{\alpha_n}\delta_{\theta\theta'}|_{\theta=\theta'} = 2^n(\sqrt{-k^2})^{n-1}$ for $n = 2l+1$, where l is a non-negative integer. Hence, from Eqs. (21) and (25), we get

$$\Gamma_2^{(1)} = \int d^5z \int \frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{-k^2}}\sum_{l=0}^{\infty}\frac{1}{2(2l+1)} \times \left(-\frac{e^2\bar{\Phi}\Phi\sqrt{-k^2}}{2\xi(k^2)^{m+1}}\right)^{2l+1}$$
$$= -\frac{1}{2}\int d^5z \int \frac{d^3k}{(2\pi)^3}\frac{1}{|k|}\arctan\left(\frac{e^2\bar{\Phi}\Phi|k|}{2\xi(k^2)^{m+1}}\right), \tag{26}$$

where we have used the fact that $\sqrt{-k^2} = i|k|$ and the identity $\arctan(x) = \frac{1}{i}\,\mathrm{arctanh}(ix)$. The integral above can be solved by induction. Then we obtain

$$\Gamma_2^{(1)} = -\int d^5z \frac{1}{16\pi}\sec\left(\frac{\pi}{2m+1}\right)\left(\frac{e^2\bar{\Phi}\Phi}{2\xi}\right)^{\frac{2}{2m+1}}. \tag{27}$$

It is worth noticing that this result is finite and does not need any renormalization, which, however, is a rather generic effect in the three-dimensional superfield theories [20]. Moreover, we note that if the operator $\hat{R}$ is of the first order in spacetime derivatives or (similarly) of the second order in spinor supercovariant derivatives, the theory is super-renormalizable, with the only possible divergences being the two-loop ones and no divergences at higher order. This is just the situation of the super-QED [23], and if the operator $\hat{R}$ is of second order in spacetime derivatives the corresponding theory is all-loop finite.

Again, the complete one-loop Kähler effective potential can be read off from the sum of Eqs. (18) and (27). As a result, we finally obtain

$$K^{(1)}(\bar{\Phi}, \Phi) = -\frac{1}{16\pi}\sec\left(\frac{\pi}{2m+1}\right)\left(\frac{e^2\bar{\Phi}\Phi}{2\xi}\right)^{\frac{2}{2m+1}} - \frac{1}{8\pi}(V^2_{\bar{\Phi}\Phi} + V_{\Phi\Phi}V_{\bar{\Phi}\bar{\Phi}}), \tag{28}$$

for $f(\Box) = \xi(-\Box)^m$ and $g(\Box) = 0$.

The result (28) is highly generic. In particular, if $m = 0$, $\xi = 1$, and $V(\bar{\Phi}\Phi) = \frac{\lambda}{2}(\bar{\Phi}\Phi)^2$, we get

$$K^{(1)}(\bar{\Phi}, \Phi) = \frac{1}{64\pi}(e^2\bar{\Phi}\Phi)^2 - \frac{5}{8\pi}\lambda^2(\bar{\Phi}\Phi)^2. \tag{29}$$

This is just the (Euclidean) one-loop Kähler effective potential for the Chern-Simons theory coupled to a self-interacting massless scalar matter without higher derivatives. Our result agrees with that obtained in Ref. [19].

IV. SUMMARY

We formulated a generic Abelian three-dimensional supergauge theory coupled to matter. In the general case, the classical action of this theory involves higher derivatives. However, despite of this, we developed a universal procedure for calculating the one-loop effective potential for this theory—which actually applies to a wide class of theories including supersymmetric Chern-Simons theory, supersymmetric QED, supersymmetric Maxwell-Chern-Simons theory, and their non-Abelian generalizations—and found that the result is rather generic for a wide class of the theories. In particular, we explicitly demonstrated that for the three-dimensional supersymmetric QED the one-loop effective potential vanishes. Also, we noted that any three-dimensional higher-derivative supersymmetric gauge theory is all-loop finite.

Studies of the higher-derivative superfield theories can have natural continuation. For example, it is interesting to look for other ways to introduce higher derivatives in the four-dimensional superfield theories that are different from those presented in Refs. [6,7].

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