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Refractive index in the viscous quark-gluon plasma

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Under the framework of the viscous chromohydrodynamics, the gluon self-energy is derived for the quark-gluon plasma with shear viscosity. The viscous chromoelectric permittivity and chromomagnetic permeability are evaluated from the gluon self-energy, through which the chromorefraction index is investigated. The numerical analysis indicates that the chromorefractive index becomes negative in some frequency range. The starting point for that frequency range is around the chromoelectric permittivity pole, and the chromomagnetic permeability pole determines the endpoint. As η/s increases, the frequency range for the negative refraction becomes wider.

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I. INTRODUCTION

Quantum chromodynamics (QCD) predicts that color degrees of freedom will be liberated at high temperature and/or high density, as a result the nuclear matter will undergo a transition to quark-gluon plasma (QGP), a microscopic system composed by liberated quarks and gluons. One main goal for the Relativistic Heavy Ion Collider (RHIC) and the Large Hadronic Collider (LHC) is to seek this new state of matter. There are two striking findings at RHIC. One is that the produced hot medium behaves as a nearly perfect fluid with a small viscosity [1–4]. Several groups have applied viscous hydrodynamics to simulate the evolution of the produced matter in heavy ion collisions. The simulations successfully fit the observables at RHIC, such as the elliptic flow, the particle spectra, etc. [5–13]. The other is the strong jet quenching, which is believed to be a potential signal for the QGP formation [14]. It should be stressed that the first LHC results also strongly support similar conclusions as seen at RHIC [15,16].

At the very early stage of the relativistic heavy ion collisions, named the glasma stage [17], and at the late stage of the evolution process in the near T_c region in the so-called magnetic scenario for the QGP [18,19], there are color-electric flux tubes which contain strong color-electric fields. Therefore, the color electromagnetic properties may play an important role in the evolution of hot and dense matter produced in heavy ion collisions. So the study of them may be helpful for understanding the nature of QGP. However, to the best of our knowledge, the main emphasis of the viscous hydrodynamic simulations has been laid on the collective flow and the particle spectra formed in heavy ion collisions, the study on the color electromagnetic properties of viscous QGP is scarce in literature. It makes sense to study how the viscosity affects them and how the viscous electromagnetic properties affect the evolution of produced matter in heavy ion collisions.

Refraction index reflects the property of light propagation in an electromagnetic medium. It can be determined in terms of the electric permittivity $\varepsilon(\omega, k)$ and magnetic permeability $\mu_M(\omega, k)$. In 1968, Veselago proposed in theory that the refraction index might be negative in some special material [20]. That kind of medium is in nature consistent with the one proposed by Mandelstam in which the electromagnetic phase velocity propagates antiparallel to the energy flow [21]. But no natural material shows such special properties. Around 2000, by manipulating the array of small and closely spaced elements, scientists have constructed the negative refraction material in the laboratory [22,23]; since then, the study on the negative refraction has attracted intensive interest. Recently, Amariti *et al.* have studied the refraction index of the strong coupled system with the string-inspired theory of AdS/CFT correspondence [24]. Then, some investigations have been carried out in strong coupled and correlation systems along that line [25–28]. It is argued that the negative refraction is a general phenomenon in some frequency range in charged fluid systems [28]. The probability for the existence of negative refraction in QGP is discussed as well in that literature [28]. Later, Juan Liu *et al.* extended the study of the refractive index of light to the weak coupled quark-gluon system within the framework of the hard thermal loop perturbative theory [29].

In this paper, we will make a first step to study the refraction index of gluon in the viscous QGP with the viscous chromohydrodynamics. A gluon is the QCD counterpart of a photon. In addition, jet quenching has been proposed as a potential signal for the QGP and has become an active field in heavy ion collisions in last three decades, which is relevant to the parton propagation in the hot medium. So the study of gluon refraction in QGP may be

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helpful for the understanding of the nature of the QGP. Since we are going to study the chromodynamic properties of QGP here, for consistency we add the prefix "chromo" to the electromagnetic quantities in QGP, such as chromoelectric permittivity, chromomagnetic permeability, and chromorefraction in the following whenever a chromodynamic quantity occurs.

According to Refs. [30–34], viscosity will modify the distribution functions of the constituents of the QGP, thus it will affect the gluon self-energy through which the chromoelectric permittivity and chromomagnetic permeability can be derived. Therefore, viscosity will have an impact on the chromorefraction index.

It is argued that chromohydrodynamics can describe the polarization effect as the kinetic theory [35]. In a recent paper [36], some authors have extended the ideal chromohydrodynamics [37–40] to the viscous one in terms of the QGP kinetic theory and the distribution function modified by the shear viscosity. Under that framework, the polarization tensor is derived and the chromoelectric permittivity in the viscous QGP is studied in detail [36]. Based on the chromoelectric permittivity, the induced color charge distribution [41] and the corresponding wake potential [42] induced by the fast parton traveling through the viscous QGP have been investigated later.

In the present paper, by following the gluon polarization tensor derived from the viscous chromohydrodynamics, we will derive the chromomagnetic permeability and then study the chromorefraction index in the QGP associated with shear viscosity. Our main results are as follows. Within the framework of the viscous chromohydrodynamics, the chromorefraction index in the viscous QGP becomes negative in some frequency range. The starting point of that frequency range is around the chromoelectric permittivity pole, while the chromomagnetic permeability pole determines the endpoint. In addition, with the increase of η/s, the frequency range for the existence of the negative refraction becomes broadening.

The paper is organized as follows. In Sec. II, we will briefly review the formalism of electromagnetic properties in an electromagnetic plasma, which can be applied to study chromoelectromagnetic properties of QGP. In Sec. III, according to the polarization tensor derived from the viscous chromohydrodynamics, we will evaluate the chromorefraction index and discuss the viscous effect on it. Section IV is a summary and remarks.

The natural units $k_B = \hbar = c = 1$, the metric $g_{\mu\nu} = (+, -, -, -)$, and the following notations $K = (\omega, \mathbf{k})$ are used in the paper.

II. THE ELECTROMAGNETIC PROPERTIES IN PLASMA

In order to describe the electric and magnetic properties in plasma covariantly, it is convenient to introduce a pair of four-vectors $\tilde{E}^\mu$, $\tilde{B}^\mu$ in terms of the fluid four-velocity u^ν

$$\tilde{E}^\mu = u_\nu F^{\nu\mu}, \qquad \tilde{B}^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\nu\lambda}u_\rho \tag{1}$$

and

$$F^{\mu\nu} = u^\mu \tilde{E}^\nu - \tilde{E}^\mu u^\nu + \epsilon^{\mu\nu\lambda\rho}\tilde{B}_\lambda u_\rho, \tag{2}$$

where the Greek index μ is not confused with the magnetic permeability μ_M. According to Eqs. (1) and (2), one can obtain the Fourier-transformed free action

$$S_0 = -\frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\{\tilde{E}^\mu(K)\tilde{E}_\mu(-K) - \tilde{B}^\mu(K)\tilde{B}_\mu(-K)\}. \tag{3}$$

Taking into account the interaction between the constituents of plasma, the correction to the action is

$$S_{\rm int} = -\frac{1}{2}\int \frac{d^4K}{(2\pi)^4} A^\mu(-K)\Pi_{\mu\nu}(K)A^\nu(K), \tag{4}$$

where $A^\mu(K)$ is a vector-boson field in momentum space, and $\Pi_{\mu\nu}(K)$ is the polarization tensor which embodies the medium effects in plasma. In homogeneous and isotropic medium, the polarization tensor can be divided into longitudinal and transverse parts $\Pi_{\mu\nu}(K) = \Pi_L(K)P^L_{\mu\nu}(K) + \Pi_T(K)P^T_{\mu\nu}(K)$ with projectors defined as $P^T_{00} = P^T_{0i} = P^T_{i0} = 0$ [43,44],

$$P^T_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}, \qquad P^L_{\mu\nu} = \frac{k_\mu k_\nu}{K^2} - g_{\mu\nu} - P^T_{\mu\nu}.$$

Thus, the effective action including medium effects is

$$S_{\rm eff} = S_0 + S_{\rm int}, \tag{5}$$

which also can be described as

$$S_{\rm eff} = -\frac{1}{2}\int \frac{d^4K}{(2\pi)^4}\left[\varepsilon \tilde{E}^\mu(K)\tilde{E}_\mu(-K) - \frac{1}{\mu_M}\tilde{B}^\mu(K)\tilde{B}_\mu(-K)\right]. \tag{6}$$

In (6), ε and μ_M represent the electric permittivity and magnetic permeability, respectively, which can describe the difference of the electric and magnetic properties of the vector field in the medium and in the vacuum. According to Eqs. (3), (4), and (6), one can get the electric permittivity and magnetic permeability in an electromagnetic plasma as follows:

$$\varepsilon(\omega, k) = 1 - \frac{\Pi_L(\omega, k)}{K^2}, \tag{7}$$

$$\frac{1}{\mu_M(\omega, k)} = 1 + \frac{K^2\Pi_T(\omega, k) - \omega^2\Pi_L(\omega, k)}{k^2K^2}. \tag{8}$$

We have briefly reviewed the electric and magnetic properties in homogeneous and isotropic plasma, the detailed derivation also can be found in Refs. [29,45,46].

In addition, the extension of the discussion to the anisotropic medium has also been addressed in Ref. [29].

The refraction index is generally defined by the electric permittivity and magnetic permeability as $n^2 = \varepsilon(\omega, k)\mu_M(\omega, k)$ which is a square definition and not sensitive to the simultaneous change of signs of ε and μ_M. But it is proposed by Veselago that the simultaneous change of positive ε and μ_M to negative $-\varepsilon$ and $-\mu_M$ corresponds to the transformation of the refraction index from one branch $n = \sqrt{\varepsilon(\omega, k)\mu_M(\omega, k)}$ to the other $n = -\sqrt{\varepsilon(\omega, k)\mu_M(\omega, k)}$, i.e., the turn from the general refraction index to the negative one. The physical nature of the negative refraction is that the electromagnetic phase velocity propagates opposite to the energy flow. For the detailed discussion on the physics of negative refraction, please refer to Refs. [20,21,47]. The criterion for the negative refraction is $\varepsilon < 0$ and $\mu < 0$ simultaneously for real electric permittivity and magnetic permeability medium.

If dissipation is taken into account, the situation is complicated. The electric permittivity, magnetic permeability, and the refraction index n are generally complex functions with both real and imaginary parts. For example, $\varepsilon(\omega, k)$ and $\mu_M(\omega, k)$ can be expressed as the form $\varepsilon(\omega, k) = \varepsilon_r + i\varepsilon_i$, $\mu_M(\omega, k) = \mu_r + i\mu_i$. According to the phase velocity propagating antiparallel to the energy flow, some authors have derived the condition for negative refraction in dissipative medium and found that it is not necessary for $\varepsilon_r < 0$ and $\mu_r < 0$ simultaneously [48]. Later, another simple, convenient, and widely adopted condition has been derived as [49]

$$n_{\mathrm{eff}} = \varepsilon_r|\mu_M| + \mu_r|\varepsilon| < 0, \tag{9}$$

where n_{eff} is called the Depine-Lakhtakia index. $n_{\mathrm{eff}} < 0$ implies $\mathrm{Re}n < 0$, otherwise we will have a normal refraction index.

III. THE REFRACTION INDEX IN THE VISCOUS QUARK-GLUON PLASMA

In this section, according to the formalism presented in Sec. II, we will study the chromoelectromagnetic properties in the viscous QGP. At first, we will briefly review the derivation of the viscous chromohydrodynamics applicable to QGP with shear viscosity. Then, we will solve fluid equations to obtain the gluon polarization tensor. According to the gluon self-energy, the chromoelectric permittivity and chromomagnetic permeability are determined, through which the chromorefraction index will be investigated.

A. Kinetic theory

The kinetic equations for quarks, antiquarks, and gluons are given by [50,51]

$$p^\mu D_\mu Q(p,x) + \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu Q(p,x)\} = C[Q, \bar{Q}, G], \tag{10}$$

$$p^\mu D_\mu \bar{Q}(p,x) - \frac{g}{2} p^\mu \{F_{\mu\nu}(x), \partial_p^\nu \bar{Q}(p,x)\} = \bar{C}[Q, \bar{Q}, G], \tag{11}$$

$$p^\mu \mathcal{D}_\mu G(p,x) + \frac{g}{2} p^\mu \{\mathcal{F}_{\mu\nu}(x), \partial_p^\nu G(p,x)\} = C_g[Q, \bar{Q}, G]. \tag{12}$$

$Q(p,x)$, $\bar{Q}(p,x)$, and $G(p,x)$ denote the distribution functions of quark, antiquark, and gluon, respectively. $Q(p,x)$ $[\bar{Q}(p,x)]$ is a $N_c \times N_c$ Hermitian matrix in the color space for the color $SU(N_c)$ group and transforms under the local gauge transformation as $Q(p,x)(\bar{Q}(p,x)) \to U(x)Q(p,x)(\bar{Q}(p,x))U^\dagger(x)$. While the gluon distribution function is a $(N_c^2 - 1) \times (N_c^2 - 1)$ Hermitian matrix, and it transforms as $G(p,x) \to M(x)G(p,x)M(x)^\dagger$ where $M_{ab}(x) = \mathrm{Tr}[\tau_a U(x)\tau_b U^\dagger(x)]$ with τ_a $(a = 1, \ldots, N_c^2 - 1)$ being the $SU(N_c)$ group generators. ∂_p^ν represents the four-momentum derivative and $\{\cdots, \cdots\}$ is the anticommutator. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ represents the strength tensor in the fundamental representation, and $\mathcal{F}_{\mu\nu}$ is its counterpart in the adjoint representation. D_μ and $\mathcal{D}_\mu$ represent the covariant derivatives

$$\begin{aligned} D_\mu &= \partial_\mu - ig[A_\mu(x), \cdots], \\ \mathcal{D}_\mu &= \partial_\mu - ig[\mathcal{A}_\mu(x) \cdots]. \end{aligned} \tag{13}$$

A_μ and $\mathcal{A}_\mu$ denote four-potentials in the fundamental and adjoint representations, respectively,

$$A_\mu(x) = A_{\mu,a}(x)\tau^a, \qquad \mathcal{A}_\mu(x) = T^a A_{\mu,a}(x), \tag{14}$$

where τ^a and T^a are the generators of group $SU(N_c)$ in the corresponding representations; C, $\bar{C}$, and C_g denote the collision terms.

The transport equations are supplemented by the Yang-Mills equation,

$$D_\mu F^{\mu\nu}(x) = j^\nu(x), \tag{15}$$

the color current $j^\nu(x)$ is given in the fundamental representation as

$$\begin{aligned} j^\nu(x) = -\frac{g}{2}\int_p p^\nu\Big[Q(p,x) - \bar{Q}(p,x) - \frac{1}{3}\mathrm{Tr}[Q(p,x) \\ - \bar{Q}(p,x)] + 2\tau^a \mathrm{Tr}[T^a G(p,x)]\Big], \end{aligned} \tag{16}$$

where

$$\int_p = \int \frac{d^4p}{(2\pi)^3} 2\Theta(p_0)\delta(p^2).$$

Equations (10)–(12), (15), and (16) make up the fundamental equations of the kinetic theory for the quark-gluon plasma. In the linear approximation of QCD transport equation, by using the ideal, equilibrium distribution functions of constituents of the QGP, one can obtain the gluon self-energy [44,50,51]

$$\Pi_L(\omega,k) = m_D^2\left(1-\frac{\omega^2}{k^2}\right)\left[1-\frac{\omega}{2k}\log\left[\frac{\omega+k}{\omega-k}\right]\right], \quad (17)$$

and

$$\Pi_T(\omega,k) = \frac{1}{2}m_D^2\left[\frac{\omega^2}{k^2}+\left(1-\frac{\omega^2}{k^2}\right)\frac{\omega}{2k}\log\left[\frac{\omega+k}{\omega-k}\right]\right], \quad (18)$$

where m_D is the Debye mass. Equations (17) and (18) are consistent with those obtained in the hard thermal loop (HTL) approximation at finite temperature field theory [43,44,46]. By combining with the HTL photon self-energy and Eqs. (7)–(9), Juan Liu *et al.* have studied the refraction index of light in the QGP and found that it becomes negative in some frequency range [29].

B. Viscous chromohydrodynamics

Viscosity will modify the distribution function of the constituents of a microscopic system [30–32,52,53]. If only shear viscosity is taken into account, the modified distribution function can be written as

$$Q = Q_0 + \delta Q = Q_0 + \frac{c'}{2T^3}\frac{\eta}{s}Q_0(1\pm Q_0)p^\mu p^\nu\langle\nabla_\mu u_\nu\rangle. \quad (19)$$

Such a result is obtained by matching the macroscopic conserved quantities from usual hydrodynamics to those derived from Boltzmann kinetic theory by expanding the distribution function around local equilibrium [30–34,52–54]. That ansatz of distribution function is widely used to investigate phenomenology of ultrarelativistic heavy ion collisions [5–13,32–34,54,55]. In Eq. (19), "+" is for boson, while "−" is for fermion. $c' = \pi^4/90\zeta(5)$ and $c' = 14\pi^4/1350\zeta(5)$ are for massless boson [12,32] and massless fermion [55], respectively. $\langle\nabla_\mu u_\nu\rangle = \nabla_\mu u_\nu + \nabla_\nu u_\mu - \frac{2}{3}\Delta_{\mu\nu}\nabla_\gamma u^\gamma$, $\nabla_\mu = (g_{\mu\nu} - u_\mu u_\nu)\partial^\nu$, and $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$; η, s, T, and Q_0 represent the shear viscosity, the entropy density, the temperature of the system, and the equilibrium distribution function of boson or fermion.

Asakawa *et al.* have found that turbulent gauge fields will induce an anomalous shear viscosity, which will result in the perturbation of the distribution function in an expanding quark-gluon plasma [56–58]. According to the kinetic theory (Vlasov-Boltzmann equation), they derived the perturbation of the distribution function due to the shear viscosity induced by the fluctuations of the gauge fields [56–58]

$$f(p,r) = f_0(p) + \delta f(p,r) = f_0(p) + f_0(p)(1\pm f_0(p))f_1(p,r) \quad (20)$$

with

$$f_1(p,r) = -\frac{\tilde{\triangle}(p)}{E_pT^2}p^ip^j(\nabla u)_{ij}. \quad (21)$$

Here f_0 is the distribution function in local equilibrium and $(\nabla u)_{ij} = \frac{1}{2}(\nabla_i u_j + \nabla_j u_i) - \frac{1}{3}\nabla\cdot\mathbf{u}$. $\tilde{\triangle}(p)$ is a scalar function of the momentum p and includes the information of fluctuations of the gauge fields, which correlates to the shear viscosity by comparing the microscopic definition of stress tensor with macroscopic hydrodynamic definition of the viscous stress. It should be stressed that distribution function correction (20) and (21) due to shear viscosity from turbulent gauge fields is similar to the one from collisions [30,31], i.e., Eq. (19). It may imply that no matter what the origin the shear viscosity is, it modifies the distribution function with the similar manner $\sim c_1' f_0(p)(1\pm f_0(p))p^\mu p^\nu\langle\nabla_\mu u_\nu\rangle$ (for another related discussion please refer to Sec. 2.4 in Ref. [54]); c_1' connects to the shear viscosity and it contains the dynamical information of the system.

Under our consideration, the modified distribution function due to shear viscosity is a starting point to investigate the chromoelectromagnetic properties in a viscous quark-gluon plasma. Based on previous discussion, we speculate that the formula of the correction to the distribution function due to shear viscosity Eq. (19) is also applicable to the quark-gluon system. But unlike the usual microscopic system, Q_0 and u^μ are now matrices in the color space.

It is very difficult to evaluate the gluon self-energy with the QGP kinetic theory associated with the distribution function modified by shear viscosity Eq. (19). Fortunately, the fluid equations are rather simpler than the kinetic theory and are usually used to study the plasma properties. In the present paper, we will derive the viscous chromohydrodynamic equations in terms of the QGP transport equations and the modified distribution function due to shear viscosity. According to those equations we will focus on physics related to the colored fluctuations which exist in the QGP at a time scale shorter than the average temporal separation of collisions between partons. At large time scale, the color degrees of freedom in plasma will be whitened quickly due to the dynamical processes associated with the existence of Ohmic currents [59]. Therefore, in the following in the derivation of the viscous chromohydrodynamics, we will neglect the collision terms in the QGP transport equations.

It should be noted that the neglect of the collision terms in the QGP transport equations does not imply that there are no interactions between particles in plasma. Plasma particles interact with each other through the mean field which is generated consistently by the current [60], which also can result in dissipation in plasma. In recent years, Asakawa *et al.* have found that the turbulent plasma fields will induce an anomalous shear viscosity which may dominate the total shear one and be very small in magnitude [56–58]. That mechanism provides an alternative explanation for the nearly perfect fluidity of matter produced at the RHIC without the assumption that it is a strong coupled state. Along that line, the bulk viscosity induced by the turbulent color fields in an expanding quark-gluon plasma has also been investigated recently [61,62].

For brevity and as an example, the further derivation of the viscous chromohydrodynamic equations only focus on quarks. The inclusion of antiquarks and gluons is straightforward by using the same tactics. By expanding the collisionless kinetic equation (10) in momenta moments and truncating the expansion at the second level in terms of the quark distribution function modified by shear viscosity equation (19), we will arrive the viscous chromohydrodynamic equations which can describe the QGP in the presence of shear viscosity [36]:

$$D_\mu n^\mu = 0, \quad D_\mu T^{\mu\nu} - \frac{g}{2}\{F^\nu_\mu, n^\mu(x)\} = 0 \tag{22}$$

with

$$n^\mu(x) = \int_p p^\mu Q(p,x), \qquad T^{\mu\nu}(x) = \int_p p^\mu p^\nu Q(p,x). \tag{23}$$

Because the distribution function and u^μ are matrices in the color space, in terms of Eq. (23), the four-flow n^μ and energy momentum tensor $T^{\mu\nu}$ are also matrices in the color space and can be expressed in the form [36,40]

$$\begin{aligned} n^\mu &= n(x)u^\mu, \\ T^{\mu\nu} &= \frac{1}{2}(\epsilon(x) + p(x))\{u^\mu, u^\nu\} - p(x)g^{\mu\nu} + \pi^{\mu\nu}, \end{aligned} \tag{24}$$

where

$$\begin{aligned} \pi^{\mu\nu} &= \eta\langle\nabla^\mu u^\nu\rangle \\ &= \eta\Big\{(g^{\mu\rho} - u^\mu u^\rho)\partial_\rho u^\nu + (g^{\nu\rho} - u^\nu u^\rho)\partial_\rho u^\mu \\ &\quad - \frac{2}{3}(g^{\mu\nu} - u^\mu u^\nu)\partial_\sigma u^\sigma\Big\}. \end{aligned} \tag{25}$$

Because we only focus on the quark sector, the color current equation (16) reads

$$j^\mu(x) = -\frac{g}{2}\Big(nu^\mu - \frac{1}{3}\,\mathrm{Tr}[nu^\mu]\Big). \tag{26}$$

Equations (22), (24), and (26) make up the basic set of equations of the viscous chromohydrodynamics. In those equations n, ϵ, and p represent the particle density, the energy density, and pressure, respectively. As u^μ these quantities are also $N_c \times N_c$ matrices in the color space [40]. Usually, hydrodynamic quantities have both colorless and colorful parts, as an example, the particle density can be written as

$$n^\mu_{\alpha\beta} = n^\mu_0 I_{\alpha\beta} + \frac{1}{2} n^\mu_a \tau^a_{\alpha\beta}, \tag{27}$$

where $\alpha, \beta = 1, 2, 3$ are color indices and I is the identity matrix [40].

If $\eta = 0$, the distribution function remains the ideal form, $\pi^{\mu\nu}$ will be absent in (24) and the chromohydrodynamic equations will turn to the ideal ones [40].

C. Gluon self-energy

Linearizing the hydrodynamic quantities around the stationary, colorless, and homogeneous state which is described by $\bar{n}, \bar{u}^\mu, \bar{p}$, and $\bar{\epsilon}$, as an example, the particle density is written as

$$n(x) = \bar{n} + \delta n(x). \tag{28}$$

The fluctuation quantities should be diagonalized to be comparable to the corresponding stationary quantities [37,39,40] $\delta n \ll \bar{n}$ and $D_\mu \bar{n} = 0$. The corresponding parts of other quantities have the similar properties. The color current $j^\mu(x)$ vanishes in the stationary state. As Eq. (27) demonstrates, all the fluctuations of the hydrodynamics quantities can contain both colorless and colorful components, for example,

$$\delta n_{\alpha\beta} = \delta n_0 I_{\alpha\beta} + \frac{1}{2}\delta n_a \tau^a_{\alpha\beta}. \tag{29}$$

Substituting the linearized hydrodynamic quantities like Eq. (28) into Eq. (24) and their corresponding conservation equations (22) and projecting them on $\bar{u}^\mu$ and $(g^{\mu\nu} - \bar{u}^\mu\bar{u}^\nu)$, then, considering only the equations for colorful parts of fluctuations and performing the Fourier transformation, one can obtain equations which can describe color phenomena in the viscous QGP [36]

$$\bar{n}k_\mu \delta u^\mu_a + k_\mu \delta n_a \bar{u}^\mu = 0, \tag{30}$$

$$\bar{u}^\mu k_\mu \delta\epsilon_a + (\bar{\epsilon} + \bar{p})k_\mu \delta u^\mu_a = 0, \tag{31}$$

$$\begin{aligned} &(\bar{\epsilon} + \bar{p})(\bar{u}\cdot K)\delta u^\nu_a + (-k^\nu + \bar{u}^\nu(\bar{u}\cdot K))\delta p_a \\ &\quad + \eta\Big\{(K^2 - (K\cdot\bar{u}))\delta u^\nu_a + (k^\mu k^\nu - k^\mu \bar{u}^\nu)\delta u_{\mu,a} \\ &\quad + \frac{2}{3}(\bar{u}^\nu(K\cdot\bar{u}) - k^\nu)k_\rho \delta u^\rho_a\Big\} \\ &= ig\bar{n}\bar{u}_\mu F^{\mu\nu}_a(K). \end{aligned} \tag{32}$$

We introduce an equation of state (EoS) $\delta p_a = c_s^2 \delta\epsilon_a$ to complete the fluid equations, the explicit formalism for c_s will be introduced later. According to Eqs. (30)–(32) and the introduced EoS, we can obtain the color fluctuations of hydrodynamic quantities δn_a, $\delta u_{\nu,a}$, and $\delta\epsilon_a$. Due to the color fluctuations of the hydrodynamic quantities, the color current fluctuation is given by

$$\delta j^\mu_a = -\frac{g}{2}\Big(\bar{n}\delta u^\nu_a + \delta n_a \bar{u}^\mu - \frac{1}{3}\,\mathrm{Tr}[\bar{n}\delta u^\mu_a + \delta n_a \bar{u}^\mu]\Big). \tag{33}$$

Substituting into the solved δn_a and δu^μ_a, according to the relation between the current and the gauge field in the linear response theory $\delta j_a^\mu(K) = -\Pi_{ab}^{\mu\nu}(K)A_{\nu,b}(K)$, one can abstract the polarization tensor $\Pi_{ab}^{\mu\nu}(K)$ [36]

$$\Pi_{ab}^{\mu\nu}(\omega,k) = -\delta_{ab}\Big\{\omega_p^2\cdot\frac{1}{1+D(K^2-(K\cdot\bar{u})^2)}\cdot\frac{1}{(K\cdot\bar{u})^2} \cdot[(K\cdot\bar{u})(\bar{u}^\mu k^\nu + k^\mu\bar{u}^\nu) - K^2\bar{u}^\mu\bar{u}^\nu - (K\cdot\bar{u})^2 g^{\mu\nu} + (B+E) \cdot[K^2(K\cdot\bar{u})(\bar{u}^\mu k^\nu + k^\mu\bar{u}^\nu) - k^\mu k^\nu (K\cdot\bar{u})^2 - K^4\bar{u}^\mu\bar{u}^\nu]]\Big\}, \tag{34}$$

where

$$\omega_p^2 = \frac{g^2\bar{n}^2}{2(\bar{\epsilon}+\bar{p})}$$

is the square of the plasma frequency and

$$B = -\frac{c_s^2}{\omega^2 - c_s^2k^2}, \qquad D = \frac{\eta}{sT\omega}, \qquad E = -\frac{\frac{\eta\omega}{sT}\left(1+4\frac{c_s^2k^2}{\omega^2-c_s^2k^2}\right)}{3\omega^2 - 3c_s^2k^2 - 4\frac{\eta\omega k^2}{sT}}. \tag{35}$$

It is easy to test that $\Pi_{ab}^{\mu\nu} = \Pi_{ab}^{\nu\mu}$ and $k_\mu\Pi_{ab}^{\mu\nu} = 0$. In the further considerations, we suppress the color indices a, b.

According to the projector, we can obtain the longitudinal and transverse gluon self-energy

$$\Pi_L(K) = \frac{K^2}{k^2}\Pi^{00}(K), \tag{36}$$

$$\Pi_T(K) = \frac{1}{2}(\delta_{ij} - \hat{k}_i\hat{k}_j)\Pi^{ij}(K), \tag{37}$$

with $\hat{k}_i = k_i/k$.

We have briefly reviewed the determination of the gluon self-energy in the QGP associated with shear viscosity with the viscous chromohydrodynamic approach. For details please refer to Ref. [36]. Through the derivation, shear viscosity is encoded in the gluon self-energy. Combining with Eqs. (36), (37), and (7)–(9), we can study the chromorefraction index in the viscous QGP.

D. Numerical results

Before we do a further analysis on the chromorefraction index, we should determine the sound speed c_s first. Mannarelli and Manuel have investigated collective unstable modes of QGP with the ideal chromohydrodynamic approach [40] as well as the kinetic theory [35]. They found that when one uses "the effective speed of sound"

$$c_s = \sqrt{\frac{1}{3(1+\frac{1}{2y}\log\frac{1-y}{1+y})} + \frac{1}{y^2}} \qquad \left(y = \frac{k}{\omega}\right),$$

the results in the chromohydrodynamic approach agree well with those in the kinetic theory in the same setting (see the discussion in the Appendix in Ref. [35]). In this paper, we also use the effective speed of sound.

In this paper, we cannot determine shear viscosity coefficient itself in viscous chromohydrodynamics, but regard it as an input parameter to study the viscous effect on the chromorefraction index of the QGP. A small value of the ratio for shear viscosity to entropy density $\eta/s \le 0.2$ has been deduced from comparison of casual viscous hydrodynamic simulation results with the RHIC data [13], which is less than three times of the famous bound result $\eta/s = \frac{1}{4\pi}$ of the strongly coupled conformal field theory determined by the AdS/CFT correspondence [63]. Numerical results of the chromorefraction index are presented with those explicit values of η/s. In addition, in numerical analysis, such scales $k = 0.2\omega_p$ and $T = \omega_p$ are used to study the ω-dependent behavior of the chromorefraction index.

In terms of the relation between Eqs. (36) and (7), one can obtain the chromoelectric permittivity in soft momentum approximation [36]

$$\varepsilon(\omega,k) = 1 + \frac{3\omega_P^2}{k^2}\left[1 - \frac{\omega}{2k}\left(\log\left|\frac{\omega+k}{\omega-k}\right| - i\pi\Theta(k^2-\omega^2)\right)\right] - \frac{12\omega_P^2}{k^2}\frac{\eta\omega}{sT}\times\Big\{1 - \frac{\omega}{k}\log\left|\frac{\omega+k}{\omega-k}\right| + \frac{\omega^2}{4k^2}\left(\log\left|\frac{\omega+k}{\omega-k}\right|\right)^2 - \frac{\omega^2}{4k^2}\pi^2\Theta(k^2-\omega^2) + i\left(\frac{\omega}{k}\pi - \frac{\omega^2}{2k^2}\pi\log\left|\frac{\omega+k}{\omega-k}\right|\right)\Theta(k^2-\omega^2)\Big\}, \tag{38}$$

where Θ is the step function. It should be noted that the same result has been obtained in nonlinear viscous chromohydrodynamics derived from the non-Abelian kinetic theory and the distribution function determined by entropy production principle (EPP) in a recent literature [64]. Here, we plot the real and imaginary parts of the chromoelectric permittivity in the viscous QGP in Fig. 1. The main findings are as follows. First, there is a frequency pole for the real part at $\omega_d = k$, which is just the inflexion of the imaginary part. Second, if such relation $m_D^2 = 3\omega_P^2$ is adopted [65], when $\eta/s = 0$, chromoelectric permittivity $\varepsilon(\omega,k)$ recovers the HTL result obtained by the kinetic theory or finite temperature field theory [43,44,46,50,51]. Third, the viscous corrections to both the real and imaginary parts of $\varepsilon(\omega,k)$ are small. For detailed discussion, please refer to Ref. [36].

According to the relation between Π_L, Π_T, and $\mu_M(\omega,k)$, the chromomagnetic permeability in the viscous QGP can be derived from Eqs. (36), (37), and (8)

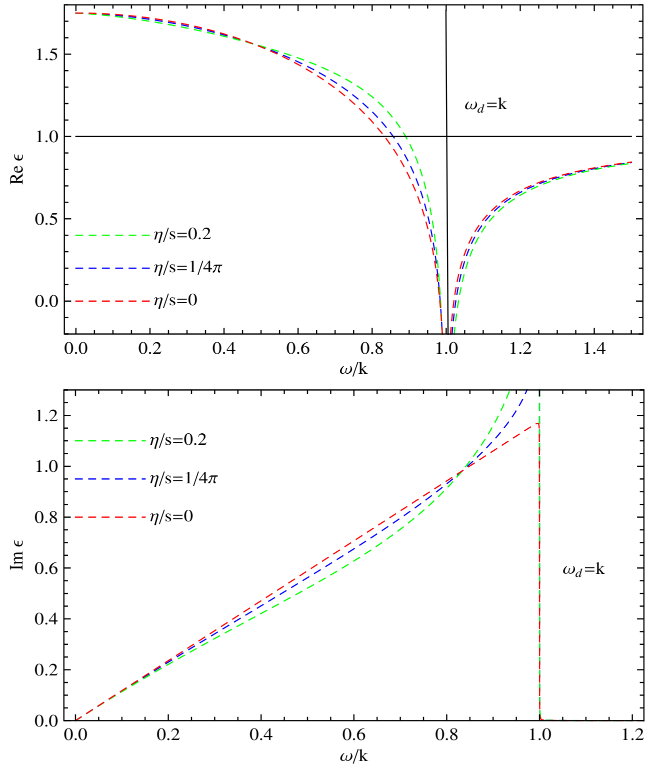

FIG. 1 (color online). The chromoelectric permittivity in the viscous QGP. Top panel: the real part. Bottom panel: the imaginary part. The dashed red, blue, and green curves are for the cases of $\eta/s = 0$, $1/4\pi$, and 0.2, respectively.

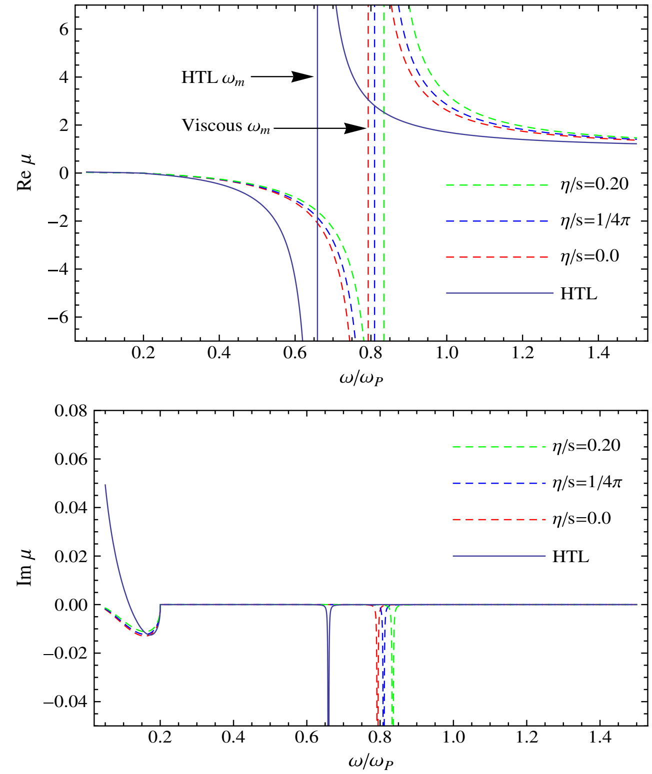

FIG. 2 (color online). The chromomagnetic permeability in the QGP. Top panel: the real part. Bottom panel: the imaginary part. The dashed red, blue, and green curves are for the cases of $\eta/s = 0$, $1/4\pi$, and 0.2, respectively, while the solid blue curve is for the HTL case.

$$\mu_M(\omega, k) = \frac{1}{1 + \frac{\omega_p^2}{k^2}\cdot\frac{1}{1-\frac{\eta}{s}\cdot\frac{k^2}{T\omega}} + \frac{\omega^2}{k^2}\cdot(\varepsilon(\omega, k) - 1)}. \quad (39)$$

We present the real and imaginary parts of chromomagnetic permeability of the viscous QGP in the top and bottom panels, respectively in Fig. 2. For comparison, we also display the HTL results as well. The dashed red, blue, and green curves are for the viscous cases of $\eta/s = 0$, $1/4\pi$, 0.2, respectively, while the blue solid curves are for the HTL results. Both the real and imaginary parts of chromomagnetic permeability show a frequency pole ω_m. Its position is around $0.65\omega_P$ for the HTL results, but around $0.8\omega_P$ for the viscous cases. In addition, it is easy to see that the frequency pole shifts to large frequency region with the increase of η/s.

From Eqs. (38), (39), and (9) we can determine the Depine-Lakhtakia index $n_{\rm eff}$ in the viscous QGP. We display its numerical results with different values of η/s in Fig. 3 as well as the HTL result evaluated from Eqs. (17), (18), and (7)–(9). As shown in Fig. 3, there is a quite large frequency range for $n_{\rm eff} < 0$. In terms of the discussion in Sec. II, in that frequency range chromorefraction index becomes negative, i.e., $\mathrm{Re}\,n < 0$. With the increase of η/s, the frequency range for the negative refraction becomes wider. In addition, the frequency range for negative refraction in the viscous QGP is much wider than that of the HTL case.

In Fig. 3, one can see that, with the increase of the frequency, there is an inflexion for $n_{\rm eff}$ where $n_{\rm eff}$ changes from positive to negative value when the frequency ω is around ω_d, and $n_{\rm eff}$ is negative until $\omega = \omega_m$.

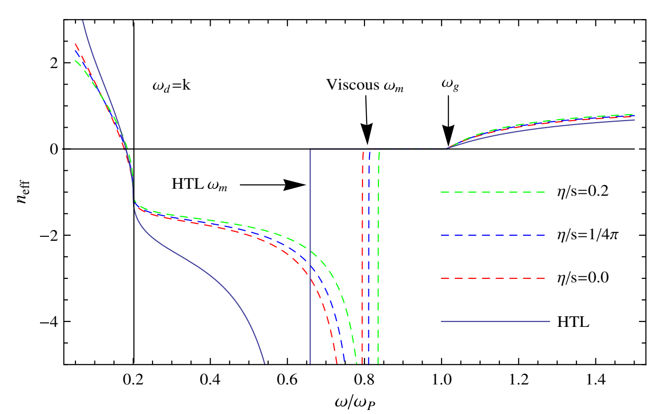

FIG. 3 (color online). The Depine-Lakhtakia index $n_{\rm eff}$ for the QGP. The dashed red, blue, and green curves are for the cases of $\eta/s = 0$, $1/4\pi$, and 0.2, respectively, while the solid blue curve is for the HTL case.

The numerical analysis in Figs. 1–3 shows that the starting point of the frequency region for negative refraction is around the pole of the chromoelectric permittivity ω_d, while the chromomagnetic permeability pole ω_m determines the endpoint. Note that the chromoelectric permittivity ω_d poles in the viscous cases superpose each other, whose position coincides with that of the HTL result, as shown in Fig. 1. Therefore, the starting points for negative refraction show no appreciable distinction among all curves in Fig. 3. From Fig. 2, one can see that the chromomagnetic permeability pole shifts to large frequency region with the increase of η/s, which leads to an enlargement of the frequency range for negative refraction. The chromomagnetic permeability pole is around $0.65\omega_P$ for the HTL result, but around $0.8\omega_P$ for the viscous cases, which results in the fact that the frequency range for negative refraction in the viscous QGP is much wider than that of the HTL case.

From Fig. 3, it is shown that both viscous curves and the HTL curve intersect one point at $\omega = \omega_g$. When $\omega > \omega_g$, $n_{\rm eff} > 0$ for all curves, which implies a general chromorefraction index. A frequency gap $\omega \in [\omega_m, \omega_g]$ is illustrated for $n_{\rm eff} = 0$, in which $n^2 < 0$ as shown in Fig. 4. It is argued that that result has not been reported in earlier literature [29] and the light does not propagate in that frequency gap because the refraction index is pure imaginary and the electromagnetic wave is damped severely [29].

To obtain the chromorefraction index n, one has to study the square root of $n^2 = \varepsilon\mu_M$. The complex value number $n^2 = \varepsilon\mu_M$ possesses two square roots,

$$n = \pm\sqrt{|n^2|}e^{i(\phi/2)}. \quad (40)$$

In (40), ϕ is the argument of $n^2 = \varepsilon\mu_M$, which can be expressed as

$$\phi = \phi_\varepsilon + \phi_\mu, \quad (41)$$

where $\phi_\varepsilon = \varepsilon_i/\varepsilon_r$ and $\phi_\mu = \mu_i/\mu_r$ are the arguments of ε and μ_M, respectively. But which root in Eq. (40) is to be chosen? In terms of the criterion (9) that $n_{\rm eff} < 0$ implying Re$n < 0$, otherwise Re$n > 0$, and the arguments of ε (38) and μ_M (39), we present the real and imaginary parts of n in the QGP with different η/s values in Fig. 5. One can see that the properties of the real part of n are qualitatively consistent with the corresponding ones of $n_{\rm eff}$. For both the real and imaginary parts of n, the differences for different viscosities are prominent only when the frequency is

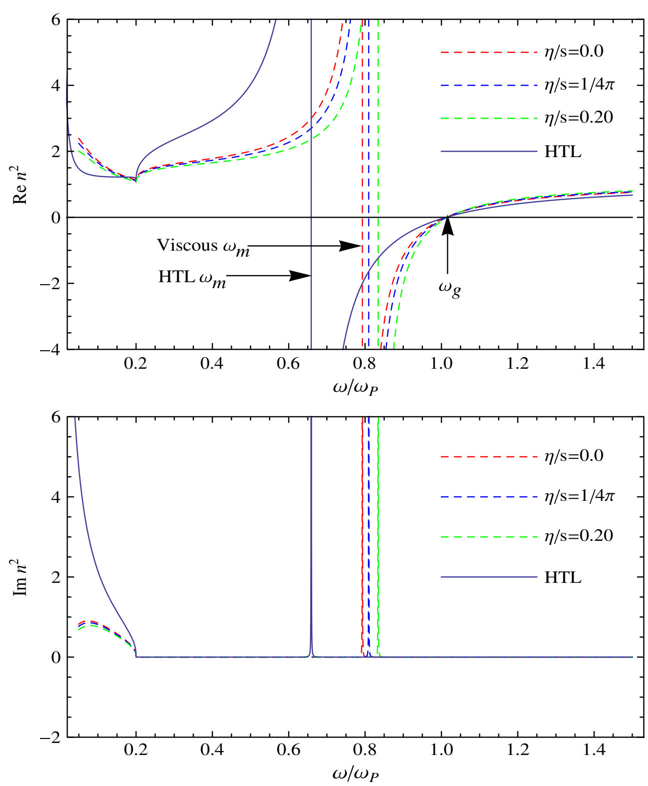

FIG. 4 (color online). The real and imaginary parts of n^2 in the QGP. Top panel: the real part. Bottom panel: the imaginary part. The dashed red, blue, and green curves are for the cases of $\eta/s = 0$, $1/4\pi$, and 0.2, respectively, while the solid blue curve is for the HTL case.

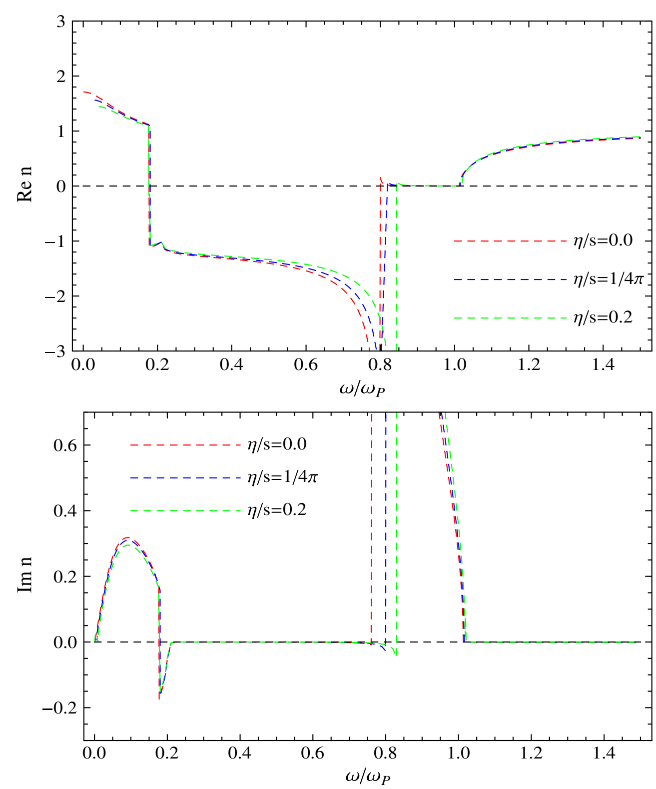

FIG. 5 (color online). Chromorefraction index in the viscous QGP, Top panel: the real part. Bottom panel: the imaginary part. The dashed red, blue, and green curves are for the cases of $\eta/s = 0$, $1/4\pi$, and 0.2, respectively.

around the chromomagnetic permeability pole. Otherwise, the viscous correction to them is very trivial. The HTL results of n are also demonstrated in Ref. [29].

Because of the simplicity and applicability in describing the polarization effect, we have applied the viscous chromohydrodynamics, which is derived from the QGP kinetic theory and the viscosity-modified distribution function, to determine the polarization tensor and investigate the chromorefraction index in the viscous QGP. It should be noted that some dynamical information will be lost during the derivation from the kinetic theory to the chromohydrodynamics [35,40,64]. However, in many cases the discrepancies between both approaches of the kinetic theory and the chromohydrodynamics can be alleviated by using effective parameters as inputs in the hydrodynamic formalisms [35,64]. Nevertheless, such phenomenological model of the chromohydrodynamics could capture the some correct physics of the QGP [40,64]. In view of the difficulty in investigating the viscous effect on the chromoelectromagnetic properties of QGP in microscopic kinetic theory description, we expect that we could obtain some insight on the physics of the problem by applying the viscous chromohydrodynamics.

IV. SUMMARY

In this paper, within the framework of the viscous chromohydrodynamics, the gluon self-energy has been evaluated in the QGP associated with shear viscosity, through which the chromoelectric permittivity and chromomagnetic permeability have been derived. Based on the viscous $\varepsilon(\omega, k)$ and $\mu_M(\omega, k)$, we have investigated the Depine-Lakhtakia index $n_{\rm eff}$ and the chromorefraction index n in the viscous QGP. For comparison, we have also presented the corresponding HTL results. $n_{\rm eff} < 0$ implies the negative real part of n, which signifies the negative refraction in the medium. The numerical analysis shows that (i) the chromorefraction index becomes negative in some frequency range; (ii) the starting point of that frequency range is around the pole of chromoelectric permittivity $\varepsilon(\omega, k)$, and the chromomagnetic permeability pole determines the endpoint; and (iii) with the increase of the η/s, the frequency range for the negative refraction becomes broader. In addition, the frequency range for negative refraction in the viscous chromohydrodynamics is wider than that of the HTL perturbation theory. The numerical analysis also indicates that viscous properties of poles for ε and μ_M are responsible for that difference.

The criterion (9) $n_{\rm eff} < 0$ has been widely used to judge the existence of the negative refraction in a medium. It is interesting to study the interplay between the modes propagation (Ren) and dissipation (Imn) in the frequency region for negative refraction. For instance, in a strongly coupled system with the framework of AdS/CFT correspondence, it was found that there exists strong dissipation in the frequency region for negative refraction, only around $\omega \to 0$, propagation may dominate over dissipation [24]. How the shear viscosity impacts the mode propagation in QGP deserves further comprehensive studies.

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