Note on the nonuniqueness of the massive Fierz-Pauli theory and spectator fields

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It is possible to show that there are three independent families of models describing a massive spin-2 particle via a rank-2 tensor. One of them contains the massive Fierz-Pauli model, the only case described by a symmetric tensor. The three families have different local symmetries in the massless limit and can not be interconnected by any local field redefinition. We show here, however, that they can be related with the help of a decoupled and nondynamic (spectator) field. The spectator field may be either an antisymmetric tensor $B_{\mu\nu} = -B_{\nu\mu}$, a vector A_μ or a scalar field φ, corresponding to each of the three families. The addition of the extra field allows us to formulate master actions which interpolate between the symmetric Fierz-Pauli theory and the other models. We argue that massive gravity models based on the Fierz-Pauli theory are not expected to be equivalent to possible local self-interacting theories built up on top of the two new families of massive spin-2 models. The approach used here may be useful to investigate dual (nonsymmetric) formulations of higher-spin particles.

I. INTRODUCTION

Speculations about a possible massive gravity theory were raised long ago. In particular, the problems of mass discontinuity and the appearance of ghosts have been pointed out in Refs. [1–3], respectively. In the last few years the interest in massive gravity has increased; see e.g. the review work [4] and references therein. Those works were driven by both the accelerated expansion of the universe and more recently by the discovery [5–7] of appropriate mass terms in spin-2 theories which furnish a correct counting of degrees of freedom. The fact that those works are all based on the usual massive Fierz-Pauli (FP) [8] theory, described by a symmetric rank-2 tensor, impels us to search for other descriptions of massive spin-2 particles.

In particular, a weak-field expansion in a frame-like formulation of gravity $e_{\mu a} = \eta_{\mu a} + h_{\mu a}$ naturally leads to a nonsymmetric field $h_{\mu a} \neq h_{a\mu}$. The general case of a second-order (in derivatives) Lagrangian for an arbitrary rank-2 tensor $e_{\mu\nu}$ has been investigated in the past in Refs. [9–13]. They concluded that the massive FP theory is the only possibility which avoids ghosts. In Refs. [14,15] other possibilities were found, which have motivated our previous work [16] where we revisited the classification of all possible (second-order) descriptions of a massive spin-2 particle in $D = 3 + 1$. We conclude that there are three ghost-free one-parameter families of solutions. In two of those families the auxiliary[1] fields $e_{[\mu\nu]}$ are required: only in the FP family is there a special case with a purely symmetric

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[1]We use basically the same notation of Ref. [16], in particular, $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$, $e_{(\alpha\beta)} = (e_{\alpha\beta} + e_{\beta\alpha})/2$ and $e_{[\alpha\beta]} = (e_{\alpha\beta} - e_{\beta\alpha})/2$.

tensor $e_{(\mu\nu)}$ with only one auxiliary field, the trace $e = \eta^{\mu\nu} e_{\mu\nu}$. In the next sections the results of Ref. [16] are confirmed in a rather simple way and the connection between the new models and the symmetric FP theory is clarified by means of interpolating master actions [17].

II. THREE FAMILIES

In Ref. [16] we considered a general Lorentz-covariant second-order quadratic action for a rank-2 tensor $e_{\mu\nu}$ with ten arbitrary real constants. Requiring that the propagator contains only one massive pole in the spin-2 sector, with positive residue (no ghost), we have obtained, up to the field redefinitions $e_{\mu\nu} \rightarrow e_{\mu\nu} + a\eta_{\mu\nu}e$ and $e_{\mu\nu} \rightarrow Ae_{\mu\nu} + (1 - A)e_{\nu\mu}$, three one-parameter families of models, which are displayed in Eqs. (5), (8), and (10). All three families lead on-shell to the Fierz-Pauli conditions,

$$e_{[\alpha\beta]} = 0, \tag{1}$$

$$e = 0, \tag{2}$$

$$\partial^\mu e_{\mu\nu} = 0, \tag{3}$$

$$(\Box - m^2)e_{\mu\nu} = 0. \tag{4}$$

The first family depends on the arbitrary real constant d_-,

$$\mathcal{L}_{\text{FP}}(d_-) = \mathcal{L}_{\text{FP}}[e_{(\alpha\beta)}] + d_- \frac{m^2}{2} e_{[\alpha\beta]}^2, \tag{5}$$

and contains the usual ($d_- = 0$) massive FP theory,

$$\mathcal{L}_{\text{FP}}[e_{(\alpha\beta)}] = -\frac{1}{2}\partial^\mu e^{(\alpha\beta)}\partial_\mu e_{(\alpha\beta)} + \frac{1}{4}\partial^\mu e\partial_\mu e$$
$$+ \left[\partial^\alpha e_{(\alpha\beta)} - \frac{1}{2}\partial_\beta e\right]^2 - \frac{m^2}{2}[e_{(\alpha\beta)}^2 - e^2]. \tag{6}$$

The massless limit of $\mathcal{L}_{\text{FP}}(d_-)$ is invariant under

$$\delta e_{\mu\nu} = \partial_\nu \xi_\mu + \Lambda_{\mu\nu}, \tag{7}$$

where $\Lambda_{\mu\nu} = -\Lambda_{\nu\mu}$. We remark that although d_- is completely arbitrary in the free theory (5), it has been argued in Ref. [18], based upon a Stueckelberg-like formulation, that one should fix $d_- = 1$.

The second family of models [15] depends on the free parameter c and is given by

$$\mathcal{L}_{\text{nFP}}(c) = -\frac{1}{2}\partial^\mu e^{(\alpha\beta)}\partial_\mu e_{(\alpha\beta)} + \frac{1}{6}\partial^\mu e\partial_\mu e + [\partial^\alpha e_{(\alpha\beta)}]^2$$
$$- \frac{1}{3}[(\partial^\alpha e_{\alpha\beta})^2 + (\partial^\alpha e_{\beta\alpha})^2]$$
$$- \frac{m^2}{2}(e_{\mu\nu}e^{\nu\mu} + ce^2). \tag{8}$$

The acronym nFP stands for non-Fierz-Pauli, since the mass term does not need to fit in the Fierz-Pauli form ($c = -1$). Analogous to $\mathcal{L}_{\text{FP}}(d_-)$, the massless limit of $\mathcal{L}_{\text{nFP}}(c)$ describes a massless spin-2 particle (see Refs. [15,19]), which is invariant under linearized reparametrizations plus Weyl transformations,

$$\delta e_{\mu\nu} = \partial_\nu \xi_\mu + \eta_{\mu\nu}\phi. \tag{9}$$

The Weyl symmetry can be extended to the whole massive theory if we choose $c = -1/4$, in which case we get rid of the trace $e = \eta^{\mu\nu}e_{\mu\nu}$ such that we only have $e_{[\mu\nu]}$ as auxiliary fields.

The third and last family depends on the arbitrary real constant a_1 introduced in Ref. [16],

$$\mathcal{L}_{a_1} = -\frac{1}{2}\partial^\mu e^{(\alpha\beta)}\partial_\mu e_{(\alpha\beta)} + \left(a_1 + \frac{1}{4}\right)\partial^\mu e[\partial_\mu e - 2\partial^\alpha e_{(\alpha\mu)}]$$
$$+ [\partial^\alpha e_{(\alpha\beta)}]^2 + \left(a_1 - \frac{1}{4}\right)(\partial^\alpha e_{\alpha\beta})^2$$
$$- \frac{m^2}{2}(e_{\mu\nu}e^{\nu\mu} - e^2). \tag{10}$$

Differently from $\mathcal{L}_{\text{FP}}(d_-)$ and $\mathcal{L}_{\text{nFP}}(c)$, the massless limit of $\mathcal{L}_{a_1}$ describes a massless spin-2 particle plus a massless scalar field (scalar-tensor). The massless theory is unitary if $a_1 \leq -1/12$ or $a_1 \geq 1/4$; see Ref. [16]. At $a_1 = 1/4$ the family $\mathcal{L}_{a_1}$ becomes the massive FP theory with $d_- = 1$, while at $a_1 = -1/12$ it becomes $\mathcal{L}_{\text{nFP}}(c = -1)$. These are the only intersecting points of $\mathcal{L}_{a_1}$ with the other two families.

The massless limit of $\mathcal{L}_{a_1}$ is invariant only under linearized reparametrizations in general,

$$\delta e_{\mu\nu} = \partial_\nu \xi_\mu, \tag{11}$$

except at $a_1 = 1/4$ where the massless symmetry is enlarged by antisymmetric shifts, as in Eq. (7), and at $a_1 = -1/12$ where a Weyl symmetry shows up, as in Eq. (9). The case $a_1 = -1/4$ corresponds to the model of Ref. [14] where it was coupled to a maximally symmetric background. In the next section we interconnect the models (8) and (10) with the symmetric massive FP theory $\mathcal{L}_{\text{FP}}(d_- = 0)$ via master actions.

III. MASTER ACTIONS

There is no local field redefinition which relates Eqs. (8) or (10) to the symmetric massive FP model. The difficulty lies in the presence of the antisymmetric tensor $e_{[\mu\nu]}$ in the derivative terms of Eqs. (8) and (10). However, since all three families have the same particle content (for nonzero mass) one should be able to interconnect them somehow. Since the FP family (5) is obtained from the usual FP theory by the addition of a pure (arbitrary) mass term for a decoupled nondynamic field $e_{[\mu\nu]}$, we might try to add to the symmetric FP model ($d_- = 0$) a mass term for some other field. This is what we do in the next two subsections.

A. Scalar spectator

First, let us add a scalar field and define

$$\mathcal{L}_b = \mathcal{L}_{\text{FP}}[h_{\alpha\beta}] - bm^2\varphi^2 + h_{\alpha\beta}T^{\alpha\beta}, \tag{12}$$

where $\mathcal{L}_{\text{FP}}[h_{\alpha\beta}]$ is given in Eq. (6) and $h_{\alpha\beta} = h_{\beta\alpha}$ is some symmetric tensor. We have also added a symmetric external source $T^{\alpha\beta}$. The additional decoupled mass term does not change the particle content of the massive FP theory. After a shift with an arbitrary real constant s,

$$h_{\mu\nu} \rightarrow h_{\mu\nu} - s\eta_{\mu\nu}\varphi - \frac{2s}{m^2}\partial_\mu\partial_\nu\varphi. \tag{13}$$

The Lagrangian $\mathcal{L}_b$ becomes

$$\mathcal{L}_b = \mathcal{L}_{\text{FP}}[h_{\alpha\beta}] + m^2(6s^2 - b)\varphi^2 - 3s^2\partial^\mu\varphi\partial_\mu\varphi$$
$$- 3sm^2\varphi h + h_{\alpha\beta}T^{\alpha\beta} - s\varphi T$$
$$- (2s/m^2)\varphi\partial_\mu\partial_\nu T^{\mu\nu}. \tag{14}$$

The shift (13) is defined by requiring that derivative couplings between φ and h vanish. By introducing an auxiliary vector field and integrating by parts we can rewrite Eq. (14) in a first-order form,

$$\mathcal{L}_b = \mathcal{L}_{\text{FP}}[h_{\alpha\beta}] + 3s^2m^2A^\mu A_\mu + m^2(6s^2 - b)\varphi^2$$
$$+ h_{\mu\nu}T^{\mu\nu} - s\varphi\left(6sm\partial\cdot A + 3m^2h\right.$$
$$\left. + T + \frac{2}{m^2}\partial_\mu\partial_\nu T^{\mu\nu}\right). \tag{15}$$

Due to the specific form of the usual Fierz-Pauli mass term in $\mathcal{L}_{FP}(d_- = 0)$ it is possible to generate a Maxwell Lagrangian by making another shift in $\mathcal{L}_b$ and using the identity

$$\mathcal{L}_{FP}[h_{\mu\nu} + r(\partial_\mu A_\nu + \partial_\nu A_\mu)]$$
$$= \mathcal{L}_{FP}[h_{\mu\nu}] - \frac{mr^2}{2} F_{\mu\nu}^2(A) + 2m^2 r A^\mu (\partial^\alpha h_{\alpha\mu} - \partial_\mu h). \tag{16}$$

If we choose $r = -s/m$ we cancel out the $\partial \cdot A$ term in Eq. (15). We can bring an antisymmetric field $B_{\mu\nu}$ into the game by rewriting the Maxwell term in a first-order form. We end up with a master Lagrangian which now involves three extra fields $(\varphi, A_\mu, B_{\mu\nu})$ besides $h_{\mu\nu}$,

$$\mathcal{L}_{M1} = \mathcal{L}_{FP}[h_{\mu\nu}] + 3m^2 s^2 A^\mu A_\mu - 2ms A^\mu \left(\partial^\alpha B_{\alpha\mu} + \partial^\alpha h_{\alpha\mu} \right.$$
$$\left. - \partial_\mu h - \frac{\partial^\alpha T_{\alpha\mu}}{m^2} \right) + \frac{m^2}{2} B_{\mu\nu}^2 + m^2 (6s^2 - b)\varphi^2$$
$$- s\varphi \left(3m^2 h + T + \frac{2}{m^2} \partial_\mu \partial_\nu T^{\mu\nu} \right) + h_{\mu\nu} T^{\mu\nu}. \tag{17}$$

We can define the generating function

$$Z_{M1}[T] = \int \mathcal{D}h_{\mu\nu} \mathcal{D}\varphi \mathcal{D}A_\mu \mathcal{D}B_{\mu\nu} e^{i \int d^4 x \mathcal{L}_{M1}}. \tag{18}$$

If we functionally integrate over the extra field $B_{\mu\nu}$ in Eq. (18) and reverse the shift (13), we come back to the massive FP theory with the source term we have started with, namely Eq. (12). On the other hand, if we integrate over φ and A_μ in the first place we obtain[2] the Lagrangian

$$\mathcal{L}(s, b) = \mathcal{L}_{FP}[h_{\mu\nu}] + \frac{m^2}{2} B_{\mu\nu}^2 - \frac{1}{3} \left(\partial^\alpha B_{\alpha\mu} + \partial^\alpha h_{\alpha\mu} \right.$$
$$\left. - \partial_\mu h - \frac{\partial^\alpha T_{\alpha\mu}}{m^2} \right)^2 - \frac{s^2}{4m^2 (6s^2 - b)}$$
$$\left(3m^2 h + T + \frac{2}{m^2} \partial_\mu \partial_\nu T^{\mu\nu} \right)^2. \tag{19}$$

The arbitrariness appears in front of the mass term proportional to $m^2 h^2$, as in the nFP family (8). Indeed, defining $e_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu}$, the Lagrangian $\mathcal{L}(s, b)$ can be rewritten as

$$\mathcal{L}(s, b) = \mathcal{L}_{nFP}(c) + h_{\mu\nu}^* T^{\mu\nu} + \mathcal{O}(T^2), \tag{20}$$

where $\mathcal{O}(T^2)$ stands for quadratic terms in the source, and the dual field $h_{\mu\nu}^*$ is given by

$$h_{\mu\nu}^* = e_{(\mu\nu)} - \frac{1 + c}{3} \eta_{\mu\nu} e - \frac{2(1 + c)}{3m^2} \partial_\mu \partial_\nu e$$
$$- \frac{(\partial_\mu \partial^\alpha e_{\alpha\nu} + \partial_\nu \partial^\alpha e_{\alpha\mu})}{3m^2}, \tag{21}$$

with the arbitrary parameter c defined through

$$b(1 + c) = 6s^2 (c + 1/4). \tag{22}$$

Comparing Eq. (20) with Eq. (12) we conclude that the master Lagrangian (17) interpolates between the symmetric massive FP theory (5) and the second family of one-parameter models nFP given in Eq. (8). Moreover, the correlation functions of $h_{\mu\nu}$ on the FP side are mapped into correlation functions of the dual field $h_{\mu\nu}^*$ on the dual nFP side, up to contact terms, such that we have the dual map $(h_{\mu\nu})_{FP} \leftrightarrow (h_{\mu\nu}^*)_{nFP}$.

Since on the nFP side we have both symmetric $h_{\mu\nu}$ and antisymmetric $B_{\mu\nu}$ tensors, one might ask what is the dual of $B_{\mu\nu}$ on the FP side. If we add a source term $J_{\mu\nu} B^{\mu\nu}$ to the master Lagrangian (17); the reader can check that correlation functions of $B_{\mu\nu}$ vanish up to contact terms. This is not surprising, since in the nFP theory we have on-shell $e_{[\mu\nu]} = B_{\mu\nu} = 0$ and the equations of motion are enforced at the quantum level in the correlation functions up to contact terms.[3] Likewise, we have on-shell $\eta^{\mu\nu} e_{\mu\nu} = h = 0 = \partial^\alpha e_{\alpha\beta}$, which completes the FP conditions (1)–(3). Therefore, up to contact terms, we see from Eq. (21) that $h_{\mu\nu}$ in the FP theory is mapped simply into $e_{(\mu\nu)}$ in the nFP family.

Lastly, we remark that if $b = 0$, the arbitrary parameter s disappears from Eq. (19) and we end up with the traceless nFP theory with $c = -1/4$; see Eq. (22). So the arbitrariness of the nFP family stems indeed from the arbitrary mass term in Eq. (12) and not from the arbitrariness in the shift (13). In the next subsection we use a fixed shift.

B. Vector spectator

Next we interconnect the third family (10) with the usual massive FP theory. Inspired by Eqs. (5) and (12) we add an arbitrary mass term for a vector field to the symmetric FP theory,

$$\mathcal{L}_{\tilde{b}} = \mathcal{L}_{FP}[h_{\alpha\beta}] + \tilde{b} \frac{m^2}{2} A^\mu A_\mu + h_{\alpha\beta} T^{\alpha\beta}. \tag{23}$$

After the shift

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[2]We can always assume $b \neq 6s^2$.

[3]This follows from the functional integral of a total derivative, $\int \mathcal{D}B \frac{\delta}{\delta B(x)} [e^{i \int d^4 x B \hat{K} B} B(x_1) \cdots B(x_N)] = 0$.

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + (\partial_\mu A_\nu + \partial_\nu A_\mu)/m, \qquad (24)$$

we obtain a Maxwell term [see Eq. (16)], which can be brought to first order again via an antisymmetric field $B_{\mu\nu}$, such that we derive from Eq. (23) the master Lagrangian

$$\mathcal{L}_{M2} = \mathcal{L}_{\text{FP}}[h_{\alpha\beta}] + \tilde{b}\frac{m^2}{2}A^\mu A_\mu + \frac{m^2}{2}B_{\mu\nu}^2$$
$$+ h_{\alpha\beta}T^{\alpha\beta} + 2mA^\mu\Big(\partial^\alpha B_{\alpha\mu} + \partial^\alpha h_{\alpha\mu}$$
$$- \partial_\mu h - \frac{1}{m^2}\partial^\alpha T_{\alpha\mu}\Big). \qquad (25)$$

On the one hand, if we integrate over $B_{\mu\nu}$ and reverse the shift (24) we return to our starting point, Eq. (23). On the other hand, by integrating over A_μ in the first place we deduce

$$\mathcal{L}_{M2} = \mathcal{L}_{\text{FP}}[h_{\alpha\beta}] + \frac{m^2}{2}B_{\mu\nu}^2 - \frac{2}{\tilde{b}}(\partial^\alpha B_{\alpha\mu} + \partial^\alpha h_{\alpha\mu} - \partial_\mu h)^2. \qquad (26)$$

Defining once again $e_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu}$ and identifying $\tilde{b} = -2/(a_1 - 1/4)$, we rewrite[4] $\mathcal{L}_{M2}$ in the form of the third family (10),

$$\mathcal{L}_{M2} = \mathcal{L}_{a_1}[e_{\alpha\beta}] + \tilde{h}_{\mu\nu}T^{\mu\nu} + \mathcal{O}(T^2), \qquad (27)$$

where

$$\tilde{h}_{\mu\nu} = e_{(\mu\nu)} + \Big(\frac{1}{4} - a_1\Big)[(\partial_\mu\partial^\alpha e_{\alpha\nu} + \partial_\nu\partial^\alpha e_{\alpha\mu})$$
$$- 2\partial_\mu\partial_\nu e]. \qquad (28)$$

We conclude that the master action (25) interpolates between the usual massive FP theory [see Eq. (23)] and $\mathcal{L}_{a_1}$. Since the equations of motion of $\mathcal{L}_{a_1}$ lead to $\partial^\alpha e_{\alpha\mu} = 0 = e = e_{[\mu\nu]}$, all such terms have vanishing correlation functions up to contact terms. So we have from Eq. (28) the simple map $(h_{\mu\nu})_{\text{FP}} \leftrightarrow (e_{(\mu\nu)})_{a_1}$.

Regarding the introduction of interactions, if we had nonlinear self-interacting terms $\mathcal{L}_{\text{SI}}[h_{\mu\nu}]$ in Eq. (12), after the shift (13) we would have some nonlinear φ dependence in $\mathcal{L}_{\text{SI}}[h_{\mu\nu} - s\eta_{\mu\nu}\varphi - (2s/m^2)\partial_\mu\partial_\nu\varphi]$. There is no reason *a priori* for the self-interaction to be invariant under these spin-0 transformations. Similarly, the shift $h_{\mu\nu} \rightarrow h_{\mu\nu} - s(\partial_\mu A_\nu + \partial_\nu A_\mu)/m$ would lead to some nonlinear A_μ dependence since we do not expect linearized reparametrization invariance for the full nonlinear theory. Of course, we would still be able to introduce an antisymmetric field in order to bring the Maxwell term to first order. However, the nonlinear terms in φ and A_μ in the master

action would lead to a nonlocal dual model after their functional integrals. A similar conclusion [see Eq. (24)] is drawn for the second case (23).

IV. CONCLUSION

With the help of spectator fields we have been able to interconnect via the master theories (17) and (25) the new one-parameter families of massive spin-2 models (8) and (10) with the symmetric massive Fierz-Pauli theory (6). Our master actions offer an alternative proof of the equivalence of the new models, which use a nonsymmetric tensor, with the fully symmetric FP theory. They confirm the results of Refs. [14–16] regarding the existence of other ghost-free second-order models different from the FP theory, contrary to early works [9–13].

We have remarked that nonlinear massive gravity models based on the usual FP theory are not expected to be equivalent to possible local nonlinear completions of the new models. The situation is similar to the duality between the second-order Abelian Maxwell-Chern-Simons theory [20] and the first-order self-dual model of Ref. [21]. Both models describe a helicity +1 (or −1) mode in $D = 2 + 1$. Although there is a master action [17] relating those Abelian (quadratic) models, the duality does not go through their non-Abelian (nonlinear) counterparts due to extra nonlocal terms; see a discussion in Ref. [22].

The next step is to consider nonlinear (self-interacting) completions of the new families (8) and (10) with a correct counting of degrees of freedom as expected for a massive spin-2 particle. Eventually, the consistency of the self-interacting theory may fix the arbitrary parameters c and a_1 in Eqs. (8) and (10). For the FP family (5) a Stueckelberg-like approach [18] has led to $d_- = 1$. In Ref. [23] one finds further evidence in favor of $d_- = 1$, since the linearized new massive gravity in three [24] and four dimensions [23] can be directly (at action level) deduced from $\mathcal{L}_{\text{FP}}(d_- = 1)$ by a derivative field redefinition which holds even at coinciding points (no contact terms). For instance, one can choose (see Ref. [25]) $e_{\mu\nu} = \partial^\rho \Omega_{\mu\nu\rho}$, where the mixed symmetry tensor $\Omega_{\mu\nu\rho} = -\Omega_{\mu\rho\nu}$ is traceless, $\eta^{\mu\nu}\Omega_{\mu\nu\rho} = 0$.

Finally, since each of the three dual families is obtained by the addition of a different (i.e., one with less components) kind of spectator field $B_{\mu\nu}$, φ, A_μ appearing in Eqs. (5), (12), and (23), it is expected that the approach used here could be generalized in order to find dual spin-S models not necessarily described by fully symmetric rank-S tensors $h_{\mu_1,...,\mu_S}$.

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[4]Recall that at $a_1 = 1/4$ the third family (10) becomes the massive FP theory, so we can assume without loss of generality that $a_1 \neq 1/4$

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