

Charged rotating black string in gravitating nonlinear electromagnetic fields

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We obtain a new solution of rotating black string coupled to a nonlinear electromagnetic field in the background of anti-de Sitter spaces. We consider two types of nonlinear electromagnetic Lagrangians, namely, logarithmic and exponential forms. We investigate the geometric effects of nonlinearity parameter and find that for large r , these solutions recover the rotating black string solutions of Einstein-Maxwell theory. We calculate the conserved and thermodynamic quantities of the rotating black string. We also analyze thermodynamics of the spacetime and verify the validity of the first law of thermodynamics for the obtained solutions.

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I. INTRODUCTION

The theory of nonlinear electrodynamics was first introduced in 1934 by Born and Infeld for the purpose of solving various problems of divergence appearing in the Maxwell theory [1]. In recent years, the study of nonlinear electrodynamics has got a new impetus. Strong motivation comes from developments in string/M-theory, which is a promising approach to quantum gravity [2]. It has been shown that the Born-Infeld (BI) theory naturally arises in the low energy limit of the open string theory [3,4]. Another motivation originates from the fact that most physical systems in the nature, including the field equations of the gravitational systems, are intrinsically nonlinear. The nonlinear BI electromagnetic theory was designed to regulate the self-energy of a pointlike charge [1]. Various aspects of black hole solutions coupled to nonlinear BI gauge field has been studied. Exact solutions of the Einstein BI theory with or without cosmological constant have been constructed in [5–9]. In the scalar-tensor theories of gravity, black object solutions coupled to a Born-Infeld nonlinear electrodynamics have also been studied widely in the literature [10,11].

However, BI theory is not the only nonlinear electrodynamics theory which can remove the divergence of the electric field at $r = 0$. In particular, in recent years, other types of nonlinear electrodynamics in the context of gravitational field have been introduced. In [12] exact solution for a static spherically symmetric field outside a charged point particle is found in a nonlinear $U(1)$ gauge theory with a logarithmic Lagrangian. While this particular theory appears to have no direct relation to superstring theory, it serves as a toy model illustrating that certain nonlinear field theories can produce particlelike solutions which can realize the limiting curvature hypothesis also for gauge fields [12]. In addition to BI and logarithmic types for

nonlinear gauge fields, very recently one of the present authors proposed an exponential form of nonlinear electromagnetic Lagrangian [13]. Although a logarithmic form of the electrodynamic Lagrangian, like BI electrodynamics, removes divergences in the electric field, the exponential form of nonlinear electromagnetic Lagrangian does not cancel the divergency of the electric field at $r = 0$, however, its singularity is much weaker than in the Einstein-Maxwell theory. Other studies on the gravitational systems coupled to nonlinear electrodynamics gauge fields have been carried out in [14–16].

The extension of the Maxwell field to the nonlinear electromagnetic gauge field provides powerful tools for investigation of black object solutions. In the present work, we would like to turn the investigations on the nonlinear electrodynamics to the rotating black string solutions with one rotation parameter in the anti-de Sitter (AdS) background. We will consider the 4-dimensional action of Einstein gravity with 2 kinds of BI-type nonlinear electromagnetic gauge fields.

The structure of this paper is as follows. In the next section we present the basic field equations as well as the Lagrangian of 2 types of nonlinear electrodynamic fields. We will also solve the equations for the rotating black string spacetime and study the physical properties of the solutions. In Sec. III, we calculate the conserved and thermodynamic quantities of the black string solutions and verify the first law of thermodynamics. We finish our paper with closing remarks in the last section.

II. BASIC EQUATIONS AND SOLUTIONS

We consider a model of a gravitating electromagnetic field in the presence of cosmological constant. The Lagrangian for this system is chosen as

$$\mathcal{L} = R - 2\Lambda + L(\mathcal{F}), \quad (1)$$

where R is the Ricci scalar, Λ refers to the cosmological constant, and $L(\mathcal{F})$ is a general Lagrangian of electromagnetic

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field in which $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ is Maxwell invariant, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and A_μ is the gauge potential. Here, we assume a kind of rotating metric whose $t = \text{constant}$ and $r = \text{constant}$ boundary has the topology $R \times S^1$ [17]

$$ds^2 = -f(r)(\Xi dt - ad\phi)^2 + \frac{r^2}{l^4}(adt - \Xi l^2 d\phi)^2 + \frac{dr^2}{f(r)} + \frac{r^2}{l^2} dz^2, \quad (2)$$

where the functions $f(r)$ should be determined from gravitational field, $\Xi = \sqrt{1 + a^2/l^2}$, a is the rotation parameter, $0 \leq \phi < 2\pi$ and $-\infty < z < \infty$.

In order to investigate the properties of the electromagnetic field, one may consider a suitable $L(\mathcal{F})$ and solve the Maxwell-like field equation. In this paper, we consider two new classes of nonlinear electromagnetic fields, namely exponential form of nonlinear electromagnetic (ENE) Lagrangian and logarithmic form of nonlinear electromagnetic (LNE) Lagrangian, whose Lagrangians are

$$L(\mathcal{F}) = \begin{cases} \beta^2 \left(\exp\left(-\frac{\mathcal{F}}{\beta^2}\right) - 1 \right) & \text{ENE} \\ -8\beta^2 \ln\left(1 + \frac{\mathcal{F}}{8\beta^2}\right) & \text{LNE} \end{cases} \quad (3)$$

where β is called the nonlinearity parameter. In the limit $\beta \rightarrow \infty$, the mentioned $L(\mathcal{F})$ s reduce to the Lagrangian of the standard Maxwell field

$$L(\mathcal{F})|_{\text{large}\beta} = -\mathcal{F} + O(\mathcal{F}^2). \quad (4)$$

Considering a strong electromagnetic field in regions near to point-like charges, Dirac suggested that one may have to use generalized nonlinear Maxwell theory in those regions [18]. Similar behavior may have occurred in the vicinity of neutron stars and black objects and so it is expected to consider nonlinear electromagnetic fields with an astrophysical motive [19]. In addition, within the framework of quantum electrodynamics, it was shown that quantum corrections lead to nonlinear properties of vacuum which affect the photon propagation [20–23].

Although in the context of nonlinear electrodynamics, BI theory is a specific model, the recent interest in the nonlinear electrodynamics theories is mainly due to their emergence in the context of the low-energy limit of heterotic string theory, where a quartic correction of Maxwell field strength appear [24]. In other words, it was shown that all order-loop corrections to gravity may be added up as a Born-Infeld-type Lagrangian [2,3,25]. Any nonlinear electrodynamics that satisfies the weak field limit (4) is said to be of the Born-Infeld type [26].

For completeness, we should note that working in the context of AdS/CFT correspondence, it is worth investigating the effects of nonlinear electrodynamic fields on the dynamics of the strongly coupled dual theory [27].

Motivated by the recent results mentioned above, we consider the mentioned Born-Infeld-type theory and investigate their properties.

The equation of motion for the gauge field can be written as

$$\partial_\mu(\sqrt{-g}L_{\mathcal{F}}F^{\mu\nu}) = 0, \quad (5)$$

where $L_{\mathcal{F}} = \frac{dL(\mathcal{F})}{d\mathcal{F}}$. Considering Eq. (2) with Eq. (5), we find that the consistent gauge potential is

$$A_\mu = h(r)(\Xi\delta_\mu^0 - a\delta_\mu^\phi), \quad (6)$$

in which the radial function $h(r)$ can be written as

$$h(r) = \begin{cases} \frac{-\beta r \sqrt{L_W}}{2} \left[1 + \frac{L_W}{5} F\left([1], \left[\frac{9}{4}, \frac{L_W}{4}\right]\right) \right] & \text{ENE} \\ \frac{-2q}{3r} \left[2F\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}, 1 - \Gamma^2\right] - \frac{1}{(1+\Gamma)} \right) \right] & \text{LNE} \end{cases} \quad (7)$$

where q is an integration constant which is related to the electric charge of the black string, $L_W = \text{Lambert}W\left(\frac{4q^2}{\beta^2 r^4}\right)$, which satisfies $\text{Lambert}W(x) \exp[\text{Lambert}W(x)] = x$, $F([a], [b], z)$ is hypergeometric function and $\Gamma = \sqrt{1 + \frac{q^2}{r^4 \beta^2}}$ (for more details, see [28]).

Using Eq. (6), we find that the only nonzero components of the electromagnetic field tensor are F_{tr} and $F_{\phi r}$,

$$F_{\phi r} = -\frac{a}{\Xi} F_{tr}, \quad F_{tr} = \frac{\Xi q}{r^2} \times \begin{cases} e^{-\frac{L_W}{2}}, & \text{ENE} \\ \frac{2}{\Gamma+1}, & \text{LNE} \end{cases}. \quad (8)$$

Expanding F_{tr} for large value of r , we arrive at

$$F_{tr} = \frac{\Xi q}{r^2} + -\frac{\xi \Xi q^3}{4\beta^2 r^6} + O\left(\frac{1}{r^{10}}\right), \quad (9)$$

where $\xi = 1, 8$ for LNE and ENE branches, respectively. It is shown that for large distances, the first term in Eq. (9) dominates and the electric field of Maxwell theory is recovered. We have plotted F_{tr} as a function of r in Fig. 1. From this figure it can be seen that for large values of r the electric fields vanish as one expected. Besides, as $r \rightarrow \infty$, both ENE and LNE behave like the linear Maxwell field. This implies that the nonlinearity of these fields make sense only near the origin. It is worth mentioning that, in contrast to the standard Maxwell and ENE fields, the electric field of LNE is finite at the origin. We should note that the divergency of the electric field of ENE is much slower than the divergency of the standard Maxwell field at $r = 0$.

Now, we are in a position to discuss the geometric view of spacetime. To do this, we should first obtain the metric function $f(r)$. Gravitational field equation of Einstein– Λ –nonlinear electromagnetic theory may be written as [13]

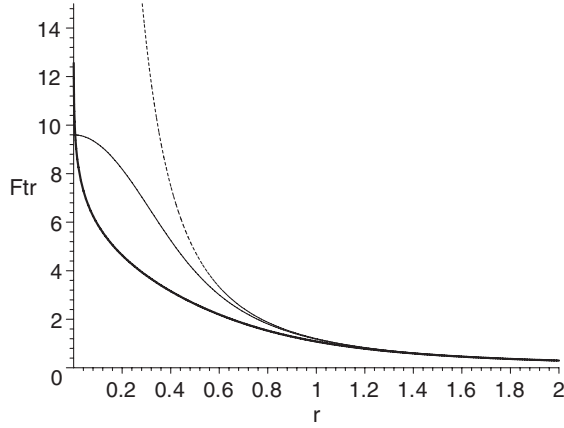


FIG. 1. The behavior of F_{tr} versus r for $q = 1$, $\beta = 4$ and $\Xi = 1.2$. ENE (Bold line), LNE (solid line), and linear Maxwell field (dotted line).

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(R - 2\Lambda) = \frac{1}{2}g_{\mu\nu}L(\mathcal{F}) - 2L_{\mathcal{F}}F_{\mu\lambda}F_{\nu}^{\lambda}. \quad (10)$$

In order to obtain $f(r)$, we should consider the field Eq. (10) with Eqs. (2), (3), and (8). It is easy to show that the rr component of Eq. (10) can be written in the following form:

$$rf'(r) + f(r) + \Lambda r^2 + \beta^2 r^2 g(r) = 0, \quad (11)$$

where

$$g(r) = \begin{cases} \frac{\exp(8\Psi)}{2} [1 - (1 - 16\Psi)] & \text{ENE} \\ 4 \ln(1 - \Psi) + \frac{8\Psi}{1 - \Psi} & \text{LNE,} \end{cases} \quad (12)$$

and $\Psi = (\frac{F_{tr}}{2\Xi\beta})^2$. After some cumbersome calculations, the solutions of Eq. (11) can be obtained as

$$f(r) = \frac{2m}{r} - \frac{\Lambda r^2}{3} + \beta W(r), \quad (13)$$

$$W(r) = \begin{cases} \frac{q}{3\sqrt{L_W}} \left[1 + L_W + \frac{4}{5} L_W^2 F\left([1], \left[\frac{9}{4}\right], \frac{L_W}{4}\right) \right] & \text{ENE} \\ \frac{8q^2 F\left(\left[\frac{1}{4}, \frac{1}{4}, \frac{11}{4}\right], 1 - \Gamma^2\right) - 4\beta r^2 \left(\Gamma - \ln\left[\frac{\Gamma^2 - 1}{2e^{-7/3}}\right] \right) - 4\beta \int r^2 \ln(\Gamma - 1) dr}{3\beta r^2} & \text{LNE,} \end{cases}$$

where m is the integration constant which is the total mass of spacetime, $F([a], [b], z)$ is hypergeometric function and the last term of LNE can be calculated as

$$\int r^2 \ln(\Gamma - 1) dr = -\frac{q^{3/2}(\Gamma - 1)^{1/4}}{2^{3/4}\beta^{3/2}} \left[\frac{14}{3} F\left(\left[\frac{1}{4}, \frac{1}{4}, \frac{11}{4}\right], \left[\frac{5}{4}, \frac{5}{4}\right], \frac{1 - \Gamma}{2}\right) - \frac{14}{25} (\Gamma - 1) F\left(\left[\frac{5}{4}, \frac{5}{4}, \frac{11}{4}\right], \left[\frac{9}{4}, \frac{9}{4}\right], \frac{1 - \Gamma}{2}\right) \right. \\ \left. - \frac{[4 + 3 \ln(\Gamma - 1)]}{9(\Gamma - 1)} F\left(\left[\frac{-3}{4}, \frac{7}{4}\right], \left[\frac{1}{4}\right], \frac{1 - \Gamma}{2}\right) + [-4 + \ln(\Gamma - 1)] F\left(\left[\frac{1}{4}, \frac{7}{4}\right], \left[\frac{5}{4}\right], \frac{1 - \Gamma}{2}\right) \right]. \quad (14)$$

We should note that obtained solutions given by Eq. (13) satisfy all the components of the field equation (10).

A. Properties of the solutions

Here we are going to study the physical properties of the solutions as well as the asymptotic behavior of the spacetime. Expanding the metric functions for large r , we have

$$f(r) = \frac{2m}{r} - \frac{\Lambda r^2}{3} + \frac{q^2}{r^2} - \frac{\xi q^4}{40r^6\beta^2} + O\left(\frac{1}{r^{10}}\right), \quad (15)$$

where for $\beta \rightarrow \infty$, one can recover the rotating black string solutions in Einstein-Maxwell gravity [17]. Next, we calculate the curvature scalars of this spacetime. It is easy to show that the Ricci scalar and the Kretschmann invariant of the spacetime are

$$R = -f''(r) - \frac{4f'(r)}{r} - \frac{2f(r)}{r^2}, \quad (16)$$

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = f'^2(r) + \left(\frac{2f'(r)}{r}\right)^2 + \left(\frac{2f(r)}{r^2}\right)^2, \quad (17)$$

where the prime denotes derivative with respect to r . Since other curvature invariants of the spacetime such as the Ricci square are only the functions of f'' , f'/r , and f/r^2 , thus we only consider the Ricci scalar and the Kretschmann invariant. Substituting the metric functions (13) in (16) and (17), we find

$$\lim_{r \rightarrow 0^+} R = \infty, \quad (18)$$

$$\lim_{r \rightarrow 0^+} R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \infty. \quad (19)$$

This indicates that we have an essential singularity located at $r = 0$. On the other side, we can expand Eqs. (16) and (17) for large r and keep the first-order nonlinear correction term

$$\lim_{r \rightarrow \infty} R = 4\Lambda + \frac{\xi Q^4}{2\beta^2 r^8} + O\left(\frac{1}{r^{12}}\right), \quad (20)$$

$$\lim_{r \rightarrow \infty} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{8}{3}\Lambda^2 + \frac{48m^2}{r^6} - \frac{96mQ^2}{r^7} + \frac{56Q^4}{r^8} - \frac{2\xi Q^4}{\beta^2 l^2 r^8} + O\left(\frac{1}{r^{11}}\right), \quad (21)$$

where we conclude that the spacetime is asymptotically anti-de Sitter.

Furthermore, such as black string with linear Maxwell source, one expects to obtain a black string with an outer and an inner horizon, an extreme black string, or a naked singularity. We should note that for the obtained black string with nonlinear source, a new interesting situation appears. This new situation appears for small values of the nonlinearity parameter in which the black string has one nonextreme horizon as it happens for Schwarzschild solution (uncharged solution). In other words, we find that there is a critical nonlinearity parameter β_c in which for $\beta < \beta_c$ the metric function may be negative near the origin. This means that the singularity is timelike for $\beta > \beta_c$, but for $\beta < \beta_c$ it is spacelike (see Fig. 2 for more details). Although we cannot obtain β_c analytically, we find that it is a function of other parameters. We obtain β_c , numerically, from the following conditions:

$$\lim_{r \rightarrow 0^+} f(r) \rightarrow \begin{cases} +\infty, & \beta > \beta_c \\ -\infty, & \beta < \beta_c \end{cases}.$$

For more clarifications, we give a numerical method for obtaining β_c . At first we should fix metric parameters m , q , and Λ , and we check that for large β the metric function is positive near the origin. Then, we reduce β until the sign of

TABLE I. $f(r)$ for $m = 1$, $q = 1$, $\Lambda = -1$, $r \rightarrow 0^+$ and $\beta_c \approx 0.171$ and 0.092 for ENE and LNE branches, respectively.

$f_{\text{ENE}}(r) \rightarrow$	$-\infty$	$+\infty$
$\beta =$	0.171167	0.171168
$f_{\text{LNE}}(r) \rightarrow$	$-\infty$	$+\infty$
$\beta =$	0.092042	0.092043

the metric function switch to negative. One can use this method to obtain β_c with ideal accuracy. Numerical calculations show that β_c change when we alter at least one of the metric parameters m , q , and Λ (see Table I for more details).

III. CONSERVED QUANTITIES AND THE FIRST LAW OF THERMODYNAMICS

At the first step, we use the definition of quasilocal energy [29–31] to compute the conserved charges of our solutions. Following the counterterm method, the divergence-free boundary stress-tensor can be written as

$$T^{ab} = \frac{\Theta^{ab} - (\Theta + \frac{2}{l})\gamma^{ab}}{8\pi}, \quad (22)$$

where the last term comes from counterterm procedure. Considering a Killing vector field ξ on the boundary \mathcal{B} , it is known that its quasilocal conserved quantity may be obtain from the following relation:

$$Q(\xi) = \int_{\mathcal{B}} d^2x \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (23)$$

where σ is the determinant of the Arnowitt-Deser-Misner form of boundary metric σ_{ij} and n^a is the unit normal vector on the boundary \mathcal{B} . It is easy to find that boundary \mathcal{B}

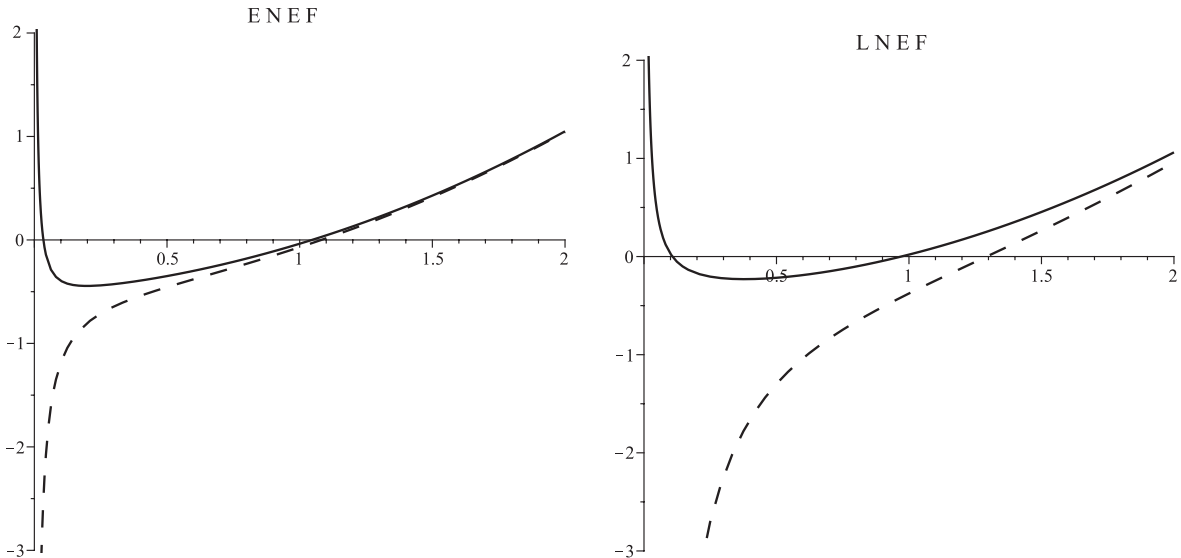


FIG. 2. $f(r)$ versus r for $\Lambda = -1$, $q = 1$, $m = 1$, and $\beta > \beta_c$ (solid line) and $\beta < \beta_c$ (dashed line) in ENE branch (left figure) and LNE branch (right figure).

has 2 killing vector fields. They are timelike ($\partial/\partial t$) and rotational ($\partial/\partial\varphi$) Killing vector fields, in which their corresponding conserved charges are the quasilocal mass and angular momentum. One can find that the mass and angular momentum per unit length of the string when the boundary \mathcal{B} goes to infinity can be calculated as

$$M = \frac{1}{16\pi l}(3\Xi^2 - 1)m, \quad (24)$$

$$J = \frac{3}{16\pi l}\Xi ma. \quad (25)$$

As one can find the angular momentum per unit length vanishes for $a = 0$ ($\Xi = 1$) and, therefore, a is the rotational parameter, correctly.

The second step is devoted to calculation of the thermodynamic quantities. It was known that the universal area law of the entropy can apply to all types of black objects in Einstein gravity [32,33]. Therefore, the entropy per unit length of the black string is

$$S = \frac{\Xi r_+^2}{4l}. \quad (26)$$

In order to obtain angular velocity Ω and Hawking temperature of the black string at the event horizon, we use the method of analytic continuation of the metric. One can obtain the Euclidean section of the metric by use of a transformation ($t \rightarrow i\tau$ and $a \rightarrow ia$) and regularity at $r = r_+$ requires that τ and ϕ should, respectively, identify with $\tau + \beta_+$ and $\phi + i\Omega\beta_+$, where β_+ is the inverse of the Hawking temperature. So, it is easy to find that

$$\Omega = \frac{a}{\Xi l^2}, \quad (27)$$

and

$$T_+ = -\frac{\Lambda r_+}{4\pi} + \begin{cases} \frac{\beta q(1-L_{W_+})}{4\pi r_+ \sqrt{L_{W_+}}} - \frac{\beta^2 r_+}{8\pi} & \text{ENE} \\ -\frac{q^2(\Gamma_+ - 2)}{\pi r_+ \Gamma_+(\Gamma_+ - 1)} + \frac{\beta^2 r_+ \left(\ln\left(\frac{\Gamma_+ - 1}{2}\right) - \frac{2}{\Gamma_+}\right)}{\pi} & \text{LNE,} \end{cases} \quad (28)$$

where $\Gamma_+ = \sqrt{1 + \frac{q^2}{r_+^4 \beta^2}}$ and $L_{W_+} = \text{LambertW}\left(\frac{4q^2}{\beta^2 r_+^4}\right)$.

In order to examine the first law of thermodynamics, we should calculate the electric charge and potential of the black string. We should use the Gauss' law and calculate the flux of the electromagnetic field at infinity to obtain the electric charge per unit length of black string

$$Q = \frac{\Xi q}{4\pi l}. \quad (29)$$

In addition, the electric potential U , measured at infinity with respect to the event horizon r_+ , is defined by [34]

$$U = A_{\mu}\chi^{\mu}|_{r \rightarrow \infty} - A_{\mu}\chi^{\mu}|_{r=r_+}, \quad (30)$$

where $\chi = \partial_t + \Omega\partial_{\phi}$ is the null generator of the event horizon. It is easy to show that

$$U = \frac{1}{\Xi} \times \begin{cases} \frac{\beta r \sqrt{L_{W_+}}}{2} \left[1 + \frac{L_{W_+}}{5} F\left([1], \left[\frac{9}{4}, \frac{L_{W_+}}{4}\right]\right) \right] & \text{ENE} \\ \frac{2q}{3r_+} \left[2F\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], 1 - \Gamma_+^2\right) - \frac{1}{(1+\Gamma_+)} \right] & \text{LNE.} \end{cases} \quad (31)$$

Now, we are in a position to check the first law of black string thermodynamics. Using the Smarr-type formula, it is straightforward to calculate the temperature, angular velocity, and electric potential in the following manner:

$$T = \left(\frac{\partial M}{\partial S}\right)_{J,Q}, \quad \Omega = \left(\frac{\partial M}{\partial J}\right)_{S,Q}, \quad U = \left(\frac{\partial M}{\partial Q}\right)_{S,J}. \quad (32)$$

One can find that the quantities calculated by Eq. (32) coincide with Eqs. (28), (27), and (31). Hence, we conclude that the first law of thermodynamics is satisfied in the following form:

$$dM = TdS + \Omega dJ + UdQ. \quad (33)$$

Since the asymptotic behavior of the solutions is the same as linear Einstein-Maxwell black string, it is expected that the nonlinearity does not affect the electrical charge, mass and angular momentum. Moreover, we find that the nonlinearity of the electromagnetic fields affects the other quantities at the horizon. Although the nonlinear electromagnetic field changes some of the conserved and thermodynamic quantities, as we expected [35] these quantities satisfy the first law of black hole mechanics.

IV. CLOSING REMARKS

Many physical systems in the nature have nonlinear behavior. Einstein field equations of general relativity is also a system of nonlinear gravitational field equations which can be applied for describing various gravitational objects. In order to solve the gravitational field equations in the presence of a matter field, one can consider either the linear gauge field such as the Maxwell electrodynamics or the nonlinear matter field such as the BI electrodynamics. Static and stationary black object solutions of these theories have been established and their thermodynamics have been studied during the past decades (see [5–9,36] and references therein). The advantages of the nonlinear electrodynamics in comparison to the Maxwell field is that it avoids the divergences at the origin and leads to a finite electric field on the point particles.

In this paper, as a new step, we considered 2 types of nonlinear electrodynamic Lagrangians as source. The first one is called the logarithmic form and the second one named the exponential form. Then, we constructed new 4-dimensional charged rotating black string solutions

with horizon topology $R \times S^1$ coupled to the nonlinear electrodynamic field. These solutions are asymptotically anti-de Sitter. If one expand the nonlinear electromagnetic fields for large r , one finds that the asymptotic behavior of them is similar to the linear Maxwell field. We also calculated the curvature invariants of the spacetime and showed that there is indeed a curvature singularity located at $r = 0$. Furthermore, we found that, unlike Einstein-Maxwell black string solutions, for small values of the nonlinearity parameter, one can obtain a black string with a nonextreme horizon. We also calculated the conserved quantities of the rotating black string, such as the mass and the angular momentum as well as the thermodynamic quantities, such as the temperature and entropy associated with the

horizon and checked that the obtained conserved and thermodynamic quantities satisfy the first law of black hole thermodynamics.

Finally, it is worthwhile to study the dynamic as well as thermodynamic stability of the solutions, and investigate the effects of nonlinearity parameter, β , on the stability of the presented solutions. We leave these problems for the future studies.

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