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Consistent null-energy-condition violation: Towards creating a universe in the laboratory

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The null energy condition (NEC) can be violated in a consistent way in models with unconventional kinetic terms, notably, in Galileon theories and their generalizations. We make use of one of these, the scale-invariant kinetic braiding model, to discuss whether a universe can in principle be created by manmade processes. We find that, even though the simplest models of this sort can have both healthy Minkowski vacuum and a consistent NEC-violating phase, there is an obstruction for creating a universe in a straightforward fashion. To get around this obstruction, we design a more complicated model and present a scenario for the creation of a universe in the laboratory.

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I. INTRODUCTION

Once it was realized that inflation can stretch a tiny region of space into the entire visible Universe, a question has been naturally raised of whether one can in principle create a new universe by manmade processes [1,2]. In the context of classical general relativity and conventional theories of matter obeying the null energy condition (NEC), the answer is negative [2,3] because of the problem with the initial singularity guaranteed by the Penrose theorem [4] (see, however, Refs. [5,6]). Widely discussed ways out are to invoke tunneling [7–16] or other quantum effects [17–19], modify gravity [20–22], and violate the NEC [23–26]. The latter option, however, has been problematic, since most of the NEC-violating theories are plagued by pathologies like ghosts, gradient instability, and/or superluminality. Yet it has been realized some time ago [27–32] that, within general relativity, the NEC violation is not necessarily accompanied by unacceptable pathologies, if one considers theories with unconventional kinetic terms. One class of examples is given by the Galileon theory [33] and its generalizations [34–41]. Indeed, by making use of the Galileon, a cosmological genesis model has been constructed [31,32], in which the evolution starts from nearly Minkowski space-time, the energy density eventually builds up, and the universe enters an epoch of rapid expansion. The NEC violation in this scenario occurs in a controllable and consistent way [32].

These developments suggest that one might be able to create a universe in the laboratory in a purely classical way and within general relativity. In this paper, we suggest a scenario of this sort, allowing ourselves not only to set up appropriate initial conditions for the field evolution but also to design a field theoretic model at our will. The idea is to construct the initial condition in a Galileon-type theory such that inside a large sphere the field is nearly homogeneous and behaves like at the initial stage of genesis, whereas outside this sphere the field tends to a constant and space-time is asymptotically Minkowskian. For these initial data, the energy density and pressure are initially small everywhere and the entire space-time is nearly Minkowskian, so that the required field configuration can in principle be prepared in the laboratory. As the field evolves from this initial state according to its equation of motion, the energy density inside the large sphere increases, space undergoes accelerated expansion there, and the region inside the sphere eventually becomes a manmade universe. Outside this sphere, the energy density remains small and asymptotes to zero at large distances; the space-time is always asymptotically Minkowskian.

Implementing this idea is not entirely trivial, however. The field theoretic model we are after should have not only a healthy genesis regime but also healthy Minkowski vacuum. The latter property is lacking in the model of Refs. [31,32]. Moreover, there must be smooth and healthy interpolation between the genesis regime inside the large sphere and asymptotic Minkowski vacuum; we will see that this requirement is particularly restrictive. For this reason, the model we end up with is rather contrived. Yet it serves the purpose of proof of principle.

This paper is organized as follows. We find it instructive to begin in Sec. II with a prototype model which actually does not work. We introduce the model and collect useful formulas in Sec. II A, consider the stability of the Minkowski vacuum in Sec. II B, and study a NEC-violating homogeneous solution in Minkowski space-time in Sec. II C. We find in Sec. II D that creating a universe in the laboratory in the way outlined above is actually impossible in the model we consider in Sec. II and, in fact, the obstruction we encounter is inherent in a class of NEC-violating theories. Yet we are able to design a working model in Sec. III by introducing an extra field whose background produces spatially inhomogeneous couplings. We present the model and discuss relevant stability issues in Sec. III A and end up with a fairly detailed scenario for the creation of a universe in the laboratory in Sec. III B. We conclude in Sec. IV.

II. PROTOTYPE MODEL

A. Preliminaries

In this section, we consider a model of kinetic braiding type [35,36] with a scalar field π and impose dilatation invariance of the action in Minkowski space-time:

$$\pi(x) \to \pi'(x) = \pi(\lambda x) + \ln \lambda. \tag{1}$$

This invariance, albeit *ad hoc*, simplifies the analysis considerably. The dilatationally invariant kinetic braiding Lagrangian is (mostly negative signature)

$$L = F(Y)e^{4\pi} + K(Y)\Box\pi \cdot e^{2\pi}, \tag{2}$$

where

$$Y = e^{-2\pi}(\partial\pi)^2 \tag{3}$$

and F and K are yet unspecified functions. Assuming that K is analytic near the origin, we set

$$K(Y = 0) = 0. \tag{4}$$

Indeed, upon integrating by parts, a constant part of K can be absorbed into the F term in (2).

The field equation is

$$4e^{4\pi}F - 2e^{2\pi}(\partial\pi)^2F' - 2\partial_\mu(e^{2\pi}F'\partial_\mu\pi) \tag{5}$$

$$+ 2e^{2\pi}\Box\pi \cdot K + \Box(e^{2\pi}K) - 2\Box\pi \cdot (\partial\pi)^2K' - 2\partial_\mu(\Box\pi \cdot K'\partial_\mu\pi) = 0, \tag{6}$$

where the prime denotes d/dY. Let $\pi_c(x)$ be a solution to this equation. We will be interested also in perturbations about π_c. To this end, let us decompose $\pi = \pi_c + \chi$ and write the quadratic Lagrangian for perturbations:

$$\begin{aligned} L^{(2)} = {} & (\partial\chi)^2F'e^{2\pi_c} + 2F''\partial_\mu\pi_c\partial_\nu\pi_c \cdot \partial_\mu\chi\partial_\nu\chi + (\partial\chi)^2[-2Ke^{2\pi_c} + 2(\partial\pi_c)^2K' + \partial_\mu(K'\partial_\mu\pi_c) + \Box\pi_c \cdot K'] \\ & + \partial_\mu\chi\partial_\nu\chi[-2\partial_\nu(K'\partial_\mu\pi_c) + 2\Box\pi_c e^{-2\pi_c}K''\partial_\mu\pi_c\partial_\nu\pi_c] + \chi^2[8Fe^{4\pi_c} - 6F'e^{2\pi_c}(\partial\pi_c)^2 - 2\partial_\mu(F'e^{2\pi_c}\partial_\mu\pi_c) \\ & + 2F''(\partial\pi_c)^4 + 2\partial_\mu(F''(\partial\pi_c)^2\partial_\mu\pi_c)] + \chi^2[\Box(e^{2\pi_c}K) + 2Ke^{2\pi_c}\Box\pi_c - \Box(K'(\partial\pi_c)^2) - 2\Box\pi_c(\partial\pi_c)^2K' \\ & + 2\Box\pi_c e^{-2\pi_c}(\partial\pi_c)^4K'' + 2\partial_\mu(\Box\pi_c e^{-2\pi_c}(\partial\pi_c)^2K''\partial_\mu\pi_c)]. \end{aligned} \tag{7}$$

We will eventually need the expression for the energy-momentum tensor. To this end, we consider minimal coupling to the metric, i.e., set $Y = e^{-2\pi}g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi$ and $\Box\pi = \nabla^\mu\nabla_\mu\pi$ in curved space-time. To calculate the energy-momentum tensor, we note that in curved space-time, the K term in $\sqrt{-g}L$ can be written, upon integrating by parts, as $\sqrt{-g}g^{\mu\nu}\partial_\mu\pi\partial_\nu(Ke^{2\pi})$. Then the variation with respect to $g^{\mu\nu}$ is straightforward, and we get

$$\begin{aligned} T_{\mu\nu} = {} & 2F'e^{2\pi}\partial_\mu\pi\partial_\nu\pi - g_{\mu\nu}Fe^{4\pi} + 2\Box\pi \cdot K'\partial_\mu\pi\partial_\nu\pi \\ & - \partial_\mu\pi \cdot \partial_\nu(Ke^{2\pi}) - \partial_\nu\pi \cdot \partial_\mu(Ke^{2\pi}) \\ & + g_{\mu\nu}g^{\lambda\rho}\partial_\lambda\pi\partial_\rho(Ke^{2\pi}). \end{aligned}$$

In what follows, we mostly consider homogeneous backgrounds, $\pi = \pi(t)$, and omit subscript c wherever possible. For a homogeneous field, equation of motion (6) reads

$$\begin{aligned} & 4e^{4\pi}F + F'e^{2\pi}(-6\dot\pi^2 - 2\ddot\pi) - 2e^{2\pi}\dot\pi F''\dot Y \\ & + Ke^{2\pi}(4\dot\pi^2 + 4\ddot\pi) + 4e^{2\pi}\dot\pi K'\dot Y + K''\dot Y(-2\dot\pi^3) \\ & + K'(-12\dot\pi^2\ddot\pi + 4\dot\pi^4) = 0, \end{aligned} \tag{8}$$

while the energy density and pressure are

$$\rho = e^{4\pi}Z, \tag{9a}$$

$$p = e^{4\pi}(F - 2YK - e^{-2\pi}K'\dot\pi\dot Y), \tag{9b}$$

where

$$Z = -F + 2YF' - 2YK + 2Y^2K'.$$

It is straightforward to see that for $\dot\pi \neq 0$, Eq. (8) is equivalent to energy conservation, $\dot\rho = 0$. Finally, in a homogeneous background the quadratic Lagrangian for perturbations, Eq. (7), simplifies to

$$L^{(2)} = U\dot\chi^2 - V(\partial_i\chi)^2 + W\chi^2, \tag{10}$$

where

$$U = e^{2\pi_c}(F' + 2YF'' - 2K + 2YK' + 2Y^2K'') = e^{2\pi_c}Z', \tag{11a}$$

$$V = e^{2\pi_c}(F' - 2K + 2YK' - 2Y^2K'') + (2K' + 2YK'')\ddot\pi_c. \tag{11b}$$

We will not need the general expression for W.

B. Minkowski vacuum

Recalling that $K(0) = 0$, we find that the Minkowski vacuum $\partial\pi = 0$ exists (cosmological constant is zero), provided that

$$F(0) = 0. \quad (12)$$

It is clear from Eq. (11) that it is stable for

$$F'(0) > 0. \quad (13)$$

There remains an issue [30,42] of the possible superluminality of perturbations about backgrounds in the neighborhood of the Minkowski vacuum, i.e., backgrounds with small $\partial\pi_c(x)$. From this viewpoint, the most dangerous terms in (7) involve $\partial_\mu\chi\partial_\nu\chi\partial_\mu\partial_\nu\pi_c$. We make these terms small by requiring that

$$K'(0) = 0. \quad (14)$$

Then the inverse effective metric for perturbations, modulo irrelevant terms, is

$$G^{\mu\nu} = \eta^{\mu\nu} + \frac{1}{F'e^{2\pi_c}}[2F''\partial^\mu\pi_c\partial^\nu\pi_c - \partial^\nu(K'\partial^\mu\pi_c) - \partial^\mu(K'\partial^\nu\pi_c)],$$

and the metric itself reads

$$G_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{F'e^{2\pi_c}}[2F''\partial_\mu\pi_c\partial_\nu\pi_c - \partial_\nu(K'\partial_\mu\pi_c) - \partial_\mu(K'\partial_\nu\pi_c)]. \quad (15)$$

A potentially dangerous situation is when the null (in the conventional sense) direction of propagation n^μ is timelike in the metric $G_{\mu\nu}$. For generic n^μ this is avoided by requiring

$$F''(0) > 0. \quad (16)$$

Indeed, the dangerous terms are of the order of $K''(\partial\pi_c)^2\partial^2\pi_c$, so the first term in square brackets is the dominant source of Lorentz violation and $G_{\mu\nu}n^\mu n^\nu < 0$ for generic n^μ.

This argument does not apply to the special direction for which $\partial_\mu\pi_c n^\mu = 0$. Let us consider this direction separately. We treat our model near the Minkowski vacuum as a low energy effective theory with a UV cutoff Λ. Consider now the background configuration [we set $\pi_c(x=0) = 0$ by using the dilatation symmetry]

$$\pi_c = q_\mu x^\mu + \frac{1}{2}A_{\mu\nu}x^\mu x^\nu,$$

and choose the wave vector $k^\mu = n^\mu k$ such that $q_\mu k^\mu = 0$. Then the effective metric (15) at distance l from the origin in the direction n^μ is

$$G_{\mu\nu} = \eta_{\mu\nu} - \frac{1}{F'(0)}(2F''(0)l^2A_{\mu\lambda}n^\lambda A_{\nu\rho}n^\rho - 2K''(0)q^2A_{\mu\nu}). \quad (17)$$

We see that $G_{\mu\nu}n^\mu n^\nu > 0$ near the origin, if $A_{\mu\nu}n^\mu n^\nu \equiv (n\cdot A\cdot n) < 0$ [assuming for definiteness that $K''(0) > 0$ and $q^2 < 0$], which signalizes the superluminality. Near the origin the correction to the propagation speed is of the order of

$$\delta c \sim \frac{K''(0)}{F'(0)}q^2(n\cdot A\cdot n).$$

This correction becomes detectable when it yields the deviation from distance traveled by light which is at least of the order of the wavelength [30]:

$$\delta c \cdot l \gtrsim k^{-1}.$$

We require that at this distance the first term in parentheses in (17), which *reduces* the speed of signal, dominates:

$$F''(0)l^2(n\cdot A\cdot n)^2 \sim \frac{F''(0)F'^2(0)}{k^2K''^2(0)q^4} > K''(0)q^2(n\cdot A\cdot n).$$

For k^2, q^2, $A_{\mu\nu} \ll \Lambda^2$, this inequality holds, provided that the functions F and K obey a constraint

$$\frac{F''(0)F'^2(0)}{K''^3(0)} \gtrsim \Lambda^{10}. \quad (18)$$

Under this constraint, the local superluminality is undetectable and, hence, not dangerous.

We conclude that the Minkowski vacuum and its neighborhood are healthy, provided that Eqs. (12)–(14), (16), and (18) are satisfied.

C. Rolling solution

With an appropriate choice of the functions F and K, Eq. (8) admits also a rolling solution, similar to that in the Galileon theory [30–32]:

$$e^\pi = \frac{1}{\sqrt{Y_*}(t_* - t)}, \quad (19)$$

where t_* is an arbitrary constant. For this solution $Y = Y_* = \text{const}$, and Y_* is determined from the equation

$$Z(Y_*) \equiv -F + 2Y_*F' - 2Y_*K + 2Y_*^2K' = 0, \quad (20)$$

where F, F', etc., are evaluated at $Y = Y_*$. For this solution, one has $T_{00} = \rho = 0$ and

$$p = \frac{1}{Y_*^2(t_* - t)^4}(F - 2Y_*K). \quad (21)$$

Thus, the rolling background violates the NEC, provided that

$$\text{NEC violation: } 2Y_*K - F > 0. \quad (22)$$

The quadratic Lagrangian for perturbations (10) reduces in this background to

$$L^{(2)} = \frac{A}{Y_*(t_* - t)^2}[\dot\chi^2 - (\partial_i\chi)^2] + \frac{B}{Y_*(t_* - t)^2}\dot\chi^2 + \frac{C}{Y_*^2(t_* - t)^4}\chi^2, \quad (23)$$

where

$$A = e^{-2\pi_c} V = F' - 2K + 4Y_* K',$$
$$B = e^{-2\pi_c}(U - V) = 2Y_* F'' - 2Y_* K' + 2Y_*^2 K'',$$
$$C = 8F - 12Y_* F' + 8Y_*^2 F'' + 8Y_* K - 8Y_*^2 K' + 8Y_*^3 K''$$

are time-independent coefficients. As a cross-check, one can derive from the latter Lagrangian the equation for homogeneous perturbation $\chi(t)$ about the rolling background and see that $\chi = \partial_t \pi_c = (t_* - t)^{-1}$ obeys this equation, as it should. Indeed, making use of Eq. (20), one finds that the coefficients of $\dot{\chi}^2$ and χ^2 in Eq. (23) are related in a simple way:

$$4(A + B) = C/Y_*.$$

Hence, homogeneous perturbation obeys a universal equation

$$-\frac{d}{dt}\left(\frac{\dot{\chi}}{(t_* - t)^2}\right) + 4\frac{\chi}{(t_* - t)^4} = 0,$$

whose solutions are $\chi = (t_* - t)^{-1}$ and $\chi = (t_* - t)^4$. This shows that the rolling background is stable against low momentum perturbations; like in the Galileon case [31], the growing perturbation $\chi = (t_* - t)^{-1} \cdot \chi_0(\mathbf{x})$ with slowly varying $\chi_0(\mathbf{x})$ can be absorbed into a slightly inhomogeneous time shift.

In fact, we can see in more general terms that the rolling background is an attractor in the class of homogeneous solutions. To this end, we use the conservation of energy (9a) to write for any homogeneous solution

$$e^{4\pi} Z = C = \text{const}. \quad (24)$$

Now, the relation $\dot{\rho} = 0$ for positive $\dot{\pi}$ can be written as

$$4\dot{\pi} Z + \dot{Y} Z' = 4e^{\pi} Y^{1/2} Z + \dot{Y} Z'$$
$$= 4\left(\frac{|C|}{|Z|}\right)^{1/4} Y^{1/2} Z + \dot{Y} Z' = 0.$$

If $Z' \neq 0$, this gives

$$\dot{Y} = -4\left(\frac{|C|}{|Z|}\right)^{1/4} Y^{1/2} \frac{Z}{Z'}, \quad (25)$$

so that

$$\dot{Z} = -4|C|^{1/4} Y^{1/2} \frac{Z}{|Z|^{1/4}}. \quad (26)$$

This shows that the rolling solution with $Z = 0$ and $\dot{\pi} > 0$ is an attractor whose basin of attraction is bounded by the points, if any, where $Z'(Y) = 0$. This is also obvious from Eq. (24): If π increases, $|Z|$ decreases.

Let us consider the stability of the rolling background and subluminality of the perturbations about it. The spatial gradient term in (23) has correct (negative) sign provided that

$$\text{no gradient instability: } A = F' - 2K + 4Y_* K' > 0. \quad (27)$$

The speed of perturbations about the rolling background is smaller than the speed of light, if the coefficient of $\dot{\chi}^2$ is greater than that of $-(\partial_i \chi)^2$, i.e.,

$$\text{subluminality: } B = 2Y_* F'' - 2Y_* K' + 2Y_*^2 K'' > 0. \quad (28)$$

We require that the latter inequality holds in a strong sense; then the perturbations about the rolling solution are strictly subluminal, and hence the perturbations about backgrounds neighboring the rolling solution are subluminal as well. When both inequalities (27) and (28) are satisfied, there are no ghosts either. The conditions (22), (27), and (28) together with Eq. (20) can be satisfied at $Y = Y_*$ by a judicious choice of the functions F and K in the neighborhood of this point, so that the NEC violation is stable and subluminal. This can be seen as follows. Equation (20) can be used to express $F(Y_*)$ in terms of $F'(Y_*)$, $K(Y_*)$, and $K'(Y_*)$, namely, $F = 2Y_* F' - 2Y_* K + 2Y_*^2 K'$. Then the inequalities (22) and (27) are satisfied, provided that $2K - 4Y_* K' < F' < 2K - Y_* K'$, which is possible for positive K'. The condition (28) can be satisfied by an appropriate choice of F'' and K''.

Obviously, the functions $F(Y)$ and $K(Y)$ can be chosen in such a way that both Minkowski vacuum and the rolling solution are stable and healthy;[1] i.e., Eqs. (12)–(14), (16), and (18) are satisfied at $Y = 0$ and Eqs. (20), (22), (27), and (28) are satisfied at $Y = Y_*$. With such a choice of $F(Y)$ and $K(Y)$, both Minkowski vacuum and the rolling solution are attractors, with nonoverlapping basins of attraction.

D. Obstruction to a simple way of creating a universe in the laboratory

It is now tempting to implement the approach outlined in Sec. I in a simple way, by considering the initial field $\pi(t, \mathbf{x})$ which slowly varies in space and interpolates between the rolling solution (19) inside a large sphere and Minkowski vacuum $\partial\pi = 0$ at spatial infinity. By slow variation in space, we mean that the spatial derivatives of π are negligible compared to temporal ones, so that at each point in space π evolves in the same way as in the homogeneous case.

An advantage of this quasihomogeneous approach is the simplicity of the analysis; a disadvantage is that it actually does not work in our prototype model. The point is that, irrespectively of the equation of motion, the term with $\dot{\chi}^2$ in (10) is proportional to $Z'(Y)$; see Eq. (11a). Thus, for configurations slowly varying in space, the absence of ghosts requires

[1] This does not mean, though, that the entire model is completely healthy: It can be a low energy effective theory of some Lorentz-invariant UV-complete theory only if perturbations about *any* allowed background are subluminal [42]. This property should hold also in the presence of gravity; cf. Ref. [43]. The analysis of this issue is beyond the scope of this paper.

$$Z'(Y) > 0$$

everywhere. For both Minkowski vacuum and the rolling solution, we have $Z = 0$, so there is no ghost-free configuration that slowly varies in space and interpolates between the two solutions as r, the distance from the center of the sphere, changes from 0 to ∞.

This obstruction to have a quasihomogeneous ghost-free configuration, interpolating between two different solutions of zero energy, is generic in theories with the following properties: (i) There is a single scalar field π; (ii) the field equation is second order; (iii) the action is invariant under dilatations (1). This class of theories includes, e.g., conformal higher order Galileons of Ref. [33] and conformal Dirac-Born-Infeld Galileons of Ref. [34]. The argument is essentially the same as above. The Noether energy-momentum tensor obeys

$$\partial_\mu T^\mu_\nu = -(\mathrm{EOM}) \cdot \partial_\nu \pi,$$

where (EOM) stands for the equation of motion. Therefore, the equation of motion for spatially homogeneous $\pi = \pi(t)$ is

$$(\mathrm{EOM}) = -\frac{1}{\dot{\pi}} \dot{\rho}. \tag{29}$$

Since the field equation is second order, $\rho = \rho(\pi, \dot{\pi})$ does not contain $\ddot{\pi}$ and higher derivatives, and by scale invariance it has the form $\rho = \exp(4\pi) Z(Y)$, where $Y = \dot{\pi}^2 \exp(-2\pi)$ [cf. Eq. (3)], and Z is a model-dependent function. It follows from Eq. (29) that the equation of motion for homogeneous perturbation about the background $\pi_c(t)$ reads

$$-\frac{1}{\dot{\pi}_c} \frac{\partial \rho}{\partial \dot{\pi}_c} \ddot{\chi} + \cdots = 0,$$

where omitted terms do not contain $\ddot{\chi}$. Hence, the kinetic part of the Lagrangian for the perturbations has the form

$$L^{(2)} \supset \frac{1}{2\dot{\pi}_c} \frac{\partial \rho}{\partial \dot{\pi}_c} \dot{\chi}^2 = \mathrm{e}^{2\pi_c} Z'(Y) \dot{\chi}^2,$$

which is the same as in (10). Both zero energy solutions have $Z = 0$, so an interpolating configuration has $Z' < 0$ somewhere in between, and thus it is not ghost-free.

One way to get around this obstacle would be to insist on slow spatial variation of the initial field configuration but give up the prescription that the field inside the large sphere is in the rolling regime (19). Instead, one would consider the field with nonzero energy density inside the sphere, so that there exists a smooth and ghost-free configuration that interpolates, as r increases, between this field and the asymptotic Minkowski vacuum. This can hardly lead to the creation of a universe, however, since Eq. (26) shows that the point $Y = 0$ is an attractor, and the field in the interior of the sphere will likely relax to it.[2] Near $Y = 0$, one has $F = F'(0)Y$ and $Z = F'(0)Y$, so that the equation of state is $p \approx \rho$ [recall that $K(0) = 0$; one can show that the last term in (9b) is negligible at small Y]. Thus, the NEC does not get violated.

Other possibilities are to consider field configurations with non-negligible spatial gradients or give up scale invariance of the action. In both cases the above no-go argument would be irrelevant, but the analysis would be more complicated. We will follow another route and complicate the model instead.

III. IMPROVED MODEL

A. Spatially inhomogeneous couplings

We do not abandon quasihomogeneity but now allow the functions F and K to depend explicitly on spatial coordinates. This can be the case if there is another field, call it φ, which determines the couplings entering these functions, and this field acts as a quasihomogeneous background, $\varphi = \varphi(\mathbf{x})$. In this case, one can consider a field configuration $\pi(t, \mathbf{x})$ which at any point in space is approximately given by the rolling solution (19) but with Y_* depending on $\mathbf{x}$ (recall that Y_* is independent of time for the homogeneous solution). We prepare the background $\varphi(\mathbf{x})$ in such a way that $Y_*(\mathbf{x})$ is constant inside the large sphere (to evolve into a manmade universe) and gradually approaches zero as $r \to \infty$. We have to check that, with an appropriate choice of the functions $F(Y; \varphi)$ and $K(Y; \varphi)$, this construction is healthy everywhere in space; i.e., there are no pathologies inside the large sphere, at spatial infinity, and in the intermediate region ("the wall").

Let $\Phi(\varphi)$ be a function of the new field, such that $Y_* = \Phi(\varphi)$ is a solution to Eq. (20). As r varies from zero to infinity, $\Phi(\mathbf{x})$ changes from some positive value Φ_0 to zero. We are going to check that the inequalities (27) and (28) can be satisfied for any $\Phi \in (0, \Phi_0)$, so that there is no ghost or gradient instability anywhere in space (including the wall region), and propagation of perturbations is subluminal, also in any region of space. To this end, we write F and K in the vicinity of $Y_* = \Phi$ as a series in $(Y - \Phi)$:

$$F = a(\Phi) + b(\Phi)(Y - \Phi) + \frac{c(\Phi)}{2}(Y - \Phi)^2, \tag{30a}$$

$$K = \kappa(\Phi) + \beta(\Phi)(Y - \Phi) + \frac{\gamma(\Phi)}{2}(Y - \Phi)^2, \tag{30b}$$

and set $\kappa(0) = 0$ without loss of generality; see Eq. (4). In these terms, Eq. (20) reads

$$a - 2\Phi b + 2\Phi\kappa - 2\Phi^2\beta = 0, \tag{31}$$

and the inequalities (27) and (28) become

$$\text{no gradient instability: } b(\Phi) - 2\kappa(\Phi) + 4\Phi\beta(\Phi) > 0, \tag{32a}$$

$$\text{subluminality: } c(\Phi) - \beta(\Phi) + \Phi\gamma(\Phi) > 0. \tag{32b}$$

[2] A loophole here is that we neglect effects of gravity.

Let us also write the pressure (21) for the rolling solution:

$$p = \frac{1}{\Phi^2(t_* - t)^4}(a - 2\kappa\Phi)$$
$$= \frac{1}{\Phi^2(t_* - t)^4} \cdot 2\Phi(b - 2\kappa + \beta\Phi), \qquad (33)$$

where we used Eq. (31). We require the NEC violation inside the large sphere, i.e.,

$$\text{NEC violation: } b(\Phi_0) - 2\kappa(\Phi_0) + \Phi_0\beta(\Phi_0) < 0. \qquad (34)$$

Finally, the stability conditions of the Minkowski vacuum, Eqs. (13) and (14), read

$$b(0) > 0, \qquad \beta(0) = 0,$$

while Eq. (18) requires that γ is sufficiently small. The condition (12) is satisfied automatically, provided that the coefficients in (30) obey Eq. (31). Note that, since $b(0) > 0$ and $\kappa(0) = 0$, pressure (33) is *positive* at small Φ. Away from the large sphere with $\Phi = \Phi_0$, the space *contracts*.

To see explicitly that all the above conditions can be satisfied, let us choose

$$b(\Phi) = u + v\Phi^2, \qquad \kappa(\Phi) = 0, \qquad \beta(\Phi) = w\Phi,$$
$$c(\Phi) > \beta(\Phi), \qquad \gamma(\Phi) = 0$$

with constant $u > 0$, $w > 0$, $v < 0$, and $w\Phi_0^2 \gg u$, while $a(\Phi)$ is given by Eq. (31). Then the only nontrivial constraints are (32a) and (34). These are satisfied by choosing

$$-4w < v < -w.$$

Thus, there is indeed a choice of $F(Y;\varphi)$ and $K(Y;\varphi)$ such that the entire setup is not pathological everywhere in space (including the wall region), at least in the quasihomogeneous case.

B. Sample scenario

Let us now sketch a concrete scenario for creating a universe. Let us assume that the field φ is a usual scalar field which has two vacua, $\varphi = 0$ and $\varphi = \varphi_0$. We prepare a spherical configuration of this field with $\varphi = \varphi_0$ inside a sphere of large enough radius R and $\varphi = 0$ outside this sphere; see Fig. 1. We assume for definiteness (although this assumption can be relaxed) that there is a source for the field φ that keeps this configuration static. Let $L \ll R$ be the thickness of the wall separating the two vacua; L is also kept time independent by the source. We require that the mass of this ball is small enough, so that $R \gg R_s$, where R_s is the Schwarzschild radius. The mass is of the order of $\mu^4 R^2 L$, where μ is the mass scale characteristic of the field φ. Hence, the latter requirement reads $\mu^4 RL \ll M_{\rm Pl}^2$. For small enough μ, both R and L can be large.

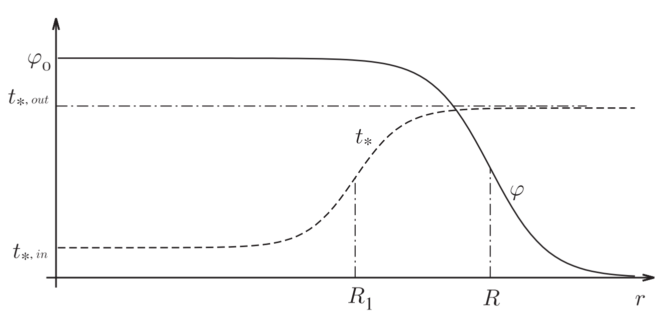

FIG. 1. The setup. Dashed and solid lines show $t_*(r)$ and $\varphi(r)$, respectively. The behavior of the function $\Phi(r) = \Phi(\varphi(r))$ is similar to that of $\varphi(r)$.

Let the function $\Phi(\varphi)$ of Sec. III A be such that $\Phi(0) = 0$, $\Phi(\varphi_0) = \Phi_0$, and $\Phi'(\varphi_0) = 0$. The latter property ensures that coupling of π to φ does not move φ out of the vacuum φ_0 inside the large sphere, whatever π does there.[3]

We prepare the initial configuration of π at $t = 0$ in such a way that it initially evolves as

$$e^{\pi} = \frac{1}{\sqrt{\Phi_0}t_*(r) - \sqrt{\Phi(r)}t}, \qquad (35)$$

where we allow the parameter t_* in (19) to vary in space, and choose a convenient parametrization. We choose $t_*(r) = t_{*,\rm in}$ inside a somewhat smaller sphere of radius $R_1 < R$ (but $R_1 \sim R$) and $t_*(r) = t_{*,\rm out} \gg t_{*,\rm in}$ at $r > R_1$ (hereafter subscripts *in* and *out* refer to the regions $r < R_1$ and $r > R_1$, respectively), as shown in Fig. 1, with the transition region of, say, the same thickness L. We take $t_{*,\rm out} \ll L$; then the characteristic time scales are smaller than the smallest length scale L inherent in the setup, so the spatial derivatives of π are negligible compared to the time derivatives. This ensures that the field π is in the quasihomogeneous regime. As $r \to \infty$, we have $\Phi(r) \to 0$ and $t_* \to$ const, so the field π tends to the Minkowski vacuum $\pi =$ const.

At the initial stage of evolution, pressure inside the sphere of radius R_1 is

$$p_{\rm in} = -\frac{M^4}{\Phi_0^2(t_{*,\rm in} - t)^4},$$

where M is the mass scale characteristic of the field π. We require that $|p_{\rm in}|R^3/M_{\rm Pl}^2 \ll R$; then the gravitational potentials are small everywhere, and gravity is initially in the linear regime. Thus, we impose a constraint

$$\frac{M^4 R^2}{\Phi_0^2 t_{*,\rm in}^4} \ll M_{\rm Pl}^2, \qquad (36)$$

which is consistent with the above conditions for $M \ll M_{\rm Pl}$ and $\Phi_0 \gtrsim M^2$.

[3]We implicitly neglect kinetic mixing between π and φ. It can be made small by considering the function Φ, which depends on $\lambda\varphi$, where λ is a small parameter.

At least at the initial stage of the evolution, the field $\pi(r, t)$ is in the quasihomogeneous regime and evolves according to (35). The metric is also quasihomogeneous:

$$ds^2 = dt^2 - a^2(r, t)(dr^2 + r^2 d\Omega^2).$$

We integrate the equation $\dot{H} = -4\pi G p$ to find that, soon after the evolution begins, the Hubble parameter inside the sphere of radius R_1 is

$$H_{\rm in} = \frac{4\pi M^4}{3M_{\rm Pl}^2 \Phi_0^2 (t_{*,\rm in} - t)^3}. \tag{37}$$

In view of (36) and $t_{*,\rm in} \ll R$, the Hubble length scale is large for some time, $H^{-1} \gg R$. This is true also at $r > R_1$, so there are no antitrapped surfaces initially.

As t approaches $t_{*,\rm in}$, pressure becomes large at $r < R_1$ and the Hubble length shrinks there to $R_1 \sim R$. The antitrapped surfaces get formed inside the sphere of radius R_1, and a new universe gets created and enters the genesis regime there. This occurs when $H_{\rm in} \sim R^{-1}$, i.e., at time t_1 such that

$$(t_{*,\rm in} - t_1) \sim \left(\frac{M^4 R}{M_{\rm Pl}^2 \Phi_0^2}\right)^{1/3}.$$

Note that at that time the energy density $\rho_{\rm in} \sim M_{\rm Pl}^2 H_{\rm in}^2$ is still relatively small:

$$\frac{\rho_{\rm in}}{|p_{\rm in}|} \sim \left(\frac{M^4}{\Phi_0^2 R^2 M_{\rm Pl}^2}\right)^{1/3} \ll 1.$$

This implies that, at time t_1, space-time is locally nearly Minkowskian. Another manifestation of this fact is that the scale factor is close to 1:

$$a_{\rm in}(t_1) = 1 + \frac{2\pi M^4}{3M_{\rm Pl}^2 \Phi_0^2 (t_{*,\rm in} - t_1)^2},$$

where the correction to 1 is of the order of $\rho_{\rm in}/|p_{\rm in}|$. Hence, our approximate solution (35) and (37) is legitimate.

Since $t_{*,\rm out} \gg t_{*,\rm in}$, the field e^{π} at time t_1 is still small at $r > R_1$, and the Hubble length scale exceeds R there. Gravity is still weak at $r > R_1$, so it is consistent to assume that the configuration of φ is not modified by that time. Note also that a black hole is not formed by then either.

At somewhat later times, the geometry of hypersurfaces $t = \text{const}$ is that of a semiclosed world: At some distance from the origin, the area of the sphere $r = \text{const}$ decreases as r increases, $\partial(ar)/\partial r < 0$. This regime begins (i.e., a neck gets formed) when there appears a solution to

$$\frac{\partial a}{\partial r} \cdot r + a \approx -\frac{4\pi M^4}{3M_{\rm Pl}^2 \Phi_0^2 [t_*(r) - t]^3} \frac{\partial t_*}{\partial r} \cdot r + 1 = 0.$$

Clearly, this happens at a place where $t_*(r) \simeq t_{*,\rm in}$, but $\partial t_*/\partial r$ starts to deviate from zero. The neck gets formed at time t_2 such that

$$\frac{M^4}{M_{\rm Pl}^2 \Phi_0^2 (t_{*,\rm in} - t_2)^3} \frac{t_{*,\rm out} R}{L} \sim 1.$$

Since we take $t_{*,\rm out} \ll L$, we have $t_2 > t_1$ indeed. Nevertheless, it is straightforward to arrange parameters in such a way that our approximate solution (35) and (37) is legitimate at time t_2 as well.

This completes the discussion of the initial stage of the creation of a new universe. To make the scenario complete, one would specify the way to design the configuration of the field φ and keep it static (or consider an evolving field φ instead). Also, one would like to trace the dynamics of the system to longer times and see what geometry develops towards the end of the genesis epoch occurring at $r < R_1$. Since the background we have studied is healthy, one does not expect surprises from this complete analysis.

IV. DISCUSSION

Because of the obstruction we encountered in Sec. II D, it is rather unlikely that simple, scale-invariant Galileon-type theories can be employed to create a universe in the laboratory. We had to make the model a lot more complicated, to the extent that the whole scenario may appear completely unrealistic. While the particular model we considered in Sec. III is indeed not very appealing, we think that the overall situation is not absolutely hopeless. First, one can think of a possibility that a model designed on paper can be implemented in the laboratory, even though this certainly sounds like fiction. Barring this possibility, let us make the second point. If there is anything like a Galileon in nature, and if the Universe experienced anything like the genesis epoch, there *must be* a smooth and consistent interpolation between the genesis regime and Minkowski vacuum, albeit in the course of cosmological evolution rather than in the radial direction in space as we need. It is not inconceivable that one may be able to use the mechanism making this interpolation healthy in cosmology for the purpose of creating a universe in the laboratory.

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