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Pure states and black hole complementarity

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The future apparent horizon of a black hole develops large stress energy due to quantum effects, unless the outgoing modes are in a thermal density matrix at the local Hawking temperature. It is shown for generic pure states that the deviation from thermality is so small that infalling observers will see no drama on their way to the stretched horizon, providing a derivation of black hole complementarity after the Page time. Atypical pure states, and atypical observers, may of course see surprises, but that is not surprising.

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I. INTRODUCTION

In the usual analysis of Hawking radiation, the problem is analyzed in either the Hartle-Hawking vacuum [1] or the Unruh vacuum [2]. The former case is appropriate for eternal black holes, supported by a thermal flux of radiation from past infinity. The latter case is not supported by such a flux, and it better models the evaporation of a black hole formed in a dynamical process. Here backreaction effects are expected to modify the spectrum of the Hawking radiation after a substantial fraction of the initial mass is lost. Each of these vacua involve exterior modes entangled with interior modes, as described in [2]. Correlation functions of local operators outside the black hole horizon may therefore be viewed as expectation values in a density matrix where at least the outgoing modes in the infinite future are in a finely tuned thermal density matrix. The most obvious attempt to modify this situation by placing such modes in their asymptotic vacuum leads to the Boulware vacuum [3], which produces a singular renormalized stress energy tensor on the future horizon (as well as the past horizon). This leads to violations of the equivalence principle for infalling observers.

In the present work our goal is to study in more detail deformations of the Unruh and Hartle-Hawking vacuum states to test the robustness of the principle of black hole complementarity [4]. Generic deformations, in particular those toward pure states, lead to time-dependent fluctuations in the radiation. Such fluctuations typically lead to divergent energy densities for a freely falling observer on either the past horizon, the future horizon, or both.

We begin by establishing that the so-called in-modes may be taken to be in an arbitrary pure state tensored with either the Hartle-Hawking or Unruh vacua. This provides us with multiparameter deformations of these vacua with finite stress energy on the future horizon. This provides evidence in favor of the black hole complementarity hypothesis, by giving examples of smooth deformations of the vacuum states. Moreover, each of these in-modes has a component outgoing at future infinity, caused by scattering off the gravitational potential. They provide examples of outgoing fluxes which do not lead to firewalls on or near the horizon.

This situation of course is not satisfactory, because these in-modes may be traced back to the selection of a non-vacuum state at past infinity. Moreover, we find that any attempt to treat similar excitations of the out-modes in the Schwarzschild background does indeed lead to divergent energies as seen by a freely falling observer near the horizon. Thus, at first sight, it seems out-modes must be locked into a purely thermal density matrix to avoid drama for an infalling observer. A finite perturbation at any frequency leads to infinite local energy densities for infalling observers at the event horizon.

The above discussion refers to a calculation that neglects backreaction of the emitted radiation on the geometry. To improve on this situation, we model the effect of backreaction using the outgoing Vaidya metric [5]. This provides a fully time-dependent metric when a null fluid is emitted from a black hole. We still find that freely falling observers see an UV divergent energy density as they approach the stretched horizon, even with backreaction included.

Lloyd [6] has pointed out that random pure states can lead to effects that mimic averaging over ensembles in statistical mechanics (see also later work by Page [7] where it was emphasized that information does not begin to emerge from a black hole until a time of order M^3, which we refer to as the Page time). In fact, there is a sense in which the convergence is much more rapid. If a reduced density matrix is constructed by tracing over a Hilbert

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subspace of dimension e^N, the error in the density matrix is of order $e^{-N/2}$. If one instead computes fluctuations in a statistical ensemble, the finite size effects are typically much larger, of order $1/\sqrt{N}$. We use this observation to show that an observer falling into a generic pure state black hole will not see any drama up to the stretched horizon. If the black hole is projected into an outgoing mode eigenstate, infallers can indeed see mild drama as they approach the stretched horizon, as noted in the previous paragraph, but projections are either nongeneric or impractical. These results provide a derivation of black hole complementarity for black holes older than the Page time.

It should be emphasized that we take care to use local unitary effective field theory only outside the stretched horizon, where it has a conventional interpretation. One may also try to build effective field theory on patches of spacetime inside the horizon; however, the interpretation there is much more problematic. Inside the horizon physical observables are inherently imprecise, and there is much room to hide highly nonlocal physics [8]. An exact fundamental description may predict nonlocal physics inside the horizon that is not captured by an approximate local unitary effective field theory in that region. Conversely, applying local effective field theory across the horizon will predict effects that are not realized in the fundamental unitary description.

II. NONROTATING BLACK HOLE EVAPORATION IN 3+1 DIMENSIONS: PROBLEMS AND SOLUTIONS

A. Mode expansions and vacua

In this section we consider a massless conformally coupled scalar field. Issues of backreaction will be ignored, and reexamined in the following section. The metric in Schwarzschild coordinates takes the form

$$ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2.$$

In these coordinates, a complete set of modes in the exterior region may be obtained by separating variables in the equation of motion and defining the tortoise radial coordinate

$$r_* = r + 2M \log\left(\frac{r}{2M} - 1\right).$$

The angular and time dependence may be handled straightforwardly, and the radial equation can be mapped into a scattering problem with a steplike potential separating the behavior at $r \to \infty$ from the region $r \to 2M$ [9]. This leads to a natural decomposition into independent modes that we refer to as ingoing and outgoing [10]:

$$\begin{aligned} u^{\text{in}}(x) &= (4\pi\omega)^{-1/2} e^{-i\omega t} R_l^{\text{in}}(\omega; r) Y_{lm}(\theta, \phi) \\ u^{\text{out}}(x) &= (4\pi\omega)^{-1/2} e^{-i\omega t} R_l^{\text{out}}(\omega; r) Y_{lm}(\theta, \phi) \end{aligned} \quad (1)$$

with

$$R_l^{\text{out}}(\omega; r) \sim \begin{cases} r^{-1} e^{i\omega r_*} + A_l^{\text{out}}(\omega) r^{-1} e^{-i\omega r_*}, & r \to 2M \\ B_l(\omega) r^{-1} e^{i\omega r_*}, & r \to \infty \end{cases}$$

$$R_l^{\text{in}}(\omega; r) \sim \begin{cases} B_l(\omega) r^{-1} e^{-i\omega r_*}, & r \to 2M \\ r^{-1} e^{-i\omega r_*} + A_l^{\text{in}}(\omega) r^{-1} e^{i\omega r_*}, & r \to \infty. \end{cases}$$

Scattering off the gravitational field leads to "grey body" factors, so a mode that is purely outgoing near infinity contains an ingoing component near the horizon, and likewise a mode that is purely ingoing near the horizon contains an outgoing component near infinity.

The Unruh vacuum is defined by requiring the modes incoming at past null infinity to be purely positive frequency with respect to t, while those outgoing from the past horizon are positive frequency with respect to the appropriate Kruskal coordinate. This vacuum corresponds to an evaporating black hole with no incoming flux at past infinity, but a thermal outgoing flux at future null infinity.

The Hartle-Hawking vacuum is defined in a similar way, except the condition at past null infinity is replaced by the condition that infalling modes on the future horizon are positive frequency with respect to the appropriate Kruskal coordinate. This corresponds to an eternal black hole with balancing ingoing and outgoing thermal fluxes at infinity.

It is also worth mentioning the Boulware vacuum where t is used to define positive frequency throughout the exterior region. This vacuum leads to singular quantum corrections at the horizon.

B. Fluctuations

In the following we will mostly be interested in the Unruh vacuum, which describes an evaporating black hole. An important set of early results in this direction developed an understanding of renormalization in this curved background [11], which led to explicit computations of the one-loop corrections to $\langle\phi^2\rangle$ and $\langle T_{\mu\nu}\rangle$ for a massless scalar field in the Schwarzschild background, in the Unruh, Hartle-Hawking and Boulware vacua [12]. In both the Unruh and Hartle-Hawking vacua, these corrections were found to be mild, leading to the expectation that backreaction near the horizon should be negligible. We will discuss these computations in more detail in the following subsection.

For the moment, let us study the behavior of the individual modes (1) near the past and future horizons. For both the ingoing and outgoing modes, the mode functions are finite as $r \to 2M$ but oscillate more and more rapidly with r as the horizon is approached. Fixed frequency oscillations appear with respect to the time coordinate t. This is illustrated in Fig. 1.

The stress energy tensor for a massless conformally coupled scalar field in a Ricci flat background is

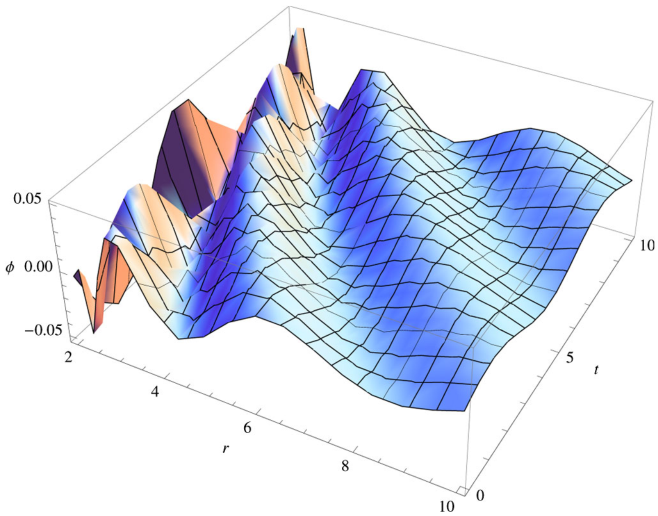

FIG. 1 (color online). Scalar mode fluctuation near the horizon. Here we set $M = 1$ and $\omega = 1$.

$$T_{\mu\nu} = \frac{2}{3}\partial_\mu\phi\partial_\nu\phi - \frac{1}{6}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi - \frac{1}{3}\phi_{;\mu\nu}\phi + \frac{1}{12}g_{\mu\nu}\phi\Box\phi.$$

To study the behavior of this quantity near the horizon, we must first contract indices with some suitably defined basis vectors. Near the future horizon, we choose a velocity 4-vector corresponding to a timelike radial ingoing geodesic,

$$u^\mu = \left(\frac{k}{1-\frac{2M}{r}}, -\sqrt{k^2 - 1 + \frac{2M}{r}}, 0, 0\right), \tag{2}$$

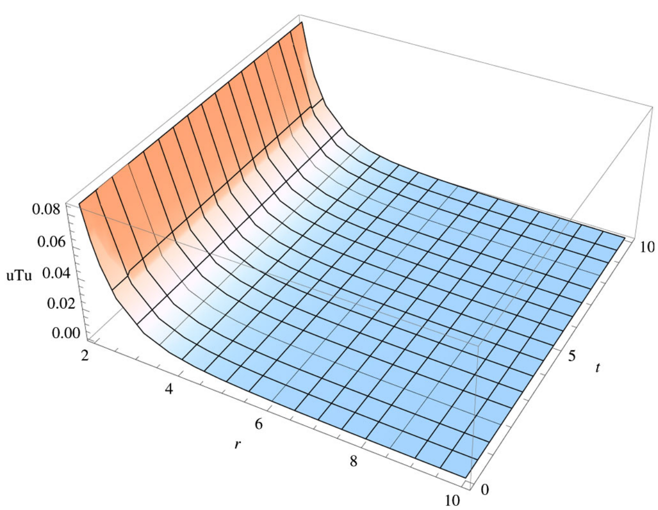

FIG. 2 (color online). The expectation value of an ingoing fluctuation contracted with the velocity of an infalling timelike geodesic near the future horizon. Here we set $M = 1$ and $\omega = 1$.

in a (t, r, θ, ϕ) basis. The result for an ingoing mode is shown in Fig. 2. The answer is finite on the horizon, and independent of time.

If we perform the same computation for an ingoing mode on the past horizon and instead choose an outgoing radial timelike geodesic, we find a double pole as $r \to 2M$ and a divergent result. The result is again independent of time.

The outgoing modes produce a stress tensor that is singular on both the past and future horizons, with rapid oscillations combined with double pole terms as $r \to 2M$. Now the stretched horizon is placed at a value of r such that the redshift to infinity is a constant, such that

$$\Lambda_{\rm UV} = \frac{M_{\rm pl}^2}{M}\frac{1}{\sqrt{1-\frac{2M}{r}}}, \tag{3}$$

where $\Lambda_{\rm UV}$ will be the ultraviolet cutoff scale for the stretched horizon theory, which may be taken to be some energy below the Planck scale. The double pole indicates that the infalling observer sees a large energy density,

$$\rho \sim T^4\frac{1}{(1-\frac{2M}{r})^2} = \Lambda_{\rm UV}^4. \tag{4}$$

As we will see in the next subsection, if we sum over modes to compute the correct one-loop contributions to the vacuum expectation values of these quantities, and correctly renormalize [11,12], there are delicate cancellations that remove the future horizon divergence, in the case of the Unruh vacuum, and for both horizons in the case of the Hartle-Hawking vacuum. It will then be our goal to model time-dependent pure state corrections to these results.

C. Correlators

Let us begin by studying the simplest quantity built out of the scalar field that receives quantum corrections and can become potentially divergent on the horizon $\langle\phi^2\rangle$. As we saw in the previous subsection, the modes themselves are finite on the horizon, but derivative operators such as $T_{\mu\nu}$ may become singular. Following [12] we can construct $\langle\phi^2\rangle$ by applying a point-splitting regularization to the tree-level propagator in the appropriate vacuum state, and then applying a local counterterm subtraction procedure.

For the Unruh vacuum $|U\rangle$, this yields

$$\langle U|\phi^2|U\rangle = \frac{1}{16\pi^2}\int_0^\infty \frac{d\omega}{\omega}\left[\sum_{l=0}^\infty (2l+1) \times \left(\coth\frac{\pi\omega}{\kappa}|R_l^{\rm out}(\omega;r)|^2 + |R_l^{\rm in}(\omega;r)|^2\right) - \frac{4\omega^2}{1-\frac{2M}{r}}\right] - \frac{4M^2}{48\pi^2 r^4(1-\frac{2M}{r})}, \tag{5}$$

where $\kappa = \frac{1}{4M}$ is the surface gravity at the horizon. The first term corresponds to the outgoing modes, the second corresponds to the ingoing modes, and the last two terms correspond to the counterterm contributions. The appearance of the coth $\frac{\pi\omega}{\kappa}$ factor is a consequence of the thermality of the outgoing modes. In the Hartle-Hawking vacuum, such a factor also appears in front of the ingoing term.

The sum over angular momenta of the outgoing term yields only a partial cancellation of the $r \to 2M$ pole, while the sum for the ingoing term is finite in this limit. Only after integrating over frequency is the $r \to 2M$ pole canceled. This requires a delicate exact cancellation between the counterterms and the thermal outgoing modes.

It is worth noting that any finite excitation of the Unruh vacuum by ingoing modes preserves the finiteness of $\langle\phi^2\rangle$. Thus there is an easily accessible collection of modifications of the Unruh vacuum obtained by tensoring in essentially arbitrary infalling pure states that leads to finite stress energy near the horizon.

However, to have a successful theory of the stretched horizon, this is necessary, but not sufficient. If the Unruh vacuum is to be replaced by a pure state built out of stretched horizon modes, and exterior modes, and the Hawking radiation is to be produced by unitary evolution, then the stretched horizon must also be capable of emitting outgoing modes in a manner that deviates from exact thermality. We turn to this question in the next subsection and examine whether backreaction ameliorates the problem.

D. Outgoing Vaidya metric

In the above we have seen that a single classical outgoing mode of definite frequency induces an infinite stress energy on the global horizon after taking into account the effects of renormalization. Let us now see if this divergence survives if we also include gravitational backreaction. This kind of problem has been studied extensively in the literature, for example, in the study of neutrino emission during stellar core collapse [13,14] in the limit of spherical symmetry. The emission of massless matter, in a so-called null fluid, may be studied analytically using the outgoing Vaidya metric [5]

$$ds^2 = -\left(1 - \frac{2M(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2,$$

with stress energy tensor

$$T^{\mu\nu} = -\frac{1}{4\pi r^2}\frac{dM}{du}k^\mu k^\nu,$$

where k^μ is a null vector directed radially outward, normalized as in [14]. Some properties of the solution are illustrated in Fig. 3.

An infalling observer will measure an energy density in his reference frame,

$$\rho_{\rm in} = T^{\mu\nu}U_\mu U_\nu,$$

where U^μ is the velocity 4-vector of the infalling observer. This is obtained by solving the equation for a timelike

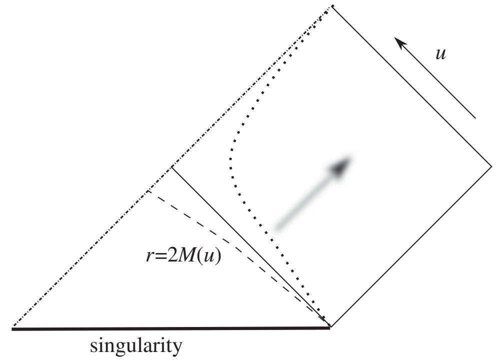

FIG. 3. The Penrose diagram for the outgoing Vaidya metric. The lower dashed line shows the apparent horizon below the global horizon. The upper dot-dashed line indicates where the metric is geodesically incomplete and may be patched onto a variety of interior solutions. The dotted timelike line shows the position of the stretched horizon.

geodesic in these coordinates. For a purely radial motion, the components of the velocity are

$$U^\mu = \left(\frac{du}{d\tau}, \frac{dr}{d\tau}, \frac{d\theta}{d\tau}, \frac{d\phi}{d\tau}\right) = \left(\frac{1}{V + \sqrt{V^2 + 1 - \frac{2M(u)}{r}}}, V, 0, 0\right).$$

The energy density seen by the infalling observer is then [14]

$$\rho_{\rm in} = -\frac{1}{4\pi r^2}\frac{dM}{du}\frac{1}{\left(V + \sqrt{V^2 + 1 - \frac{2M(u)}{r}}\right)^2}. \quad (6)$$

To model the process of interest to us, let us consider the solution corresponding to the emission of energy $M_{\rm pl}^2/M$ over a time $M/M_{\rm pl}^2$ measured at infinity. Inserting these values into (6), and taking $V < 0$ corresponding to an infalling observer, we find that near the apparent horizon the energy density becomes

$$\rho_{\rm in} = \frac{1}{4\pi}\frac{M_{\rm pl}^8}{M^4}\frac{V^2}{\left(1 - \frac{2M(u)}{r}\right)^2}. \quad (7)$$

Using formula (3) this expression may be rewritten

$$\rho_{\rm in} = \frac{V^2}{4\pi}\Lambda_{\rm UV}^4. \quad (8)$$

This shows that the UV divergence persists when the gravitational backreaction due to the outflow of energy is taken into account. This provides strong evidence that even with backreaction included, any time dependence of the outgoing radiation will lead infalling observers to effectively see a firewall as they approach the stretched horizon.

E. Near-horizon observables

We saw in the previous subsections that, while we are free to modify the infalling modes at will, delicate cancellations are needed with the outgoing modes to yield a finite renormalized $\langle\phi^2\rangle$ at the horizon. Obtaining finite stress energy involves even further cancellations. As we saw in Sec. II B even a single outgoing mode generates singular contributions to the stress energy.

To formulate these issues more sharply, let us consider a freely falling infalling observer, who is capable of measuring with a UV cutoff $\Lambda_{\rm UV}$. We model the stretched horizon theory by a surface that emits quanta of energy $M_{\rm pl}^2/M$ every M units of time t. If the infaller falls in after a finite fraction of the black hole lifetime M^3, the infaller crosses a substantial fraction of the outgoing Hawking radiation, of order $(\frac{M}{M_{\rm pl}})^2$ particles. Since a freely falling observer will hit the singularity in proper time less than of order M, the infaller sees $\frac{M}{M_{\rm pl}}$ outgoing modes per unit Planck time. Therefore, a freely falling observer cannot resolve individual Hawking modes, and his local operators will involve linear combinations of at least $M/M_{\rm pl}$ outgoing modes. If the infaller makes one local measurement every time $1/\Lambda_{\rm UV}$ in her rest frame, then the subspace of the Hilbert space accessible in her lifetime M will have dimension $n_{\rm in} = e^{M/\Lambda_{\rm UV}} \ll e^{M/M_{\rm pl}}$.

Let us denote the subspace of the Hilbert space corresponding to this outgoing radiation as A, and the subspace corresponding to the stretched horizon degrees of freedom as B. The rest of the outgoing radiation we denote by the Hilbert subspace C. According to the postulates of black hole complementarity, the combined Hilbert space $A \times B \times C$ undergoes local unitary evolution, mapping a pure state to a pure state. We expect that the dimension of A will have an upper bound of order $n_{\rm in}$, while the dimension of B and C will be of order $e^{M^2/M_{\rm pl}^2}$, assuming we are not too close to the endpoint of evaporation [15]. As time passes the dimensions of these Hilbert subspaces will shift, but the combined dimension will remain constant.

We wish to compute the expectation value of the stress energy tensor seen by an infalling observer in a generic pure state emitted after scrambling on the stretched horizon. As we have seen above, the infalling modes may be placed in an arbitrary pure state, leading to finite corrections to the expectation value. We therefore focus our attention on the contribution due to the outgoing modes.

Now fluctuations in the stress energy tensor can only become large in the limit that $r \to 2M(u)$, which follows from the $r \to 2M$ divergent terms in (5) as shown in [12]. The modes relevant for determining whether the infaller sees a large effect are those emitted within $\delta u \sim M(u)$, so even though these modes free stream from the stretched horizon, they were in relatively recent causal contact with the stretched horizon degrees of freedom. This implies that in this period of time the $A \times B$ subsystem evolves unitarily on its own, so that $\delta S_A = -\delta S_B$ and $\delta E_A = -\delta E_B$. Thus, the effective temperatures of these systems are the same,

$$\frac{1}{T} = \frac{\partial S_A}{\partial E_A} = \frac{\partial S_B}{\partial E_B}.$$

However, to within small corrections in the temperature, one can consider a time period just before the emission of A from the stretched horizon, and likewise argue that

$$\frac{1}{T} = \frac{\partial S_C}{\partial E_C} = \frac{\partial S_B}{\partial E_B}.$$

Thus, all subsystems are at the same effective temperature $T = M_{\rm pl}^2/M(u)$ to within negligible corrections, so the evolution on $A \times B \times C$ may be treated in an adiabatic approximation.

In quantum field theory, local operators are constructed to model the action of real detectors, and likewise local operators may be used to prepare initial states of interest. The resulting correlators of the local fields may be interpreted as probability amplitudes and used to predict the outcomes of experiments. As is typical in quantum mechanics, the outcome of a particular measurement is determined probabilistically, which effectively leads to a version of averaging that mimics the averaging in statistical mechanics [6]. One of the key points of that work is that expectation values in a random pure state converge much more rapidly than the ensemble averages used in ordinary statistical mechanics. It was found that fluctuations in an expectation value are typically suppressed by a factor $1/\sqrt{n}$ where n is the dimension of the Hilbert subspace that is averaged over in selecting a random pure state. This comes from integrating over a shell in the space $\mathbb{C}^n$. This is to be contrasted with the usual suppression of fluctuation from ensemble averages which are of the order $1/\sqrt{N}$ where N is the number of degrees of freedom in the system averaged over (typically $n \sim e^N$).

Unfortunately, it is difficult to make these ideas precise in a completely general context. For example, a pure state which is an eigenstate of some particular operator that commutes with the Hamiltonian will remain in that eigenstate for all time, and any effective measurements that commute with this operator will only produce that eigenvalue. This makes the definition of a complex pure state a rather basis-dependent question.

We can make these statements rather more precise in the context of measurements of the evaporation of a black hole. The natural basis for an observer far from the black hole is indeed the outgoing modes discussed above. However, such modes are highly unnatural from the viewpoint of a freely falling observer near the horizon.

Applying this to the case at hand, any operator corresponding to the detector of a freely infalling observer will average over the subspace $B \times C$. Since the operator is local, it will not probe the subspaces B and C. One may therefore compute the expectation value by tracing over the Hilbert subspaces B and C to produce the reduced density matrix ρ_A. At late times, the modes A will be maximally entangled with the earlier radiation C [16]. By the arguments of [6] this density matrix will agree with the canonical ensemble at temperature T up to corrections of order $e^{-S_C/2}$.

Let us briefly review this computation in more detail [6]. Let us assume we have a pure state on a product Hilbert space $A \times C$ described by the density matrix ρ_{AC} with total energy E. We define

$$\rho_A = \mathrm{Tr}_C \rho_{AC} = \sum_i p(E_i)\rho_{E_i}.$$

Here E_i are the energy eigenvalues of the subspace A, i labels the energy eigenstates, $p(E_i)$ is the probability of occupation of energy eigenstate E_i, and ρ_{E_i} is a density matrix on the subspace of A corresponding to eigenstates with energy E_i. Computing this for a typical pure state, one finds

$$\rho_A = \frac{1}{n}\sum_i e^{S_A(E_i)+S_C(E-E_i)}\rho_{E_i}\left(1 \pm \mathcal{O}\left(\frac{1}{e^{S_C(E-E_i)/2}}\right)\right),$$

where we have assumed $S_C \gg S_A$, and defined n as the total dimension of the Hilbert space $A \times C$. The exponential may be approximated using $S_C(E-E_i) \approx S_C(E) - E_i/T$, with $1/T = \partial S_C/\partial E$, leading to the canonical ensemble expression, up to small corrections,

$$\rho_A = \frac{1}{N}\sum_i e^{-E_i/T+S_A(E_i)}\rho_{E_i}\left(1 \pm \mathcal{O}\left(\frac{1}{e^{S_C(E)/2}}\right)\right),$$

with $1/N = e^{S_C(E)}/n$.

This density matrix may then be used to estimate

$$\langle T_{\mu\nu}\rangle U^\mu U^\nu \sim \frac{e^{-S_C/2}}{(1-\frac{2M(u)}{r})^2}, \tag{9}$$

as $r \to 2M(u)$ by viewing the correction as a classical contribution to the emitted energy in the outgoing Vaidya solution (7). While this still becomes singular very close to the global horizon, this is safely behind the stretched horizon, and in that region we do not trust conventional effective field theory. We conclude that infalling observers see no drama in their approach to the stretched horizon for a generic pure state.

F. EPR paradox in the black hole setting

The above argument suggests that the infalling observer sees smooth stress energy all the way up to the stretched horizon, beyond which it is difficult to make model-independent statements. However, we run into an apparent paradox if we suppose that an external observer far from the black hole projects it onto an eigenstate of the outgoing modes. In this case, the model of Sec. II D should provide an accurate estimate of the stress energy, and we expect the infalling observer to see a firewall.

The resolution is very similar to that of the original EPR paradox. Suppose the infalling observer is initially spacelike separated from the outside observer. His measurements are unaffected by the outside observer's measurements. But, nevertheless, the measurements can be correlated via the nonlocality of ordinary quantum mechanics. Effectively, the density matrix ρ_A corresponds to a trace over macroscopic superpositions of the states of the outside observer. Only in a generic superposition is the correlator $\langle T_{\mu\nu}\rangle U^\mu U^\nu$ finite.

A measurement in the state projected by the outside observer $\langle T_{\mu\nu}\mathcal{O}_{M^2}\rangle U^\mu U^\nu$ is expected to be large, where the operator $\mathcal{O}_{M^2}$ represents the measurement of the outside observer on $\frac{M^2}{M_{\mathrm{pl}}^2}$ Hawking particles. However, the unnaturally large value for this correlator only appears as a puzzle to the outside observer if he is able to accelerate away from the black hole and compare notes with the distant observer. It has no local significance to the infaller, except in the atypical situation when the black hole is prepared in such an eigenstate from the beginning. However, here we may rely on the fact that the likelihood of such an eigenstate is of order $e^{-M^2/M_{\mathrm{pl}}^2}$.

Another variant on this process involves an observer who stays outside the black hole for a long time to precisely measure its state, and then falls in. Perhaps not surprisingly, such an observer can predict the emission of nonthermal Hawking particles and choose to fall into the horizon to measure them. Such an observer will similarly see a large effect of order (8) near the stretched horizon. However, the practicality of these measurements seems unlikely. Such an observer would need energy and entropy with which to store all these data, comparable to those of the black hole he is reconstructing. This process would be well approximated by the collision of two black holes of similar mass. In such a collision, Planck-scale curvatures are not produced in the vicinity of the apparent horizon(s), but there is, nevertheless, a substantial fraction of the initial Bondi energy radiated in terms of gravitational radiation. It is interesting to note that gravitational effects show a tendency to smooth out would-be curvature or stress energy singularities. We conclude that, just as atypical pure states can give surprising answers, we may also have atypical observers who are surprised by their measurements.

Finally, one can try to imagine that a single Hawking particle plays the role of the observer, to parallel the arguments of [17], who instead conclude a firewall exists at the horizon. Related arguments have been made in [18,19] in the context of the fuzzball scenario. The arguments made in these works have already been rebutted in [20], and in the present work we extend and strengthen this approach. In the case of a single Hawking particle the "observer" only has access to a 1-dimensional subspace of the Hilbert space, so once again it is appropriate to trace over the other subspaces. For an infalling Hawking particle, we reproduce (9) with the velocity U^{μ} replaced by a normalized null vector. For an outgoing Hawking particle we get a negligible result. We conclude, therefore, that neither Hawking particles nor infalling observers see drama near the stretched horizon of a black hole in a generic pure state. However, it is possible to choose a special pure state, or even a special observer, where this conclusion does not hold.

III. DISCUSSION

To extend these considerations to an observer falling across the horizon, one would need to account for the fact that the mapping from the fundamental unitary description to the effective description is no longer local. The rules of unitary and locality in the bulk must then be given up. Some early work which found that local effective field theory does not predict its own demise when horizons are present appeared in [21,22]. Rather, we expect that local unitary effective field theories [23] are capable of approximately describing the measurements that may be carried out by an infalling observer. However, these will disagree with the exact answers of a unitary nonlocal holographic description of the same measurements [8]. Related ideas have been considered more recently in the context of the firewall scenario using a quantum computational model in [24]. Evidence for such a scenario has been provided using the AdS/CFT framework in [25]. This scenario has a chance of working because the finite lifetime of an infalling observer limits the measurement operations that may be carried out; thus, the effective field theory in a region inside the horizon need not give exact answers.

From the viewpoint of evolution of the stretched horizon theory, an infalling observer's degrees of freedom evolve for a time of order the scrambling time, before being reemitted in the Hawking radiation. The scrambling time, measured at infinity, thus provides a time scale at which the evolution of these degrees of freedom qualitatively changes. It is tempting to match this delay time with the proper time that the infaller takes to hit the singularity. We hope to return to this question using more specific models of the stretched horizon theory in future work, though progress has already been made [8,25].

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[1] J. Hartle and S. Hawking, Phys. Rev. D **13**, 2188 (1976).
[2] W. Unruh, Phys. Rev. D **14**, 870 (1976).
[3] D. G. Boulware, Phys. Rev. D **11**, 1404 (1975).
[4] L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **48**, 3743 (1993).
[5] P. Vaidya, Proc. Indian Acad. Sci. A **33**, 264 (1951).
[6] S. Lloyd, Ph.D. thesis, The Rockefeller University, 1988.
[7] D. N. Page, Phys. Rev. Lett. **71**, 1291 (1993).
[8] D. A. Lowe and L. Thorlacius, Phys. Rev. D **73**, 104027 (2006).
[9] B. S. DeWitt, Phys. Rep. **19**, 295 (1975).
[10] Following [11], we correct a typo that appears in [9].
[11] S. Christensen and S. Fulling, Phys. Rev. D **15**, 2088 (1977).
[12] P. Candelas, Phys. Rev. D **21**, 2185 (1980).
[13] C. W. Misner, Phys. Rev. **137**, B1360 (1965).
[14] R. Lindquist, R. Schwartz, and C. Misner, Phys. Rev. **137**, B1364 (1965).
[15] In these formulas for large numbers, such as $e^{M^2/M_{\rm pl}^2}$, of order this number should be taken to mean of order $e^{cM^2/M_{\rm pl}^2}$ where c is a finite constant.
[16] P. Hayden and J. Preskill, J. High Energy Phys. 09 (2007) 120.
[17] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, J. High Energy Phys. 02 (2013) 062; S. L. Braunstein, S. Pirandola, and K. Życzkowski, Phys. Rev. Lett. **110**, 101301 (2013).
[18] S. D. Mathur, arXiv:1108.0302.
[19] S. D. Mathur and D. Turton, arXiv:1208.2005.
[20] K. Larjo, D. A. Lowe, and L. Thorlacius, Phys. Rev. D **87**, 104018 (2013).
[21] D. A. Lowe, J. Polchinski, L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **52**, 6997 (1995).
[22] D. A. Lowe, Nucl. Phys. **B456**, 257 (1995).
[23] Note that nonlocal effective theories cannot be quantized in general.
[24] D. Harlow and P. Hayden, J. High Energy Phys. 06 (2013) 085.
[25] D. A. Lowe, Phys. Rev. D **79**, 106008 (2009).