Extra relativistic degrees of freedom without extra particles using Planck data

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A recent number of analyses of cosmological data have shown indications for the presence of extra radiation beyond the standard model at the equality and nucleosynthesis epochs, which has been usually interpreted as an effective number of neutrinos, $N_{\rm eff} > 3.046$. In this work we establish the theoretical basis for a particle physics-motivated model (bound dark matter, BDM) which explains the need for extra radiation. The BDM model describes dark matter particles which are relativistic at a scale below $a < a_c$; these particles acquire mass with an initial velocity, v_c, at scales $a > a_c$ due to nonperturbative methods (as protons and neutrons do) and this process is described by a time-dependent equation of state, $\omega_{\rm BDM}(a)$. Owing to this behavior the amount of extra radiation changes as a function of the scale factor, and this implies that the extra relativistic degrees of freedom $N_{\rm ex}$ may also vary as a function of the scale factor. This is favored by data on the cosmic microwave background and big bang nucleosynthesis (BBN) epochs. We compute the range of values of the BDM model parameters, $x_c = a_c v_c$, that explain the values obtained for the ^{4}He at BBN and $N_{\rm eff}$ at equality. Combining different analyses, we compute the values $x_c = 4.13(^{+3.65}_{-4.13}) \times 10^{-5}$ and $v_c = 0.37^{+0.18}_{-0.17}$. We conclude that we can account for the apparent extra neutrino degrees of freedom $N_{\rm ex}$ using a phase transition in the dark matter with a time-dependent equation of state without introducing extra relativistic particles.

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I. INTRODUCTION

In recent years, precision measurements have revealed an incredible amount of information used to describe the Universe, among other things, the fluctuations in the cosmic microwave background (CMB) radiation [1,2], type Ia supernova [3], large-scale structure, and the baryon acoustic oscillations (BAO) [4].

Cosmological observations are systematically consistent with the standard model of cold dark matter (CDM) with a cosmological constant driving an accelerated expansion of the universe. Nevertheless, there are few unresolved issues, aside from the nature of dark matter (DM) and energy, that could be evidence of physics beyond the paradigm of CDM. These include the large amount of substructure and the incompatible cuspy energy density profile for the galaxy DM halo predicted by the CDM framework. Another problem is the amount of radiation in the Universe and, therefore, the expansion rate of the early Universe.

One of the fundamental observables that describe the Universe is the redshift of equivalence, $z_{\rm eq}$, the moment when the energy densities of matter and radiation were equal. This equality epoch is relevant in structure formation and it also affects the early integrated Sachs-Wolfe (ISW) effect, which constrains the value of the matter-radiation equality. The ISW effect receive more CMB photons the later the equality epoch is. The early ISW effect is actually a direct constraint via the ratio from the first and third peak of the CMB spectrum [5], and therefore we can compute the amount of radiation at equality if we know the amount of matter in the Universe and vice versa.

The radiation energy density at the equality epoch is given by photons and neutrinos. The standard model neutrino species contributes $N_\nu = 3.046$ degrees of freedom [6]. However, recent analyses of the Planck [1] data, WMAP [2], Atacama Cosmology Telescope [7], the South Pole Telescope [8], the Sloan Digital Sky Survey (SDSS) data release 7 (DR7) [9,10], and several other analyses [11,12] have reported indications that the effective degrees of freedom, $N_{\rm eff} \equiv N_\nu + N_{\rm ex}$, could be greater than the expected $N_{\rm eff} > N_\nu$. The value of $N_{\rm eff}$ obtained before the Planck data was reported had an $N_{\rm ex} > 0$ at 1σ and was consistent with $N_{\rm ex} = 0$ at $2\sigma = 0$. However, the new Planck data has an $N_{\rm ex} = 0$ at 1σ but the central value hints at a small amount of extra relativistic degrees of freedom. It is worth noticing that there is a tension between the values of the Hubble constant H_o (in units of $\rm km\,s^{-1}\,Mpc^{-1}$)—with $H_o = 67.3 \pm 1.2$ for Planck [1], $H_o = 70.0 \pm 2.2$ for WMAP9 [2] and $H_o = 74.8 \pm 3.1$ for Cepheids + SNeIa [3]—and the value of $N_{\rm eff}$ depends strongly on this value. The larger the value of H_o the greater the amount of relativistic degrees of freedom that are required. In order to have a wider scope we present our results using different sets of data. This means that the amount of radiation prior to the epoch of decoupling seems to be more than the expected ($\hat{\rho}_r > \rho_r$), where $\hat{\rho}_r = \rho_r + \rho_{\rm ex}$ and $\rho_{\rm ex}$ identifies an extra relativistic component.

Additionally, recent studies find a somewhat higher ^{4}He abundance of $Y_p > 0.25$ [2,13,14], also suggesting a novel radiation during the big bang nucleosynthesis (BBN) epoch. In both cases the extra relativistic component is parametrized as extra neutrino degrees of freedom as a function of the neutrino temperature. The value of $N_{\rm eff}$

obtained from data analysis gives a larger value at BBN than that at equality. In a CDM scenario, extra relativistic particles give a constant contribution to N_{eff}.

Constraints on $N_{\mathrm{eff}} > N_{\nu}$ can be interpreted as the existence of radiation energy beyond the standard model. In the literature there are different proposals that attempt to explain the extra radiation, some of which propose the existence of new particles—for instance sterile neutrinos [15,16]—or that the radiation is a relativistic product of a massive relic particle [17–19]. We also know that N_{eff} cannot be accounted for by statistical effects alone [11].

Here we are going to propose a different interpretation of the extra radiation: we shall see that a model named bound dark matter (BDM) [20], where the DM particles go through a phase transition, can well explain the need for an extra component without introducing new particles (besides the DM particle).

The theoretical motivation behind BDM is the existence of a new asymptotic free dark gauge group [20], e.g., an $SU(Nc)$ [similar to the QCD SU(3) gauge group], whose particles interact with the standard model particles only through gravity. The fundamental particles in this dark gauge group are massless at high energies but at low energies they form neutral massive particles—similar to neutrons and protons in QCD—due to a phase transition scale at E_c. Our BDM particle is the lightest fermionic bound particle. The mass of the neutral bound states is due to the binding energy of the gauge interaction and is related to the phase transition scale E_c with $m = \mathcal{O}(E_c)$; in QCD the mass of neutrons and protons is much larger than that of the sum of the constituent quark masses, with $m \simeq 5$ and the QCD condensation scale $E_{\mathrm{QCD}} \simeq 200$ MeV, cf., Ref. [21]. The condensation or phase-transition scale is defined as the energy when the coupling constant becomes strong, $g(E_c) \gg 1$, with $E_c = E_i \exp[-8\pi^2/(\beta_0 g_i^2)]$ (see Refs. [20,22–24]), where β_0 is the one-loop beta function that counts the number of elementary particles under the group and g_i is the coupling constant at the energy E_i. The E_c parameter is closely related to the mass of our DM particle, and its value is not yet known and we require observational evidence to determine it. A similar situation takes place with the masses for all the standard model particles, which depend on the value of the Yukawa constant and are determined by experiment. We expect this transition to be between the MeV and eV scales, i.e., between the BBN and matter-radiation equality epochs. If $E_c \gg$ MeV, then BDM would reduce to CDM, and for $E_c \ll E_{\mathrm{eq}} = \mathcal{O}(\mathrm{eV})$ there would be no DM to account for structure formation.

We expect that the BDM model describes relativistic particles at the BBN epoch, and below E_c these particles go through a phase transition between radiation and matter described by a time-dependent equation of state (EoS). This particular behavior allows us to understand the inconsistency in the number of degrees of freedom of neutrinos at the equality and BBN epochs by predicting that the amount of extra radiation changes as a function of the scale factor. If this extra radiation is written in terms of the neutrino radiation this means that the neutrino relativistic degrees of freedom will also be a function of the scale factor. However, since we expect that the BDM phase transition takes place at energy scales $E_c \gg E_{\mathrm{eq}}$, the amount of extra degrees of freedom for BDM is small in accordance with recent Planck data (see Fig. 1). We also expect that the transition of the BDM particles influences the CMB power spectrum by changing the time when matter-radiation equality holds. Furthermore, BDM can reduce the amount of substructure predicted by CDM [20] and have a DM with a core galaxy profile [20,25,26], thus avoiding the CDM problems.

The paper is organized as follows. In Sec. II we present the theoretical framework to compute the matter-radiation equality for the standard CDM and BDM scenario. Current observational evidence also suggests extra radiation at the BBN epoch, and we explore the possibility that the BDM particles also account for such an excess in Sec. III. We compute the range of values of the free parameters of the BDM model using published values of N_{eff} in Sec. IV. Finally, our conclusions are discussed in Sec. V.

II. FRAMEWORK

The growth of structure formation and cosmological observational data show that the scale factor at matter-radiation equality may be different (larger) than that of the standard CDM scenario with no extra particles. This can be achieved by having extra relativistic particles in a CDM model or by having a time-dependent EoS for DM, as in our BDM model. We will present here a novel and

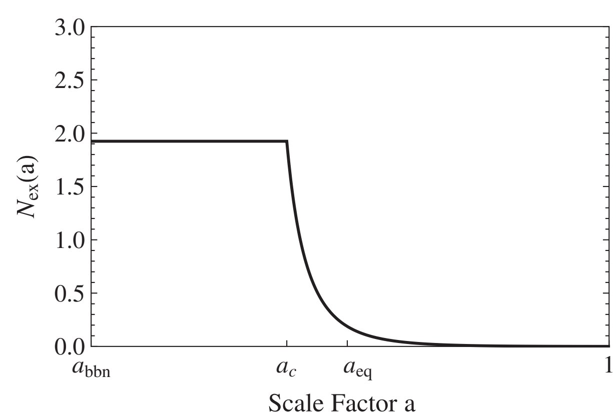

FIG. 1. Plot of the extra relativistic degrees of freedom, $N_{\mathrm{ex}}(a)$, as a function of the scale factor using data from Planck. The BDM particles goes through a transition at $a = a_c$, which is expected to be smaller than the equality $a_c < a_{\mathrm{eq}}$. Before this moment the BDM particles behaves as pure radiation. After the transition the amount of radiation given by the BDM particles is given by Eq. (12). This means that the extra degrees of freedom remain constant before the transition, and afterwards decrease as a function of the scale factor, cf., Eq. (11).

simple way to compute the epoch of matter-radiation equality, which is also valid when the EoS of DM is time dependent, as is the case for the BDM model.

A. CDM scenario

First let us present the standard CDM scenario, where DM is given by a massive particle with vanishing dispersion velocity at scales relevant for structure formation. In this scenario we have a nonrelativistic energy density given by the baryons (b) and CDM, $\rho_m = \rho_b + \rho_{\rm CDM}$, and the relativistic particles at energies below the neutrino decoupling are the photons (γ) and neutrinos (ν) with an energy density given by

$$\rho_r = (1 + \alpha N_{\rm eff})\rho_\gamma, \tag{1}$$

where we use the relation $T_\nu = (4/11)^{1/3} T_\gamma$ derived from the entropy conservation across the electron-positron annihilation and $\alpha \equiv (7/8)(4/11)^{4/3} \simeq 0.227$. If we have extra degrees of freedom it is common to parametrize them using $N_{\rm eff} \equiv N_\nu + N_{\rm ex}$, where $N_\nu = 3.046$ is the neutrino degrees of freedom and $N_{\rm ex}$ accounts for the amount of extra radiation. In the CDM framework considering no extra radiation the equality epoch is ($a_{\rm eq}/a_o = \rho_{\rm ro}/\rho_{\rm mo}$). It is clear that if we have more relativistic degrees of freedom the equality epoch $\hat{a}_{\rm eq}$ (with $\hat{a}_{\rm eq}/a_o = \hat{\rho}_{\rm ro}/\rho_{\rm mo}$) will change to

$$\frac{\hat{a}_{\rm eq}}{a_{\rm eq}} = \frac{\hat{\rho}_{\rm ro}}{\rho_{\rm ro}} = \frac{1 + \alpha N_{\rm eff}}{1 + \alpha N_\nu} = 1 + \frac{\alpha N_{\rm ex}}{1 + \alpha N_\nu}, \tag{2}$$

where $\hat{\rho}_r = \rho_r + \alpha N_{\rm ex} \rho_\gamma$ is the relativistic energy density including the extra radiation. Clearly, if the measurements give a larger scale factor at equality, $\hat{a}_{\rm eq} \geq a_{\rm eq}$, we would then have extra degrees of freedom, $N_{\rm ex} \geq 0$ ($N_{\rm eff} \geq N_\nu$), and the equality holds for $N_{\rm ex} = 0$.

Before presenting the second scenario where DM is given by BDM instead of CDM, let us determine the value of the full EoS given by the total energy density ($\rho_{\rm tot}$) and pressure ($P_{\rm tot}$), $\omega_{\rm tot} \equiv P_{\rm tot}/\rho_{\rm tot}$. For simplicity we will assume that the contribution of dark energy (DE) at matter-radiation equality is negligible and we do not expect it to play a significant role in our analysis. Of course different DE models, such as early DE, could also be studied. The equation of state, $\omega_{\rm tot}$, of a fluid consisting of relativistic and matter (cold) particles as function of the scale factor is

$$\omega_{\rm tot} = \frac{\rho_r/3}{\rho_r + \rho_m} = \frac{1}{3}\frac{a_{\rm eq}}{a_{\rm eq} + a}, \tag{3}$$

$$\hat{\omega}_{\rm tot} = \frac{\hat{\rho}_r/3}{\hat{\rho}_r + \rho_m} = \frac{1}{3}\frac{\hat{a}_{\rm eq}}{\hat{a}_{\rm eq} + a}. \tag{4}$$

Notice that $\omega_{\rm tot} = 1/6$ at equality ($a = a_{\rm eq}$) and $\hat{\omega}_{\rm tot} = 1/6$ at $a = \hat{a}_{\rm eq}$. We propose to use this alternative criterion to define the equality between the matter and radiation epoch, which is very useful as the particles do not have a constant EoS at this time. This is the case for our BDM model where the particles are in transition between $\omega_{\rm bdm} = 1/3$ and $\omega_{\rm bdm} = 0$.

B. BDM scenario

Let us now present the second scenario, where we use our BDM model as DM. The particles of the BDM model go through a nonperturbative phase transition at $a = a_c$ when $\rho_{\rm BDM}(a_c) = \rho_{\rm c} \equiv E_c^4$. At this time the particles acquire mass through nonperturbative phenomena, similar to protons and neutrons in QCD. Below the scale $a < a_c$ (or $\rho_{\rm BDM} > \rho_{\rm c}$) they are relativistic ($\omega_{\rm BDM} = 1/3$) massless particles. Above the scale $a > a_c$ the EoS of BDM is time dependent and goes from the values $\omega_{\rm BDM}(a_c) \leq 1/3$ to $\omega_{\rm BDM} \simeq 0$ for $a \gg a_c$.

For simplicity we will take these particles to have an average momentum $\langle |\bar{p}|^2 \rangle$ and average energy $\langle E \rangle$ so that the pressure becomes $P = n\langle |\bar{p}|^2 \rangle / 3\langle E \rangle$ and the energy density $\rho = \langle E \rangle n$, with n the particle number density. The EoS for BDM then becomes

$$\omega_{\rm BDM} = \frac{\langle |\bar{p}|^2 \rangle}{3\langle E \rangle^2} = \frac{v_{\rm BDM}^2}{3} = \frac{1}{3}\left(\frac{v_c a_c}{a}\right)^2, \tag{5}$$

where $v_{\rm BDM}$ is the average velocity of the BDM particles, and we have taken into account that in a Friedmann-Robertson-Walker background the velocity redshifts with the scale factor as

$$v_{\rm BDM}(a) = v_c\left(\frac{a_c}{a}\right). \tag{6}$$

The last equation contains two free parameters: the scale factor at the transition a_c and the velocity of the dark particle at that moment, v_c. The quantity v_c, with $0 \leq v_c \leq 1$, gives the initial speed of the particles after the BDM phase transition, reflecting the fact that the BDM particle mass has a nonperturbative origin and the resulting velocity may be suppressed in comparison with the speed of light, $v_{\rm BDM}(a_c) = v_c < 1$. This is one of the main differences between BDM particles and a standard relativistic particle with a (perturbative) mass m which becomes nonrelativistic at $\omega = T/m$ (e.g., at $a = a_c$) with $v = 1$. Clearly our BDM reduces to standard CDM when $a_c \to 0$ (with $v \to 0$); therefore, $\omega_{\rm BDM} \to 0$ and BDM particles become cold for $a \gg a_c$, and again we have $v \to 0$ and $\omega_{\rm BDM} \to 0$. On the other hand, BDM reduces to a standard particle becoming nonrelativistic at a_c if $v_c = 1$.

Using Eq. (5), we can integrate $\dot{\rho} = -3H(\rho + P)$ to obtain an analytic form for $\rho_{\rm BDM}(a)$ which describes the transition of a radiative fluid to matter-like particles. Thus, it allows one to easily compute the evolution of the expansion rate and cosmological distances,

$$\rho_{\rm BDM} = \rho_c\left(\frac{a}{a_c}\right)^{-4}, \qquad w = 1/3, \quad \text{for } a < a_c,$$
$$\rho_{\rm BDM} = \rho_{\rm CDM} f(a), \quad \text{for } a \geq a_c, \tag{7}$$
$$f(a) \equiv \exp\left[\frac{3}{2}\omega_{\rm BDM}(a)\left(1 - \frac{a^2}{a_o^2}\right)\right],$$

with $\rho_{\rm CDM} \equiv \rho_{\rm CDMo}(a/a_o)^{-3}$, where $\rho_{\rm BDMo} = \rho_{\rm CDMo} = \rho_{\rm DMo}$ is the DM density today, and

$$\rho_c \equiv E_c^4 \simeq \rho_{\rm BDMo}\left(\frac{a_c}{a_o}\right)^{-3} e^{v_c^2/2}, \tag{8}$$

where we have used the fact that $f(a_c) \equiv f|_{a=a_c} \simeq \exp[v_c^2/2]$ since $\omega_{\rm BDM}(a_c) = v_c^2/3$, and we have taken $a_c \ll a_o$; $f(a_c)$ is now only a function of v_c.

It is naive to think that the matter-radiation ratio at early ages can be computed by a simple extrapolation of today's values because we are proposing a DM phase transition with a time-dependent $\omega_{\rm BDM}$. Hence, we cannot say that matter-radiation equality is when $\rho_r = \rho_m$, but instead we define equality when the total EoS is $\omega_{\rm tot} = 1/6$, as discussed in Eq. (3), which overcomes the fact that $\omega_{\rm BDM}$ is a function of a. Therefore, in the case of the BDM we have

$$\omega_{\rm tot}(a) = \frac{\rho_r/3 + \omega_{\rm BDM}\rho_{\rm BDM}}{\rho_r + \rho_b + \rho_{\rm BDM}} = \frac{1/3 + \frac{a}{a_{\rm eq}}\frac{\Omega_{\rm BDMo}}{\Omega_{\rm mo}}\omega_{\rm BDM}(a) f(a)}{1 + \frac{a}{a_{\rm eq}}(1 + \frac{\Omega_{\rm BDMo}}{\Omega_{\rm mo}}(f(a) - 1))}, \tag{9}$$

where we have used $\rho_{\rm mo} = \rho_{\rm bo} + \rho_{\rm BDMo}\mu$, have again neglected DE, and $\Omega_{\rm xo}$ is today's density parameter of the x fluid. We see that $\omega_{\rm tot}$ is a function of a and x_c through $f(a, x_c)$ and $\omega_{\rm BDM}$, and this equation has to be solved numerically. We can rewrite Eq. (9) as

$$\frac{a}{a_{\rm eq}} = \frac{(1 - 3\omega_{\rm tot})/3\omega_{\rm tot}}{1 + \frac{\Omega_{\rm BDMo}}{\Omega_{\rm mo}}[f(1 - \frac{\omega_{\rm BDM}}{\omega_{\rm tot}}) - 1]}. \tag{10}$$

By defining matter-radiation equality for BDM at $\tilde{a}_{\rm eq}$ when $\omega_{\rm tot} = 1/6$, which is also valid for the limiting case of the standard model, we notice that the quantity $(1 - 3\omega_{\rm tot}(a_{\rm eq}))/3\omega_{\rm tot}(a_{\rm eq}) = 1$. Also, it is interesting to note that $\tilde{a}_{\rm eq} > a_{\rm eq}$ can be obtained without introducing extra relativistic particles, due to the time-dependent EoS of BDM particles. Notice that in Eq. (10) $\omega_{\rm BDM}$ and $f(a)$ depend on a_c and v_c only through the combination $x_c = v_c a_c$. In the limit $x_c \to 0$ we have $\omega_{\rm BDM} = 0$, $f = 1$, and $\tilde{a}_{\rm eq} = a_{\rm eq}$, as in the standard CDM scenario with no extra degrees of freedom.

The connection between BDM and extra relativistic degrees of freedom at an arbitrary scale factor a is easily achieved by using Eqs. (2) and (10), which yields

$$\tilde{N}_{\rm ex}(a) = \frac{1 + \alpha N_\nu}{\alpha}\left(\frac{a}{a_{\rm eq}}\frac{3\omega_{\rm tot}(a)}{1 - 3\omega_{\rm tot}(a)} - 1\right), \tag{11}$$

and at equality we have $\omega_{\rm tot} = 1/6$ and $\tilde{N}_{\rm ex}(\tilde{a}_{\rm eq}) = (\tilde{a}_{\rm eq}/a_{\rm eq} - 1)(1 + \alpha N_\nu)/\alpha$. Equation (11) should be interpreted as giving an apparent number of extra relativistic particles $\tilde{N}_{\rm ex}$ at equality, even though we have not introduced extra particles; this is due to the effect of a time-dependent EoS for DM, i.e., for BDM. The $N_{\rm ex}$ without the tilde is for the CDM scenario and is constant by assumption. Because the BDM particles behave as radiation before the epoch of the transition, $a < a_c$, the apparent number of extra relativistic neutrinos must remain constant. After the transition [Eq. (11)] it is a function of the scale factor. This behavior is show in Fig. 1.

We can rearrange elements of the last equation and extract the contribution of the BDM particles to the cosmic radiation, $\rho_{\rm ex}(a) = \alpha N_{\rm ex}(a)\rho_\gamma(a)$, as a function of the scale factor,

$$\rho_{\rm ex}(a) = \frac{3\omega_{\rm tot}}{1 - 3\omega_{\rm tot}}\left(\rho_{\rm CDM} - \rho_{\rm BDM}\left[1 - \frac{\omega_{\rm BDM}}{\omega_{\rm tot}}\right]\right). \tag{12}$$

We expect the phase transition to be $a_c < a_{\rm eq}$, and therefore our BDM would only be able to account for a small amount of $N_{\rm ex}$. We plot in Fig. 2 the value of $\tilde{a}_{\rm eq}/a_{\rm eq}$ and $N_{\rm ex}$ as a function of x_c. The largest amount of $N_{\rm ex}$ as a function of v_c is given at $v_c = 1$, and since $x_c = v_c a_c$ if we set $v_c = 1$ in Fig. 2 we have the upper level $x_a/a_{\rm eq}|_{v_c=1} = a_c/a_{\rm eq}$. For example, if $a_c/a_{\rm eq} = 1$ we have a maximum amount of extra relativistic degrees of freedom $N_{\rm ex} = 6.15$, for $a_c/a_{\rm eq} = 0.2$ it reduces to $N_{\rm ex} = 0.37$, while for $a_c/a_{\rm eq} = 0.1$ we find $N_{\rm ex} = 0.09$. The moment of the transition x_c can be determined by CMB observations by the amount of matter, Ω_m, and the equality epoch, $z_{\rm eq}$, and in Fig. 3 we show the 68% and 95% C.L.'s of x_c using Planck data.

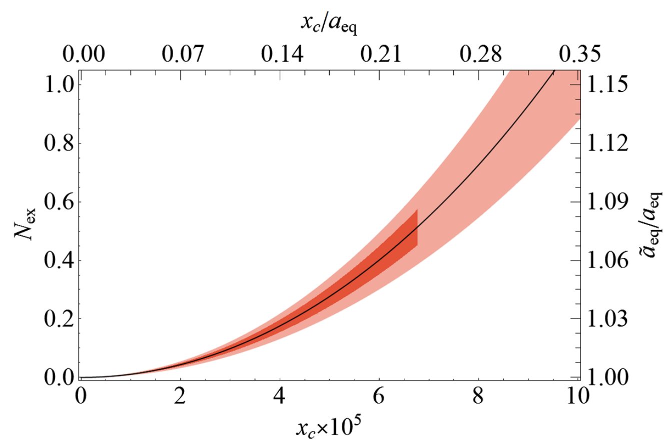

FIG. 2 (color online). Plot of $N_{\rm ex}$ and $\tilde{a}_{\rm eq}/a_{\rm eq}$ as a function of x_c [see Eqs. (10) and (11)] using Planck data, cf., Table II. The colored region represent two-dimensional (68%, 95%) contours marginalized over Ω_{mo}. The thick line represents the central value obtained with the data. We expect $a_c \ll a_{\rm eq}$ and therefore our BDM would only be able to account for a small amount of $N_{\rm ex}$.

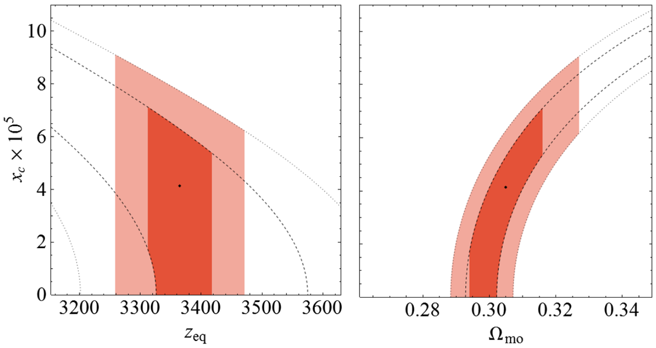

FIG. 3 (color online). In the left panel we show the marginalized two-dimensional (68%, 95%) contour in the x_c-z_{eq} plane using Planck results, cf., Table II. The C.L. are trimmed with, respectively, the 1σ and 2σ errors of z_{eq}. The dependance on x_c is due to the 1σ (dashed) and 2σ (dotted) limits on Ω_m. The point is the central value. In the right panel a similar two-dimensional contour is seen for the x_c-Ω_{mo} plane where the C.L.'s are trimmed with, respectively, the 1σ and 2σ errors of Ω_{mo}, and the dependance on x_c is due to the 1σ (dashed) and 2σ (dotted) limits on z_{eq}.

III. BBN

In the previous section we considered the observational result that extra radiation beyond the standard model is imprinted in the CMB [2]. Now, we explore the possibility that the BDM particles account for such an excess during BBN. The BDM particles can change the prediction of BBN for the abundance of the light elements, such as helium and deuterium, by changing the radiation density at that epoch, thereby increasing the expansion rate during this stage of the Universe.

The ^{4}He is very sensitive to the competition between the weak interaction rates and the expansion rate which, during the radiation-dominated evolution, is fixed by the energy density in relativistic particles. As a result, ^{4}He abundance tests the standard model and provides one of the strongest constraints on $x_c = a_c v_c$.

At the BBN epoch, before $e^{\pm}$ decoupling, the standard model of particle physics establishes that the energy density consists of an equilibrium mixture of photons, relativistic $e^{\pm}$ pairs, neutrinos, and antineutrinos. With all chemical potentials set to zero the energy densities are related by thermal equilibrium so that the total radiation density may be written in terms of the photon density as $\rho_r^{BBN} = \rho_\gamma + \rho_{e^\pm} + \rho_\nu = 43\rho_\gamma/8$, $\rho_{ex}^{BBN} = \frac{7}{8} N_{ex}^{BBN} \rho_\gamma$, and one has $T_\gamma = T_\nu$ at BBN.

It is convenient to define the nonstandard expansion rate S to account for the extra contribution to the standard model energy density for the standard CDM considering extra radiation,

$$S^2 = \left(\frac{\hat{H}}{H}\right)^2 = \frac{\hat{\rho}_r}{\rho_r}\bigg|_{BBN} = 1 + \frac{7}{43} N_{ex}^{BBN}. \tag{13}$$

This extra component is modeled just like an additional neutrino, though we emphasize that the extra need may not be for additional flavors of active or sterile neutrinos, it is just additional relativistic degrees of freedom.

The following simple fits to the ^{4}He mass fraction are quite accurate and take into account the nonstandard expansion [27,28]:

$$Y_p = 0.2485 + 0.0016[(\eta_{10} - 6) + 100(S - 1)], \tag{14}$$

where $\eta_{10} = 273.9\Omega_b h^2$ is the baryon-to-photon ratio. The last equation is the connection between the neutral hydrogen, the BDM model, and the extra relativistic degrees of freedom at the time of BBN, N_{ex}^{BBN}. If $S \neq 1$ ($N_{ex}^{BBN} > 0$), it is an indication of new physics beyond the standard model.

Using the values of Ω_m and z_{eq}, we can determine the value of $N_{ex}(z_{eq})$ at equality [cf., Eq. (2)] and with Y_p we constrain N_{ex}^{BBN} at BBN [cf., Eq. (13)]. From Table I we can see that the central values for Planck$_\star$, i.e., $\Omega_m = 0.305$, $z_{eq} = 3365$, and $Y_p = 0.26$, give $N_{ex}(z_{eq}) = 0.14$ at equality and $N_{ex}^{BBN} = 0.90$ at BBN. Clearly the values of N_{ex} at equality and BBN are quite different and in a CDM scenario one should have a constant N_{ex}, i.e., $N_{ex}^{BBN} = N_{ex}(z_{eq}) = N_{ex}(a_o)$, if the particles are still relativistic at equality and/or at present time. However, the value of N_{ex}^{BBN} is very sensitive to Y_p, and $N_{ex}^{BBN} = N_{ex}(z_{eq}) = 0.14$ requires $Y_p = 0.2505$.

Let us now study the constraints on BDM from BBN. Since we expect that the phase transition of BDM takes place after BBN, the BDM particles are relativistic during the nucleosynthesis epoch. Using Eqs. (7) and (8) we have for $a < a_c$

$$\rho_{BDM} = \rho_c \left(\frac{a}{a_c}\right)^{-4} \simeq \rho_{BDMo} \frac{a_c}{a_o} \left(\frac{a}{a_o}\right)^{-4} e^{v_c^2/2}. \tag{15}$$

Therefore, the nonstandard expansion rate becomes

$$S^2 = \left(\frac{\tilde{H}}{H}\right)^2 = \frac{\tilde{\rho}_r}{\rho_r} = 1 + \frac{8}{43} \frac{\rho_{BDM}}{\rho_\gamma}. \tag{16}$$

From Eqs. (13), (15), and (16) and using the fact that at BBN $\rho_\gamma = \rho_{\gamma o}(T_{\gamma,BBN}/T_{\gamma o})^4$ with $T_{\gamma,BBN}/T_{\gamma o} = (a_o/a)(g_o/g_{BBN})^{1/3}$, $g_o/g_{BBN} = 4/11$ is the ratio of the degrees of freedom of the relativistic components in thermal equilibrium with the photons at present time ($g_o = 2$) and just after neutrino decoupling ($g_{BBN} = 11/2$) and before e^+e^- annihilation. Therefore, we have

$$N_{ex}^{BBN} = \frac{8}{7} \frac{\rho_{BDM}}{\rho_\gamma} = \frac{1 + \alpha N_\nu}{\alpha} \frac{\Omega_{BDMo}}{\Omega_{mo}} \frac{a_c}{a_{eq}} e^{v_c^2/2}, \tag{17}$$

where we have also used $\Omega_{mo}/\Omega_{ro} = a_o/a_{eq}$ and $\rho_{ro} = (1 + \alpha N_\nu)\rho_{\gamma o}$. The BDM particles are relativistic above E_c, i.e., for $a < a_c$ the number of $N_{ex}(a \leq a_c) = N_{ex}(a_c)$ remains constant, as seen in Fig. 1. This includes the time of BBN, so BDM must have $N_{ex}^{BBN} = N_{ex}(a_c)$.

TABLE I. We present previous results of different surveys where the effective degrees of freedom of the neutrino $N_{\rm eff}$ and/or the primordial helium Y_p were considered to be free parameters. The center dots refers to the fixed values of $Y_p = 0.24$ and/or $N_{\rm eff} = 3.046$. We also show the derived value $N_{\rm eff}^{\rm BBN}$ assuming that the extra radiation was in thermal equilibrium with the photons.

	Ω_m	$z_{\rm eq}$	$N_{\rm eff}$	Y_p
Planck[a]	0.308 ± 0.010	3366 ± 39	⋯	⋯
Planck[a]+$N_{\rm eff}$	0.304 ± 0.011	3354 ± 42	3.30 ± 0.27	⋯
Planck[a]+Y_p	0.306 ± 0.011	3373 ± 40	⋯	0.267 ± 0.020
Planck$_\star$[a]+$N_{\rm eff}$ + Y_p	0.305 ± 0.011	3365 ± 53	$3.19^{+0.54}_{-0.43}$	$0.260^{+0.034}_{-0.029}$
Planck[b]+$N_{\rm eff}$	0.296 ± 0.010	3329 ± 38	3.52 ± 0.24	⋯
WMAP9[c]	$0.287^{+0.009}_{-0.009}$	3318 ± 55	$3.55^{+0.49}_{-0.48}$	$0.278^{+0.034}_{-0.032}$
ACT[d]	0.29 ± 0.01	3312 ± 78	3.50 ± 0.42	$0.255^{+0.01}_{-0.11}$
SPT[e]	0.28 ± 0.02	3267 ± 81	3.86 ± 0.42	$0.296^{+0.30}_{-0.30}$

[a]This results considered Planck + WMAP9 Polarization(WP) + high-l Planck temperature (highL) + BAO combined data [1].
[b]This results considered Planck + WP + highL + BAO + HST combined data [1].
[c]This results considered WMAP9 + ACT + SPT + BAO + HST combined data [2].
[d]Takes into account ACT + WMAP7 + SPT + BAO + HST [7].
[e]Combined data of SPT + WMAP7 + BAO + HST [8].

IV. RESULTS

In this section we compute the values of the BDM parameters x_c and v_c from current cosmological data. We use the published values of $N_{\rm eff}$ to determine x_c and v_c at equality and BBN. From Eqs. (10) and (11), with $a_{\rm eq} \ll a_o$, we see that $\rho_{\rm BDMo}$ and $N_{\rm ex}$ are determined only by the value of $x_c = a_c v_c$ and the amount of DM at present time $\Omega_{\rm BDMo}$. On the other hand, at BBN the amount of Y_p given in Eq. (17) depends on $\Omega_{\rm BDMo}$, x_c, and also on v_c. Therefore, we can constrain x_c and v_c using the value of $N_{\rm eff}$ at these different epochs.

We see from Table I that the WMAP9, ACT, and SPT results have at 1σ an $N_{\rm ex} > 0$, but the Planck data has an $N_{\rm ex} = 0$ at 1σ. However, the central value of $N_{\rm ex}$ hints at a small amount of extra relativistic degrees of freedom and its value is highly dependent on H_o [1]. For larger values of H_o more relativistic degrees of freedom are required, and therefore we present our results using different sets of data. An $N_{\rm ex} > 0$ implies the need for extra relativistic particles for a CDM cosmology or a nonvanishing value of x_c in BDM. A small value of $N_{\rm ex}$ requires a very small x_c, and for $N_{\rm ex} \leq 0.07$ x_c will be constrained to be less than $x_c < 2 \times 10^{-5}$ (for $v_c = 1$ we get $a_c/a_{\rm eq} < 0.09$) and if $N_{\rm eff} \simeq N_\nu$ then $x_c \ll 10^{-5}$, as can be seen in Fig. 2. From Eq. (10) and Fig. 4 we see that we can get the same x_c with a combination of different values of Ω_m and $a_{\rm eq}$. Hence, we use the results with the strongest constraints on Ω_m, namely, we use results from the combined data analysis of CMB, BAO, and H_o when available, cf., Table I.

Using the relation between $N_{\rm ex}$ and x_c in Eq. (11) at the equality epoch and the Planck$_\star$ results, the cosmological observations give

$$x_c = 4.13^{+3.65}_{-4.13} \times 10^{-5}. \tag{18}$$

Figure 3 shows the 68% and 95% C.L.'s of x_c using Planck data. The moment of the transition x_c can be determined by CMB observations by the amount of matter, Ω_m, and the equality epoch, $z_{\rm eq}$. The contours lie on the expected linear correlation between $\Omega_{\rm m}$ and a_c given by Eq. (3), for which we take the value shown in Table I.

We now constrain from BBN the value of $x_c = a_c v_c$ and v_c using Eqs. (17) and (14), which gives

$$\frac{x_c}{v_c} e^{v_c^2/2} = 4.17^{+6.86}_{-4.17} \times 10^{-5}. \tag{19}$$

If we take the previous result for x_c [Eq. (18)] at equality we can determine the value of v_c. Notice that the

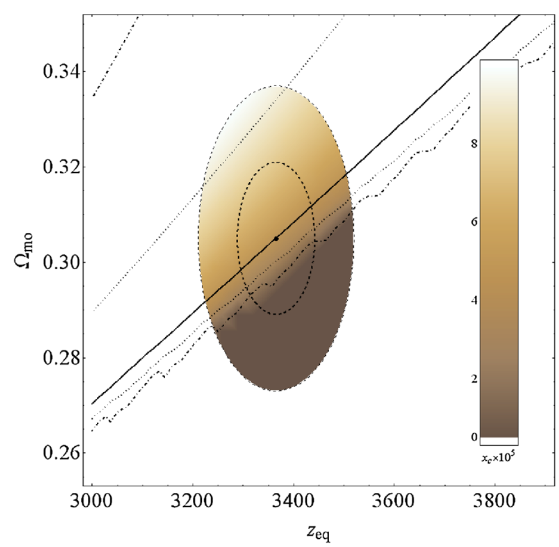

FIG. 4 (color online). In this plot we show the degeneracy between $\Omega_{\rm mo}$ and $z_{\rm eq}$; in other words, different values of $\Omega_{\rm mo}$ and $z_{\rm eq}$ can give the same x_c. The black line represents $x_c = 4.13 \times 10^{-5}$ while the dotted green line represents the $\pm 1\sigma$ of x_c. The small (big) dotted circle represents the $1\sigma(2\sigma)$ C.L. between $z_{\rm eq}$ and $\Omega_{\rm mo}$. The color gradient represents the different values that one gets with different combinations of $\Omega_{\rm mo}$ and $z_{\rm eq}$, ranging from $x_c > 10^{-8}$ (blue) to $x_c < 10^{-4}$ (black). The black dot represents the central value.

dependence on v_c in Eq. (19) is given by the quantity $g(v_c) \equiv e^{v_c^2/2}/v_c$, which has a lower limit $e^{1/2} = 1.64 \leq g(v_c = 1)$ since the velocity must be between $0 \leq v_c \leq 1$. Therefore, the central value of Eq. (19) gives an upper value of $x_c < 2.7 \times 10^{-4}$ for $v_c = 1$, which is an order of magnitude larger than x_c in Eq. (18). However, we expect to have $v_c < 1$ if the BDM mass is due to nonperturbative physics, as is suggested for BDM.

The consistency of BDM requires that the apparent number of extra degrees of freedom at BBN is the same as at the time of the BDM at the transition a_c, since BDM particles are relativistic for $a \leq a_c$. At a fixed value of x_c we have that $\tilde{N}_{\rm ex}(a_c)$ and $N_{\rm ex}^{\rm BBN}$ in Eqs. (11) and (17) are only functions of v_c. In Fig. 5 we plot the values of $\tilde{N}_{\rm ex}(a_c)$ and $N_{\rm ex}^{\rm BBN}$ as functions of v_c, and the result for v_c at $\tilde{N}_{\rm ex}(a_c) = N_{\rm ex}^{\rm BBN}$ is shown in Table II for the different data. The extra radiation due to the BDM particles changes the amount of neutral hydrogen produced at the BBN epoch.

The value of Y_p can be predicted for the BDM model assuming that the apparent number of extra degrees of freedom evolves as in Eq. (11). By replacing $\tilde{N}_{\rm ex}(a_c)$ in Eq. (17) we are able to constrain the value of v_c and Y_p simultaneously knowing only the moment of equality. Therefore, BDM relates the amount of neutral hydrogen produced at BBN with the equality epoch, and vice versa. In this case, taking Planck$_\star$ data ($z_{\rm eq} = 3365$) gives an $N_{\rm ex}^{\rm BBN} = 1.92$ and $v_c = 0.53$, and BDM requires an $Y_p = 0.272$ (within 1σ C.L.); see Fig. 5. We would like to emphasize that the value of Y_p and $N_{\rm ex}^{\rm BBN}$ is quite sensitive to x_c, and, for example, if we take $x_c = 10^{-5}$ then we get $Y_p = 0.256$ and $N_{\rm ex}^{\rm BBN} = 0.62$.

In Table II we summarize the constraints on the BDM parameters x_c and v_c obtained directly from the $N_{\rm eff}$ and

TABLE II. We present the constraints on x_c and v_c as discussed in Secs. II and III using different results for $N_{\rm eff}$ and Y_p (cf., Table I). We also present the moment (z_c) and the energy when the transition happens (E_c). Notice that in some cases the transition $z_c < z_{\rm eq}$; however, we do not expect these cases to be valid in order to account for structure formation. We present the minimum value for z_c and E_c for the Planck$_\star$ data because no extra radiation is contained at 1σ, and therefore the moment of the transition should be consistent with $x_c \to 0$ at 1σ.

	Planck$_\star$[a]	WMAP9	ACT	SPT
$x_c \times 10^5$	$4.13^{+3.65}_{-4.13}$	$6.64^{+2.46}_{-3.54}$	$6.34^{+2.56}_{-4.16}$	$8.25^{+2.92}_{-4.07}$
$N_{\rm ex}^{\rm BBN}$	$0.9^{+1.5}_{-0.9}$	$2.4^{+1.6}_{-1.4}$	$0.6^{+1.8}_{-0.6}$	$4.2^{+1.6}_{-1.5}$
v_c	$0.37^{+0.18}_{-0.17}$	$0.54^{+0.09}_{-0.10}$	$0.26^{+0.13}_{-0.14}$	$0.64^{+0.09}_{-0.10}$
z_c	≥ 24217	$15060^{+2\times10^4}_{-1\times10^4}$	$15781^{+3\times10^4}_{-6\times10^3}$	$6313^{+10^4}_{-3\times10^3}$
E_c [eV]	≥ 3.89	$2.75^{+2.12}_{-1.77}$	$2.38^{+3.49}_{-1.03}$	$1.32^{+2.60}_{-0.55}$
$E(a_c)$ [eV]	≥ 9.01	$5.93^{+5.77}_{-3.9}$	$6.17^{+9.87}_{-2.34}$	$2.65^{+6.32}_{-1.13}$

[a]These results consider Planck + WP + highL + BAO combined data [1].

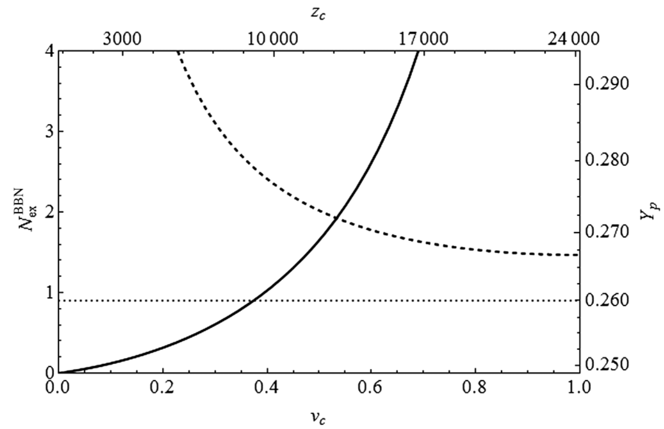

FIG. 5. Plot of the extra relativistic degrees of freedom as a function of v_c. The thick line is $\tilde{N}_{\rm ex}(a_c)$ evaluated at the time of the transition [Eq. (11)]. The dashed line is $N_{\rm ex}^{\rm BBN}$ [Eq. (17)]. The dotted line is the extra relativistic degree of freedom corresponding to the reported value of Y_p, cf., Eq. (13). The first line is derived from the extra radiation at the time of equality that makes $\omega_{\rm tot} = 1/6$. The second line is from the constraints of the nonstandard expansion rate at the BBN epoch. In all cases we assume a fixed value for $x_c = 4.13 \times 10^{-5}$. The BDM model predicts a value for $Y_p = 0.272$, which is within the 1σ error of the reported $Y_p = 0.26$; see the discussion in Secs. III and V for more details.

^{4}He using different previous results, such as Planck$_\star$, WMAP9, ACT, and SPT. We also show the derived parameters, such as the moment, $1 + z_c = a_c^{-1}$, the energy of the BDM particles, $E_c = \rho_c^{1/4}$ [cf., Eq. (8)], and the energy of the Universe $E \equiv \rho^{1/4}$, with ρ the total energy density, the last two quantities being at the moment of the transition. Notice that with Eq. (19) and the constraint on x_c [cf., Eq. (18)] we are able to derive the constraints of the

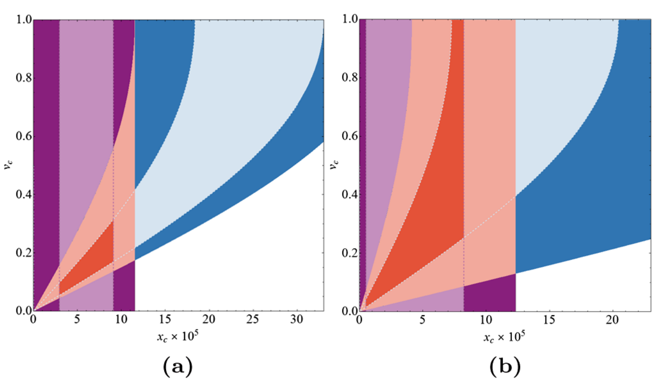

FIG. 6 (color online). In this plot we show the range of values valid for x_c and v_c given the constraints of ^{4}He and $z_{\rm eq}$ and using (a) WMAP9 (left panel) and (b) Planck$_\star$ (right panel) data (cf., Table II). The purple and blue regions represent the C.L. (68% and 96%) obtained with $N_{\rm eff}$ (cf., Sec. IV) and ^{4}He data, respectively. The orange region is where the results of both analyses overlap. Notice that we cannot constrain the value of v_c using only the $N_{\rm eff}$ data.

central value of $x_c = 4.13 \times 10^{-5}$, $v_c = 0.37$. However, at 1σ the Planck data have that $N_{\rm ex} = 0$, and therefore $x_c = 0$ is valid at 1σ. Hence, the moment of the transition would only be constrained to $z_c \geq 6445$ and the energy of the Universe to $E(a_c) \geq 2.65$ eV. In Fig. 6 we show the range of values at 1σ and 2σ (68% and 95% C.L. region) that are valid for x_c and v_c by combining the two pieces of evidence for extra radiation: the one stemming from the equality epoch and the other from the amount of primordial ^{4}He.

V. CONCLUSION

Cosmological observations suggest the existence of extra radiation, $N_{\rm eff} > N_\nu$, in order to explain CMB and ^{4}He measurements. Motivated by this lack of radiation in the standard CDM framework we have considered the BDM model, which may explain the need for an extra relativistic component without introducing new particles.

The BDM particles behave as radiation for scales $a < a_c$, while for $a > a_c$ these particles become nonrelativistic due to a phase transition in which the particles acquire a mass due to nonperturbative methods when $v_c < 1$, similarly to protons and neutrons. We expect this phase transition to be between BBN $[E = \mathcal{O}(\rm MeV)]$ and the matter-radiation equality $[E = \mathcal{O}(\rm eV)]$. If $E \gg$ MeV then our BDM will be indistinguishable from CDM, and for $E <$ eV BDM would not be able to account for structure formation. The evolution of the BDM energy density during this process is described by a time-dependent EoS, $\omega_{\rm BDM}(a)$. The amount of radiation due to the transition of the BDM particles changes as a function of the scale factor $\rho_{\rm ex}(a)$, cf., Eq. (12). If this extra radiation is modeled as neutrino radiation this means that the neutrino relativistic degrees of freedom will also be a function of the scale factor $N_{\rm ex}(a)$.

Since we have a time-dependent EoS we cannot simply use $\rho_r = \rho_m$ to determine the matter-radiation equality. Instead, we define equality when the total EoS is $\omega_{\rm tot} = 1/6$, which overcomes the fact that $\omega_{\rm BDM}$ is a function of a and is also valid in the limiting case of the standard model. We conclude that the apparent number of relativistic particles, $N_{\rm ex}$, is explained by a time-dependent EoS of the DM without introducing new particles, cf., Sec. II. For a phase transition $a_c \ll a_{\rm eq}$ the amount of apparent extra relativistic degrees of freedom in our BDM model is small, and for $N_{\rm ex} \leq 0.07$ one requires $a_c/a_{\rm eq} \leq 0.09$ if $v_c = 1$.

The BDM particles also change the prediction of BBN for the abundance of the light elements, such as helium, by changing the radiation density, thereby increasing the expansion rate of the early Universe. Incidentally, observation also shows an excess during BBN which can be explained by the BDM particles.

We computed the range of values for the transition epoch $x_c = a_c v_c$ and v_c using cosmological data, which predict extra radiation; Table II summarizes the results. Using the latest result from Planck$_\star$, we concluded that the order of the transition should be $x_c = 4.13(^{+3.65}_{-4.13}) \times 10^{-5}$ in order to explain the evidence of extra radiation at matter-radiation equality $a_{\rm eq}$. Using the ^{4}He results of BBN we obtained equivalent constraints for $N_{\rm ex}^{\rm BBN} = 0.9^{+1.5}_{-0.9}$. By combining both previous results we were able to constrain the velocity $v_c = 0.37^{+0.18}_{-0.17}$, and therefore $z_c > 24217$ and $E(a_c) \geq 9.01$. However, if the value of $N_{\rm eff}$ becomes close to N_ν, i.e., $N_{\rm eff} \simeq N_\nu$, then $x_c \ll 10^{-5}$ and $z_c \gg 10^5$.

The BDM model is also able to explain the inconsistency between the apparent extra degrees of freedom at the equality and BBN epochs, $N_{\rm ex}(a_{\rm eq}) \neq N_{\rm ex}^{\rm BBN}$, and to predict the amount of ^{4}He given the moment of equality $z_{\rm eq}$, and vice versa. From the assumption that equality occurs when $\omega_{\rm tot} = 1/6$ we were able to compute how $\tilde{N}_{\rm ex}$ is dependent of the scale factor [Eq. (11)]. Combining this equation with the one obtained from BBN $N_{\rm ex}^{\rm BBN}$, Eq. (17), we were able to predict that the amount of ^{4}He is consistent with $z_{\rm eq} = 3365$ and $x_c = 4.13 \times 10^{-5}$ should be $Y_p = 0.272$, which is conciliable within the 1σ error of the reported $Y_p = 0.26$, but a slightly smaller $x_c = 10^{-5}$ gives $N_{\rm ex}^{\rm BBN} = 0.62$ and $Y_p = 0.256$.

We conclude that we can account for the apparent extra $N_{\rm ex}$ at the equality and BBN epochs using only the BDM particles, which have a time-dependent EoS $\omega_{\rm BDM}(a)$, with no need to introduce extra relativistic particles. However, further analysis will provide us with a better understanding of dark matter and the possibility that the dark matter mass is due to nonperturbative physics.

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