Neutrinos and dark energy constraints from future galaxy surveys and CMB lensing information

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We explore the possibility of obtaining better constraints from future astronomical data by means of the Fisher information matrix formalism. In particular, we consider how cosmic microwave background (CMB) lensing information can improve our parameter error estimation. We consider a massive neutrino scenario and a time-evolving dark energy equation of state in the Λ cold dark matter framework. We use Planck satellite experimental specifications together with the future galaxy survey Euclid in our forecast. We found improvements in almost all studied parameters considering Planck alone when CMB lensing information is used. In this case, the improvement with respect to the constraints found without using CMB lensing is of 93% around the fiducial value for the neutrino parameter. The improvement on one of the dark energy parameters reaches 4.4%. When Euclid information is included in the analysis, the improvements on the neutrino parameter constraint are of approximately 128% around its fiducial value. The addition of Euclid information provides smaller errors on the dark energy parameters as well. For Euclid alone, the figure of merit is a factor of $\sim$29 higher than that from Planck alone even considering CMB lensing. Finally, the consideration of a nearly perfect CMB experiment showed that CMB lensing cannot be neglected, especially in more precise future CMB experiments, since it provided in our case a six-times-better figure of merit with respect to the unlensed CMB analysis.

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I. INTRODUCTION

The discovery of the accelerated expansion of the Universe [1,2] can be interpreted by introducing in the cosmological model a negative pressure component, termed "dark energy." The simplest dark energy candidate is a cosmological constant Λ, having constant equation of state $w_{\rm de} = P_{\rm de}/\rho_{\rm de} = -1$. Together with a pressureless cold dark matter (CDM) component, this constitutes the standard ΛCDM model. Although this "concordance model" is in very good agreement with a variety of cosmological observations [3,4], different candidates of dark energy cannot be discarded yet. Moreover, the basic cosmological constant scenario has two difficulties known as "fine-tuning" and cosmic coincidence problems (see, e.g., [5]). To overcome these problems, alternative candidates for the dark energy have been proposed, such as the quintessence [6] that allows the possibility of a time-dependent equation of state [7]. In this paper, we will assume a redshift-dependent equation of state for the dark energy,

$$w_{\rm de}(z) = \frac{P_{\rm de}(z)}{\rho_{\rm de}(z)}, \tag{1}$$

and adopt the well-known Chevalier-Plarsky-Linder parametrization [7,8]

$$w_{\rm de}(a) = w_0 + (1-a)w_a. \tag{2}$$

Cosmological observation can in principle be used to constrain the neutrinos' masses. It was shown by neutrino oscillation experiments that neutrinos have nonzero masses (see [9] and references therein). However, these experiments can only constrain the neutrinos' mass-squared differences and not their individual values (for a review in neutrino masses, see de Gouvea [10]). On the other hand, cosmological probes are most sensitive to the total neutrino masses, $\sum m_\nu$. Using cosmic microwave background (CMB) radiation data only, from the Planck satellite, an upper limit to $\sum m_\nu$ of 0.933 eV at 95% C.L. was found [11]:

$$\Omega_\nu = \frac{m_\nu}{94h^2\ \text{eV}}. \tag{3}$$

However, the dark energy equation of state and the neutrinos' total mass parameters are degenerated (see, e.g., [12]). Some work has already been done to constrain both parameters simultaneously in a few dark energy scenarios, such as for models with a constant and time-varying equations of state [13–16].

Our goal is to forecast the constraint in the total mass of neutrinos in a time-evolving dark energy model, using the CMB temperature and polarization power spectrum from the Planck satellite experimental setup (also including CMB lensing information), as well as the large-scale matter distribution that can be observed by the Euclid survey. We emphasize the usage of Planck CMB polarization information since its temperature data has been recently released [11]. We assume a geometrically flat ΛCDM model with

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two massive neutrinos with identical mass and one massless in an inverted hierarchy mass splitting, that being $m_\nu = 0.125\,\mathrm{eV}$ for each massive neutrino. The fiducial parameters are $h^2\omega_b = 0.02219$, $h^2\omega_c = 0.1122$, $h^2\omega_\nu = 0.0027$, $h = 0.65$, $n_s = 0.952$. We normalize the CMB power spectra to COBE. For similar approaches see Joudaki and Kaplinghat [16], Marsh *et al.* [17], Hollenstein *et al.* [18], Namikawa *et al.* [19], Das *et al.* [20], Hall and Challinor [21], Hamann *et al.* [22]. The paper is organized as follows: we give a small introduction on CMB lensing in Sec. II. In Sec. III, we briefly review the Fisher information matrix formalism for the CMB (with and without lensing information) and for a galaxy survey. Finally we present our results in Sec. IV, followed by our discussion and conclusions in Sec. V.

II. CMB LENSING

A small effect that can be observed in the CMB power spectrum regards the deflection of photons, during their travel between the last scattering surface and the observer, by gravitational potentials Ψ due to clusters of galaxies. Smith *et al.* [23] detected the CMB lensing signal for the first time by cross correlating WMAP data to radio galaxy counts in the NRAO VLA sky survey (NVSS). Recently, the detection of the gravitational lensing using CMB temperature maps alone and the measurement of the power spectrum of the projected gravitational potential were done using the Atacama Cosmology Telescope and the South Pole Telescope [24,25].

The lensing potential is defined as

$$\psi(\hat{\mathbf{n}}) \equiv -2\int_0^{\chi^*} d\chi \frac{\chi^* - \chi}{\chi^* \chi}\Psi(\chi\hat{\mathbf{n}}; \eta_0 - \chi), \tag{4}$$

where χ^* is the comoving distance and $\eta_0 - \chi$ is the conformal time at which the photon was at position $\chi\hat{\mathbf{n}}$.

The lensing effect remaps the temperature and polarization fields as

$$\frac{\Delta\tilde{\mathrm{T}}(\hat{\mathbf{n}})}{T} = \frac{\Delta\mathrm{T}(\hat{\mathbf{n}}')}{T} = \frac{\Delta\mathrm{T}(\hat{\mathbf{n}} + d)}{T}, \tag{5}$$

$$[Q + iU](\hat{\mathbf{n}}) = [Q + iU](\hat{\mathbf{n}} + d), \tag{6}$$

where in the case of the temperature field, the temperature T of the lensed CMB in a direction $\hat{\mathbf{n}}$ is equal to the unlensed CMB in a different direction $\hat{\mathbf{n}}'$. Both of these directions, $\hat{\mathbf{n}}$ and $\hat{\mathbf{n}}'$, differ by the deflection angle d. To first order, the deflection angle is simply the lensing potential gradient, $d = \nabla\psi$. In the same way, the effect of lensing in CMB polarization is written in terms of the Stokes parameters $Q(\hat{\mathbf{n}})$ and $U(\hat{\mathbf{n}})$ (for a review in CMB polarization theory, see Lewis and Challinor [26], Cabella and Kamionkowski [27]).

CMB lensing has important quantitative contributions that should be taken into account; therefore, a lot of work has been done to CMB lensing reconstruction techniques (e.g., Bucher *et al.* [28], Carvalho and Tereno [29], Hu [30], Okamoto and Hu [31], Smith *et al.* [32]). In this paper, we use the CAMB software package [33] to obtain the numerical lensed and unlensed power spectra (C^{TT}, C^{EE}, C^{BB}, C^{TE} and C^{dd}, C^{Td}, C^{Ed}) for our cosmological model with $l \leq 2749$. We then use these predictions to forecast how CMB lensing information will help us to constrain our model.

III. METHOD

We apply the Fisher information matrix formalism to a Planck-like experiment [34], considering both temperature and polarization for the lensed and unlensed CMB spectrum, and to an experiment such as the future Euclid survey (for a different approach considering Markov Chain Monte Carlo to forecast the total neutrino mass for an Euclid-like galaxy or cluster number counts surveys combined with Planck see Audren *et al.* [35], Cerbolini *et al.* [36]). We forecast the dark energy and the massive neutrino parameters in our fiducial model. In addition, we check the impact of CMB lensing information on the constraints of the mentioned parameters in a nearly perfect CMB experiment.

A. Information from CMB

The Fisher information matrix for the CMB temperature anisotropy and polarization is given by [37]

$$F_{ij} = \sum_l \sum_{XY} \frac{\partial C_l^X}{\partial p_i} (\mathrm{Cov}_l^{-1})_{XY} \frac{\partial C_l^Y}{\partial p_j}, \tag{7}$$

where C_l^X is the power in the lth multipole, X stands for TT (temperature), EE (E-mode polarization), BB (B-mode polarization), and TE (temperature and E-mode polarization cross correlation). We will not include primordial B-modes in the analysis since the measurement of the primordial C_l^{BB} by Planck is expected to be noise dominated. Our covariance matrix therefore becomes

$$\mathrm{Cov}_l = \frac{2}{(2l+1)f_{\mathrm{sky}}} \begin{bmatrix} \Xi_{TTTT} & \Xi_{TTEE} & \Xi_{TTTE} \\ \Xi_{EETT} & \Xi_{EEEE} & \Xi_{EETE} \\ \Xi_{TETT} & \Xi_{TEEE} & \Xi_{TETE} \end{bmatrix}. \tag{8}$$

Explicit expressions for the matrix elements are given in Appendix A.

For the lensed case we performed a correction in the covariance matrix elements taking into consideration the power spectrum of the deflection angle and its cross correlation with temperature and E-polarization, C_l^{Td} and C_l^{Ed}. We used the same procedure introduced in [38] to obtain the covariance matrix elements using the new information of the C_l^{Ed} power spectrum (see Appendix B).

We also change in this case the unlensed CMB power spectra, C_l^X, for the lensed ones, $\tilde{C}_l^X$. Note that in this case we are taking into consideration the B-mode polarization generated by the CMB gravitational lensing from the E-mode polarization.

When we include these corrections, the covariance matrix becomes

$$\mathrm{Cov}_l = \frac{2}{(2l+1)f_{\rm sky}} \begin{bmatrix} \xi_{TTTT} & \xi_{TTEE} & \xi_{TTTE} & \xi_{TTTd} & \xi_{TTdd} & \xi_{TTEd} & 0 \\ \xi_{TTEE} & \xi_{EEEE} & \xi_{EETE} & \xi_{EETd} & \xi_{EEdd} & \xi_{EEEd} & 0 \\ \xi_{TTTE} & \xi_{EETE} & \xi_{TETE} & \xi_{TETd} & \xi_{TEdd} & \xi_{TEEd} & 0 \\ \xi_{TTTd} & \xi_{EETd} & \xi_{TETd} & \xi_{TdTd} & \xi_{Tddd} & \xi_{TdEd} & 0 \\ \xi_{TTdd} & \xi_{EEdd} & \xi_{TEdd} & \xi_{Tddd} & \xi_{dddd} & \xi_{ddEd} & 0 \\ \xi_{TTEd} & \xi_{EEEd} & \xi_{TEEd} & \xi_{TdEd} & \xi_{ddEd} & \xi_{EdEd} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \xi_{BBBB} \end{bmatrix}. \tag{9}$$

$$\xi_{TTTT} = (\tilde{C}_l^{TT} + N_l^{TT})^2 \tag{10}$$

$$\xi_{EEEE} = (\tilde{C}_l^{EE} + N_l^{PP})^2, \tag{11}$$

$$\xi_{dddd} = (C_l^{dd} + N_l^{dd})^2, \tag{12}$$

$$\xi_{BBBB} = (\tilde{C}_l^{BB} + N_l^{PP})^2, \tag{13}$$

$$\xi_{TETE} = \frac{1}{2}[(\tilde{C}_l^{TE})^2 + (\tilde{C}_l^{TT} + N_l^{TT})(\tilde{C}_l^{EE} + N_l^{PP})], \tag{14}$$

$$\xi_{TdTd} = \frac{1}{2}[(C_l^{Td})^2 + (\tilde{C}_l^{TT} + N_l^{TT})(C_l^{dd} + N_l^{dd})], \tag{15}$$

$$\xi_{EdEd} = \frac{1}{2}[(C_l^{Ed})^2 + (C_l^{dd} + N_l^{dd})(\tilde{C}_l^{EE} + N_l^{PP})], \tag{16}$$

$$\xi_{TTEE} = (\tilde{C}_l^{TE})^2, \tag{17}$$

$$\xi_{TTdd} = (C_l^{Td})^2, \tag{18}$$

$$\xi_{EEdd} = (C_l^{Ed})^2, \tag{19}$$

$$\xi_{TEdd} = C_l^{Ed} C_l^{Td}, \tag{20}$$

$$\xi_{EETd} = C_l^{Ed} C_l^{TE}, \tag{21}$$

$$\xi_{TTEd} = C_l^{Td} C_l^{TE}, \tag{22}$$

$$\xi_{TTTE} = \tilde{C}_l^{TE}(\tilde{C}_l^{TT} + N_l^{TT}), \tag{23}$$

$$\xi_{EETE} = \tilde{C}_l^{TE}(\tilde{C}_l^{EE} + N_l^{PP}), \tag{24}$$

$$\xi_{TTTd} = C_l^{Td}(\tilde{C}_l^{TT} + N_l^{TT}) \tag{25}$$

$$\xi_{Tddd} = C_l^{Td}(C_l^{dd} + N_l^{dd}), \tag{26}$$

$$\xi_{ddEd} = C_l^{Ed}(C_l^{dd} + N_l^{dd}), \tag{27}$$

$$\xi_{EEEd} = C_l^{Ed}(\tilde{C}_l^{EE} + N_l^{PP}), \tag{28}$$

$$\xi_{TETd} = \frac{1}{2}[C_l^{Td}\tilde{C}_l^{TE} + C_l^{Ed}(\tilde{C}_l^{TT} + N_l^{TT})], \tag{29}$$

$$\xi_{TEEd} = \frac{1}{2}[(\tilde{C}_l^{EE} + N_l^{PP})C_l^{Td} + C_l^{Ed}\tilde{C}_l^{TE}], \tag{30}$$

$$\xi_{TdEd} = \frac{1}{2}[C_l^{Ed}C_l^{Td} + (C_l^{dd} + N_l^{dd})\tilde{C}_l^{TE}]. \tag{31}$$

In these equations, N_l^{TT} and N_l^{PP} are the Gaussian random detector noises for temperature and polarization, respectively, whose expression is written using the window function, $B_l^2 = \exp[-l(l+1)\theta_{\rm beam}^2/8\ln 2]$, and the inverse square of the detector noise level for temperature and polarization, w_T and w_P. The full width half maximum (FWHM), $\theta_{\rm beam}$, is used in radians and $w = (\theta_{\rm beam}\sigma)^{-2}$ is the weight given to each considered Planck channel [39]. We tested two types of experimental setups that can be checked in Tables I and II:

$$N_l^{TT} = [(w_T B_l^2)_{100} + (w_T B_l^2)_{143} + (w_T B_l^2)_{217} + (w_T B_l^2)_{353}]^{-1} \tag{32}$$

$$N_l^{PP} = [(w_P B_l^2)_{100} + (w_P B_l^2)_{143} + (w_P B_l^2)_{217} + (w_P B_l^2)_{353}]^{-1}. \tag{33}$$

TABLE I. Planck specifications[a]. We used $f_{\rm sky} = 0.65$.

Frequency (GHz)	$\theta_{\rm beam}$	$\sigma_T(\mu K - \mathrm{arc})$	$\sigma_P(\mu K - \mathrm{arc})$
100	9.5′	6.82	10.9120
143	7.1′	6.0016	11.4576
217	5.0′	13.0944	26.7644
353	5.0′	40.1016	81.2944

[a]See the Planck mission blue book at http://www.rssd.esa.int/SA/PLANCK/docs/Bluebook-ESA-SCI(2005)1_V2.pdf.

TABLE II. Nearly perfect experiment suggested by Okamoto and Hu [31], Hu and Okamoto [40]. We used $f_{\rm sky} = 0.90$.

$\theta_{\rm beam}$	$\sigma_T(\mu K - \mathrm{arc})$	$\sigma_P(\mu K - \mathrm{arc})$
4.0′	0.093×10^{-6}	0.13×10^{-6}

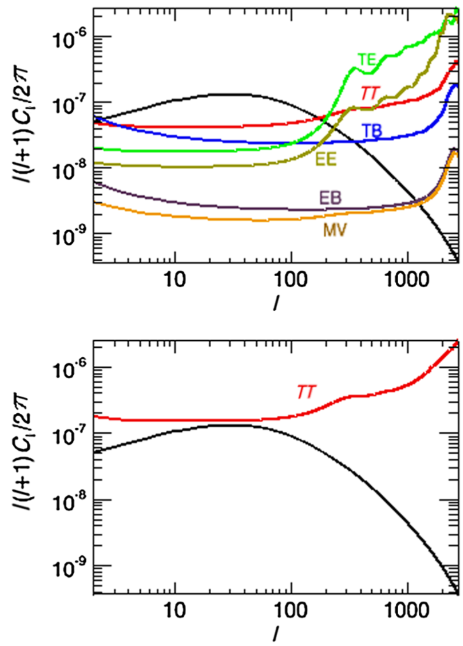

FIG. 1 (color online). The CMB deflection field and its quadratic estimator noises for the nearly ideal experiment (up) and for Planck specifications (down).

Here we used four channels (100, 143, 217, and 353 GHz) of the Planck experiment, as can be seen from Eqs. (32) and (33).

In addition, N_l^{dd} is the optimal quadratic estimator noise of the deflection field (we consider only the TT quadratic estimator noise for the planck experiment since it provides the best estimator). For the nearly ideal experiment we consider the minimum variance (MV) estimator noise written as a combination of the noises TB, TT, TE, EE, and EB of the quadratic estimators (for a review in the topic, see Okamoto and Hu [31], Hu and Okamoto [40]). Figure 1 shows the quadratic estimator noises for our fiducial model considering Planck and the nearly ideal experiment.

B. Information from galaxy survey: Baryonic acoustic oscillation

We show here how the baryonic acoustic oscillation (BAO) information can be used to forecast errors in the dark energy parameters using the Fisher formalism. It was shown by [41] that the Hubble parameter $H(z)$ and the angular diameter distance $Da(z)$ can be measured very precisely by using the BAO information present in the matter power spectrum. $H(z)$ and $Da(z)$ are expected to be determined as a function of redshift by future galaxy surveys. The goal is then to propagate the errors on $H(z)$ and $Da(z)$ to the constraints of dark energy parameters.

We start defining the observed galaxy power spectrum in a reference cosmology (in our case we use the ΛCDM model), distinguished by the subscript "ref" (different from the true spectrum, written with no subscript) that will be used to derive the cosmological parameter constraints using a galaxy survey that covers a wide range of redshifts. Following [41],

$$P_{\rm obs}(k_{\rm ref\perp}, k_{\rm ref\parallel}) = \frac{Da(z)^2_{\rm ref} \times H(z)}{Da(z)^2 \times H(z)_{\rm ref}} P_g(k_{\rm ref}, k_{\rm ref}) + P_{\rm shot}, \tag{34}$$

where the Hubble parameter $H(z)$ in a flat Universe is related to the dark energy parameters through

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_d e(1+z)^{3(1+w_0+w_a)}\exp\left(3w_a(a-1)\right)}, \tag{35}$$

and the angular diameter distance is defined as

$$D_a(z) = \frac{c}{1+z}\int_0^z \frac{dz}{H(z)}. \tag{36}$$

$P_{\rm shot}$ is the unknown Poisson shot noise.

The wave numbers across and along the line of sight are denoted by $k_\perp$ and $k_\parallel$. It is important to point out that the wave numbers in the reference cosmology are related to the ones in the true cosmology by

$$k_{\rm ref\perp} = k_\perp \frac{Da(z)}{Da(z)_{\rm ref}} \qquad k_{\rm ref\parallel} = k_\parallel \frac{H(z)_{\rm ref}}{H(z)}. \tag{37}$$

Moreover, we define the galaxy power spectrum, P_g, including the redshift distortions:

$$P_g(k_{\rm ref\perp}, k_{\rm ref\parallel}) = b^2(z)(1+\beta\mu^2)^2 \times \left(\frac{G(z)}{G(0)}\right)^2 P_{{\rm matter},z=0}(k) e^{-k^2\mu^2\sigma_r^2}, \tag{38}$$

where $\mu = \mathbf{k}\cdot\hat{\mathbf{r}}/k$, $\hat{\mathbf{r}}$ being the unit vector along the line of sight, and the linear matter power spectrum, $P_{{\rm matter},z=0}(k)$, was generated using the CAMB software package [33] and COBE normalized. The $k_{\rm max}$ is chosen in a way to exclude information from the nonlinear regime where Eq. (34) is inaccurate (see [41]). For an approach considering the nonlinear regime see, for example, [42–45]. Moreover, for the impact of precisely modeling systematic effects, such as the nonlinear clustering and redshift space distortions, in the evolution of BAO, see [46–48]. The exponential damping factor is due to redshift uncertainties, where $\sigma_r = c\sigma_z/H(z)$. $G(z)$, $\beta(z)$, and $b(z)$ are the growth function, the linear redshift space distortion parameter, and the linear galaxy bias, respectively. We use a growth factor dependent on the dark energy parameter and a massive neutrino effect computed by [49]. The growth rate of perturbations is defined as

$$f \equiv \frac{d\ln G}{d\ln a}, \tag{39}$$

where the growth function $G(z)$ is related to the density of matter. In a matter dominated Universe $f \approx \Omega_m(z)^{0.6}$, with $\Omega_m(z) = H_0^2\Omega_m(1+z)^3/H^2(z)$. More generally, we use $f = \nu\Omega_m(z)^\alpha$ with

$$\begin{aligned} \alpha &= \alpha_0 + \alpha_1[1-\Omega_m(z)], \\ \alpha_0 &= \frac{3}{5-\frac{w}{1-w}}, \\ \alpha_1 &= \frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^3}. \end{aligned} \tag{40}$$

ν is a numerical function dependent on Ω_{de} and $f_\nu = \Omega_\nu/\Omega_m$ (see Eq. (17) and Table 5 of [49]).

The linear redshift space distortion is also defined as a function of the growth rate and the galaxy bias:

$$\beta(z) \equiv \frac{f}{b(z)}. \tag{41}$$

1. Fisher formalism

The Fisher information matrix for the matter power spectrum obtained from galaxy surveys is given by [50]

$$F_{ij} = \int_{-1}^{1}\int_{k_{\min}}^{k_{\max}} \frac{\partial \ln P(k,\mu)}{\partial p_i}\frac{\partial \ln P(k,\mu)}{\partial p_j} V_{\rm eff}(k,\mu) \times \frac{2\pi k^2 dk d\mu}{2(2\pi)^3}. \tag{42}$$

The effective volume of the survey for a constant comoving number density is given by

$$V_{\rm eff}(k,\mu) = \left[\frac{\bar{n}(r)P_g(k,\mu)}{1+\bar{n}(r)P_g(k,\mu)}\right]^2 V_{\rm survey}. \tag{43}$$

We use the information from a Euclid-like survey with an area of 20000 deg ^{2}, redshift accuracy of $\sigma_z/(1+z) = 0.001$, and a redshift range $0.5 \le z \le 2.1$. Finally we divided our forecast into 15 redshift slices of $\Delta z = 0.1$ centered in z_i. We chose the initial set of parameters $P = \{h^2\Omega_b, h^2\Omega_c, h^2\Omega_\nu, H(z_i), Da(z_i), G(z_i), \beta(z_i), P^i_{\rm shot}\}$. For each redshift bin we use the specifications on Table III (see [51,52] and references therein).

To obtain the constraints on our final set of parameters $Q = \{h^2\Omega_b, h^2\Omega_c, h^2\Omega_\nu, w_0, w_a\}$, first we marginalize our first Fisher matrix over $G(z_i)$, $\beta(z_i)$, $P^i_{\rm shot}$ and use this submatrix to change into the desired variables as

TABLE III. Values of $k_{\max}$, the galaxy bias, and the galaxy density for each redshift bin.

z_i	$K_{\max}$ $(h\,{\rm Mpc}^{-1})$	$b(z)$	$n(z)\times 10^{-3}(h/{\rm Mpc})^3$
0.55	0.144	1.0423	3.56
0.65	0.153	1.0668	3.56
0.75	0.163	1.1084	2.42
0.85	0.174	1.1145	2.42
0.95	0.185	1.1107	1.81
1.05	0.197	1.1652	1.81
1.15	0.2	1.2262	1.44
1.25	0.2	1.2769	1.44
1.35	0.2	1.2960	0.99
1.45	0.2	1.3159	0.99
1.55	0.2	1.4416	0.55
1.65	0.2	1.4915	0.55
1.75	0.2	1.4973	0.29
1.85	0.2	1.5332	0.29
1.95	0.2	1.5705	0.15

$$F_{DE,ij} = \sum_{\alpha,\beta}\frac{\partial P_\alpha}{\partial Q_i}F^{\rm sub}_{\alpha\beta}\frac{\partial P_\beta}{\partial Q_j}. \tag{44}$$

C. Information from galaxy survey: Weak lensing

In this subsection, we show how weak lensing (WL) information can improve the constraints on cosmological parameters, in our case especially for dark energy and neutrino density parameters using the Fisher formalism. The observable, in weak lensing surveys, is the convergence power spectrum. In the analysis presented in this paper, we use an extension of the CAMB software with the Halofit approximation (recently updated according to [53]) to generate the convergence power spectra P_{ij}, where the subscripts i and j stand for the lensed galaxy redshift bins. The Fisher matrix for weak lensing is then given by [54]

$$F_{\alpha\beta} = f_{\rm sky}\sum_l \frac{(2l+1)}{2}\frac{\partial P_{ij}}{\partial p_\alpha}(C^{-1})_{jk}\frac{\partial P_{km}}{\partial p_\beta}(C^{-1})_{mi}. \tag{45}$$

The covariance matrix is defined as

$$C_{jk} = P_{jk} + \delta_{jk}\langle\gamma^2_{\rm int}\rangle n_j^{-1}, \tag{46}$$

$\gamma_{\rm int}$ being the rms intrinsic shear and n_j the number of galaxies per steradian in the jth bin

$$n_j = 3600d\left(\frac{180}{\pi}\right)^2\hat{n}_j. \tag{47}$$

In the equation above, d is the number of galaxies per square arc minute and $\hat{n}_j$ is the fraction of sources belonging to a certain bin. We compute our calculations considering a Euclid-like experiment following [55], with $f_{\rm sky} = 1/2$, $d = 40$, and $\langle\gamma^2_{\rm int}\rangle^{1/2} = 0.22$. We take the range $0.5 \le z \le 2.0$ and divide it into four equal-galaxy-number bins. We also consider $10 \le l \le 10000$.

TABLE IV. Marginalized errors for ΛCDM model with two massive neutrinos with identical mass and one massless in an inverted hierarchy mass splitting ($m_\nu = 0.125$ eV).

Parameter	Fiducial value	Planck T + P	Planck T + P + lens	EUCLID (BAO + WL)	Planck + EUCLID
$h^2\Omega_b$	0.02219	0.00012	0.00012	0.00034	7.9×10^{-05}
$h^2\Omega_c$	0.01122	0.00080	0.00070	0.00011	8.2×10^{-05}
$h^2\Omega_\nu$	0.0027	0.0036	0.0011	0.00035	0.00013
w_0	−0.95	0.084	0.042	0.0027	0.0015
w_a	0	0.084	0.057	0.036	0.015
FOM	⋯	8.21	25.81	732.97	2909.13
Relative FOM [a]	⋯	1	3.14	89.3	354.34

[a] Relative FOM with respect to Planck (T + P) without CMB lensing.

In this case, we tested the analysis for a maximum multipole of 3000 and no significant change was found, confirming that both larger and smaller multipoles do not give a significant contribution to the results (see [18]). The photo-z error is assumed to be normally distributed with variance $\sigma_z = 0.005(1 + z)$. It is important to point out that non-Gaussian errors can be significant in the measurements of weak lensing, degrading the signal-to-noise ratio of the convergence power spectrum [56] and therefore the marginalized errors on individual parameters by a few percent [57].

IV. RESULTS

We performed the forecast for Planck alone, with and without considering CMB lensing. Moreover, we introduced the Euclid forecast, combining the results approximately as

$$F_{ij}^{\text{Total}} = F_{ij}^{\text{Planck}} + F_{ij}^{\text{Euclid (BAO)}} + F_{ij}^{\text{Euclid (WL)}}. \quad (48)$$

It was shown by Hollenstein *et al.* [18] that the covariance between the measurements of cosmic shear tomography and the CMB lensing can be safely neglected since

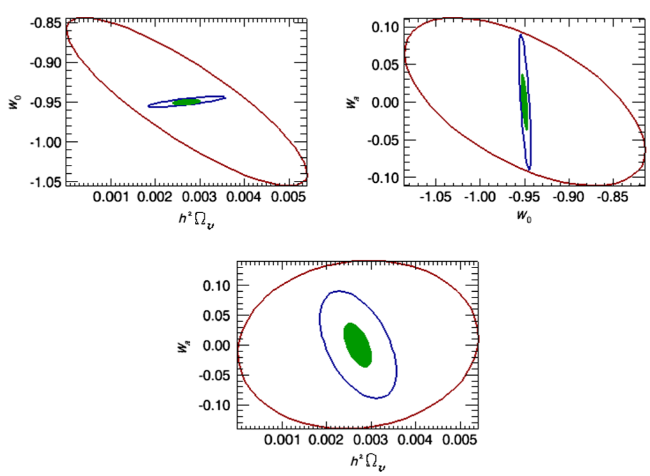

FIG. 2 (color online). Fisher contours for our fiducial model. The contours represent 95.4% C.L. for CMB (red outer ellipse), for the Euclid galaxy survey (blue), and for the combination of CMB + Euclid (filled green inner ellipse) (see Table IV).

the redshifts in which they are probed are quite distinct from each other.

We found the best limits for the neutrino density, w_0 and w_a in the combined Planck (with lensing) + Euclid (BAO + WL). We have that $0.00244 < h^2\Omega_\nu < 0.00296$, $-0.953 < w_0 < -0.947$, and $-0.03 < w_a < 0.03$ (95% C.L.) as can be inferred from column 6 of Table IV. The figure of merit (FOM), described as the reciprocal of the 95% confidence limit's area of the error ellipse from the plane w_0-w_a [58], of Euclid (BAO + WL), is a factor of ∼29 higher than the FOM for Planck alone, even when CMB lensing is considered. Euclid will be able to strongly constrain the late-Universe parameters. The combined result Planck (with lensing) + Euclid (BAO + WL) still improves the FOM with respect to Euclid (BAO + WL) in a factor of ∼3. In Fig. 2 we show the two sigma Fisher contours.

In Table V we show how CMB lensing could affect the parameter constraints for an almost ideal experiment. In this case, we see a substantial improvement of about six times in the FOM when CMB lensing is considered. Comparing also with the Planck experiment, the nearly ideal experiment improves the FOM by a factor of 5 without considering lensing information in any case. For the neutrino parameter, we have that $h^2\Omega_\nu < 0.005872$ without lensing and $h^2\Omega_\nu < 0.003113$ when CMB lensing is considered. On the other hand, the use of the cross power spectrum between the deflection angle and the E mode polarization makes no significant impact on any cosmological parameter constraint. Figure 3 shows the Fisher contours for the unlensed and lensed analysis. Note that the two sigma contour obtained when we use C^{dd} and C^{td} power spectra overlaps the two sigma contour when we also add the C^{ed} power spectrum.

V. DISCUSSION AND CONCLUSIONS

We are considering massive neutrinos and a time-evolving equation of state in the ΛCDM model. Using the Fisher formalism, we obtained the best constraints possible for $h^2\Omega_\nu$, w_0, and w_a considering the Planck and Euclid surveys.

TABLE V. Marginalized errors for ΛCDM model with two massive neutrinos with identical mass and one massless in an inverted hierarchy mass splitting ($m_\nu = 0.125$ eV) for a nearly perfect experiment.

Parameter	Fiducial value	T + P (unlensed)	T + P + lens (C^{dd} and C^{td})	T + P lens (C^{dd}, C^{td} and C^{ed})
$h^2\Omega_b$	0.02219	2.4778×10^{-05}	2.2296×10^{-05}	2.2285×10^{-05}
$h^2\Omega_c$	0.01122	0.0004420	0.0003073	0.0003071
$h^2\Omega_\nu$	0.0027	0.0015860	0.0002065	0.0002064
w_0	−0.95	0.0338432	0.0142514	0.0142416
w_a	0	0.0338432	0.0238881	0.0238811
FOM	...	41.83	255.43	255.63
Relative FOM	...	1	6.106	6.111

One of the goals of this work has been to quantify the influence of CMB lensing information in the constraints of the parameters of interest, especially $h^2\Omega_\nu$, w_0, and w_a. We saw on columns 3 and 4 from Table IV that we improve the constraints in basically all the studied cosmological parameters. The improvement found on $h^2\Omega_\nu$ for the Planck one sigma error alone varied from approximately 133% to 40% from its fiducial value without using CMB lensing and using CMB lensing information, respectively. For w_0 the error is 4.4% smaller when CMB lensing is taken into consideration. The two sigma constraint on w_a from Planck alone varies from $-0.168 < w_a < 0.168$ (without CMB lensing) and $-0.114 < w_a < 0.114$.

When we add Euclid information to Planck information, we get an impressive improvement of 128.2% on the 1 sigma error of $h^2\Omega_\nu$ considering the results from Planck without lensing and Planck (+lensing) + Euclid (BAO + WL). An improvement of approximately 9% in the error of w_0 was found when including Euclid information to the Planck forecast.

In the case of a nearly perfect CMB experiment, as mentioned before, CMB lensing improved all the constraints of the tested parameters. It is clear from the analysis that CMB lensing can play an important role in constraining cosmological parameters in future CMB experiments and must be taken into account. On the other hand, the C^{ed} power spectra can be safely neglected in near future CMB experiments since their contribution to the parameters constraints is minimum.

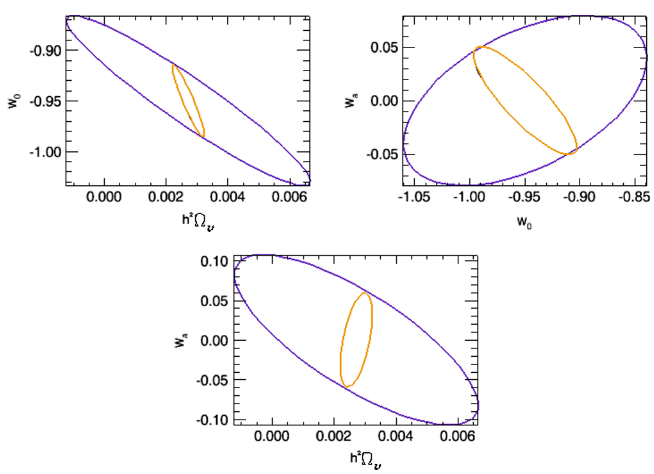

FIG. 3 (color online). Fisher contours with and without lensing information in orange (inner ellipse) and purple (outer ellipse), respectively. The contours represent 95.4% C.L. for CMB (see Table V).

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APPENDIX A: ELEMENTS OF THE UNLENSED COVARIANCE MATRIX

The elements of the covariance matrix in the unlensed case are

$$\Xi_{\mathrm{TTTT}} = (C_l^{\mathrm{TT}} + N_l^{\mathrm{TT}})^2, \tag{A1}$$

$$\Xi_{\mathrm{EEEE}} = (C_l^{\mathrm{EE}} + N_l^{\mathrm{PP}})^2, \tag{A2}$$

$$\Xi_{\mathrm{BBBB}} = (C_l^{\mathrm{BB}} + N_l^{\mathrm{PP}})^2, \tag{A3}$$

$$\Xi_{\mathrm{TETE}} = (C_l^{\mathrm{TE}})^2 + (C_l^{\mathrm{TT}} + N_l^{\mathrm{TT}})(C_l^{\mathrm{EE}} + N_l^{\mathrm{PP}}), \tag{A4}$$

$$\Xi_{\mathrm{TTEE}} = (C_l^{\mathrm{TE}})^2, \tag{A5}$$

$$\Xi_{\mathrm{TTTE}} = C_l^{\mathrm{TE}}(C_l^{\mathrm{TT}} + N_l^{\mathrm{TT}}), \tag{A6}$$

$$\Xi_{\mathrm{EETE}} = C_l^{\mathrm{TE}}(C_l^{\mathrm{EE}} + N_l^{\mathrm{PP}}), \tag{A7}$$

$$\Xi_{\mathrm{TTBB}} = \Xi_{\mathrm{EEBB}} = \Xi_{\mathrm{TEBB}} = 0. \tag{A8}$$

APPENDIX B: ELEMENTS OF THE LENSED COVARIANCE MATRIX

First of all, we make use of the effective χ^2 defined in Eq. (3.3) of [38]:

$$\chi^2_{\rm eff} = \sum_l (2l+1)\left(\frac{D}{|C|} + \ln\frac{|C|}{|\hat{C}|} - 3\right), \tag{B1}$$

where is our case D is defined to be

$$\begin{aligned} D = {} & \hat{C}^{\rm TT}C^{\rm EE}C^{dd}C^{\rm BB} + C^{\rm TT}\hat{C}^{\rm EE}C^{dd}C^{\rm BB} + C^{\rm TT}C^{\rm EE}\hat{C}^{dd}C^{\rm BB} + C^{\rm TT}C^{\rm EE}C^{dd}\hat{C}^{\rm BB} + 2(\hat{C}^{\rm TE}C^{Ed}C^{Td}C^{\rm BB} \\ & + C^{\rm TE}\hat{C}^{Ed}C^{Td}C^{\rm BB} + C^{\rm TE}C^{Ed}\hat{C}^{Td}C^{\rm BB} + C^{\rm TE}C^{Ed}C^{Td}\hat{C}^{\rm BB}) - C^{Ed}(\hat{C}^{\rm TT}C^{\rm BB}C^{Ed} \\ & + C^{\rm TT}\hat{C}^{\rm BB}C^{Ed} + 2C^{\rm TT}C^{\rm BB}\hat{C}^{Ed}) - C^{\rm TE}(\hat{C}^{dd}C^{\rm BB}C^{\rm TE} + C^{dd}\hat{C}^{\rm BB}C^{\rm TE} + 2C^{dd}C^{\rm BB}\hat{C}^{\rm TE}) \\ & - C^{Td}(\hat{C}^{\rm EE}C^{\rm BB}C^{Td} + C^{\rm EE}\hat{C}^{\rm BB}C^{Td} + 2C^{\rm EE}C^{\rm BB}\hat{C}^{Td}, \end{aligned} \tag{B2}$$

and $|C|$, $|\hat{C}|$ are the determinants of the theoretical and observed data covariance matrices

$$|\hat{C}| = \hat{C}^{\rm TT}\hat{C}^{\rm EE}\hat{C}^{dd}\hat{C}^{\rm BB} + 2\hat{C}^{\rm TE}\hat{C}^{Ed}\hat{C}^{Td}\hat{C}^{\rm BB} - \hat{C}^{\rm TT}\hat{C}^{\rm BB}(\hat{C}^{Ed})^2 - \hat{C}^{dd}\hat{C}^{\rm BB}(\hat{C}^{\rm TE})^2 - \hat{C}^{\rm EE}\hat{C}^{\rm BB}(\hat{C}^{Td})^2, \tag{B3}$$

$$|C| = C^{\rm TT}C^{\rm EE}C^{dd}C^{\rm BB} + 2C^{\rm TE}C^{Ed}C^{Td}C^{\rm BB} - C^{\rm TT}C^{\rm BB}(C^{Ed})^2 - C^{dd}C^{\rm BB}(C^{\rm TE})^2 - C^{\rm EE}C^{\rm BB}(C^{Td})^2. \tag{B4}$$

The theoretical covariance matrix M is given by

$$M = \begin{bmatrix} C^{\rm TT} & C^{\rm TE} & C^{Td} & 0 \\ C^{\rm TE} & C^{\rm EE} & C^{Ed} & 0 \\ C^{Td} & C^{Ed} & C^{dd} & 0 \\ 0 & 0 & 0 & C^{\rm BB} \end{bmatrix}. \tag{B5}$$

The Fisher matrix information is then derived from the second order derivative of the likelihood function, L, from an observing data set x given the real parameters $p_1, p_2, p_3, \ldots, p_n$:

$$F_{ij} = -\left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle_x. \tag{B6}$$

Knowing that $\chi^2_{\rm eff} \equiv -2\ln L$, we derived the new elements for the covariance matrix Eq. (9).

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