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$H \to Z\gamma$ in gauge-Higgs unification

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In our previous paper [N. Maru and N. Okada, Phys. Rev. D **87**, 095019 (2013)], we have investigated effects of a simple gauge-Higgs unification model on the diphoton signal events from the Higgs boson production at the Large Hadron Collider. We have found that in this model the effective Higgs-to-diphoton coupling can be enhanced by 1-loop corrections with Kaluza-Klein (KK) modes of bulk fields. This result can be an explanation for the observed excess of the signal strength in the Higgs-to-diphoton decay channel. In this paper, we investigate KK-mode effects on another Higgs boson decay mode, $H \to Z\gamma$. One naturally expects that the KK modes also contribute to the effective H-Z-γ coupling and can cause some deviation for the $H \to Z\gamma$ decay mode from the Standard Model prediction. Revealing a correlation between the KK-mode effects on the Higgs-to-diphoton and H-Z-γ couplings is an interesting topic in terms of a possibility to discriminate a variety of models for physics beyond the Standard Model. We show a very striking result for the gauge-Higgs unification model, namely, the absence of KK-mode contributions to the H-Z-γ coupling at the 1-loop level. This is a very specific and general prediction of the gauge-Higgs unification scenario.

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As announced on July 4, 2012, the long-sought Higgs boson was finally discovered by ATLAS [1] and CMS [2] collaborations at the Large Hadron Collider. The discovery is based on the Higgs boson search with a variety of Higgs boson decay modes. Although the observed data were mostly consistent with the Standard Model (SM) expectations, the diphoton decay mode showed the signal strength considerably larger than the SM prediction. Since the effective Higgs-to-diphoton coupling is induced at the quantum level even in the SM, a certain new physics can significantly affect the coupling. This fact motivated many recent studies for a possible explanation of the excess in the Higgs-to-diphoton decay mode by various extensions of the SM with supersymmetry [3] or without supersymmetry [4,5]. Although the updated CMS analysis [6] gives a much lower value for the signal strength of the diphoton events than the previous one, the updated ATLAS analysis [7] is still consistent with their earlier result. The excess may persist in future updates.

Gauge-Higgs unification (GHU) [8] is one of the fascinating scenarios for physics beyond the SM, which can provide us a solution to the gauge hierarchy problem without invoking supersymmetry. In this scenario, the SM Higgs doublet is identified with an extra spatial component of a gauge field in higher dimensional gauge theory. Nevertheless the scenario is nonrenormalizable, the higher dimensional gauge symmetry allows us to predict various finite physical observables such as Higgs potential [9,10], $H \to gg$, $\gamma\gamma$ [11,12], the anomalous magnetic moment $g - 2$ [13], and the electric dipole moment [14].

In our previous paper [4], we have shown that the excess of the Higgs-to-diphoton decay mode can be explained by 1-loop corrections via Kaluza-Klein (KK) modes in a simple extension of the five-dimensional minimal GHU model supplemented by color-singlet bulk fermions with a half-periodic boundary condition and appropriately chosen electric charges. Through analysis of the renormalization group equation of the Higgs quartic coupling with the so-called gauge-Higgs boundary condition [15], it has been shown that the bulk fermions have also played a crucial role in achieving the observed Higgs boson mass around 125 GeV. Since the KK modes have the electroweak charges, we naturally expect that 1-loop diagrams with the KK modes also affect the effective H-Z-γ coupling and deviate the Higgs boson partial decay width to $Z\gamma$ from the SM prediction. It is an interesting topic to reveal a correlation between the KK-mode effects on the decay modes of $H \to \gamma\gamma$ and $H \to Z\gamma$, because the correlation may show a model-dependent specific property. If the excess of the Higgs-to-diphoton decay mode persists, the correlation between the deviations from the SM predictions for the two decay modes can be a clue to distinguish scenarios beyond the SM, providing a significant improvement of the sensitivity for the Higgs boson signals in the future.

The purpose of this paper is to study the KK-mode contributions to the Higgs-to-$Z\gamma$ decay in the context of GHU, which has not been addressed in the previous paper [4]. Our result is very striking; namely, contributions via the KK modes at the 1-loop level do not exist and hence the effective Higgs-to-$Z\gamma$ coupling remains unchanged, irrespective of the effective Higgs-to-diphoton coupling. This result is quite specific for the GHU scenario, which we have never seen in models beyond the SM. The reason is the following. In the GHU scenario, the electroweak symmetry breaking causes a mass splitting between degenerate KK modes, which one may think analogous to the left-right mixing between sfermions in supersymmetric theories. As we will explicitly show below, in the mass eigenstate basis, the Z boson always has couplings with two different mass eigenstates corresponding to the splitting mass eigenvalues, while the Higgs boson and photon couple with the same mass eigenstates. As a result, there is no 1-loop diagram with the KK modes for the effective Higgs-to-$Z\gamma$ coupling. This specific structure originates from the basic structure of the GHU scenario, where the SM gauge group is embedded in a larger gauge group in extra dimensions and the SM Higgs doublet is identified with the higher dimensional component of the bulk gauge field.

Now we study the KK-mode contributions to the effective Higgs-to-$Z\gamma$ coupling. In order to show essential points of our discussion, let us consider a toy model of five-dimensional $SU(3)$ GHU with an orbifold S^1/Z_2 compactification for the fifth dimension. Although the following discussion is only for this toy model, we expect that our conclusion is applicable to any model of the GHU scenario. In the toy model, the $SU(3)$ gauge symmetry is broken to the electroweak gauge group $SU(2)\times U(1)$ by orbifolding on S^1/Z_2 and adopting a nontrivial Z_2 parity assignment on five-dimensional bulk $SU(3)$ gauge field. The remaining gauge symmetry $SU(2)\times U(1)$ is supposed to be radiatively broken by the vacuum expectation value of the zero mode of A_5, which is identified with the SM Higgs doublet field.

In the toy model, we introduce a bulk fermion of an $SU(3)$ triplet (Ψ) and the basic Lagrangian of the model is simply expressed as

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}(F_{MN}F^{MN}) + i\bar{\Psi}\not{D}\Psi, \tag{1}$$

where the gamma matrix in five-dimensional theory is defined as $\Gamma^M = (\gamma^\mu, i\gamma^5)$,

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig_5[A_M, A_N] \quad (M, N = 0,1,2,3,5), \tag{2}$$

$$\not{D} = \Gamma^M(\partial_M - ig_5 A_M) \left(A_M = A_M^a \frac{\lambda^a}{2} (\lambda^a\text{: Gell-Mann matrices})\right), \tag{3}$$

$$\Psi = (\psi_1, \psi_2, \psi_3)^T. \tag{4}$$

Here, the periodic boundary condition along S^1 is imposed for all fields.[1] The nontrivial Z_2 parities are assigned for each field using the orbifolding matrix $P = \mathrm{diag}(-,-,+)$,

$$A_\mu(-y) = PA_\mu(y)P^{-1}, \qquad A_5(-y) = -PA_5(y)P^{-1}, \qquad \Psi(-y) = P\gamma^5\Psi(y). \tag{5}$$

By these boundary conditions, the $SU(3)$ gauge symmetry is broken to the electroweak gauge group of $SU(2)\times U(1)$.

For the zero mode of the bosonic sector, we obtain exactly what we need for the SM:

$$\begin{aligned}
A_\mu^{(0)} &= \frac{1}{2}\begin{pmatrix} W_\mu^3 + 2t_W\left(\frac{1}{6}\right)B_\mu & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + 2t_W\left(\frac{1}{6}\right)B_\mu & 0 \\ 0 & 0 & 2t_W\left(-\frac{1}{3}\right)B_\mu \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} \left(c_W - \frac{s_W t_W}{3}\right)Z_\mu + \frac{4}{3}s_W\gamma_\mu & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -\left(c_W + \frac{s_W t_W}{3}\right)Z_\mu - \frac{2}{3}s_W\gamma_\mu & 0 \\ 0 & 0 & \frac{s_W t_W}{3}Z_\mu - \frac{2}{3}s_W\gamma_\mu \end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix} \frac{2}{\sqrt{3}}\gamma_\mu & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -Z_\mu - \frac{1}{\sqrt{3}}\gamma_\mu & 0 \\ 0 & 0 & Z_\mu - \frac{1}{\sqrt{3}}\gamma_\mu \end{pmatrix},
\end{aligned} \tag{6}$$

$$A_5^{(0)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & h^+ \\ 0 & 0 & h^0 \\ h^- & h^{0*} & 0 \end{pmatrix}, \tag{7}$$

[1] It is possible to impose a half-periodic boundary condition for the bulk fermion. For both cases with the periodic and the half-periodic boundary conditions, the structure of the couplings between the Z boson and KK modes are the same, leading us to the same conclusion. Thus, in the following, we explicitly show our results only for the bulk fermion with the periodic boundary condition.

where W^3_μ, $W^\pm_\mu$ and B_μ are the $SU(2)$ and $U(1)$ gauge bosons in the SM; $h = (h^+, h^0)^T$ is the Higgs doublet field; and $t_W \equiv \tan\theta_W$, $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, with the weak mixing angle θ_W. Note that in this toy model the $SU(2)\times U(1)$ gauge groups are unified into a single $SU(3)$ and an unrealistic weak mixing angle, $\sin^2\theta_W = \frac{3}{4}$, is predicted. The zero mode of the bulk fermion is decomposed into an $SU(2)$ doublet left-handed fermion and an $SU(2)$ singlet right-handed fermion. Their $U(1)$ charges are the same as those of the quark $SU(2)$ doublet and the $SU(2)$ singlet down-type quark. Thus, if the bulk fermion is a QCD color triplet, we may express it as $\Psi^{(0)} = (u_L, d_L, d_R)^T$. However, for our purpose, such an identification of the zero-mode fermion with a SM fermion is not important. We refer, for example, to Ref. [16] for a realistic setup of a five-dimensional GHU scenario.

We are interested in a four-dimensional effective Lagrangian for the KK-mode fermions derived from Eq. (1). The bulk field with an even (odd) parity is expanded by the KK-mode function $\cos(ny/R)$ $[\sin(ny/R)]$ with an integer n and the radius R of the S^1. Substituting the KK-mode expansion into Eq. (1) and integrating the KK-mode functions over the fifth dimensional coordinate (y), we obtain a four-dimensional effective Lagrangian. The four-dimensional effective Lagrangian relevant to our discussion is given by

$$\mathcal{L}^{(4D)}_{\text{fermion}} \supset \sum_{n=1}^{\infty}\left\{ i(\bar\psi_1^{(n)}\bar\psi_2^{(n)}\bar\psi_3^{(n)})\gamma^\mu\partial_\mu\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix} + \frac{g_4}{2}(\bar\psi_1^{(n)},\bar\psi_2^{(n)},\bar\psi_3^{(n)})\begin{pmatrix}\frac{2}{\sqrt3}\gamma_\mu & \sqrt2 W^+_\mu & 0\\ \sqrt2 W^-_\mu & -Z_\mu-\frac{1}{\sqrt3}\gamma_\mu & 0\\ 0 & 0 & Z_\mu-\frac{1}{\sqrt3}\gamma_\mu\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}\right.$$
$$\left. -(\bar\psi_1^{(n)},\bar\psi_2^{(n)},\bar\psi_3^{(n)})\begin{pmatrix}m_n & 0 & 0\\ 0 & m_n & -m\\ 0 & -m & m_n\end{pmatrix}\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix}\right\}, \tag{8}$$

where $\psi_i^{(n)}$ is the nth KK-mode Dirac fermion, $m_n = \frac{n}{R}$ is the KK-mode mass, $g_4 = \frac{g_5}{\sqrt{2\pi R}}$ is the four-dimensional gauge coupling, and $m = \frac{g_4 v}{2} = m_W$ is the mass associated with the electroweak symmetry breaking by the vacuum expectation value of the Higgs doublet (v). In this toy model, m is identical to the W-boson mass (m_W) because of the unification of the gauge and Yukawa interactions in the GHU scenario. In deriving the mass matrix in Eq. (8), we have used a chiral rotation

$$\psi_{1,2,3} \to e^{-i\frac{\pi}{4}\gamma_5}\psi_{1,2,3} \tag{9}$$

in order to get rid of $i\gamma_5$.

We easily diagonalize the mass matrix for the KK-mode fermions by use of the mass eigenstates $\tilde\psi_2^{(n)}$, $\tilde\psi_3^{(n)}$,

$$\begin{pmatrix}\psi_1^{(n)}\\ \tilde\psi_2^{(n)}\\ \tilde\psi_3^{(n)}\end{pmatrix} = U\begin{pmatrix}\psi_1^{(n)}\\ \psi_2^{(n)}\\ \psi_3^{(n)}\end{pmatrix},\qquad U = \frac{1}{\sqrt2}\begin{pmatrix}\sqrt2 & 0 & 0\\ 0 & 1 & -1\\ 0 & 1 & 1\end{pmatrix}, \tag{10}$$

as

$$U\begin{pmatrix}m_n & 0 & 0\\ 0 & m_n & -m\\ 0 & -m & m_n\end{pmatrix}U^\dagger = \begin{pmatrix}m_n & 0 & 0\\ 0 & m_n+m & 0\\ 0 & 0 & m_n-m\end{pmatrix}. \tag{11}$$

In terms of the mass eigenstates for the KK modes, the Lagrangian is described as

$$\mathcal{L}^{(4D)}_{\text{fermion}} \supset \sum_{n=1}^{\infty}\left\{ (\bar\psi_1^{(n)}\bar{\tilde\psi}_2^{(n)}\bar{\tilde\psi}_3^{(n)})\begin{pmatrix}i\gamma^\mu\partial_\mu - m_n & 0 & 0\\ 0 & i\gamma^\mu\partial_\mu-(m_n+m) & 0\\ 0 & 0 & i\gamma^\mu\partial_\mu-(m_n-m)\end{pmatrix}\begin{pmatrix}\psi_1^{(n)}\\ \tilde\psi_2^{(n)}\\ \tilde\psi_3^{(n)}\end{pmatrix}\right.$$
$$\left. +\frac{g_4}{2}(\bar\psi_1^{(n)},\bar{\tilde\psi}_2^{(n)},\bar{\tilde\psi}_3^{(n)})\begin{pmatrix}\frac{2\gamma_\mu}{\sqrt3} & W^+_\mu & W^+_\mu\\ W^-_\mu & -\frac{1}{\sqrt3}\gamma_\mu & -Z_\mu\\ W^-_\mu & -Z_\mu & -\frac{1}{\sqrt3}\gamma_\mu\end{pmatrix}\gamma^\mu\begin{pmatrix}\psi_1^{(n)}\\ \tilde\psi_2^{(n)}\\ \tilde\psi_3^{(n)}\end{pmatrix}\right\}. \tag{12}$$

Now we can see that a Z boson (W boson) couples with two different mass eigenstates corresponding to the splitting mass eigenvalues $m_n \pm m$,[2] while the photon couples to the same mass eigenstates as expected from the electromagnetic $U(1)$ gauge symmetry. The coupling between the Higgs boson and the mass eigenstates is found by the replacement $m \to m + \frac{m}{v} H$, where H is the physical Higgs boson. Similarly to the photon coupling, the Higgs boson couples to the same mass eigenstates. From the structure of the couplings of the KK-mode mass eigenstates with the Z boson, photon and Higgs boson, we conclude that there is no KK fermion contribution to the effective Higgs-to-$Z\gamma$ coupling at the 1-loop level. This result is very typical for the GHU scenario, distinguishable from other models beyond the SM. Although calculations are more involved, we can show that the structure of the KK-mode couplings with the Z boson, photon and Higgs boson is the same also for bulk fermions of higher dimensional $SU(3)$ representations.

Our next interest is focused on the contribution by the KK-mode W bosons to the effective Higgs-to-$Z\gamma$ couplings. There are two types of interactions of the KK W bosons involving one Z boson. One is the three-point vertex between two KK W bosons and one Z boson, and the other is the four-point vertex among two KK W bosons, one Z boson and one photon. In order to give explicit expressions for the vertices, we first need to find mass eigenstates of the KK W bosons. For this purpose, we write a matrix form of the five-dimensional $SU(3)$ gauge boson as

$$A_\mu = \frac{1}{2}\begin{pmatrix} \frac{2}{\sqrt{3}}\gamma_\mu & \sqrt{2}W^+_\mu & \sqrt{2}A^+_\mu \\ \sqrt{2}W^-_\mu & -Z_\mu - \frac{1}{\sqrt{3}}\gamma_\mu & * \\ \sqrt{2}A^-_\mu & * & Z_\mu - \frac{1}{\sqrt{3}}\gamma_\mu \end{pmatrix}, \tag{13}$$

where the (1, 3) and (3, 1) elements are parity-odd charged gauge bosons whose KK modes are mixed with the KK W boson to yield the mass eigenstates. In the same way, the (2, 3) and (3, 2) elements are parity-odd neutral gauge bosons corresponding to the KK modes of Z and photon. Since they are irrelevant to our discussion, we have omitted them by the symbol "*" in the matrix.

Using the KK-mode decomposition, we extract the mass terms for the KK W and A gauge bosons from the Lagrangian with the gauge fixing term as (after the electroweak symmetry breaking)

[2]Because of the coupling of the $Z(W)$ boson to fermions with two different mass eigenstates, one might worry about large contributions to electroweak precision measurements; for instance, the KK fermion contributes to the T parameter. However, such contributions are suppressed by a factor $(m_W/m_{KK})^2$ and can be safely neglected with a typical KK-mode mass scale m_{KK} being of $\mathcal{O}(1\ \mathrm{TeV})$.

$$\int_{-\pi R}^{\pi R} dy\Big[-\frac{1}{2}(F^a_{\mu 5})^2 - \frac{1}{2\xi}(\partial^\mu A^a_\mu - \xi\partial^5 A^a_5)^2\Big] \supset -\sum_{n=0}^{\infty}(m_n^2 + m_W^2)W^{\mu+(n)}W^{-(n)}_\mu - \sum_{n=1}^{\infty}(m_n^2 + m_W^2) \times A^{\mu+(n)}A^{-(n)}_\mu - i2m_W\sum_{n=1}^{\infty} m_n(W^{\mu+(n)}A^{-(n)}_\mu - W^{\mu-(n)}A^{+(n)}_\mu). \tag{14}$$

The mass matrix for the KK modes of $W^{\pm(n)}_\mu$ and $A^{\pm(n)}_\mu$,

$$(W^{\mu-(n)} A^{\mu-(n)})\begin{pmatrix} m_n^2 + m_W^2 & 2im_W m_n \\ -2im_W m_n & m_n^2 + m_W^2 \end{pmatrix}\begin{pmatrix} W^{+(n)}_\mu \\ A^{+(n)}_\mu \end{pmatrix}, \tag{15}$$

can be easily diagonalized by the following unitary matrix:

$$V = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \tag{16}$$

as

$$(P^{\mu-(n)} N^{\mu-(n)})\begin{pmatrix} (m_n + m_W)^2 & 0 \\ 0 & (m_n - m_W)^2 \end{pmatrix}\begin{pmatrix} P^{+(n)}_\mu \\ N^{+(n)}_\mu \end{pmatrix}. \tag{17}$$

Here, the mass eigenstates are defined as

$$\begin{pmatrix} P^{\pm(n)}_\mu \\ N^{\pm(n)}_\mu \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} W^{\pm(n)}_\mu \\ A^{\pm(n)}_\mu \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} W^{\pm(n)}_\mu + iA^{\pm(n)}_\mu \\ iW^{\pm(n)}_\mu + A^{\pm(n)}_\mu \end{pmatrix}. \tag{18}$$

Due to the electroweak symmetry breaking, the KK-mode mass eigenvalue m_n splits into $m_n \pm m_W$.

Now we express the three-point and four-point vertices in terms of these mass eigenstates. For the three-point vertex, we find

$$\int_{-\pi R}^{\pi R} dy \mathrm{Tr}[(\partial_\mu A_\nu - \partial_\nu A_\mu)[A_\mu, A_\nu]] \supset 4Z^\mu\sum_{n=1}^{\infty}[W^{-(n)}_{\mu\nu}W^{\nu+(n)} - W^{+(n)}_{\mu\nu}W^{\nu-(n)} + A^{+(n)}_{\mu\nu}A^{\nu-(n)} - A^{-(n)}_{\mu\nu}A^{\nu+(n)}] = 4iZ^\mu\sum_{n=1}^{\infty}(P^{+(n)}_{\mu\nu}N^{\nu-(n)} - P^{-(n)}_{\mu\nu}N^{\nu+(n)} + N^{-(n)}_{\mu\nu}P^{\nu+(n)} - N^{+(n)}_{\mu\nu}P^{\nu-(n)}), \tag{19}$$

while for the four-point vertex,

$$\int_{-\pi R}^{\pi R} dy\,\mathrm{Tr}[A_\mu, A_\nu]^2 \supset 8\sqrt{3}\sum_{n=1}^{\infty}[\gamma^\nu Z^\mu (W_\mu^{+(n)}W_\nu^{-(n)} + W_\mu^{-(n)}W_\nu^{+(n)}) - 4\gamma^\mu Z_\mu(-W^{\nu+(n)}W_\nu^{-(n)} + A^{\nu+(n)}A_\nu^{-(n)})$$
$$+ 2\gamma^\nu Z^\mu (A_\mu^{+(n)}A_\nu^{-(n)} + A_\mu^{-(n)}A_\nu^{+(n)})]$$
$$= 8\sqrt{3}i\sum_{n=1}^{\infty}[\gamma^\nu Z^\mu(-P_\mu^{-(n)}N_\nu^{+(n)} + N_\mu^{-(n)}P_\nu^{+(n)} - P_\mu^{+(n)}N_\nu^{-(n)} + N_\mu^{+(n)}P_\nu^{-(n)})$$
$$+ 2\gamma^\mu Z_\mu (P^{\nu-(n)}N_\nu^{+(n)} - N^{\nu-(n)}P_\nu^{+(n)})]. \quad (20)$$

Here, $P_{\mu\nu}^{\pm(n)} \equiv \partial_\mu P_\nu^{\pm(n)} - \partial_\nu P_\mu^{\pm(n)}$ and $N_{\mu\nu}^{\pm(n)} \equiv \partial_\mu N_\nu^{\pm(n)} - \partial_\nu N_\mu^{\pm(n)}$. Therefore, the three-point and four-point vertices with one Z boson always involve two different mass eigenstates $P_\mu^{\pm(n)}$ and $N_\mu^{\pm(n)}$ corresponding to the splitting mass eigenvalues, as in the case of the KK fermions.

In the same way, we express the three-point vertex with one photon and the four-point vertex with two photons in terms of the mass eigenstates. The three-point vertex is given by

$$\int_{-\pi R}^{\pi R} dy\,\mathrm{Tr}[(\partial_\mu A_\nu - \partial_\nu A_\mu)[A_\mu, A_\nu]] \supset 4\sqrt{3}\gamma^\mu \sum_{n=1}^{\infty}[W_{\mu\nu}^{-(n)}W^{\nu+(n)} - W_{\mu\nu}^{+(n)}W^{\nu-(n)} + A_{\mu\nu}^{-(n)}A^{\nu-(n)} - A_{\mu\nu}^{+(n)}A^{\nu+(n)}]$$
$$= 4\sqrt{3}\gamma^\mu \sum_{n=1}^{\infty}[P_{\mu\nu}^{-(n)}P^{\nu+(n)} - P_{\mu\nu}^{+(n)}P^{\nu-(n)} + N_{\mu\nu}^{-(n)}N^{\nu+(n)} - N_{\mu\nu}^{+(n)}N^{\nu-(n)}], \quad (21)$$

while for the four-point vertex,

$$\int_{-\pi R}^{\pi R} dy\,\mathrm{Tr}[A_\mu, A_\nu]^2 \supset 12\Big[2\gamma^\mu\gamma_\mu \sum_{n=1}^{\infty}(W^{\nu+(n)}W_\nu^{-(n)} + A^{\nu+(n)}A_\nu^{-(n)}) + \gamma^\mu\gamma^\nu \sum_{n=1}^{\infty}(W_\mu^{+(n)}W_\nu^{-(n)} + W_\mu^{-(n)}W_\nu^{+(n)}$$
$$+ A_\mu^{+(n)}A_\nu^{-(n)} + A_\mu^{-(n)}A_\nu^{+(n)})\Big]$$
$$= 12\Big[2\gamma^\mu\gamma_\mu \sum_{n=1}^{\infty}(P^{\nu-(n)}P_\nu^{+(n)} + N^{\nu-(n)}N_\nu^{+(n)}) + \gamma^\mu\gamma^\nu \sum_{n=1}^{\infty}(P_\mu^{+(n)}P_\nu^{-(n)} + P_\mu^{-(n)}P_\nu^{+(n)}$$
$$+ N_\mu^{+(n)}N_\nu^{-(n)} + N_\mu^{-(n)}N_\nu^{+(n)})\Big]. \quad (22)$$

In contrast with the three-point and four-point vertices with one Z boson, the resultant vertices involve only the same mass eigenstates.

The three-point vertex of the mass eigenstates with the Higgs boson is obtained by the replacement $m_W \to m_W + \frac{m_W}{v}H$ in Eq. (17), and hence the structure is the same as the three-point vertex with one photon. Therefore, we arrive at the same conclusion as in the KK fermion case that there is no 1-loop Feynman diagram with the KK gauge bosons for the contribution to the effective Higgs-to-$Z\gamma$ coupling.

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