Bilocal baryon interpolating fields with three flavors

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Fierz identities follow from permutations of quark indices and thus determine which chiral multiplets of baryon fields are Pauli allowed and which are not. In a previous paper we investigate the Fierz identities of baryon fields with two light flavors and find that all bilocal fields that can be constructed from three quarks are Pauli allowed. That does not mean that all possible chiral multiplets exist; however, some chiral multiplets do not appear among structures with a given spin in the local limit, say $J = 1/2$. One such chiral multiplet is the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, which is necessary for a successful chiral mixing phenomenology. In the present paper we extend those methods to three light flavors, i.e., to $SU_F(3)$ symmetry and explicitly construct all three necessary chiral $SU_L(3) \times SU_R(3)$ multiplets, viz. $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$, and $[(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$ that are necessary for a phenomenologically successful chiral mixing. We complete this analysis by considering some bilocal baryon fields that are sufficient for the construction of the "missing" spin-1/2 baryon interpolating fields. Bilocal baryon fields have definite total angular momentum only in the local limit. The physical significance of these results lies in the fact that they show that there is no need for higher Fock space components, such as the $q^4\bar{q}$, in the baryon chiral mixing framework, for the purpose of fitting the observed axial couplings and magnetic moments: all of the sufficient "mirror components" exist as bilocal fields.

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I. INTRODUCTION

It is by now fairly well known that the $SU_L(3) \times SU_R(3)$ chiral symmetry multiplets' mixing successfully describes several basic properties of $J^P = (\frac{1}{2})^+$ baryons, including their Abelian and non-Abelian axial couplings, and their magnetic moments [1–4]. For the phenomenological mixing to work one only needs a few (three, to be precise) out of (five "naive" plus five "mirror" =) ten possible chiral multiplets built from three-quark interpolating fields. Not all ten chiral multiplets exist in the local triquark baryon field limit [1,5,6], however, due (a) to the fact that some chiral structures are not associated with all values of spin and (b) to the Pauli exclusion principle implemented by way of Fierz identities that annihilate certain (local) interpolators corresponding to Pauli-forbidden states. As one relaxes the restriction from strictly local fields [1,5,6], to bilocal [7], and finally trilocal fields [8], one may use the additional spatial degree of freedom to antisymmetrize with, and thus one finds that some previously Pauli-forbidden two-flavor chiral multiplets are allowed in the nonlocal case. In this manner we found that all chiral structures available for a particular "value of spin" are Pauli allowed in the bilocal two-flavor baryon sector. Strictly speaking, rather than the spin it is the Lorentz group representation (LGR) that is important here, as for spins higher than 1/2, there is usually more than one LGR that corresponds to that particular value of spin, Ref. [9].

Moreover, some chiral multiplets appear more than once in the nonlocal case, whereas in the local limit, they were explicitly shown as identical by way of Fierz identities. And yet, it is not always possible to construct all of the "naive," or "mirror" multiplets from three nonlocal quark fields, although generally this can be accomplished using five-quark, i.e., $q^4\bar{q}$ fields. Now, some of the "missing multiplets" can be obtained as by-products of unphysical (spin) degrees of freedom from higher-spin fields' "projecting out" procedure. For example, as a by-product of projecting out the spin-3/2 component from the Rarita-Schwinger (RS) [LGR $(1, 1/2)$] fields, one obtains a spin-1/2 field component with chiral properties that are "opposite"/mirror to those of the spin-3/2 component. This provides the (phenomenologically absolutely necessary) chiral $[(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)]$ multiplet in the $J^P = (\frac{1}{2})^+$ baryon sector, whereas the nonlocal fields provide only the "mirror" chiral $[(\frac{1}{2}, 1) \oplus (1, \frac{1}{2})]$ multiplet.

With three light flavors there is a bigger variety of both flavor and chiral multiplets than with two. For this reason one cannot readily generalize our two-flavor results to three flavors. So, the question remains if all of the phenomenologically necessary $SU_L(3) \times SU_R(3)$ chiral multiplets exist in the three-quark nonlocal case. In particular the question of the so-called "mirror" multiplets' existence is important, as they can be (easily) constructed from (3q + meson) fields but not necessarily from three quarks. If such "mirror" fields exist only in the (3q + meson)

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TABLE I. Structure of all three-quark baryon fields in the local limit, together with their LGR, spin, Young diagram, chiral $SU(2)$ and $SU(3)$ representations, axial $U(1)_A$ charge g_A^0, and their Fierz transformation equivalent fields or vanishing for Pauli-forbidden fields.

Lorentz	Spin	Young diagram for chiral rep.	Chiral $SU(2)$	Chiral $SU(3)$	g_A^0	Fields	Fierz and local lim.
		$([111],\cdots)\oplus(\cdots,[111])$		$(\mathbf{1},\mathbf{1})$	3	$\Lambda_1+\Lambda_2$	0
		$([21],\cdots)\oplus(\cdots,[21])$	$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	3	N_1+N_2 M_5	N_1+N_2
$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	1/2	$([1],[11])\oplus([11],[1])$		$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	-1	$(\Lambda_1-\Lambda_2, N_1-N_2)$ (Λ_3, N_3-M_4)	(Λ_1, N_1-N_2)
		$([1],[2])\oplus([2],[1])$	$(\frac{1}{2},1)\oplus(1,\frac{1}{2})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})$	-1	$(N_3+\frac{1}{3}M_4, \Delta_4)$	0
		$([3],\cdots)\oplus(\cdots,[3])$	$(\frac{3}{2},0)\oplus(0,\frac{3}{2})$	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$	3	Δ_5	0
$(\frac{1}{2},1)\oplus(1,\frac{1}{2})$	1/2 & 3/2	$([11],[1])\oplus([1],[11])$	$(0,\frac{1}{2})\oplus(\frac{1}{2},0)$	$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$	1	$(\Lambda_3^\mu, N_3^\mu-M_4^\mu)$	0
		$([2],[1])\oplus([1],[2])$	$(1,\frac{1}{2})\oplus(\frac{1}{2},1)$	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$	1	$(N_3^\mu+\frac{1}{3}M_4^\mu, \Delta_4^\mu)$ (M_5^μ, Δ_5^μ)	(N_3^μ, Δ_4^μ)
$(\frac{3}{2},0)\oplus(0,\frac{1}{2})$	3/2	$([21],\cdots)\oplus(\cdots,[21])$	$(\frac{1}{2},0)\oplus(0,\frac{1}{2})$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	3	$M_5^{\mu\nu}$	0
		$([3],\cdots)\oplus(\cdots,[3])$	$(\frac{3}{2},0)\oplus(0,\frac{3}{2})$	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$	3	$\Delta_5^{\mu\nu}$	$\Delta_5^{\mu\nu}$

form, then that would be the first indication of a nonexotic "pentaquark" Fock component in the nucleon's wave function. In the present paper we answer that question for $J^P=(\frac{1}{2})^+$ baryons; higher spin objects will not be dealt with here systematically, except for the explicit purpose of providing spin-1/2 components.

In a series of previous papers, Refs. [5–8], we have investigated the Fierz identities and chiral $SU_L(2)\times SU_R(2)$ transformation properties of bilocal baryon fields with two light flavors. In the present paper we extend those methods and results to three light flavors, i.e., to $SU(3)_F$ symmetry.

We note here that this extension to three flavors introduces only a mathematical change to the analogous two-flavor analysis, Refs. [4,10]: the fact that the $SU(3)_F$ symmetry is explicitly broken does not play a role here, because the quark mass difference does not enter into considerations of the permutation symmetry. Rather, it is the very existence of the third flavor that makes the difference. Needless to say, the most remarkable consequences are in the flavor-singlet channel that does not exist with two flavors. Another place where the difference between two and three flavors is pronounced is the flavor-octet chiral multiplets $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ and $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$, both of which are "reduced to" the two-flavor chiral multiplet $[(\frac{1}{2},0)\oplus(0,\frac{1}{2})]$.

Whereas the $SU(3)$ algebra is considerably more complicated than the $SU(2)$ one, the physical results are largely determined by the overall permutation symmetry properties (i.e., the Fierz identities) of the baryon operators, which, in turn, are determined by the chiral $SU_L(3)\times SU_R(3)$ or $SU_L(2)\times SU_R(2)$ multiplets. As the $SU_L(3)\times SU_R(3)$ multiplets contain (several smaller) $SU_L(2)\times SU_R(2)$ multiplets within them that have already been examined in Refs. [7,8], it should come as no surprise that the $SU_L(3)\times SU_R(3)$ "completions" of chiral $SU_L(2)\times SU_R(2)$ multiplets exist as well. Indeed, one may adopt a chiral multiplet nomenclature based on the Young diagrams/tableaux, see Table I, rather than the actual dimensionality of the multiplet, that shows the full analogy of chiral multiplets with different flavor numbers. There is (only) one exception to this $SU(3)$ completion "rule": the flavor-singlet $[(\mathbf{1},\mathbf{1})]$, Λ hyperon that is antisymmetric in flavor space and does not exist with two flavors. It can either belong to a chiral $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ multiplet or to a chiral singlet.

The primary question is then: which chiral multiplets do these ("new") bilocal operators belong to? We investigate all cases and classify the bilocal three-flavor baryon interpolators according to their chiral transformations. Before doing that, we would like to note that the bilocal or trilocal fields have components overlapping with more than one orbital angular momentum L states. To project out definite-J components from these fields, one needs to specify the three-body dynamics. For example, if one wishes to use such fields on the lattice, one can use the Euclidean space version, and the corresponding spin projection methods, such as that in Ref. [11]. However, these operators have definite total angular momentum only in the limit of local fields, and so we shall assume our nonlocal fields have spins $J=1/2$ or $J=3/2$ in the following analysis.

We find three new spin-1/2 chiral multiplets that do not exist in the local-operator limit: one $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$, one $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$, and one $[(\mathbf{1},\mathbf{1})]$, and several other multiplets that used to be related ("identical") by Fierz identities to others, that are independent in the nonlocal case. The chiral transformations do not depend on the (non)locality of the operator but the Fierz identities do. For this reason we concentrate only on the latter in this paper—the $SU_L(3)\times SU_R(3)$ chiral transformations have been worked out in some detail in Ref. [1] and are briefly reviewed in the Appendix. The physical significance of our results is that they show an absence of need for $q^4\bar{q}$ components when fitting the observed axial couplings and magnetic moments in the chiral mixing framework: all of the "mirror components" exist as bilocal fields.

This paper consists of four sections and is organized as follows. After the (present) Introduction in Sec. II, we firstly define all possible "straightforward bilocal extensions" of local baryon operators. There we classify the baryon operators according to the representations of the Lorentz and the flavor groups, viz. the Dirac, the RS, and the antisymmetric tensor (AST) Bargmann-Wigner (BW) fields. Then in Sec. III, we define the "nonstraightforward bilocal extensions" of local baryon operators, such as the derivative-contracted RS and AST fields that appear as by-products of spin-3/2 projecting out. The final Sec. IV is a summary and an outlook to possible future extensions and applications. The Appendix, we define the Abelian and non-Abelian chiral transformations of the baryon operators as functions of the quarks' chiral transformation parameters.

II. STRAIGHTFORWARD THREE-FLAVOR BILOCAL THREE-QUARK FIELDS

Three-quark baryon interpolating fields in QCD have well-defined $SU_L(3) \times SU_R(3)$ and $U_A(1)$ chiral transformation properties, see Table I,

$$[(\mathbf{3},\mathbf{1})\oplus(\mathbf{1},\mathbf{3})]^3 \sim [(\mathbf{1},\mathbf{1})]\oplus[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]\oplus[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})] \oplus[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]\oplus[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})], \quad (1)$$

viz. $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$, $[(\mathbf{1},\mathbf{1})]$, $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$, $[(\mathbf{10},\mathbf{1}) \oplus (\mathbf{1},\mathbf{10})]$, and their "mirror" images, Ref. [1]. It has been shown (phenomenologically) in Ref. [2] that mixing of the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ chiral multiplet with one ordinary ("naive") $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ and one "mirror" field $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$ multiplet can be used to fit the values of the isovector ($g_A^{(3)}$) and the flavor-singlet (isoscalar) axial coupling ($g_A^{(0)}$) of the nucleon and then predict the axial F and D coefficients, or vice versa, in reasonable agreement with the experiment. Moreover, this mixing can be reproduced by a chirally symmetric interaction Lagrangian with observed baryon masses used as the input for unknown coupling constants, Ref. [3], and the anomalous magnetic moments of baryons can be introduced in accordance with chiral symmetry and experimental observations, Ref. [4]. For this reason it is vital that all three of these chiral multiplets are not forbidden by the Pauli principle in the three-quark interpolators. Yet, the original analysis of local three-quark fields, Ref. [1], allowed only one out of three: the ("naive") $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$. In the following we shall explicitly construct the other two interpolators. For that purpose we shall need both the straightforward and the not-so-straightforward extensions of local fields, as the straightforward method yields only the "mirror" field $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$, whereas the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ chiral multiplet appears only as a remnant of the spin-projection procedure in Rarita-Schwinger fields.

Before doing that, we would like to note that the bilocal or trilocal fields contain in general (infinitely many) components overlapping with more than one orbital angular momentum L state. Consequently, these operators have definite total angular momentum J only in the limit of local fields, though individual J components might be extracted by a suitable spin projection. Such a spin-projection technique has been devised for three-quark fields on a Euclidean lattice space-time, Ref. [11], though in a continuum Minkowski space-time, one is better suited by projecting out good-J states in matrix elements, e.g., using the Jacob-Wick formalism, rather than in operators themselves. In order to project out the good-J operators and thus address this "theoretical uncertainty," one has to specify the three-body dynamics explicitly, which is well beyond the scope of this paper.

At any rate, such a total angular momentum projection would not change the Dirac structure of the composite fields, and their chiral properties would remain unchanged as well. Moreover, the existence of the two lowest values ($J = 1/2$ or $J = 3/2$) of the total angular momentum J components in our nonlocal fields is beyond doubt anyway.

A. Dirac fields

In this section we investigate independent baryon fields for each LGR which is formed by three quarks. The Clebsch-Gordan series for the irreducible decomposition of the direct product of three $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$ representations of the Lorentz group (the three-quark Dirac fields) is

$$\left(\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right)^3 \sim \left(\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right)\oplus\left(\left(1,\frac{1}{2}\right)\oplus\left(\frac{1}{2},1\right)\right) \oplus\left(\left(\frac{3}{2},0\right)\oplus\left(0,\frac{3}{2}\right)\right), \quad (2)$$

where we have ignored the different multiplicities of the representations on the right-hand side. Three LGRs $((\frac{1}{2},0) \oplus (0,\frac{1}{2}))$, $((1,\frac{1}{2}) \oplus (\frac{1}{2},1))$, $((\frac{3}{2},0) \oplus (0,\frac{3}{2}))$ describe the Dirac spinor field, the Rarita-Schwinger's vector-spinor field, and the antisymmetric-tensor-spinor field, respectively. In order to establish independent fields we employ the Fierz transformations for the color, flavor, and Lorentz (spin) degrees of freedom, which is essentially equivalent to the Pauli principle for three quarks. Here we demonstrate the essential idea for the simplest case of the Dirac spinor, $(\frac{1}{2},0) \oplus (0,\frac{1}{2})$.

It is convenient to introduce a "tilde-transposed" quark field $\tilde{q}$ as follows:

$$\tilde{q} = q^T C\gamma_5, \quad (3)$$

where $C = i\gamma_2\gamma_0$ is the Dirac field charge conjugation operator.

1. Flavor singlet baryon

Let us start by writing down five trilocal baryon fields that contain a "diquark" operator formed by one of five sets of (products of) Dirac matrices, 1, γ_5, γ_μ, $\gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}$,

$$\begin{aligned}
\Lambda_1(x,y,z) &= \epsilon_{abc}\epsilon^{ABC}(\tilde{q}_A^a(x)q_B^b(y))q_C^c(z),\\
\Lambda_2(x,y,z) &= \epsilon_{abc}\epsilon^{ABC}(\tilde{q}_A^a(x)\gamma_5 q_B^b(y))\gamma_5 q_C^c(z),\\
\Lambda_3(x,y,z) &= \epsilon_{abc}\epsilon^{ABC}(\tilde{q}_A^a(x)\gamma_\mu q_B^b(y))\gamma^\mu q_C^c(z),\\
\Lambda_4(x,y,z) &= \epsilon_{abc}\epsilon^{ABC}(\tilde{q}_A^a(x)\gamma_\mu\gamma_5 q_B^b(y))\gamma^\mu\gamma_5 q_C^c(z),\\
\Lambda_5(x,y,z) &= \epsilon_{abc}\epsilon^{ABC}(\tilde{q}_A^a(x)\sigma_{\mu\nu} q_B^b(y))\sigma_{\mu\nu} q_C^c(z).
\end{aligned} \tag{4}$$

Here and in the following we use the notation and conventions of Sec. II in Ref. [1], where the capital roman letter indices, e.g., A, B, $C = 1, 2, 3$ denote the $SU(3)$ flavor degrees of freedom of a quark, and ϵ^{ABC} is the (Levy-Cività) totally AST. The AST in color space ϵ_{abc} ensures that the baryons are color singlets. Our results are not affected by taking nonlocal baryon operators with path-ordered phase factors

$$\begin{aligned}
B(x_1,x_2,x_3) \sim \epsilon_{abc}(\tilde{q}_{a'}(x_1)q_{b'}(x_2))q_{b'}(x_3)
&\times\left[P\exp\left(ig\int_{x_1}^{z}A_\mu(y_1)dy_1^\mu\right)\right]_{a'a}\\
&\times\left[P\exp\left(ig\int_{x_2}^{z}A_\mu(y_2)dy_2^\mu\right)\right]_{b'b}\\
&\times\left[P\exp\left(ig\int_{x_3}^{z}A_\mu(y_3)dy_3^\mu\right)\right]_{c'c}
\end{aligned} \tag{5}$$

that ensure local $SU(3)$ color invariance, cf. Ref. [12], instead of the straightforward ones, such as those in Eq. (4). As these factors are always assumed to be present, we shall omit them from now on, but we note that they give an extra minus sign when performing a color $SU(3)$ Fierz transformation.

Due to the nonlocality of these operators, the Pauli principle does not forbid any one of these *a priori*. For each one of the five trilocal operators $\Lambda_i(x,y,z)$ in Eq. (4), there are three possible fields with bilocal (functions of two position four-vectors x and y) operators:

$$\Lambda_i(x,x,y), \qquad \Lambda_i(x,y,x), \qquad \Lambda_i(y,x,x). \tag{6}$$

The latter two sets can be related to each other by simply interchanging the positions of the first and second quark fields, for example,

$$q_A^{aT}(x)\gamma_5 q_B^b(y) = -q_B^{bT}(y)\gamma_5 q_A^a(x). \tag{7}$$

The last two are also related to the first set through the Fierz transformation:

$$\Lambda_j(x,y,x) = T_{ij}^{S1}\Lambda_i(x,x,y), \tag{8}$$

where the transition matrix $\mathbf{T}^{S1}$ is

$$\mathbf{T}^{S1} = \frac{1}{4}\begin{pmatrix} -1 & -1 & -1 & -1 & \frac{1}{2}\\ -1 & -1 & 1 & 1 & \frac{1}{2}\\ -4 & 4 & 2 & -2 & 0\\ 4 & -4 & 2 & -2 & 0\\ -12 & -12 & 0 & 0 & -2 \end{pmatrix}. \tag{9}$$

The Pauli principle does eliminate some local diquarks, however, and one quickly finds that

$$\Lambda_4(x,x,y) = \Lambda_5(x,x,y) = 0. \tag{10}$$

Therefore, only three of the original 15 operators are independent. They are $\Lambda_1(x,x,y)$, $\Lambda_2(x,x,y)$, and $\Lambda_3(x,x,y)$.

2. The flavor-decuplet baryons

There are also five decuplet baryon fields formed from five different combinations of γ matrices:

$$\begin{aligned}
\Delta_1^P &= S_P^{ABC}(\tilde{q}_A q_B)q_C,\\
\Delta_2^P &= S_P^{ABC}(\tilde{q}_A\gamma_5 q_B)\gamma_5 q_C,\\
\Delta_3^P &= S_P^{ABC}(\tilde{q}_A\gamma_\mu q_B)\gamma^\mu q_C,\\
\Delta_4^P &= S_P^{ABC}(\tilde{q}_A\gamma_\mu\gamma_5 q_B)\gamma^\mu\gamma_5 q_C,\\
\Delta_5^P &= S_P^{ABC}(\tilde{q}_A\sigma_{\mu\nu} q_B)\sigma_{\mu\nu} q_C.
\end{aligned} \tag{11}$$

Here S_P^{ABC} is the totally symmetric $SU(3)$ tensor with components listed in Table II. Index $P = 1, \ldots, 10$ denotes the $SU(3)$ flavor label of a decuplet state. Here also we have three sets of bilocal fields that are related to each other by Fierz identities:

$$\begin{aligned}
\Delta_i^P(y,x,x) &\leftrightarrow \Delta_j^P(x,y,x),\\
\Delta_j^P(x,y,x) &= T_{ij}^{D1}\Delta_i^P(x,x,y),
\end{aligned}$$

where the flavor-decuplet matrix $\mathbf{T}^{D1}$ is identical to the flavor-singlet matrix $\mathbf{T}^{S1}$ given in Eq. (9),

$$\mathbf{T}^{D1} = \mathbf{T}^{S1}. \tag{12}$$

Due to the Pauli principle, we find that

$$\Delta_1^P(x,x,y) = \Delta_2^P(x,x,y) = \Delta_3^P(x,x,y) = 0. \tag{13}$$

Therefore, only two of the original 15 bilocal Δ operators are independent. They are $\Delta_4^P(x,x,y)$ and $\Delta_5^P(x,x,y)$.

TABLE II. Nonzero components of S_P^{ABC}.

P	1	2	3	4	5	6	7	8	9	10
ABC	111	112	122	222	113	123	223	133	233	333
Baryons	Δ^{++}	Δ^+	Δ^0	Δ^-	Σ^{*+}	Σ^{*0}	Σ^{*-}	Ξ^{*0}	Ξ^{*-}	Ω^-
Normalization	1	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1	$\frac{1}{\sqrt{3}}$	$\sqrt{6}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1

3. The flavor-octet baryon fields

We start once again with five trilocal fields

$$\begin{aligned} &N_1^N = \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A q_B)q_C, \qquad N_2^N = \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_5 q_B)\gamma_5 q_C, \qquad N_3^N = \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_\mu q_B)\gamma^\mu q_C, \\ &N_4^N = \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\gamma_\mu\gamma_5 q_B)\gamma^\mu\gamma_5 q_C, \qquad N_5^N = \epsilon^{ABD}\lambda_{DC}^N(\tilde{q}_A\sigma_{\mu\nu} q_B)\sigma_{\mu\nu} q_C. \end{aligned} \tag{14}$$

The index $N = 1, \ldots, 8$ labels the flavor $SU(3)$ states in an octet. Here λ_{DC}^N is the Dth column, Cth row component of the Nth Gell-Mann matrix. There are, however, two other kinds of baryon octet fields with the flavor $SU(3)$ structures $\epsilon^{BCD}\lambda_{DA}^N$ and $\epsilon^{CAD}\lambda_{DB}^N$:

$$\begin{aligned} &N_6^N = \epsilon^{BCD}\lambda_{DA}^N(\tilde{q}_A q_B)q_C, \qquad N_7^N = \epsilon^{BCD}\lambda_{DA}^N(\tilde{q}_A\gamma_5 q_B)\gamma_5 q_C, \qquad N_8^N = \epsilon^{BCD}\lambda_{DA}^N(\tilde{q}_A\gamma_\mu q_B)\gamma^\mu q_C, \\ &N_9^N = \epsilon^{BCD}\lambda_{DA}^N(\tilde{q}_A\gamma_\mu\gamma_5 q_B)\gamma^\mu\gamma_5 q_C, \qquad N_{10}^N = \epsilon^{BCD}\lambda_{DA}^N(\tilde{q}_A\sigma_{\mu\nu} q_B)\sigma_{\mu\nu} q_C, \qquad N_{11}^N = \epsilon^{CAD}\lambda_{DB}^N(\tilde{q}_A q_B)q_C, \\ &N_{12}^N = \epsilon^{CAD}\lambda_{DB}^N(\tilde{q}_A\gamma_5 q_B)\gamma_5 q_C, \qquad N_{13}^N = \epsilon^{CAD}\lambda_{DB}^N(\tilde{q}_A\gamma_\mu q_B)\gamma^\mu q_C, \qquad N_{14}^N = \epsilon^{CAD}\lambda_{DB}^N(\tilde{q}_A\gamma_\mu\gamma_5 q_B)\gamma^\mu\gamma_5 q_C, \\ &N_{15}^N = \epsilon^{CAD}\lambda_{DB}^N(\tilde{q}_A\sigma_{\mu\nu} q_B)\sigma_{\mu\nu} q_C. \end{aligned} \tag{15}$$

We have to consider all three sets of bilocal fields; they are related through the Fierz relation:

$$N_i^N(y,x,x) \leftrightarrow N_i^N(x,y,x), \qquad N_i^N(x,y,x) = T_{ij}^{O1} N_i^N(x,x,y),$$

where the transition matrix $\mathbf{T}^{O1}$ is obtained from the Fierz transformation

$$\mathbf{T}^{O1} = \frac{1}{4}\left(\begin{array}{ccccc|ccccc|ccccc} 0&0&0&0&0&-1&-1&-1&-1&\frac{1}{2}&0&0&0&0&0\\ 0&0&0&0&0&-1&-1&1&1&\frac{1}{2}&0&0&0&0&0\\ 0&0&0&0&0&-4&4&2&-2&0&0&0&0&0&0\\ 0&0&0&0&0&4&-4&2&-2&0&0&0&0&0&0\\ 0&0&0&0&0&-12&-12&0&0&-2&0&0&0&0&0\\ \hline 0&0&0&0&0&0&0&0&0&0&-1&-1&-1&-1&\frac{1}{2}\\ 0&0&0&0&0&0&0&0&0&0&-1&-1&1&1&\frac{1}{2}\\ 0&0&0&0&0&0&0&0&0&0&-4&4&2&-2&0\\ 0&0&0&0&0&0&0&0&0&0&4&-4&2&-2&0\\ 0&0&0&0&0&0&0&0&0&0&-12&-12&0&0&-2\\ \hline -1&-1&-1&-1&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0\\ -1&-1&1&1&\frac{1}{2}&0&0&0&0&0&0&0&0&0&0\\ -4&4&2&-2&0&0&0&0&0&0&0&0&0&0&0\\ 4&-4&2&-2&0&0&0&0&0&0&0&0&0&0&0\\ -12&-12&0&0&-2&0&0&0&0&0&0&0&0&0&0 \end{array}\right). \tag{16}$$

Together with the Jacobi-type identity

$$\epsilon^{ABD}\lambda_{DC}^N + \epsilon^{BCD}\lambda_{DA}^N + \epsilon^{CAD}\lambda_{DB}^N = 0, \tag{17}$$

and the Pauli principle, we obtain (only) five of the original 15 operators that are independent. Here we choose them as $N_1(x,x,y)$, $N_2(x,x,y)$, $N_3(x,x,y)$, and

$$M_4(x,x,y) = N_9(x,x,y) - N_{14}(x,x,y), \tag{18}$$

$$M_5(x,x,y) = N_{10}(x,x,y) - N_{15}(x,x,y). \tag{19}$$

Other octet baryons can be related to these five; here we only show the equations for $N_6(x,x,y)$, $N_7(x,x,y)$, and $N_8(x,x,y)$:

$$N_6(x,x,y) = -\frac{1}{2}N_1(x,x,y), \tag{20}$$

$$N_7(x,x,y) = -\frac{1}{2}N_2(x,x,y), \tag{21}$$

$$N_8(x,x,y) = -\frac{1}{2}N_3(x,x,y). \tag{22}$$

B. Rarita-Schwinger fields

1. Flavor-singlet baryon

We start by writing down three trilocal baryon fields

$$\begin{aligned}\Lambda_{3\mu} &= \epsilon^{ABC}(\tilde{q}_A\gamma_\nu q_B)\Gamma^{\mu\nu}_{3/2}\gamma_5 q_C,\\ \Lambda_{4\mu} &= \epsilon^{ABC}(\tilde{q}_A\gamma_\nu\gamma_5 q_B)\Gamma^{\mu\nu}_{3/2} q_C,\\ \Lambda_{5\mu} &= \epsilon^{ABC}(\tilde{q}_A\sigma_{\alpha\beta} q_B)\Gamma^{\mu\alpha}_{3/2}\gamma^\beta\gamma_5 q_C\end{aligned} \tag{23}$$

that contain diquarks formed from three sets of Dirac matrices, γ_μ, $\gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}$. Here $\Gamma^{\mu\nu}_{3/2}$ is the projection operator for the Rarita-Schwinger fields:

$$\Gamma^{\mu\nu}_{3/2} = g^{\mu\nu} - \frac{1}{4}\gamma^\mu\gamma^\nu. \tag{24}$$

Here, again, we have three sets of bilocal fields that are related to each other through the Fierz transformation:

$$\begin{aligned}\Lambda_{i\mu}(y,x,x) &\leftrightarrow \Lambda_{j\mu}(x,y,x),\\ \Lambda_{i\mu}(x,y,x) &= T^{S2}_{ij}\Lambda_{j\mu}(x,x,y),\end{aligned}$$

where the transition matrix $\mathbf{T}^{S2}$ is

$$\mathbf{T}^{S2} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1\\ -1 & 1 & -1\\ -2 & -2 & 0\end{pmatrix}. \tag{25}$$

Due to the Pauli principle, we find vanishing of two fields

$$\Lambda_{4\mu}(x,x,y) = \Lambda_{5\mu}(x,x,y) = 0, \tag{26}$$

leaving the $\Lambda_{3\mu}(x,x,y)$ as the only nonvanishing bilocal $\Lambda_\mu(x,x,y)$ field. Therefore, only one of the original nine operators is independent.

2. The flavor-decuplet baryons

Let us start by writing down three baryon fields which contain a diquark formed by three sets of Dirac matrices, γ_μ, $\gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}$,

$$\begin{aligned}\Delta^P_{3\mu} &= S^{ABC}_P(\tilde{q}_A\gamma_\nu q_B)\Gamma^{\mu\nu}_{3/2}\gamma_5 q_C,\\ \Delta^P_{4\mu} &= S^{ABC}_P(\tilde{q}_A\gamma_\nu\gamma_5 q_B)\Gamma^{\mu\nu}_{3/2} q_C,\\ \Delta^P_{5\mu} &= S^{ABC}_P(\tilde{q}_A\sigma_{\alpha\beta} q_B)\Gamma^{\mu\alpha}_{3/2}\gamma^\beta\gamma_5 q_C.\end{aligned} \tag{27}$$

We have also three sets of bilocal fields that are related through the Fierz transformation:

$$\begin{aligned}\Delta^P_{i\mu}(y,x,x) &\leftrightarrow \Delta^P_{j\mu}(x,y,x),\\ \Delta^P_{i\mu}(x,y,x) &= T^{D2}_{ij}\Delta^P_{j\mu}(x,x,y),\end{aligned}$$

where the flavor-decuplet Fierz matrix $\mathbf{T}^{D2}$ is identical to the flavor-singlet Fierz matrix, Eq. (25)

$$\mathbf{T}^{D2} = \mathbf{T}^{S2}. \tag{28}$$

Due to the Pauli principle, we immediately find

$$\Delta^P_{3\mu}(x,x,y) = 0. \tag{29}$$

Therefore, only two [$\Delta^P_{4\mu}(x,x,y)$ and $\Delta^P_{5\mu}(x,x,y)$] of the original nine operators are independent.

3. The flavor-octet baryon fields

Again, we start by writing down three trilocal baryon fields

$$\begin{aligned}N^N_{3\mu} &= \epsilon^{ABD}\lambda^N_{DC}(\tilde{q}_A\gamma_\nu q_B)\Gamma^{\mu\nu}_{3/2}\gamma_5 q_C,\\ N^N_{4\mu} &= \epsilon^{ABD}\lambda^N_{DC}(\tilde{q}_A\gamma_\nu\gamma_5 q_B)\Gamma^{\mu\nu}_{3/2} q_C,\\ N^N_{5\mu} &= \epsilon^{ABD}\lambda^N_{DC}(\tilde{q}_A\sigma_{\alpha\beta} q_B)\Gamma^{\mu\alpha}_{3/2}\gamma^\beta\gamma_5 q_C\end{aligned} \tag{30}$$

that contain a diquark formed with one of three sets of Dirac matrices, γ_μ, $\gamma_\mu\gamma_5$, and $\sigma_{\mu\nu}$. There are, however, two other kinds of octet baryons with the flavor structures $\epsilon^{BCD}\lambda^N_{DA}$ and $\epsilon^{CAD}\lambda^N_{DB}$:

$$\begin{aligned}N^N_{8\mu} &= \epsilon^{BCD}\lambda^N_{DA}(\tilde{q}_A\gamma_\nu q_B)\Gamma^{\mu\nu}_{3/2}\gamma_5 q_C,\\ N^N_{9\mu} &= \epsilon^{BCD}\lambda^N_{DA}(\tilde{q}_A\gamma_\nu\gamma_5 q_B)\Gamma^{\mu\nu}_{3/2} q_C,\\ N^N_{10\mu} &= \epsilon^{BCD}\lambda^N_{DA}(\tilde{q}_A\sigma_{\alpha\beta} q_B)\Gamma^{\mu\alpha}_{3/2}\gamma^\beta\gamma_5 q_C,\\ N^N_{13\mu} &= \epsilon^{CAD}\lambda^N_{DB}(\tilde{q}_A\gamma_\nu q_B)\Gamma^{\mu\nu}_{3/2}\gamma_5 q_C,\\ N^N_{14\mu} &= \epsilon^{CAD}\lambda^N_{DB}(\tilde{q}_A\gamma_\nu\gamma_5 q_B)\Gamma^{\mu\nu}_{3/2} q_C,\\ N^N_{15\mu} &= \epsilon^{CAD}\lambda^N_{DB}(\tilde{q}_A\sigma_{\alpha\beta} q_B)\Gamma^{\mu\alpha}_{3/2}\gamma^\beta\gamma_5 q_C.\end{aligned} \tag{31}$$

Considering all three sets of bilocal fields, we find that they are related through the Fierz transformation:

$$\begin{aligned}N^N_{i\mu}(y,x,x) &\leftrightarrow N^N_{i\mu}(x,y,x),\\ N^N_i(x,y,x) &= T^{O2}_{ij}N^N_i(x,x,y),\end{aligned}$$

where the flavor-octet Fierz matrix $\mathbf{T}^{O2}$ is

$$\mathbf{T}^{O2} = \frac{1}{4}\left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & -2 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & -2 & 0 \\ \hline -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right). \tag{32}$$

Together with the identity Eq. (17) and the Pauli principle we find that two fields vanish identically:

$$N^N_{4\mu}(x, x, y) = N^N_{5\mu}(x, x, y) = 0. \tag{33}$$

Therefore, only three of the original 15 operators are independent. Here we choose them as $N^N_{3\mu}(x, x, y)$ and

$$M_{4\mu}(x, x, y) = N_{9\mu}(x, x, y) - N_{14\mu}(x, x, y), \tag{34}$$

$$M_{5\mu}(x, x, y) = N_{10\mu}(x, x, y) - N_{15\mu}(x, x, y). \tag{35}$$

Other bilocal octet baryons can be related to these three; here we only show the representative equation for $N_{8\mu}(x, x, y)$:

$$N_{8\mu}(x, x, y) = -\frac{1}{2}N_{3\mu}(x, x, y). \tag{36}$$

C. Antisymmetric tensor (Bargmann-Wigner) fields

1. Flavor-singlet baryon

We start by writing down the trilocal baryon field

$$\Lambda_{5\mu\nu} = \epsilon^{ABC}(\tilde{q}_A\sigma_{\alpha\beta}q_B)\Gamma^{\mu\nu\alpha\beta}_{3/2}q_C, \tag{37}$$

which contains a diquark formed with the AST matrices $\sigma_{\mu\nu}$. Here $\Gamma^{\mu\nu\alpha\beta}$ is the Bargmann-Wigner projection operator defined as

$$\Gamma^{\mu\nu\alpha\beta} = \left(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\nu\beta}\gamma^{\mu}\gamma^{\alpha} + \frac{1}{2}g^{\mu\beta}\gamma^{\nu}\gamma^{\alpha} + \frac{1}{6}\sigma^{\mu\nu}\sigma^{\alpha\beta}\right). \tag{38}$$

We have also three sets of bilocal fields that are related through the Fierz transformation:

$$\Lambda_{5\mu\nu}(y, x, x) \leftrightarrow \Lambda_{5\mu\nu}(x, y, x),$$
$$\Lambda_{5\mu}(x, y, x) = T^{S2}_{ij}\Lambda_{5\mu\nu}(x, x, y),$$

where the flavor-singlet Fierz (1×1 matrix) number $\mathbf{T}^{S3}$ is unity

$$\mathbf{T}^{S3} = 1. \tag{39}$$

The Pauli principle leads immediately to

$$\Lambda_{5\mu\nu}(x, x, y) = 0. \tag{40}$$

Thus, we have obtained the result that all flavor-singlet bilocal AST fields vanish due to the Pauli principle.

2. The flavor-decuplet baryons

Let us start with writing down the baryon field which contain a diquark formed by the Dirac matrices $\sigma_{\mu\nu}$:

$$\Delta^P_{5\mu\nu} = S^{ABC}_P(\tilde{q}_A\sigma_{\alpha\beta}q_B)\Gamma^{\mu\nu\alpha\beta}_{3/2}q_C. \tag{41}$$

We have also three sets of bilocal fields, and they are related to each other through the Fierz transformation:

$$\Delta_{5\mu\nu}(y, x, x) \leftrightarrow \Delta_{5\mu\nu}(x, y, x),$$
$$\Delta_{5\mu\nu}(x, y, x) = T^{D3}_{ij}\Delta_{5\mu\nu}(x, x, y),$$

where the flavor-decuplet Fierz (1×1) matrix $\mathbf{T}^{D3}$ is equivalent to the flavor-singlet Eq. (39)

$$\mathbf{T}^{D3} = 1. \tag{42}$$

Therefore, the only original operator is Pauli allowed.

3. The flavor-octet baryon fields

Start by writing down the flavor-octet trilocal baryon field

$$N^N_{5\mu\nu} = \epsilon^{ABD}\lambda^N_{DC}(\tilde{q}_A\sigma_{\alpha\beta}q_B)\Gamma^{\mu\nu\alpha\beta}_{3/2}q_C, \tag{43}$$

which contains a diquark formed by the $\sigma_{\mu\nu}$ matrices. There are, however, also two other kinds of octet baryons with the flavor structures $\epsilon^{BCD}\lambda^N_{DA}$ and $\epsilon^{CAD}\lambda^N_{DB}$:

$$\begin{aligned} N^N_{10\mu\nu} &= \epsilon^{BCD}\lambda^N_{DA}(\tilde{q}_A\sigma_{\alpha\beta}q_B)\Gamma^{\mu\nu\alpha\beta}_{3/2}q_C, \\ N^N_{15\mu\nu} &= \epsilon^{CAD}\lambda^N_{DB}(\tilde{q}_A\sigma_{\alpha\beta}q_B)\Gamma^{\mu\nu\alpha\beta}_{3/2}q_C. \end{aligned} \tag{44}$$

Considering all three sets of bilocal fields that are related through the Fierz transformation:

$$N^N_{i\mu\nu}(y, x, x) \leftrightarrow N^N_{i\mu\nu}(x, y, x),$$
$$N^N_{i\mu\nu}(x, y, x) = T^{O3}_{ij}N^N_{i\mu\nu}(x, x, y),$$

where the flavor-octet Fierz matrix $\mathbf{T}^{O3}$ is

$$\mathbf{T}^{O3} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{45}$$

together with the relation Eq. (17) and the Pauli principle we find

$$N^N_{5\mu\nu}(x, x, y) = 0. \tag{46}$$

Therefore, only one of the original three flavor-octet operators is independent. Here we choose it as $M^N_{5\mu\nu}(x, x, y) = N^N_{10\mu\nu}(x, x, y) - N^N_{15\mu\nu}(x, x, y)$.

D. Summary of straightforward bilocal fields

We have investigated the chiral multiplets consisting of bilocal three-quark baryon operators, where we took into account the Pauli principle by way of the Fierz transformation. All spin-1/2 and -3/2 baryon operators were classified according to their Lorentz and isospin group representations, where spin and flavor projection operators were employed in Table I. We have derived the nontrivial Fierz relations among various baryon operators and thus found the independent baryon fields, see Tables III, IV, and V.

Thus, for example, in the spin-1/2 sector, three flavor-singlet fields ("Λ's"), five octet fields ("nucleons"), and two decimet fields ("Δ's") are independent in the bilocal limit, in stark contrast to the local limit where there are (only) two nucleons and no Δ, see Ref. [6]. We see in Table III, that five out of 12 entries in Table I vanish in the local-operator limit $x \to y$, and other Fierz identities reduce the number of independent chiral multiplets from seven to four. The baryon fields ($\Lambda_1 - \Lambda_2$, $N_1 - N_2$) and (Λ_3, $N_3 - M_4$) form two independent $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral multiplets; $N_1 + N_2$ and M_5 form two independent $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ chiral multiplets; ($N_3 + \frac{1}{3}M_4$, Δ_4) form one $[(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})]$ chiral multiplet; Δ_5 also forms a separate $[(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$ chiral multiplet.

In the spin-$\frac{3}{2}$ sector, the Rarita-Schwinger fields (Λ_3^μ, $N_3^\mu - M_4^\mu$) form an independent $[(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$ chiral multiplet, and ($N_3^\mu + \frac{1}{3}M_4^\mu$, Δ_4^μ) and (M_5^μ, Δ_5^μ) form two $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ chiral multiplets, see Table IV. Similarly, Lorentz representation $(\frac{3}{2}, 0)$ Bargmann-Wigner fields $M_5^{\mu\nu} \in [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$, $\Delta_5^{\mu\nu} \in [(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$ are also independent, see Table V. This is again in contrast with the local limit where there is only one independent nucleon field and two independent Δ's, [6].

This exhausts all chiral multiplets obtained from straightforward three-quark interpolators, so that relaxing the bilocal limit and going to the trilocal case would not yield new chiral multiplets. Note, however, that some chiral multiplets are repeated (doubled), whereas their mirror image(s) do not appear: why? The answer to this

TABLE III. The Abelian and the non-Abelian axial charges (+ sign indicates "naive," − sign indicates "mirror" transformation properties) and the non-Abelian chiral multiplets of spin-$\frac{1}{2}$, Lorentz representation $(\frac{1}{2}, 0)$ nucleon N, delta resonance Δ, and Λ hyperon fields. All fields are independent and Fierz invariant. In the last column we show the Fierz-equivalent/identical field in the local limit ($x \to y$).

	$U_A(1)$	$SU(3)_F$	$SU_L(3) \times SU_R(3)$	$\text{Fierz}(x \to y)_{\text{locallim.}}$
$\Lambda_1 - \Lambda_2$	−1	1	$(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	Λ_3
Λ_3	−1	1	$(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	$\Lambda_1 - \Lambda_2$
$N_1 - N_2$	−1	8	$(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	$N_3 - M_4$
$N_3 - M_4$	−1	8	$(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$	$N_1 - N_2$
$N_3 + \frac{1}{3}M_4$	−1	8	$(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$	0
Δ_4	−1	10	$(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})$	0
$\Lambda_1 + \Lambda_2$	+3	1	$(\mathbf{1}, \mathbf{1})$	0
$N_1 + N_2$	+3	8	$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	M_5
M_5	+3	8	$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	$N_1 + N_2$
Δ_5	+3	10	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	0

TABLE IV. The Abelian and the non-Abelian axial charges and the non-Abelian chiral multiplets of spin-$\frac{3}{2}$, Lorentz representation $(1, \frac{1}{2})$ nucleon, and Δ fields. All of the fields are independent and Fierz invariant. In the last column we show the Fierz-equivalent/identical field in the local limit ($x \to y$).

	$U_A(1)$	$SU(3)_F$	$SU_L(3) \times SU_R(3)$	$\text{Fierz}(x \to y)_{\text{locallim.}}$
Λ_3^μ	+1	1	$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	0
$N_3^\mu - M_4^\mu$	+1	8	$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	0
$N_3^\mu + \frac{1}{3}M_4^\mu$	+1	8	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	N_5^μ
Δ_4^μ	+1	10	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	Δ_5^μ
M_5^μ	+1	8	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	$N_3^\mu + \frac{1}{3}N_4^\mu$
Δ_5^μ	+1	10	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	Δ_4^μ

TABLE V. The Abelian and the non-Abelian axial charges and the non-Abelian chiral multiplets of spin-$\frac{3}{2}$, Lorentz representation $(\frac{3}{2}, 0)$ nucleon, and Δ fields. All of the fields are independent and Fierz invariant. In the last column we show the Fierz-equivalent/identical field in the local limit $(x \rightarrow y)$.

	$U_A(1)$	$SU(3)_F$	$SU_L(3) \times SU_R(3)$	$\mathrm{Fierz}(x \rightarrow y)_{\mathrm{locallim.}}$
$M_5^{\mu\nu}$	+3	8	$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	0
$\Delta_5^{\mu\nu}$	+3	10	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	$\Delta_5^{\mu\nu}$

question has to do with the number (even/odd) of Dirac γ matrices that appear in the field itself. With one new four-vector [the $(x-y)_\mu$] available, this problem is (very) easily solved: contracting the spin-$\frac{3}{2}$ fields with this four-vector yields new spin-$\frac{1}{2}$ fields.

III. NONSTRAIGHTFORWARD THREE-FLAVOR BILOCAL THREE-QUARK FIELDS

Thus far we have straightforwardly extended the local field analysis to the bilocal case and thus ignored new, less straightforward possibilities: besides the (center-of-mass variable x) derivative ∂_μ, we have one new four-vector [the $(x-y)_\mu$] available. Contracting the various spin-$\frac{3}{2}$ fields with these four-vectors yields new spin-$\frac{1}{2}$ fields.

Once again we would like to note that the bilocal fields constructed in this section may have components overlapping with more than one angular momentum J state. Their chiral properties are independent of the exact value of J, however.

A. Derivative-contracted fields

Contraction with the [center-of-mass variable x in the local limit, or $\frac{1}{3}(2x+y)$ in the bilocal case] derivative ∂_μ is obligatory, as the true Rarita-Schwinger fields must satisfy the auxiliary condition $\partial^\mu \Psi_\mu = 0$, which is not automatically satisfied by Ioffe's three-quark interpolators with one Lorentz index μ [13,14]. Thus, one must subtract the (generally nonvanishing) $\partial_\mu \partial^\nu B_\nu \frac{1}{\partial_\nu \partial^\nu}$ from the original (unsubtracted) Ioffe fields B_ν in order to obtain genuine Rarita-Schwinger fields

$$\Psi_\mu = B_\mu - \partial_\mu \frac{\partial^\nu B_\nu}{\partial_\nu \partial^\nu}. \tag{47}$$

That leaves us with $\Psi \simeq i\partial^\nu B_\nu$ as a new Dirac field interpolator. One look at Table VII reveals that these new fields have precisely the "mirror" properties to those of the "usual" or "naive" Dirac field interpolators in Table VI. Note, however, that the chiral multiplets $[(\mathbf{1}, \mathbf{1})]$, $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$, and $[(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$ and their "mirror fields" are still missing from this list of Rarita-Schwinger fields.

The same holds for Rarita-Schwinger fields obtained from the (local) Bargmann-Wigner fields [15] by contraction with one derivative ∂^ν:

$$\Psi_\mu = \partial^\nu B_{\nu\mu}. \tag{48}$$

This takes care of the $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ and $[(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$ chiral multiplets by way of Bargmann-Wigner fields $\partial_\nu M_5^{\mu\nu} \in [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$, $\partial_\nu \Delta_5^{\mu\nu} \in [(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$, but not of their mirror images, which are still missing from this list of Rarita-Schwinger fields, as is the $[(\mathbf{1}, \mathbf{1})]$ field. Moreover, this procedure does not produce new Bargmann-Wigner fields with chiral properties not seen thus far, see Table VI. We also note that one cannot obtain new Dirac field interpolators from Bargmann-Wigner fields due to the identity $\partial^\nu \partial^\mu B_{\nu\mu} = \partial^\mu \partial^\nu B_{\nu\mu} = 0$.

In a short summary, the derivative-contracted fields produce new Dirac fields $(\partial_\mu \Lambda_3^\mu, \partial_\mu (N_3^\mu - M_4^\mu)) \in [(\bar{\mathbf{3}}, \mathbf{3}) \oplus$

TABLE VI. The Abelian and the non-Abelian axial charges and the non-Abelian chiral multiplets of spin-$\frac{1}{2}$, Lorentz representation $(\frac{1}{2}, 0)$, nonstraightforward "nucleon" N octet, delta resonance Δ decuplet, and Λ hyperon singlet fields. All fields are independent and Fierz invariant.

	$U_A(1)$	$SU(3)_F$	$SU_L(3) \times SU_R(3)$	$\mathrm{Fierz}(x \rightarrow y)_{\mathrm{locallim.}}$
$\partial_\mu \Lambda_3^\mu$	+1	1	$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	0
$\partial_\mu (N_3^\mu - M_4^\mu)$	+1	8	$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	0
$\partial_\mu (N_3^\mu + \frac{1}{3} M_4^\mu)$	+1	8	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	$\partial_\mu M_5^\mu$
$\partial_\mu \Delta_4^\mu$	+1	10	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	$\partial_\mu \Delta_5^\mu$
$\partial_\mu M_5^\mu$	+1	8	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	$\partial_\mu (N_3^\mu + \frac{1}{3} M_4^\mu)$
$\partial_\mu \Delta_5^\mu$	+1	10	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	$\partial_\mu \Delta_4^\mu$
$(x-y)_\mu \Lambda_3^\mu$	+1	1	$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	0
$(x-y)_\mu (N_3^\mu - M_4^\mu)$	+1	8	$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$	0
$(x-y)_\mu (N_3^\mu + \frac{1}{3} M_4^\mu)$	+1	8	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	0
$(x-y)_\mu \Delta_4^\mu$	+1	10	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	0
$(x-y)_\mu M_5^\mu$	+1	8	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	0
$(x-y)_\mu \Delta_5^\mu$	+1	10	$(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$	0
$(x-y)_\mu \partial_\nu M_5^{\mu\nu}$	+3	8	$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	0
$(x-y)_\mu \partial_\nu \Delta_5^{\mu\nu}$	+3	10	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	0

TABLE VII. The Abelian and the non-Abelian axial charges and the non-Abelian chiral multiplets of spin-$\frac{3}{2}$, Lorentz representation $(1, \frac{1}{2})$ "nucleon" octet, and "Δ" decuplet nonstraightforward fields.

	$U_A(1)$	$SU(3)_F$	$SU_L(3) \times SU_R(3)$	Fierz$(x \to y)_{\text{locallim.}}$
$\partial_\nu M_5^{\mu\nu}$	$+3$	8	$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	0
$\partial_\nu \Delta_5^{\mu\nu}$	$+3$	10	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	$\partial_\nu \Delta_5^{\mu\nu}$
$(x-y)_\nu M_5^{\mu\nu}$	$+3$	8	$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$	0
$(x-y)_\nu \Delta_5^{\mu\nu}$	$+3$	10	$(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$	0

$(\mathbf{3}, \bar{\mathbf{3}})]$, $(\partial_\mu(N_3^\mu + \frac{1}{3}M_4^\mu), \partial_\mu \Delta_4^\mu) \in [(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, and $(\partial_\mu M_5^\mu, \partial_\mu \Delta_5^\mu) \in [(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, and new Rarita-Schwinger fields $\partial_\nu M_5^{\mu\nu} \in [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ and $\partial_\nu \Delta_5^{\mu\nu} \in [(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$, which do not vanish, see Tables VI and VII.

B. Nonderivative-contracted fields

Similarly to the previous subsection, we can contract Rarita-Schwinger fields with the four-vector $(x-y)_\mu$ to obtain new Dirac fields,

$$\Psi = (x-y)^\nu B_\nu. \tag{49}$$

We can also contract Bargmann-Wigner fields with the four-vector $(x-y)_\mu$ to obtain new Rarita-Schwinger fields,

$$\Psi_\mu = (x-y)^\nu B_{\nu\mu}. \tag{50}$$

Again we cannot obtain new Dirac field interpolators from Bargmann-Wigner fields due to $(x-y)^\nu (x-y)^\mu B_{\nu\mu} = (x-y)^\mu (x-y)^\nu B_{\nu\mu} = 0$.

In a short summary, the nonderivative-contracted fields produce new Dirac fields $((x-y)_\mu \Lambda_3^\mu, (x-y)_\mu \times (N_3^\mu - M_4^\mu))$, $((x-y)_\mu (N_3^\mu + \frac{1}{3}M_4^\mu), (x-y)_\mu \Delta_4^\mu)$, and $((x-y)_\mu M_5^\mu, (x-y)_\mu \Delta_5^\mu)$, and new Rarita-Schwinger fields $(x-y)_\nu M_5^{\mu\nu}$ and $(x-y)_\nu \Delta_5^{\mu\nu}$, see Tables VI and VII. The chiral representations of these fields are the same as the corresponding derivative-contracted fields. Fierz identities show that all of these fields vanish in the local limit $x \to y$.

C. Mixed-contracted fields

Together with the derivative and the four-vector $(x-y)_\mu$, we obtain new Dirac fields from Bargmann-Wigner fields

$$\Psi = (x-y)^\nu \partial^\mu B_{\nu\mu}. \tag{51}$$

The other three $(x-y)^\mu \partial^\nu B_{\nu\mu}$, $\partial^\nu (x-y)^\mu B_{\nu\mu}$, and $\partial^\mu (x-y)^\nu B_{\nu\mu}$ can be related to this one, and so this is the only independent field. Therefore, the mixed-contracted fields only produce the Dirac fields $(x-y)_\mu \partial_\nu M_5^{\mu\nu} \in [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ and $(x-y)_\mu \partial_\nu \Delta_5^{\mu\nu} \in [(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$, all of which vanish in the local limit $x \to y$, see Table VI.

IV. SUMMARY AND CONCLUSIONS

We have investigated the chiral multiplets consisting of bilocal three-quark baryon operators, where we took into account the Pauli principle by way of the Fierz transformation. All spin-$\frac{1}{2}$ and some -$\frac{3}{2}$ baryon operators were classified in Tables III, IV, V, VI, and VII according to their Lorentz and flavor symmetry group representations. Again we would like to note that these baryon fields have definite total angular momentum only in the local limit. We have employed the standard flavor $SU(3)$ formalism instead of the explicit expressions in terms of different flavored quarks in the flavor components of the baryon fields that are commonplace in this line of work.

In doing so, we have been able to systematically derive the Fierz identities and chiral transformations of the baryon fields. More specifically, we have derived all nontrivial Fierz relations among various baryon bilocal operators and thus found the independent bilocal baryon fields. We have shown that the Fierz transformation connects only those bilocal baryon interpolating fields with identical chiral group-theoretical properties, i.e., those belonging to the same chiral multiplet, just as in the case of local baryon operators.

For example, in the spin-$\frac{1}{2}$ sector, five flavor-singlet fields ("Λ's"), 12 octet fields ("nucleons"), and seven decimet fields ("Δ's") were independent in the bilocal limit, in stark contrast to the local limit where there was (only) one Λ, two nucleons, and no Δ's, Ref. [6]. One can see that 14 out of 24 entries in the Tables III and VI vanished in the local-operator limit $x \to y$, and another three Fierz identities reduced the number of independent fields from ten to five.

The $(\Lambda_1 + \Lambda_2)$ formed one independent $[(\mathbf{1}, \mathbf{1})]$ chiral multiplet, $(\Lambda_1 - \Lambda_2, N_1 - N_2)$ and $(\Lambda_3, N_3 - M_4)$ formed two independent $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral multiplets, $(N_1 + N_2)$ and M_5 formed two independent $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ chiral multiplets, $(N_3 + \frac{1}{3}M_4, \Delta_4)$ formed one $[(\mathbf{3}, \mathbf{6}) \oplus (\mathbf{6}, \mathbf{3})]$ chiral multiplet, and the independent field Δ_5 also formed a separate $[(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$ chiral multiplet.

The derivative-contracted fields produced new nonvanishing Dirac fields $(\partial_\mu \Lambda_3^\mu, \partial_\mu (N_3^\mu - M_4^\mu)) \in [(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$, $(\partial_\mu (N_3^\mu + \frac{1}{3}M_4^\mu), \partial_\mu \Delta_4^\mu) \in [(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, and $(\partial_\mu M_5^\mu, \partial_\mu \Delta_5^\mu) \in [(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$. The nonderivative-contracted fields produced $((x-y)_\mu \Lambda_3^\mu, (x-y)_\mu \times (N_3^\mu - M_4^\mu))$, $((x-y)_\mu (N_3^\mu + \frac{1}{3}M_4^\mu), (x-y)_\mu \Delta_4^\mu)$, and $((x-y)_\mu M_5^\mu, (x-y)_\mu \Delta_5^\mu)$, and the mixed-contracted fields produced $(x-y)_\mu \partial_\nu M_5^{\mu\nu} \in [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ and $(x-y)_\mu \partial_\nu \Delta_5^{\mu\nu} \in [(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$, see Table VI.

In the spin-$\frac{3}{2}$ sector, the $(\Lambda_3^\mu, N_3^\mu - M_4^\mu)$ formed an independent $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral multiplet, whereas $(N_3^\mu + \frac{1}{3}M_4^\mu, \Delta_4^\mu) \in [(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ and $(M_5^\mu, \Delta_5^\mu) \in [(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ were also independent, again in contrast with the local limit where there was only one independent nucleon field and two independent Δ's, [6]. The derivative-contracted fields produced new Rarita-Schwinger fields $\partial_\nu M_5^{\mu\nu} \in [(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ and $\partial_\nu \Delta_5^{\mu\nu} \in [(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})]$. The nonderivative-contracted fields produced $(x-y)_\nu M_5^{\mu\nu}$ and $(x-y)_\nu \Delta_5^{\mu\nu}$, see Table VII.

This increase of the number of independent fields was in line with expectations based on the nonrelativistic quark model, where the number of Pauli-allowed three-quark states in the $L^P = 1^-$ shell sharply rose from the corresponding number in the ground state. Indeed, there was a deep analogy between the Pauli principle acting in the nonrelativistic quantum formalism, where the flavor-spin group $SU(6)_{FS}$ played the role of the chiral symmetry group $SU_L(3) \times SU_R(3)$ in the relativistic formalism. Of course, the chiral symmetry group $SU_L(3) \times SU_R(3)$ was a subgroup of some ("bigger") $SU(6)$, but that was not the flavor-spin group $SU(6)_{FS}$ [16]. This analogy is at the present still (only) empirical: we do not have a set of clear and simple rules that determine the allowed chiral multiplets in this relativistic approach that would correspond to the rules leading to the allowed $SU_{FS}(6)$ multiplets in the nonrelativistic approach. Rather, we had to rely on the (rather involved) present analysis.

The physical significance of our present work was that it showed that there was no need to introduce $q\bar{q}$ components in addition to the three-quark "core," so as to agree with the observed axial couplings and magnetic moments: the phenomenologically necessary $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ chiral component and the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ "mirror" component existed as bilocal fields [23]. Thus, we have shown that there is no need for "meson cloud" or (nonexotic) "pentaquark" components in the Fock expansion of the baryon wave function to explain (at least) the axial currents and magnetic moments, contrary to established opinion, Ref. [24]. This goes to show that the algebraic complexity of three Dirac quark fields is such that it can mimic the presence of $q\bar{q}$ pairs, at least in certain observables. For us this was a surprise.

The framework presented here holds in standard approaches to QCD, such as the QCD sum rules [13,14] and lattice QCD [11], under the proviso that chiral symmetry is observed by the approximation used. There is another (sub)field of QCD where it ought to make an impact: in the class of fully relativistic three-body models, such as those based on the three-body Salpeter, Refs. [25–27] or Bethe-Salpeter equation approaches to chiral quark models Refs. [28–31]. One potential application of our results is to classify various components in the Salpeter or Bethe-Salpeter amplitudes (wave functions), instead of the nonrelativistic $SU(6)_{FS}$ multiplets that have been used so far, and thus to try and determine the baryons' chiral mixing coefficients (angles), Refs. [2,3,32,33], starting from an underlying chiral model. Model calculations like that could give one insight into structural questions that cannot be (reasonably) expected to be answered by lattice QCD. For example, why do certain chiral multiplets not appear in the baryons?

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APPENDIX: CHIRAL TRANSFORMATIONS

Here, we briefly review the $SU_L(3) \times SU_R(3)$ chiral transformations of three-quark baryon operators, which are determined by their Dirac matrix structure, see Ref. [1]. Under the $U_V(1) = U_B(1)$ (baryon number), $U_A(1)$ (axial baryon number), $SU_V(3) = SU_F(3)$ [flavor $SU(3)$], and $SU_A(3)$ [axial flavor $SU(3)$] transformations, the quark field $q = q_L + q_R$ transforms as

$$\begin{aligned} \mathbf{U_V(1)}&: q \to \exp(ia^0)q = q + \delta q, \\ \mathbf{SU_V(3)}&: q \to \exp(i\vec{\lambda}\cdot\vec{a})q = q + \delta^{\vec{a}} q, \\ \mathbf{U_A(1)}&: q \to \exp(i\gamma_5 b^0)q = q + \delta_5 q, \\ \mathbf{SU_A(3)}&: q \to \exp(i\gamma_5\vec{\lambda}\cdot\vec{b})q = q + \delta_5^{\vec{b}} q, \end{aligned} \tag{A1}$$

where $\vec{\lambda}$ are the eight Gell-Mann matrices, a^0 is the infinitesimal parameter for the $U_V(1)$ "vector" transformation, $\vec{a}$ are the octet of $SU_V(3)$ group parameters, b^0 is the infinitesimal parameter for the $U_A(1)$ γ_5 transformation, and $\vec{b}$ are the octet of $SU_A(3)$ γ_5 transformation parameters.

The $U_V(1)$ baryon number ("vector") transformation is simple, while the $SU_V(3)$ flavor-symmetry (vector) transformations are also well known:

(1) for any singlet baryon field Λ, we have

$$\delta^{\vec{a}}\Lambda = 0; \tag{A2}$$

(2) for any octet baryon field N^M, we have

$$\delta^{\vec{a}} N^M = 2a^N f_{NMO} N^O; \tag{A3}$$

(3) for any decuplet baryon field Δ^P, we have

$$\delta^{\vec{a}}\Delta^P = 2ia^N \mathbf{F}^N_{PQ}\Delta^Q, \tag{A4}$$

where the coefficients d^{NMO} and f^{NMO} are the standard symmetric and antisymmetric structure constants of $SU(3)$; the transition matrices $\mathbf{F}^N_{PQ}$ as well as $\mathbf{T}^{\dagger N}_{PM}$ in the following subsections are listed in Ref. [2].

1. Dirac fields (spin $\frac{1}{2}$)

Under the Abelian chiral transformation the rule, we have

$$\delta_5(\Lambda_1+\Lambda_2)=3ib\gamma_5(\Lambda_1+\Lambda_2), \tag{A5}$$

$$\delta_5(\Lambda_1-\Lambda_2)=-ib\gamma_5(\Lambda_1-\Lambda_2), \tag{A6}$$

$$\delta_5\Lambda_3=-ib\gamma_5\Lambda_3, \tag{A7}$$

and

$$\delta_5\Delta_4^P=-ib\gamma_5\Delta_4^P, \tag{A8}$$

$$\delta_5\Delta_5^P=3ib\gamma_5\Delta_5^P, \tag{A9}$$

and

$$\delta_5(N_1^N+N_2^N)=3ib\gamma_5(N_1^N+N_2^N), \tag{A10}$$

$$\delta_5(N_1^N-N_2^N)=-ib\gamma_5(N_1^N-N_2^N), \tag{A11}$$

$$\delta_5 N_3^N=-ib\gamma_5 N_3^N, \tag{A12}$$

$$\delta_5 M_4^N=-ib\gamma_5 M_4^N, \tag{A13}$$

$$\delta_5 M_5^N=3ib\gamma_5 M_5^N. \tag{A14}$$

Under the $SU_A(3)$ chiral transformation the rule, we have

$$\delta_5^{\vec b}(\Lambda_1+\Lambda_2)=0, \tag{A15}$$

$$\delta_5^{\vec b}(\Lambda_1-\Lambda_2)=2ib^N\gamma_5(N_1^N-N_2^N), \tag{A16}$$

$$\delta_5^{\vec b}\Lambda_3=-ib^N\gamma_5(N_3^N-M_4^N), \tag{A17}$$

and

$$\delta_5^{\vec b}\Delta_4^P=ib^N\gamma_5\mathbf{T}^{\dagger N}_{PM}\left(N_3^M+\frac{1}{3}M_4^M\right)-\frac{2}{3}ib^N\gamma_5\mathbf{F}^N_{PQ}\Delta_4^Q, \tag{A18}$$

$$\delta_5^{\vec b}\Delta_5^P=2ib^N\gamma_5\mathbf{F}^N_{PQ}\Delta_5^Q, \tag{A19}$$

and

$$\delta_5^{\vec b}(N_1^M+N_2^M)=2b^N\gamma_5 f^{NMO}(N_1^O+N_2^O), \tag{A20}$$

$$\delta_5^{\vec b}(N_1^M-N_2^M)=\frac{4}{3}ib^N\gamma_5(\Lambda_1-\Lambda_2)+2ib^N\gamma_5 d^{NMO}(N_1^O-N_2^O), \tag{A21}$$

$$\delta_5^{\vec b}\left(N_3^M+\frac{1}{3}M_4^M\right)=\frac{16}{3}ib^N\gamma_5\mathbf{T}^N_{MP}\Delta_4^P+ib^N\gamma_5\left(-2d^{NMO}+\frac{4}{3}if^{NMO}\right)\times\left(N_3^O+\frac{1}{3}M_4^O\right), \tag{A22}$$

$$\delta_5^{\vec b}(N_3^M-M_4^M)=-\frac{8}{3}ib^N\gamma_5\Lambda_3+2ib^N\gamma_5 d^{NMO}(N_3^O-M_4^O), \tag{A23}$$

$$\delta_5^{\vec b}M_5^M=2b^N\gamma_5 f^{NMO}M_5^O. \tag{A24}$$

2. Rarita-Schwinger fields (spin $\frac{1}{2}$ and $\frac{3}{2}$)

Under the Abelian chiral transformation, we have

$$\delta_5\Lambda_{3\mu}=ib\gamma_5\Lambda_{3\mu}, \tag{A25}$$

and

$$\delta_5\Delta^P_{4\mu}=ib\gamma_5\Delta^P_{4\mu}, \tag{A26}$$

$$\delta_5\Delta^P_{5\mu}=ib\gamma_5\Delta^P_{5\mu}, \tag{A27}$$

and

$$\delta_5 N^N_{3\mu}=ib\gamma_5 N^N_{3\mu}, \tag{A28}$$

$$\delta_5 M^N_{4\mu}=ib\gamma_5 M^N_{4\mu}, \tag{A29}$$

$$\delta_5 M^N_{5\mu}=ib\gamma_5 M^N_{5\mu}. \tag{A30}$$

Under the $SU_A(3)$ chiral transformations, we have

$$\delta_5^{\vec b}\Lambda_{3\mu}=ib^N\gamma_5(N^N_{3\mu}-M^N_{4\mu}), \tag{A31}$$

and

$$\delta_5^{\vec b}\Delta^P_{4\mu}=-ib^N\gamma_5\mathbf{T}^{\dagger N}_{PM}\left(N^M_{3\mu}+\frac{1}{3}M^M_{4\mu}\right)+\frac{2}{3}ib^N\gamma_5\mathbf{F}^N_{PQ}\Delta^Q_{4\mu}, \tag{A32}$$

$$\delta_5^{\vec b}\Delta^P_{5\mu}=\frac{2}{3}ib^N\gamma_5\mathbf{T}^{\dagger N}_{PM}M^M_{5\mu}+\frac{2}{3}ib^N\gamma_5\mathbf{F}^N_{PQ}\Delta^Q_{5\mu}, \tag{A33}$$

and

$$\delta_5^{\vec b}\left(N^M_{3\mu}+\frac{1}{3}M^M_{4\mu}\right)=-\frac{16}{3}ib^N\gamma_5\mathbf{T}^N_{MP}\Delta^P_{4\mu}+ib^N\gamma_5\left(2d^{NMO}-\frac{4}{3}if^{NMO}\right)\times\left(N^O_{3\mu}+\frac{1}{3}M^O_{4\mu}\right), \tag{A34}$$

$$\delta_5^{\vec b}(N^M_{3\mu}-M^M_{4\mu})=\frac{8}{3}ib^N\gamma_5\Lambda_{3\mu}-2ib^N\gamma_5 d^{NMO}(N^O_{3\mu}-M^O_{4\mu}), \tag{A35}$$

$$\delta_5^{\vec b}M^M_{5\mu}=8ib^N\gamma_5\mathbf{T}^N_{MP}\Delta^P_{5\mu}+ib^N\gamma_5\left(2d^{NMO}-\frac{4}{3}if^{NMO}\right)\times\left(N^O_{3\mu}+\frac{1}{3}M^O_{4\mu}\right). \tag{A36}$$

3. Antisymmetric tensor fields (spin $\frac{3}{2}$)

Under the Abelian chiral transformation, we have

$$\delta_5 \Delta^P_{5\mu\nu} = 3ib\gamma_5 \Delta^P_{5\mu\nu}, \tag{A37}$$

and

$$\delta_5 M^N_{5\mu\nu} = 3ib\gamma_5 M^N_{5\mu\nu}. \tag{A38}$$

Under the $SU_A(3)$ chiral transformations, we have

$$\delta_5^{\vec{b}} \Delta^P_{5\mu\nu} = ib^N \gamma_5 \mathbf{F}^N_{PQ} \Delta^Q_{5\mu\nu}, \tag{A39}$$

and

$$\delta_5^{\vec{b}} M^M_{5\mu\nu} = 2b^N \gamma_5 f^{NMO} M^O_{5\mu\nu}. \tag{A40}$$

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