Analysis of single-photon and linear amplifier detectors for microwave cavity dark matter axion searches

S. K. Lamoreaux*

Department of Physics, Yale University, P.O. Box 208120, New Haven, Connecticut 06520-8120, USA

K. A. van Bibber

Department of Nuclear Engineering, University of California Berkeley, Berkeley, California 94720, USA

K. W. Lehnert

JILA and Department of Physics, University of Colorado and National Institute of Standards and Technology, Boulder, Colorado 80309, USA

G. Carosi

Physical and Life Sciences Directorate, Lawrence Livermore National Laboratory, Livermore, California 94550, USA

We show that at higher frequencies, and thus higher axion masses, single-photon detectors become competitive and ultimately favored, when compared to quantum-limited linear amplifiers, as the detector technology in microwave cavity experimental searches for galactic halo dark matter axions. The crossover point in this comparison is of order 10 GHz (~ 40 μeV), not far above the frequencies of current searches.

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I. INTRODUCTION

The next generation of microwave cavity experimental searches for axions constituting the dark matter halo of our Galaxy will possess the sensitivity to find or exclude favored axion models over a significant fraction of their allowed mass ranges. These experiments will benefit from new tunable cavity designs of higher frequency and possibly much higher quality factor Q with the use of thin-film superconducting coatings. They will also operate with much lower intrinsic noise owing to the development of quantum-limited amplifiers, such as microstrip-coupled SQUID amplifiers and Josephson parametric amplifiers (JPA). The purpose of this paper is to demonstrate that photon detectors will eventually win over linear amplifiers at high frequencies, and possibly not far above where the Axion Dark Matter Experiment (ADMX) and ADMX High Frequency (ADMX-HF) will soon begin taking data (~ 1 and ~ 5 GHz, respectively). Given that the microwave cavity experiments owe their extraordinary sensitivities to being both resonant and spectrally resolved, the possible utility of detectors which sacrifice phase information, all or in part, bears some discussion. Here we are concerned with *fundamental* detection limits that cannot be improved upon, and we do not address excess technical noise that can be eliminated with careful experimental design. We do note, however, that technical noise problems tend to be more simply solved at higher frequencies.

In Sikivie's microwave cavity experiment, axions resonantly convert into a very weak quasimonochromatic microwave signal in a high-Q cavity permeated by a strong magnetic field (Fig. 1) [1]. The axion-photon conversion signal power is given by

$$P_{a \to \gamma} = \eta g_{a\gamma\gamma}^2 \left[\frac{\rho_a}{m_a} \right] B_0^2 V C Q_c, \qquad (1)$$

where m_a and ρ_a are the mass and local density of the axion, respectively, $g_{a\gamma\gamma}$ is the axion-photon coupling, B_0 is the strength of the magnetic field, V is the cavity volume, and C is a mode-dependent form factor. The loaded (coupled) quality factor of the cavity is designated by $Q_c = Q/(1 + \beta)$, Q being the unloaded or intrinsic quality factor, and β the ratio of power coupled out by the antenna to power dissipated by cavity wall losses. The fraction of
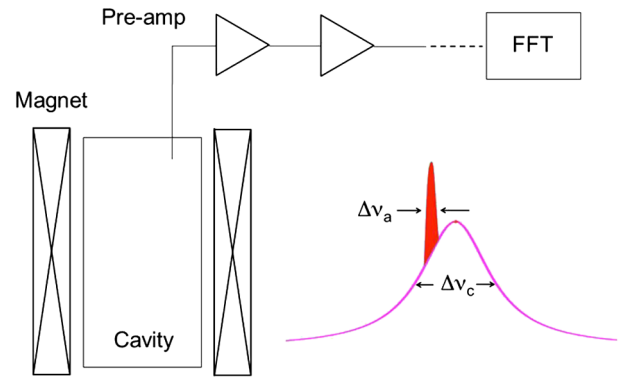

FIG. 1 (color online). Simplified schematic of the microwave cavity search for halo axions. The insert depicts the virialized axion signal ($Q_a \equiv E/\Delta E \sim 10^6$) within the Lorentzian bandpass of the cavity [$Q_c = Q/(1 + \beta)$].

*steve.lamoreaux@yale.edu

the converted power which is coupled out to the amplifier or other detector is thus $\eta = \beta/(1 + \beta)$.

The integration time required to achieve a desired signal-to-noise (SNR) ratio is given by the Dicke radiometer equation [2],

$$\text{SNR} = \frac{P_{a \to \gamma}}{k_B T} \sqrt{\frac{t}{\Delta \nu}}, \tag{2}$$

where the system noise temperature T is the sum of the physical temperature plus the equivalent noise temperature of the device $T = T_P + T_N$, k_B is Boltzmann's constant, t is the integration time, and $\Delta \nu$ is the signal bandwidth. Here linear amplifiers and single-photon detectors differ in two important respects. The first is the relevant bandwidth determining the integration time, which is $\Delta \nu_a \sim \nu/Q_a$ in the case of a spectral receiver. A single-photon detector would measure the power over the entire cavity bandpass, for which the relevant bandwidth is $\Delta \nu_c = \nu/Q_c$. Given that in practice $Q_a/Q_c \sim 10$–50 even for good copper cavities, a single-photon detector might not immediately suggest itself. However, recent research and development on the deposition of type-II superconducting thin films (e.g., $Nb_x Ti_{1-x} N$) on the axial surfaces of microwave cavities, i.e., barrel and tuning rods, holds out promise that hybrid normal-superconducting cavities would have a bandpass approximately the width of the virialized axion signal, $Q_a/Q_c \sim 3$ [3].

More importantly, linear receivers are subject to quantum fluctuations, leading to a standard quantum limit (SQL) in their performance. This limitation is parametrized as an effective minimum temperature, $T_{\text{SQL}} = h\nu/k_B$, for any linear amplifier [4,5]. The origin of the SQL can be understood as follows. Consider a well-defined mode of an electromagnetic field. In analogy with the harmonic oscillator and following Ref. [5], we introduce conjugate variables p_0 and q_0 as operators that determine the field; the operators commute as $[p_0, q_0] = i\hbar/2$, which is true for any field. Let us now apply a linear amplification to the zero point field, which increases each operator, hence the field, by a factor G, which motivates writing new field operators p_f, q_f as

$$[p_f, q_f] = \frac{i\hbar}{2} = [Gp_0, Gq_0] + [p_g, q_g]$$
$$= \frac{iG^2\hbar}{2} + [p_g, q_g], \tag{3}$$

where $[p_g, q_g]$ is introduced because linear amplification modifies the commutation relation. We see immediately that

$$[p_g, q_g] = \frac{i(1 - G^2)\hbar}{2}. \tag{4}$$

By use of the generalized Heisenberg uncertainty relation, the bounds on the fluctuations on p_g and q_g are determined by the value of their commutator,

$$\langle |\Delta p_g|^2 \rangle \langle |\Delta q_g|^2 \rangle \geq \frac{(G^2 - 1)^2 \hbar^2}{4} \tag{5}$$

(the zero point is a minimum uncertainty state, so the equality holds). The first term on the right-hand side of Eq. (3) results in a final field energy of $G^2 h\nu/2$, while the second term results in, by analogy, a field energy of $(G^2 - 1)h\nu/2$. Referring the final field energy to the initial field energy before amplification,

$$\frac{1}{G^2} \left[\frac{G^2 h\nu}{2} + \frac{(G^2 - 1)h\nu}{2} \right] = \frac{h\nu}{2} + \frac{(G^2 - 1)h\nu}{2G^2}$$
$$\approx 2 \times \frac{h\nu}{2} \tag{6}$$

in the limit of large G. This result shows final state fluctuations that imply twice the zero point energy for the initial state; this is the SQL of a linear amplifier. A principal implication is that the final state is, as might be expected, not a minimum uncertainty, or coherent, state.

In contrast to linear receivers, single-photon detectors—which provide a strongly coupled measurement of a quantum nondemolition variable—can in principle be arbitrarily noiseless. In the extreme case, $T \sim T_P \ll T_{\text{SQL}}$, it is easily seen that the scanning rate for single-photon detection relative to that for linear amplifiers improves exponentially with frequency, by a factor of $\approx (Q_c/Q_a) \cdot \exp(2h\nu/k_B T_P)$ (in the low temperature limit) and will be overwhelmingly advantageous at the high end of the open mass range. The main point of this paper, however, is to examine quantum noise in the cavity and amplifier more rigorously and to conclude that single-photon detectors may be favorable even at frequencies not far above the current search region, i.e. <10 GHz.

II. DETAILED ANALYSIS

Here we present a detailed analysis of the ultimate sensitivity of two schemes for the detection of electromagnetic field energy and a discussion of the conditions under which the sensitivity of one prevails over the other. Numerical estimates will be drawn from the example of the ADMX-HF experiment nearing commissioning. This platform is designed specifically to search for axions in the 20–100 μeV range (5–25 GHz), as well as serving as a test-bed for new cavity and detector concepts [6].

A linear detector provides a direct and coherent measurement of the electromagnetic field and is thus subject to phase/amplitude zero point fluctuation noise. In contrast, a single-photon detector is sensitive only to the photon number, not the signal phase. Examples of single-photon detectors are photomultiplier tubes and techniques that determine the photon number in a cavity [e.g., Rydberg atoms which respond to the square of the radio-frequency (RF) field amplitude]. However, at nonzero temperatures, single-photon detectors are subject to noise from fluctuations in the number of detected thermal photons. We further

assume that the detected axion signal is the same for all detection techniques, so we need only compare the noise power for the different techniques. Finally, we assume that the noise temperature $T_N \ll T_P$ so the temperature T is the same as the physical temperature of the system.

The detected noise power P_ℓ from a linear amplifier system is given by the Dicke radiometer equation, which gives the fluctuations in the average noise power that is detected,

$$P_\ell = k_B T \sqrt{\frac{\Delta \nu_a}{t}}, \qquad (7)$$

where the parameters have been defined in relation to Eq. (2). For all axion cavity experiments to date, Q_a associated with the axion is much larger than that of the cavity Q_c, typically for ADMX-HF, $Q_a \approx 10^6$ compared to the cavity $Q_c \approx 5 \times 10^4$. The axion Q_a is determined by the virialized axion de Broglie wavelength (around 100 meters). This means that the axion field will produce a coherent signal with a coherence time longer than the cavity lifetime; we think of the cavity in the applied magnetic field as a converter of the axion field to a radiofrequency field, with the cavity being a passive intermediary.

In the case of low temperatures, the Dicke radiometer equation must be modified to account for zero point fluctuations. The photon occupation number of a cavity resonant at frequency ν at temperature T is

$$n(\nu, T) = \frac{1}{e^{h\nu/k_B T} - 1} + \frac{1}{2} \equiv \bar{n} + \frac{1}{2}. \qquad (8)$$

For a linear amplifier, the zero point fluctuations contribute twice to the quantum-limited output noise [4,5]. This motivates an *ad hoc* modification of the Dicke radiometer equation as

$$P_\ell = h\nu(\bar{n} + 1) \sqrt{\frac{\Delta \nu_a}{t}}, \qquad (9)$$

which produces the SQL of a linear amplifier system and produces the high temperature limit given in Eq. (3).

Consider an axion search at 5 GHz (e.g., ADMX-HF), operating at realistic physical temperature of 20 mK. The average thermal photon number for this temperature is $\bar{n} \approx 6 \times 10^{-6}$. Using a linear amplifier operating at the SQL, where the effective photon number is $\bar{n} + 1$, the thermal noise in completely negligible, and there is no significant advantage to operate at this or lower temperatures. On the other hand, for photon counting experiments where the cavity photon number is detected, the zero point fluctuations that lead to the SQL do not contribute noise; photon-counting experiments are subject primarily to shot noise on the detected thermal photons. Given a detection efficiency η that depends on the detection method and is of order unity, the rate of photon detection is

$$\dot{n} = \eta \Gamma \bar{n}, \qquad (10)$$

where $\Gamma = 1/\tau_c$, and τ_c is the loaded cavity lifetime. (Here it is assumed that there is no excess noise, i.e., there are no fluctuations in detection efficiency, no ''dark'' noise with single photon detection, and that the excess noise temperature is zero.)

In an observation time t, N photons (both associated with the axion and with the thermal excitations) are detected, where

$$N = \dot{n} t = \eta \bar{n} \Gamma t, \qquad (11)$$

which has fluctuations due to counting statistics of

$$\delta N = \sqrt{N} = \sqrt{\eta \bar{n} \Gamma t}. \qquad (12)$$

Note that even though $\bar{n}$ is small, N is almost always large enough to use Poisson statistics: again taking 5 GHz, 20 mK, and $t \approx 100$ sec, $\Gamma \approx 10^5$ sec^{-1}, and $\bar{n} \approx 6 \times 10^{-6}$, $N \approx 60$ which is sufficiently large to use the $\sqrt{n}$ approximation for Poisson statistics.

As an aside, and for comparison, the detected axion signal itself will have shot noise, for example, if $P_{a \to \gamma} = 10^{-24}$ W, as expected in the range of models and of operating parameters for ADMX-HF, corresponding to 0.3 photons/second at 5 GHz, or 30 photons in a 100 second averaging period. This implies an intrinsic signal-to-noise ratio of $\approx \sqrt{30} = 5.5$, or for the present example, a total signal-to-noise ratio with thermal photons of $30/\sqrt{30 + 60} = 3.2$. This numerical example brings up an interesting issue: in the absence of thermal photons, the probability to observe at least one axion-generated photon over the observation time t should be at the 95% confidence limit. From Poisson statistics, the probability to observe zero events given a mean observation number of λ is $P(0, \lambda) = \exp(-\lambda)$, which is 0.05 for $\lambda \approx 3$. This can be taken as the minimum detectable axion signal for photon counting at zero temperature. For a nonzero temperature, the minimum detectable power can be determined from the usual criterion that the signal-to-noise ratio be at least 5. Thus, for the present numerical example, the signal-to-noise ratio for a 10^{-24} W signal is only 3.2, which is slightly below the minimum detection limit criterion.

Returning now to the principal issue of the photon counting shot noise fluctuations, assuming only thermal photons, Eq. (8) leads directly to an equivalent photon counting noise power,

$$P_{\text{sp}} = \frac{h\nu \delta N}{t} = h\nu \sqrt{\frac{\eta \bar{n} \Gamma}{t}}. \qquad (13)$$

Comparing the photon shot noise power and linear amplifier noise power,

$$\frac{P_\ell}{P_{\text{sp}}} = \frac{\bar{n} + 1}{\sqrt{\bar{n}}} \sqrt{\frac{\Delta \nu_a}{\eta \Gamma}}. \qquad (14)$$

This can be cast into a simpler form by noting that $\Gamma = 2\pi\nu/Q_c$; also, the optimum bandwidth for linear detection of the axion signal is $\Delta \nu_a \approx \nu/Q_a$. Therefore, in terms of the thermal photon occupation number, Q_a, and Q_c,

$$\frac{P_\ell}{P_{\mathrm{sp}}} = \left[\sqrt{\bar{n}} + \frac{1}{\sqrt{\bar{n}}} \right] \sqrt{\frac{Q_c}{2\pi\eta Q_a}}. \tag{15}$$

(η can be different for the two detection methods, and in general it is not the noise powers that need to be compared, but the SNRs. Given that $\eta \sim 1$, the comparison of the noise powers is adequate for the present discussion.)

In the case of ADMX-HF, operating at a modest temperature of 100 mK, $\bar{n} = .1$, and conservatively $Q_a/Q_c \approx 20$ so (with $\eta \approx 1$)

$$\frac{P_\ell}{P_{\mathrm{sp}}} \approx \left(\frac{1}{.3} + .3 \right) \times \frac{1}{9.5} \approx \frac{1}{3}, \tag{16}$$

and it appears that linear detection is better than photon counting detection for these parameters.

On the other hand, if we pick a readily achievable temperature $T = 10$ mK, then $\bar{n} \approx 3 \times 10^{-11}$ then $P_\ell/P_{\mathrm{sp}} \sim 17,000$, and clearly single photon detection has lower noise (and we need to include dark count rates and other relevant contributors to the noise assessment). Similarly, for a conservative temperature $T = 100$ mK, but with an improved cavity quality factor, $Q_a/Q_c = 3$, $P_\ell/P_{\mathrm{sp}} \sim 0.5$ so linear detection will contribute lower noise. With both lower temperature and enhanced quality factor together, single-photon detection is obviously superior, $P_\ell/P_{\mathrm{sp}} \sim 30,000$. In these limits careful attention needs to be paid to the dark count rate of single photon detection (see the summary below).

III. PROSPECTS FOR SINGLE-PHOTON AND SQUEEZED-STATE DETECTION

We briefly discuss in turn the status of actual devices such as Rydberg atom single-photon detectors and prospects of more conventional quantum devices such as JPAs. We are concerned here only with the fundamentals of the detection process and do not discuss the technical issues relating to their implementation in ADMX or ADMX-HF.

The possibility of single-photon detection in an ADMX-type experiment has already been demonstrated in the CARRACK experiment, which used a Rydberg atom single-photon detector [7]. The temperature dependence of blackbody photons in the cavity at 2.527 GHz, measured by the resonant absorption of the $111s_{1/2} \rightarrow 111p_{3/2}$ transition in a ^{85}Rb beam and subsequent selective field ionization, was in perfect agreement with the theoretical occupation number down to $T = 67$ mK, a factor of two below the SQL noise temperature. While sustained operation as an axion search was not achieved at that time, no fundamental limitations were encountered beyond the overall complexity of the technique.

Recent advances have simplified Rydberg atom microwave electrometry [8]. These new methods have the electric field sensitivity required to detect the presence of a single photon in the ADMX-HF cavity and are tunable over the 1–100 GHz range. However, achieving this level of sensitivity requires a bandwidth of order 1 Hz. Because a single photon will exist as a transient with the loaded cavity lifetime, the single photon detection bandwidth needs to be comparable to the cavity bandwidth, and this will require about a factor of 1000 improvement in signal-to-noise ratio for these new methods, which might be possible.

JPAs, such as used presently in ADMX-HF, naturally generate squeezed microwave fields, and the technology of parametric amplification has advanced sufficiently that it is realistic to imagine injecting a squeezed field generated by one JPA into the axion cavity and then detecting the field emerging from the cavity with a second JPA [9]. The virtue of detecting a squeezed field with a JPA can be seen by writing the cavity's state as a quantum mechanical generalization of a voltage phasor,

$$V_{\mathrm{cav}} \propto [\hat{q}_0(t) \cos(\omega_c t) + \hat{p}_0(t) \sin(\omega_c t)] \tag{17}$$

The two quadratures $\hat{q}_0$ and $\hat{p}_0$ neither commute with the Hamiltonian nor with each other. Separately these non-commutations each contribute $h\nu/2$ and thus a total noise $h\nu$ to the system. However, if the cavity is prepared not in its vacuum state, but in a squeezed state with reduced fluctuations in $\hat{q}_0$ (and consequently amplified fluctuations in $\hat{p}_0$), and one measures $\hat{q}_0$ only, the noise power could be suppressed to arbitrarily low values dependent on the degree of squeezing. Presently, a receiver's system noise can be reduced to about $h\nu/4$, limited by the power losses in microwave components and cables [10]. Such a modest improvement does not justify the additional complexity of the receiver at this time, but with improvements in the efficiency of microwave components, squeezed state devices may soon be an attractive option.

Superconducting circuits designed to act as quantum bits (qubits) seem well-poised to act as microwave photodetectors in an axion experiment. Indeed, superconducting qubits have been operated as detectors of single microwave photons analogous to optical photomultiplier tubes [11], but as yet the dead time and bandwidth of these detectors are not well suited to the axion signal. More promising would be to adapt the systems that embed a superconducting qubit inside of a microwave cavity [12] [so-called circuit quantum electrodynamics (cQED)] to the task of photodetection. These cQED systems have demonstrated an astounding ability to control and detect microwave fields inside the cQED cavity itself. Specifically, the presence of a single microwave photon inside a cavity is sufficient to shift the qubit transition frequency by many qubit linewidths. Consequently, one can test for the presence of a single photon by applying a microwave pulse—resonant with the qubit but not the cavity—that brings the qubit to its excited state if and only if the cavity contains a single photon. By subsequently measuring the state of the qubit, one has answered the question "is there a photon in the cavity?" Remarkably, the whole procedure can be accomplished

faster than a photon decays from the cavity [13]. As such, one can envision coupling the radiation field emerging from the axion cavity into the cQED cavity and interrogating the cQED cavity sufficiently rapidly to match the axion cavity bandwidth. While many design challenges must be overcome to incorporate a state-of-the-art cQED experiment into an axion search, the scheme would be technically homogenous and naturally cryogenic.

IV. SUMMARY

In conclusion, the axion signal has a narrow bandwidth compared to the cavity where the axions are converted to RF photons, and the relative narrowness is of advantage in a linear detection system because we need only consider the cavity noise over a small bandwidth, whereas single photon (or bolometric) detection experiences the full cavity noise bandwidth resulting in shot noise in the detected thermal photons. Photon counting techniques become favorable at sufficiently low temperatures (10 mK for ADMX-HF) and higher Q_c RF cavities. However, one must bear in mind that increasing $Q_c > Q_a$ will result in a loss of axion signal, implying an upper limit to the improvement by increasing Q_c.

In comparing the noise performance of single-photon vs linear detection, the parameters of interest are the ratio of the axion and cavity Q's and the number of thermal photons in the cavity; in the limit of $k_B T / h\nu \ll 1$,

$$\left[\frac{P_\ell}{P_{\rm sp}}\right] \approx \sqrt{\frac{Q_c}{2\pi Q_a}} e^{h\nu/k_B T}, \qquad (18)$$

and when this number is large, cavity photon counting can have a lower noise than linear detection. At conservative values of $T = 100$ mK and $Q_a/Q_c = 20$, the crossover point occurs at a surprisingly low frequency (~ 10 GHz), not far above where the current round of experiments will be running. The device technology to support such a strategy may already be at hand.

One must bear in mind, however, that there remains a fundamental limit to the detectable axion photon power. As discussed, a minimum number of three photons that must be detected to give 95% confidence for the presence of an axion signal. Thus, the minimum power, given a frequency ν and observation time t, is (which are 5 GHz and 100 sec for ADMX-HF)

$$P_{\rm min} = \frac{3h\nu}{t} = 10^{-25}\ {\rm W}\left[\frac{\nu}{5\ {\rm GHz}}\frac{100\ {\rm sec}}{t}\right]. \qquad (19)$$

Furthermore, to achieve this limit, the dark count rate must be suppressed to below one count during t at the 95% confidence level. Therefore, to observe zero photons with 95% probability, $P(0, \lambda) = \exp(-\lambda) = 0.95$ or $\lambda \approx 0.05 = 1/20$, so that

$$n\Gamma t = \frac{1}{20} = \Gamma t e^{-h\nu/k_B T_{\rm max}}. \qquad (20)$$

This implies the physical temperature requirement

$$T < T_{\rm max} = \frac{h\nu}{k_B \log(20\Gamma t)} \approx 14.5\ {\rm mK}\left[\frac{\nu}{5\ {\rm GHz}}\right], \qquad (21)$$

using typical parameters for ADMX-HF ($Q_c \approx 40,000$, $\nu = 5$ GHz, $t = 100$ sec). These are quite fundamental limits on the minimum detectable axion power and on the maximum physical temperature for single photon counting to be fully effective.

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