Polarization in $\chi_{cJ} \to \phi\phi$ decays

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To understand the enhanced partial decay width for the decay of χ_{c1} to the vector-meson pair $\phi\phi$ recently observed by the BESIII collaboration, we suggest measuring the polarization parameter of the ϕ meson by analyzing the jointed angular distribution in the cascade decay $e^+e^- \to \psi(2S) \to \gamma\chi_{cJ} \to \gamma\phi\phi \to \gamma 2(K^+K^-)$. The formulas to measure polarization parameters are presented, and they are estimated in the framework of perturbative QCD (pQCD) and the quark-pair-creation model. In pQCD we obtain results consistent with the expectation of the helicity selection rule. In the quark-pair-creation model, the parameter associated with the violation of the helicity selection rule is enhanced relative to that estimated in pQCD.

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I. INTRODUCTION

Since the discovery of the J/ψ particle, the study of the decay of charmonia has continued to be an active field in understanding the nature of strong interactions [1–5]. In the charmonium energy region $3 \sim 5$ GeV—even though the charm quark is not as heavy as the b- or t-quark in high-energy processes—some perturbative QCD (pQCD) asymptotic behaviors can be expected. For example, the so-called helicity selection rule (HSR) was obtained in early pQCD analyses [1,2], which states that if the mass of a light quark produced from $c\bar{c}$ annihilation is neglected, the vector-gluon coupling conserves quark helicity. Thus the amplitude will vanish if the spin of the light quark is reversed by the gluon scattering.

For a charmonium ψ decay to light hadrons $h_1(\lambda_1)$ and $h_2(\lambda_2)$, the asymptotic behavior of the branching fraction is given by pQCD calculations [2,6],

$$\mathrm{Br}[\psi(\lambda) \to h_1(\lambda_1)h_2(\lambda_2)] \sim \left(\frac{\Lambda^2_{\mathrm{QCD}}}{m_c^2}\right)^{|\lambda_1+\lambda_2|+2}, \quad (1)$$

where λ, λ_1, and λ_2 denote the helicity of the corresponding hadrons, m_c is the charm-quark mass, and Λ_{QCD} is the QCD energy-scale factor. If the light-quark mass is neglected, the vector-gluon coupling conserves quark helicity and this leads to the helicity selection rule [1]: $\lambda_1 + \lambda_2 = 0$. If the helicity configuration does not satisfy this relation, the branching fraction will be suppressed.

In the quark model, the $\chi_{cJ}(J = 0, 1, 2)$ states are assigned as P-wave charmonium states with spin parity and charge conjugation J^{++}. The decay of $\chi_{cJ} \to \phi\phi$ is a golden mode used to test the HSR prediction since the helicity combination of the $\phi\phi$ pair can provide us with the polarization measurement of at most nine spin configurations. The pQCD calculation [7] shows that the partial decay width with the helicity combination $\lambda_1 + \lambda_2 \neq 0$ is highly suppressed due to the HSR, so this leads to the problem of a calculated decay width for $\chi_{c0/2} \to \phi\phi$ smaller than the measured value. Furthermore, the decay of χ_{c1} to $\phi\phi$ is forbidden at the leading order due to the Landau-Yang theorem [8].

However, the BESIII collaboration [9] has recently observed for the first time the decay of χ_{c1} to a vector-meson pair VV ($VV = \phi\phi$, $\omega\omega$, and $\omega\phi$) with branching fraction at the same order as $\chi_{c0,2}$ decays. The measurement indicates the evasion of HSR in this energy region [6], and poses a challenge for the χ_{cJ} two-gluon decay mechanism. In the interplay of the perturbative and nonperturbative regions, there might be another mechanism contributing to the HSR decays, such as the charm octet state, the higher-order Fock state of light hadrons, the final-state interaction, or other long-distance effects. To account for the nonperturbative effects, the role of the charm-meson loop in $\chi_{c1} \to VV$ decays has been investigated [6,10]. The results show that the charm-meson loop can produce the branching fractions for $\chi_{c1} \to \phi\phi$, $\omega\omega$ with a large uncertainty, but it fails to explain the double Okubo-Zweig-Iizuka decay $\chi_{c1} \to \omega\phi$. To consistently understand the χ_{cJ} decay mechanism, more experimental information is desirable, such as the polarization information of the vector meson.

Studies on the polarization parameter in the decay of χ_{cJ} to the vector-meson pair $\phi\phi$ will shed light on the underlying structure of χ_{cJ} states and their decay mechanisms. Experimentally, decays of $\chi_{cJ} \to \phi\phi$ can be easily reconstructed using $\phi \to K\bar{K}$ or $\pi^+\pi^-\pi^0$ decay modes with low-level backgrounds. These decays facilitate the measurement of ϕ polarization by analyzing the jointed angular distribution. In this work, we present the formula for the joint angular distribution for $\chi_{cJ} \to \phi\phi \to 2(K^+K^-)$, which can be used in data analysis. The polarization parameters are studied in the pQCD scheme and the quark-pair-creation model.

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II. POLARIZATION PARAMETERS

We consider the decay $\chi_{cJ} \to \phi\phi \to 2(K^+K^-)$, where the $\chi_{cJ}(J = 0, 1, 2)$ states are produced from the transitions $\psi(2S) \to \gamma\chi_{cJ}$, which are characterized by the spin density matrix $\rho_{(J)}$. The ϕ polarization parameter can be measured through the analysis of the angular distribution. The joint amplitudes for the sequential decays are constructed with helicity angles as follows.

(i) For the first decay $\chi_{cJ} \to \phi\phi$, the solid angle of the ϕ meson $[\Omega_1(\theta_1, \varphi_1)]$ refers to the χ_{cJ} rest frame, where the z axis is taken along the outgoing direction of the χ_{cJ} state in the $\psi(2S)$ rest frame.

(ii) Similarly, for the decay $\phi \to K^+K^-$, the solid angle of the kaon $[\Omega_2(\theta_2, \varphi_2)]$ (Ω_3 for another ϕ decay) refers to the ϕ meson rest frame, where the z axis is taken along the outgoing direction of the ϕ state in the χ_{cJ} rest frame.

Given ϕ helicity states, the amplitudes for these decays are denoted by

$$B^{(J)}_{\lambda_1,\lambda_2} \quad \text{for } \chi_{cJ}(m_1) \to \phi(\lambda_1)\phi(\lambda_2)(\Omega_1), \tag{2}$$

$$b \quad \text{for } \phi(\lambda_1) \to K^+K^-(\Omega_2), \tag{3}$$

$$b \quad \text{for } \phi(\lambda_2) \to K^+K^-(\Omega_3), \tag{4}$$

where $\lambda_{1,2}$ denotes the ϕ helicity value, and $\Omega_i(\theta_i, \varphi_i)$ $(i = 1, 2, 3)$ stands for the solid angle for the corresponding decay.

The joint amplitude for the $\chi_{cJ}(J = 0, 1, 2)$ sequential decay reads

$$|M|^2_J \propto \sum_{m_1,m_1',\lambda_i} \rho^{(m_1,m_1')}_{(J)} D^J_{m_1,\lambda_1-\lambda_2}(\Omega_1) D^{J*}_{m_1',\lambda_1'-\lambda_2'}(\Omega_1) \times B^{(J)}_{\lambda_1,\lambda_2} B^{(J)*}_{\lambda_1',\lambda_2'} D^1_{\lambda_1,0}(\Omega_2) D^{1*}_{\lambda_1',0}(\Omega_2) \times D^1_{\lambda_2,0}(\Omega_3) D^{1*}_{\lambda_2',0}(\Omega_3)|b|^4. \tag{5}$$

Here the spin-density matrix $\rho_{(J)}$ describing the χ_{cJ} production can be estimated from the $e^+e^- \to \psi(2S) \to \gamma\chi_{cJ}$ process, which is determined by

$$\rho^{(\lambda_0,\lambda_0')}_{(J)} = \int d\cos\theta_0 d\phi_0 \sum_{M,\lambda_\gamma=\pm1} D^1_{M,\lambda_\gamma-m_1}(\theta_0, \phi_0) \times D^{1*}_{M,\lambda_\gamma-m_1'}(\theta_0, \phi_0) A^{(J)}_{\lambda_\gamma,m_1} A^{(J)*}_{\lambda_\gamma,m_1'}, \tag{6}$$

where M, λ_γ, and m_1 are the helicity values for the $\psi(2S)$, photon, and χ_{cJ} states, respectively, and $A^{(J)}_{\lambda_\gamma,m_1}$ is the helicity amplitude for $\psi(2S) \to \gamma\chi_{cJ}$ with the helicity angle $\Omega_0(\theta_0, \phi_0)$. The sum over M takes $M = \pm1$ since the $\psi(2S)$ is produced from the e^+e^- annihilation. Recent measurements have shown that the contribution of high magnetic and electric multipoles to χ_{cJ} production is negligible, and the $E1$ transition dominates this process [11]. Hence, the components of the helicity amplitude are chosen to satisfy the $E1$ relation [12], namely, $A^{(1)}_{1,1} = A^{(1)}_{1,0}$ for χ_{c1} and $A^{(2)}_{1,2} = \sqrt{2}A^{(2)}_{1,1} = \sqrt{6}A^{(2)}_{1,0}$ for χ_{c2}. Further, by considering the requirement of parity conservation, one has the relation $A^{(J)}_{-\lambda_\gamma,-m_1} = (-1)^J A^{(J)}_{\lambda_\gamma,m_1}$. Thus the spin densities are determined to be $\rho_{(J=0)} = 1$, $\rho_{(J=1)} \propto \text{diag}\{1, 2, 1\}$, and $\rho_{(J=2)} \propto \text{diag}\{2, 1, 2/3, 1, 2\}$.

The decay rate for the decay of χ_{cJ} to a $\phi\phi$ pair is expected to be dominated by the strong interaction, by which $c\bar{c}$ quarks are annihilated into gluons and then materialize into the final state. In the strong decay the parity is conserved such that the helicity amplitude satisfies the relation $B^{(J)}_{-\lambda_1,-\lambda_2} = (-)^J B^{(J)}_{\lambda_1,\lambda_2}$. Thus one has the relations $B^{(1)}_{1,1} = -B^{(1)}_{-1,-1}$, $B^{(1)}_{1,0} = -B^{(1)}_{-1,0}$, $B^{(1)}_{0,1} = -B^{(1)}_{0,-1}$, and $B^{(1)}_{0,0} = 0$ for χ_{c1} decays, while for $\chi_{c0,2}$ decays, one has the relation $B^{(J)}_{0,0} \neq 0$ with $J = 0, 2$.

By taking the identical particle symmetry into consideration, the helicity amplitude for the decay of χ_{cJ} to a $\phi\phi$ pair satisfies the relation $B^{(J)}_{\lambda_1,\lambda_2} = (-1)^J B^{(J)}_{\lambda_2,\lambda_1}$, which indicates that the structure of the amplitude for the decay $\chi_{c1} \to \phi\phi$ is antisymmetric by exchanging two ϕ mesons, and hence an amplitude with $\lambda_1 = \lambda_2$ will vanish, while a nonvanishing amplitude satisfies the relation $B^{(1)}_{1,0} = -B^{(1)}_{0,1}$. As for the $\chi_{c0,2} \to \phi\phi$ decay, all amplitudes are nonvanishing and symmetric in terms of exchanging two ϕ mesons. Thus one can obtain the relation $B^{(2)}_{1,0} = B^{(2)}_{0,1}$ for χ_{c2} decays.

Using these relations, one obtains the jointed angular distribution for the $\chi_{c0} \to \phi\phi$ decay,

$$\frac{d|M|^2_0}{d\cos\theta_1 d\cos\theta_2 d\cos\theta_3} \propto 2\cos^2\theta_2\cos^2\theta_3 + x\sin^2\theta_2\sin^2\theta_3, \tag{7}$$

where the variable $x = |B^{(0)}_{1,1}|^2/|B^{(0)}_{0,0}|^2$ measures the fraction of transverse over longitudinal polarization.

For the $\chi_{c1} \to \phi\phi$ decay, one has

$$\frac{d|M|^2_1}{d\cos\theta_1 d\cos\theta_2 d\cos\theta_3} \propto (2 + \sin^2\theta_1)(\sin^2\theta_2\cos^2\theta_3 + u_1\cos^2\theta_2\sin^2\theta_3) + u_2(1 + \cos^2\theta_1)\sin^2\theta_2\sin^2\theta_3, \tag{8}$$

with the variables $u_1 = |B^{(1)}_{0,1}|^2/|B^{(1)}_{1,0}|^2$ and $u_2 = |B^{(1)}_{1,1}|^2/|B^{(1)}_{1,0}|^2$. The requirement of identical particle symmetry leads to the results $u_1 = 1$ and $u_2 = 0$.

For the $\chi_{c2} \to \phi\phi$ decay, one has

$$\frac{d|M|_2^2}{d\cos\theta_1 d\cos\theta_2 d\cos\theta_3} \propto (7 - 3\cos 2\theta_1)\cos^2\theta_2\cos^2\theta_3 + w_1(9 - 3\cos^2\theta_1)\cos^2\theta_2\sin^2\theta_3 + w_2(3 + 3\cos^2\theta_1)\sin^2\theta_2\sin^2\theta_3 + w_3(9 - 3\cos^2\theta_1)\sin^2\theta_2\cos^2\theta_3 + w_4(5 - 3\cos^2\theta_1)\sin^2\theta_2\sin^2\theta_3, \quad (9)$$

where the variables $w_1 = |B_{0,1}^{(2)}|^2/|B_{0,0}^{(2)}|^2$, $w_2 = |B_{1,-1}^{(2)}|^2/|B_{0,0}^{(2)}|^2$, $w_3 = |B_{1,0}^{(2)}|^2/|B_{0,0}^{(2)}|^2$, and $w_4 = |B_{1,1}^{(2)}|^2/|B_{0,0}^{(2)}|^2$ measure fractions of transverse over longitudinal polarization. The requirement of identical particle symmetry leads to the relation $w_1 = w_3$.

In the χ_{cJ} helicity frame, the polar angular distribution of the ϕ meson is reduced to

$$\frac{d|M|^2}{d\cos\theta_1} \propto (1 + \alpha\cos^2\theta_1), \quad (10)$$

with

$$\alpha = \begin{cases} 0 & \text{for } \chi_{c0}, \\ \frac{2|B_{1,1}^{(1)}|^2 - |B_{0,1}^{(1)}|^2 - |B_{1,0}^{(1)}|^2}{3(|B_{1,0}^{(1)}|^2 + |B_{0,1}^{(1)}|^2) + 2|B_{1,1}^{(1)}|^2} & \text{for } \chi_{c1}, \\ -\frac{6|B_{1,1}^{(2)}|^2 + 3|B_{1,0}^{(2)}|^2 - 6|B_{1,-1}^{(2)}|^2 + 3|B_{0,1}^{(2)}|^2 + 3|B_{0,0}^{(2)}|^2}{10|B_{1,1}^{(2)}|^2 + 9|B_{1,0}^{(2)}|^2 + 6|B_{1,-1}^{(2)}|^2 + 9|B_{0,1}^{(2)}|^2 + 5|B_{0,0}^{(2)}|^2} & \text{for } \chi_{c2}, \end{cases} \quad (11)$$

where α is the angular distribution parameter, which indicates that χ_{c0} decays into a $\phi\phi$ pair yield a flat polar angle distribution due to its zero spin, while for χ_{c1} decays the requirement of identical particle symmetry yields $\alpha = -1/3$.

The above formula can be generalized to the decay $\chi_{c0,1,2} \to \omega\omega \to 2(\pi^+\pi^-\pi^0)$. For the three-body decay $\omega^{(J=1)} \to \pi^+(E_1)\pi^-(E_2)\pi^0(E_3)$, following the convention used in Ref. [13], the amplitude is given by

$$A(m) = \sqrt{\frac{2J+1}{8\pi^2}} F_\mu^J(E_1E_2E_3) D_{m\mu}^{J*}(\alpha, \beta, \gamma), \quad (12)$$

where m is the eigenvalue of the component of the angular momentum operator J along an axis chosen to define the ω helicity, μ is the eigenvalue of angular momentum along the normal to the three-pion-decay plane, $E_i (i = 1, 2, 3)$ denotes the pion energy measured in the ω rest frame, $J = 1$ is the spin of the ω particle, and α, β, γ are the Euler angles, which are defined as the angles to rotate the normal to the three-pion-decay plane along the ω outgoing direction seen in the χ_{cJ} rest frame. $F_\mu^J(E_i)$ is the helicity amplitude, which satisfies the relation required by parity conservation,

$$F_\mu^J(E_1E_2E_3) = (-1)^\mu F_\mu^J(E_1E_2E_3). \quad (13)$$

So one has $F_\pm^J(E_1E_2E_3) = 0$, and the amplitude is reduced to

$$A(m) = \sqrt{\frac{2J+1}{8\pi^2}} F_0^1(E_1E_2E_3) D_{m0}^{1*}(\alpha, \beta, 0). \quad (14)$$

Compared to the $\chi_{cJ} \to \phi\phi \to 2(K^+K^-)$ decay, the joint angular distribution for $\chi_{cJ} \to \omega\omega$ can be obtained by making the replacement with $\theta_i \to \beta_{i-1} (i = 2, 3)$ in Eqs. (7)–(9).

III. $\chi_{cJ} \to \phi\phi$ DECAYS IN THE PQCD SCHEME

We first estimate the ϕ-meson polarization parameters in the pQCD scheme. To the lowest order, the decay of the χ_{cJ} meson proceeds via two steps. First, the $\chi_{c0,2}$ states annihilate into two gluons (gg), and then materialize into two outgoing (anti)quarks, as illustrated in Fig. 1(a). The

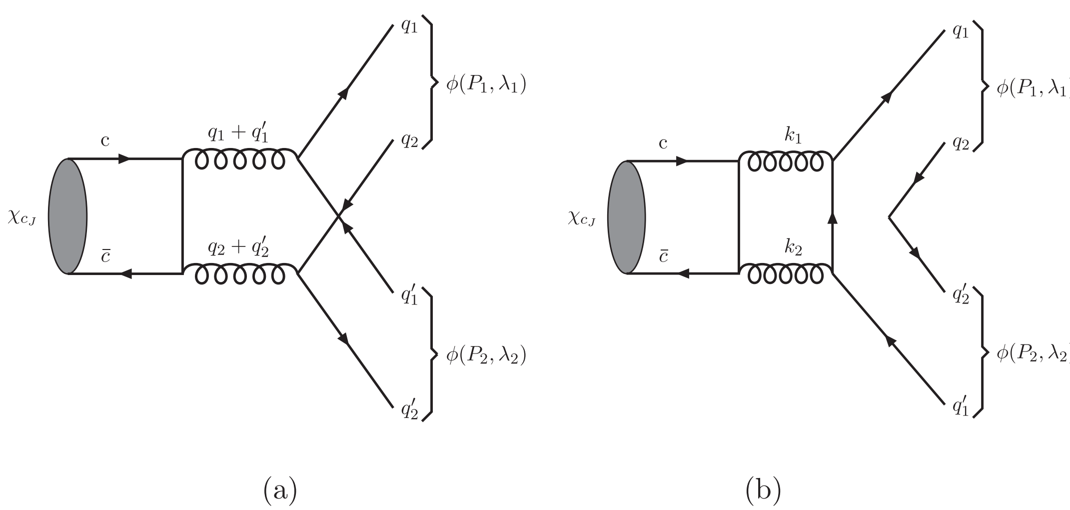

FIG. 1. Mechanisms for $\chi_{cJ} \to \phi\phi$: (a) The perturbative QCD scheme and (b) the $^3P_0$ quark-pair-creation scheme.

next leading order contributes via another two additional gluon exchanges between the two pairs of strange quarks. These amplitudes are suppressed by a factor $(\frac{\alpha_s}{\pi})^4$. In our previous work, it was found that the leading-order approximation can produce a comparable branching fraction for $\chi_{c2} \to \phi\phi$ [7]. As per the usual treatment, the helicity amplitude can be decomposed into a hard-scattering part multiplied by the wave functions of two-ϕ mesons. Thus one has the helicity amplitude for $\chi_{cJ}(\lambda) \to \phi(\lambda_1)\phi(\lambda_2)$,

$$\begin{aligned}\mathcal{M}(\lambda, \lambda_1, \lambda_2) &\equiv \langle \psi(P_1, \lambda_1)\psi(P_2, \lambda_2)|T_J|\chi_{cJ}\rangle \\ &= \int\left(\prod_{i=1}^{2} d^3\mathbf{q}_i d^3\mathbf{q}_i'\right)\langle \psi(P_1, \lambda_1)\psi(P_2, \lambda_2) \\ &\quad \times |q_i, s_i, q_i', s_i'\rangle\langle q_i, s_i, q_i', s_i'|T_J|\chi_{cJ}(\lambda)\rangle \\ &\quad \times \delta^3(\mathbf{P}_1 - \mathbf{q}_1 - \mathbf{q}_2)\delta^3(\mathbf{P}_2 - \mathbf{q}_1' - \mathbf{q}_2'),\end{aligned} \tag{15}$$

where $\psi(P_i, \lambda_i)$ $(i = 1, 2)$ is the wave function of the ϕ meson with momentum P_i and helicity value λ_i. The hard-scattering amplitude $\langle q_i, s_i, q_i', s_i'|T_J|\chi_{cJ}\rangle$ can be obtained according to standard Feynman rules for the process $\chi_{cJ} \to gg \to (s\bar{s})(s\bar{s})$, as shown in Fig. 1(a).

If we neglect dynamical contributions from $c\bar{c}$ quarks, and parametrize the decay of $\chi_{cJ} \to gg$ into a decay constant $f_J (J = 0, 2)$, then one obtains

$$\begin{aligned}&\langle q_i, s_i, q_i', s_i'|T_0|\chi_{c0}\rangle \\ &= \frac{f_0 c_a \alpha_s g_{\mu\nu}\bar{u}(q_1, s_1)\gamma^\mu v(q_1', s_1')\bar{u}(q_2', s_2')\gamma^\nu v(q_2, s_2)}{(q_1 + q_1')^2(q_2 + q_2')^2}\end{aligned} \tag{16}$$

for χ_{c0} decays, where $u(q_i, s_i)$ and $v(q_i, s_i)$ are free Dirac spinors for a quark and antiquark with momentum q_i and spin s_i, respectively. They are normalized as $u^+(q_i, s_i)u(q_i, s_i') = -v^+(q_i, s_i)v(q_i, s_i') = \delta_{ss'}$. α_s is a strong coupling constant, and c_a is the color factor for the process shown in Fig. 1(a) with $c_a = \frac{1}{3}[\frac{\lambda_a}{2}]_{ik} \times [\frac{\lambda_a}{2}]_{jl}\delta_{ij}\delta_{kl} = 1/3$, where λ_a is the color SU(3) matrix.

As for χ_{c2} decays, we have

$$\begin{aligned}&\langle q_i, s_i, q_i', s_i'|T_2|\chi_{c2}(\lambda)\rangle \\ &= \frac{f_2 c_a \alpha_s \Phi_{\mu\nu}(\lambda)\bar{u}(q_1, s_1)\gamma^\mu v(q_1', s_1')\bar{u}(q_2', s_2')\gamma^\nu v(q_2, s_2)}{(q_1 + q_1')^2(q_2 + q_2')^2},\end{aligned} \tag{17}$$

where f_2 is a decay constant for $\chi_{c2} \to gg$, and $\Phi_{\mu\nu}(\lambda)$ is the covariant spin wave function of χ_{c2} with the helicity value λ, which can be built out of the polarization vector from the relation

$$\Phi_{\mu\nu}(\lambda) = \sum_{m_1, m_2}\langle 1m_1, 1m_2|2\lambda\rangle\epsilon_\mu(m_1)\epsilon_\nu(m_2), \tag{18}$$

where $\epsilon(m_i)$ is the polarization vector for the spin-1 particle with helicity value m_i.

IV. $\chi_{cJ} \to \phi\phi$ DECAYS IN THE $^3P_0$ MODEL

Figure 1(b) shows the decay of χ_{cJ} to a $\phi\phi$ pair via the mechanism of the $^3P_0$ quark-pair creation. The decay is assumed to proceed via two steps. First, the $c\bar{c}$ quarks annihilate into two gluons, then two gluons materialize into a strange-quark pair. Due to the quark-gluon coupling, another quark-antiquark pair $(q\bar{q})$ is allowed to be produced from the QCD vacuum with the quantum number $J^{PC} = 0^{++}$, which corresponds to the $q\bar{q}$ state of $^3P_0$. Generally, a quark pair with any flavor and color can be generated anywhere in space, but only those whose flavor-color wave function and spacial wave function overlap with those of outgoing ϕ mesons can make a contribution to the partial decay width. Following the usual procedure, the Hamiltonian for the created quark pair in the $^3P_0$ model [14] is expressed in terms of the quark and antiquark creation operators b^+ and d^+ as

$$\begin{aligned}H_I = &\sum_{i,j,\alpha,\beta,s,s'} 2m_q\gamma\int d^3k[\bar{u}(\vec{k}, s)v(-\vec{k}, s')]b^+_{\alpha,i}(\vec{k}s) \\ &\times d^+_{\beta,j}(-\vec{k}, s')\delta_{\alpha\beta}\hat{C}_I,\end{aligned} \tag{19}$$

where $\alpha(\beta)$ and $i(j)$ are the flavor and color indices of the created quark(antiquark), and $u(k, s)$ and $v(k', s')$ are free Dirac spinors for the quark and antiquark, respectively. $\hat{C}_I = \delta_{ij}$ is the color operator for $q\bar{q}$ and γ is the strength of the quark-pair creation independent of flavor, which is assumed as a constant in χ_{cJ} decays. Note that the amplitude for the $q\bar{q}$ pair created in the $^3P_0$ model is free of the HSR suppression, and hence suggests the evasion of the HSR.

The four-vector momentum for gluons and the (anti) quark involved in the $^3P_0$ model [see Fig. 1(b)] are defined as follows:

$$\begin{aligned}\mathbf{q}_i = -\mathbf{q}_i', \qquad q_i^0 = q_i'^0, \\ k_1 + k_2 = (M_{\chi_c}, \vec{0}), \qquad k_1 - k_2 = (0, 2\mathbf{k}).\end{aligned} \tag{20}$$

Similarly, the helicity amplitude in the $^3P_0$ model is constructed with the relation given in Eq. (15), and the hard-scattering amplitude of the χ_{c0} decay reads

$$\begin{aligned}&\langle q_i, s_i, q_i', s_i'|T_0|\chi_{c_0}\rangle \\ &= f_0 c_b \alpha_s 2m\gamma\int\frac{d^4k}{(2\pi)^4}g_{\mu\nu}A^{\mu\nu}\bar{u}(q_2's_2')v(q_2s_2)\frac{1}{k_1^2k_2^2} \\ &\quad \times \delta^3(\mathbf{q}_2 + \mathbf{q}_2') + (q_1 \leftrightarrow q_2, q_1' \leftrightarrow q_2') \\ &= -f_0 c_b \alpha_s 2m\gamma\int\frac{d\Omega_{k_1}}{8\pi^2}g_{\mu\nu}A^{\mu\nu}\bar{u}(q_2's_2')v(q_2s_2) \\ &\quad \times \delta^3(\mathbf{q}_2 + \mathbf{q}_2') + (q_1 \leftrightarrow q_2, q_1' \leftrightarrow q_2'),\end{aligned} \tag{21}$$

with

$$A^{\mu\nu} = \bar{u}(q_1 s_1)\gamma^\mu \frac{\not{q}_1 - \not{k}_1 + m}{(q_1 - k_1)^2 - m^2}\gamma^\nu v(q_1' s_1'), \quad (22)$$

where γ is the strength of the $^3P_0$ quark-pair creation, m is the strange-quark mass, $c_b = 4/3$ is the color factor, and Ω_{k_1} is the solid angle for $\mathbf{k}_1$. In the above equation, we have used Cutkosky's cutting rule [15] and made the replacement for gluonic propagators, i.e., $1/(k_1^2 k_2^2) \to (2\pi i)^2\delta(k_1^2)\delta(k_2^2)$.

The hard-scattering amplitude of the $\chi_{c2} \to \phi\phi$ decay is given by

$$\begin{aligned}\langle q_i, s_i, q_i', s_i'|T_2|\chi_{c_2}(\lambda)\rangle &= -f_2 c_b \alpha_s 2m\gamma \int \frac{d\Omega_{k_1}}{8\pi^2}\Phi_{\mu\nu}(\lambda)A^{\mu\nu}\bar{u}(q_2' s_2')v(q_2 s_2) \\ &\quad \times \delta^3(\mathbf{q}_2 + \mathbf{q}_2') + (q_1 \leftrightarrow q_2, q_1' \leftrightarrow q_2'), \quad (23)\end{aligned}$$

where $\Phi_{\mu\nu}$ is the χ_{c2} polarization vector defined in Eq. (18).

V. NUMERICAL RESULTS

The calculation of the χ_{cJ} polarization parameters in our model requires the knowledge of the ϕ-meson properties. However, from perturbative QCD theory, little information is known about the nonperturbative properties of the light-meson structure, since the light hadron lies out of the asymptotic region of pQCD. We simply account for the properties of the $s\bar{s}$ bound state by explicitly including the bound-state wave function in the naive quark model. The flavor-spin wave function of the ϕ meson is constructed in the representation of the SU(6) group, and the total wave function ψ for the ϕ meson is

$$\psi(\mathbf{q}, \lambda) = \rho(\lambda)\phi_F \phi_R(2\mathbf{q}), \quad (24)$$

where ϕ_F and $\rho(\lambda)$ are the flavor and spin wave functions with a helicity value λ, respectively. They are taken as $\phi_F = |s\bar{s}\rangle$, and $\rho(1) = |\uparrow\uparrow\rangle$, $\rho(0) = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$, and $\phi(-1) = |\downarrow\downarrow\rangle$. The spacial wave function of ϕ_R is taken as

$$\phi_R(2\mathbf{q}) = \frac{1}{\pi^{3/4}\beta^{3/2}} e^{-\frac{q^2}{2\beta^2}}, \quad (25)$$

where β is the harmonic-oscillator parameter, and $2\mathbf{q}$ is the momentum difference of the $s\bar{s}$ quarks in the ϕ rest frame.

The polarization parameters are independent of the χ_{cJ} decay constants f_J and the strong coupling constant α_s, since they are canceled in the calculation of these parameters, x, $w_i (i = 1, 2, 3, 4)$, as defined in Eqs. (7)–(9). The strength of quark-antiquark creation in the $^3P_0$ model is found to be roughly flavor independent in light-meson decays involving the pair production of $u\bar{u}$, $d\bar{d}$, and $s\bar{s}$. A global fit [14] to the light-meson decay widths yields $\gamma = 0.506$ with the harmonic-oscillator parameter $\beta \simeq 0.4$ GeV. The mass of the constituent s-quark is taken as one half of the ϕ-meson mass, namely, $m = 0.5$ GeV. For the estimation of the uncertainty for the s-quark mass, the interval of $m = 0.45 \sim 0.55$ GeV is investigated.

For $\chi_{c0} \to \phi\phi$, the polarization parameter x can be numerically estimated either in the pQCD or $^3P_0$ process by directly calculating the ratio $x = |\mathcal{M}^{(i)}(0, 1, 1)|^2/|\mathcal{M}^{(i)}(0, 0, 0)|^2$ with $i = a, b$ for the process shown in Figs. 1(a) and 1(b), respectively. For $\chi_{c2} \to \phi\phi$, the polarization parameters are calculated by the definition in Eq. (9), $w_1 = w_3 = |\sum_\lambda \mathcal{M}^{(i)}(\lambda, 0, 1)|^2/\sum_\lambda|\mathcal{M}^{(i)}(\lambda, 0, 0)|^2$, $w_2 = |\sum_\lambda \mathcal{M}^{(i)}(\lambda, 1, -1)|^2/\sum_\lambda|\mathcal{M}^{(i)}(\lambda, 0, 0)|^2$, and $w_4 = |\sum_\lambda \mathcal{M}^{(i)}(\lambda, 1, 1)|^2/\sum_\lambda|\mathcal{M}^{(i)}(\lambda, 0, 0)|^2$ with $i = a, b$. We also investigate the mixing effect between the pQCD process and the $^3P_0$ process, and the polarization parameters are calculated by replacing $|\mathcal{M}^{(i)}(\lambda, \lambda_1\lambda_2)|^2$ with $|\mathcal{M}^{(a)}(\lambda, \lambda_1\lambda_2) + \mathcal{M}^{(b)}(\lambda, \lambda_1\lambda_2)|^2$ in the above relations.

For $\chi_{c1} \to \phi\phi$, the polarization parameters cannot be estimated since the decay of $\chi_{c1} \to gg$ is prohibited due to the Landau-Yang theorem [8] at the leading order of the Feynman diagram. From the helicity-amplitude analysis, it turns out that the polarization parameters of this decay are strongly constrained by the requirement of identical particle symmetry. One gets $u_1 = |B_{0,1}^{(1)}|^2/|B_{1,0}^{(1)}|^2 = 1$ and $u_2 = |B_{1,1}^{(1)}|^2/|B_{1,0}^{(1)}|^2 = 0$. This indicates that a nonvanishing polarization parameter requires that the two ϕ mesons must have different polarization configurations. For example, if one observes a longitudinally polarized ϕ, another ϕ must be transversely polarized, and vice versa.

Table I lists numerical calculation results of the polarization parameters for $\chi_{c0,2} \to \phi\phi$ decays. For $\chi_{c0} \to \phi\phi$

TABLE I. Numerical results of the polarization parameters defined in Eqs. (7) and (9) in the pQCD [see Fig. 1(a)] and $^3P_0$ [see Fig. 1(b)] schemes, where the central values are calculated with the parameters $m = 0.5$ GeV, $\beta = 0.4$ GeV, and $\gamma = 0.506$, while the uncertainties cover the interval of the s-quark mass $|m - 0.5| < 0.05$ GeV. Values in the last row are calculated in the pQCD scheme with the ϕ wave function up to twist-3 [16] as a comparison.

Mode	$\chi_{c0} \to \phi\phi$	$\chi_{c2} \to \phi\phi$			
Parameter	x	w_1	w_2	w_3	w_4
Fig. 1(a)	0.086 ± 0.010	0.660 ± 0.015	2.714 ± 0.112	0.660 ± 0.015	0.118 ± 0.007
Fig. 1(b)	0.265 ± 0.015	1.957 ± 0.812	0.942 ± 0.267	1.957 ± 0.812	0.165 ± 0.007
Figs. 1(a) and 1(b)	0.375 ± 0.171	3.004 ± 2.140	1.694 ± 1.196	3.004 ± 2.140	0.265 ± 0.052
Twist-3 [16]	0.259	0.217	0.749	0.217	0.136

decay, the polarization parameter x is about a few percent in the pQCD process, while it receives a significant contribution from the $^3P_0$ quark-creation mechanism, and the mixture of these two processes yields a higher value, $x = 0.375 \pm 0.171$. For $\chi_{c2} \to \phi\phi$ decay, in pQCD the parameter w_2 is roughly one order of magnitude larger than the other parameters, since the helicity amplitude associated with this parameter does not violate the HSR. The $^3P_0$ process yields a w_2 value approximately equal to w_1 within the model parameter uncertainty. The polarization parameters w_1, w_3, and w_4 estimated in the mixture scheme are larger than those estimated in pQCD; this indicates that these amplitudes receive a significant contribution from the $^3P_0$ process, which suggests an explanation for the evasion of HSR in the χ_{cJ} decays into vector-meson pairs.

To check the consistency with the nonrelativistic quark-model calculation for $\chi_{c2} \to \phi\phi$, one takes $q_1 = q_2 = P_1/2$ and $q_1' = q_2' = P_2/2$ in Fig. 1(a), and the calculation yields $w_1 = w_3 = 0.55$, $w_2 = 3.44$, and $w_4 = 0.06$, which are consistent with the nonrelativistic approximation [7]. In Ref. [16], the processes of $\chi_{cJ} \to \phi\phi$ were investigated considering high-twist contributions from the ϕ light-cone distribution amplitude. We quote the helicity decay widths to estimate the parameters x, w_1, w_2, and w_3, which are given in Table I. From this comparison, one can see that the polarization parameters associated with the break of the helicity selection rule in $\chi_{cJ} \to \phi\phi$ decays receive significant contributions from the $^3P_0$ process.

Figure 2 shows the helicity-angle distributions obtained from Monte Carlo simulations for the χ_{cJ}, ϕ, and K^+ in the cascade decay $e^+e^- \to \psi(2S) \to \gamma\chi_{cJ} \to \gamma\phi\phi \to \gamma 2(K^+K^-)$. The decay $\psi(2S) \to \gamma\chi_{cJ}$ is an $E1$ transition, whose angular distribution is determined to be $dN/d\cos\theta \propto (1 + \alpha\cos^2\theta_0)$ with $\alpha = 1, \ -1/3, \ 1/13$ for χ_{c0}, χ_{c1}, and χ_{c2}, respectively [17]. The angular distributions for $\chi_{c0,2}$ decays are produced with the polarization parameters estimated with the pQCD, $^3P_0$, and a mixture of the two schemes, as shown in Table I. For the decay $\chi_{c1} \to \phi\phi$, the angular distributions are produced with the parameters $u_1 = 1$ and $u_2 = 0$. For the decay $\chi_{c2} \to \phi\phi$, the distributions of $\cos\theta_1$ and $\cos\theta_2$ appear quite different in the pQCD and $^3P_0$ models.

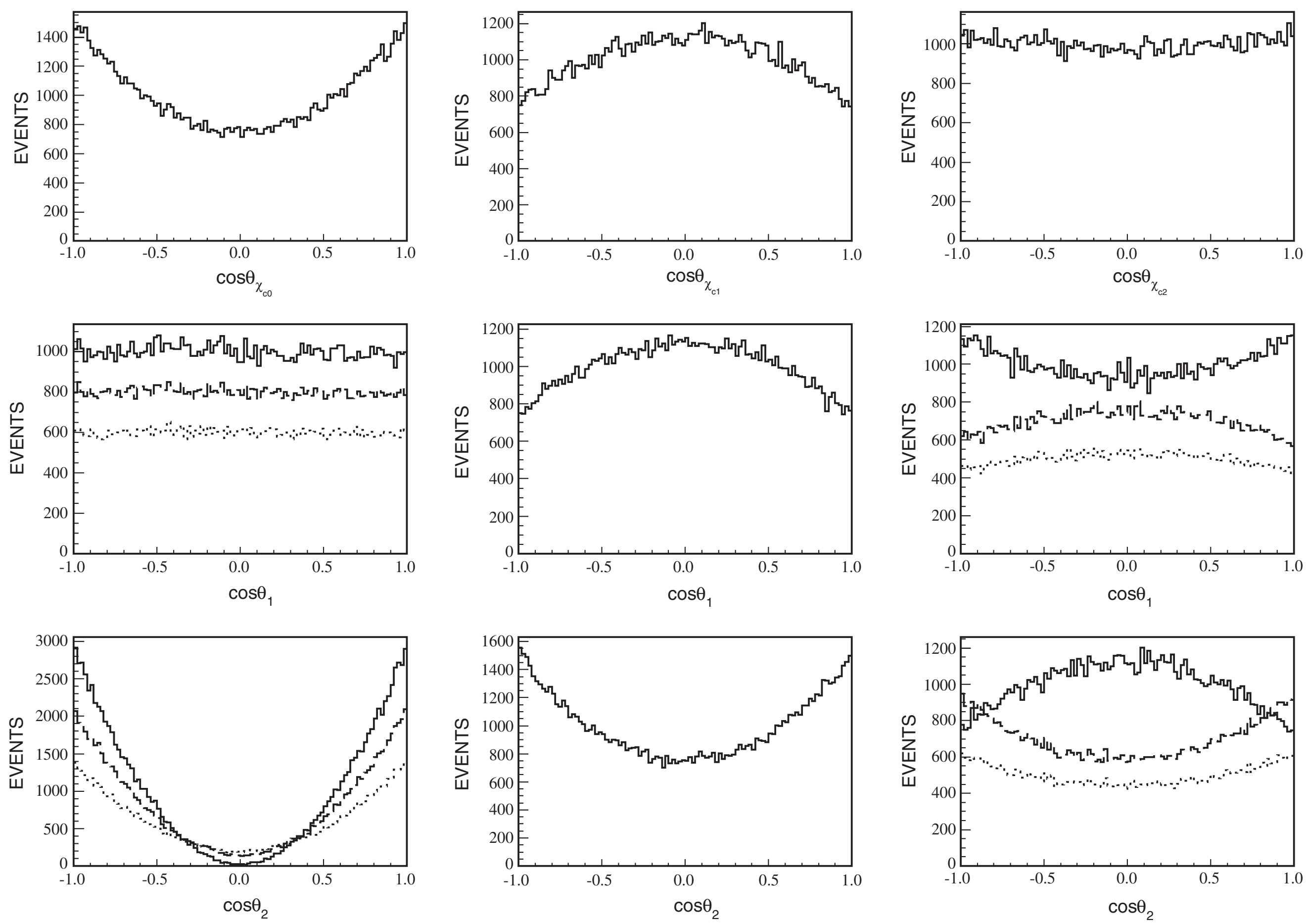

FIG. 2. Angular distributions for χ_{cJ} (top row), ϕ (middle row), and one K^+ (bottom row), where θ_1 and θ_2 are defined in Eqs. (7) and (9), and $\theta_{\chi_{cJ}}$ is the angle for χ_{cJ} states. The left, middle, and right columns are for χ_{c0}, χ_{c1}, and χ_{c2} decays, respectively. In the plots, the line, dashed, and dotted histograms are produced based on the polarization parameters obtained in the pQCD, $^3P_0$, and a mixture of the two schemes, respectively. The superimposed histograms are plotted with arbitrary units.

VI. CONCLUSION

To understand the enhanced partial decay width for the decay of χ_{cJ} to a vector-meson pair $\phi\phi$, we suggest measuring the ϕ-meson polarization in the decay $\chi_{cJ} \to \phi\phi \to 2(K^+K^-)$. The measurement will shed light on the underlying structure of the χ_{cJ} state and its decay mechanism.

The spin-parity analysis shows that the polarization parameters for the decay of χ_{c1} to $\phi\phi$ can be explicitly determined by the requirement of parity conservation and identical particle symmetry. The structure of the antisymmetric helicity amplitude of this decay requires that the two ϕ mesons must take different polarization states. If the polarization vector of one ϕ meson is longitudinal, another ϕ meson must be transversely polarized, and vice versa.

The polarization parameters for the decay of $\chi_{c0,2}$ to $\phi\phi$ were estimated in the pQCD scheme and the $^3P_0$ model. In the pQCD scheme, the results are consistent with the expectation of the helicity selection rule, and the polarization parameters associated with the s-quark spin reversal are suppressed. In the $^3P_0$ model, the polarization parameters associated with the break of HSR are enhanced, which indicates that the $^3P_0$ model provides an explanation for the evasion of HSR in the decay of $\chi_{c0,2}$ to the vector-meson pair $\phi\phi$. Based on the estimated values of the polarization parameters, the angular distributions for the decay $\psi(2S) \to \gamma\chi_{cJ} \to \gamma\phi\phi \to \gamma 2(K^+K^-)$ were produced. The angular distributions appear quite different in the pQCD and $^3P_0$ decay mechanisms, and they will help us to identify the χ_{cJ} decay mechanism if comparisons with experimental measurements are available in the future.

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[1] S. J. Brodsky and G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).
[2] V. L. Chernyak and A. R. Zhitnitsky, Nucl. Phys. **B201**, 492 (1982); **B214**, 547(E) (1983).
[3] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
[4] N. Brambilla *et al.* (Quarkonium Working Group), CERN Yellow Report No. CERN-2005-005 (CERN, Geneva, 2005).
[5] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008).
[6] X.-H. Liu and Q. Zhao, Phys. Rev. D **81**, 014017 (2010).
[7] H. Q. Zhou, R. G. Ping, and B. S. Zou, Phys. Lett. B **611**, 123 (2005).
[8] L. D. Landau, Dokl. Akad. Nauk Ser. Fiz. **60**, 207 (1948); C. N. Yang, Phys. Rev. **77**, 242 (1950).
[9] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **107**, 092001 (2011).
[10] D.-Y. Chen, J. He, X.-Q. Li, and X. Liu, Phys. Rev. D **81**, 074006 (2010).
[11] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. D **84**, 092006 (2011).
[12] G. Karl, J. Meshkov, and J. L. Rosner, Phys. Rev. D **13**, 1203 (1976).
[13] S. M. Berman and M. Jacob, Phys. Rev. **139**, B1023 (1965).
[14] E. S. Ackleh, T. Barnes, and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996).
[15] R. E. Cutkosky, J. Math. Phys. (Cambridge, Mass.) **1**, 429 (1960).
[16] L.-B. Chen and C.-F. Qiao, J. High Energy Phys. 11 (2012) 168.
[17] G. R. Liao, R. G. Ping, and Y. X. Yang, Chin. Phys. Lett. **26**, 051101 (2009).