First estimates of nonleptonic $B_c \to AT$ weak decays

Rohit Dhir* and C. S. Kim†

Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea

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We obtain the first estimates of branching ratios for the weak decays of B_c meson decaying to 2 orbitally excited mesons namely, axial-vector meson (A) and tensor meson (T) in the final state. We calculate $B_c \to T$ transition form factors using the Isgur-Scora-Grinstein-Wise II framework and, consequently, predict branching ratios of $B_c \to AT$ decays in Cabibbo-Kobayashi-Maskawa favored and suppressed modes.

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I. INTRODUCTION

The B_c meson was first discovered through the observation of semileptonic decay, $B_c \to J/\psi l^+ \nu X$ by the CDF Collaboration [1]. Later on, more precise measurements of its lifetime and mass are confirmed to be (0.453 ± 0.042) ps and (6277 ± 6) MeV, respectively [2,3]. A few of the B_c decay channels have also been observed experimentally. A new era of B_c has started with an expected production cross section of $\sim 0.4\ \mu$b at center-of-mass energy $\sqrt{s} = 7$ TeV at the LHC [4]. Very recently, the LHC-b reported an observation of decays like $B_c^+ \to \psi(2S)\pi^+$, $B_c^+ \to J/\psi\pi^+\pi^+\pi^-$, and $B_c^+ \to J/\psi\pi^+$ [3,4]. As a result, the investigation of B_c meson has become one of the most interesting current topic. Moreover, the LCH-b is expected to produce $\mathcal{O}(10^{10})$ events per year [3–7], which would provide a rich amount of information regarding B_c meson.

B_c meson is the standard model particle composed of 2 flavors of heavy quarks, charm (c) and beauty (b). The $B_c(b\bar{c})$ meson being flavor asymmetric behaves differently from the heavy quarkonia ($b\bar{b}$, $c\bar{c}$). It only decays via weak interactions as compared to heavy quarkonium states which can decay via strong interactions and/or electromagnetic interactions. The decay processes of the B_c meson involve decay of any constituent quarks b and c with the other being a spectator. They can also annihilate weakly to produce leptons or lighter mesons which, being relatively suppressed, are ignored in the present analysis. The 2-body weak decays of B_c may provide a good scenario for understanding QCD dynamics both in the perturbative and nonperturbative regime, to test QCD-motivated theories/models, and to study physics beyond the standard model.

Hadronic 2-body decays of B_c mesons have broadly been studied in recent years. There exist comprehensive literature on phenomenological works based on different approaches to study semileptonic and nonleptonic decays of B_c meson emitting s-wave mesons in the final state [6–19]. Recently, the focus has shifted towards the p-wave emitting decays of B_c meson [20–29]. In our previous work [29], we have studied the B_c meson decaying to two p-wave in the final state, i.e., $B_c \to AA$ decays, and shown that branching ratios of these decays are comparable to $B_c \to VA$ modes. Also, the Particle Data Group [2] has reported many single p-wave meson emitting decays having branching ratios of $\mathcal{O}(10^{-6})$. This motivated us to carry forward our analysis to nonleptonic 2-body B_c decays considering that both mesons in final state are orbitally excited mesons (or p-wave mesons). Particularly, we study B_c decaying to an axial vector (A) and tensor meson (T), which could compete with $B_c \to VT$ modes and their branching fractions could be measured in near future at the LHC-b experiment and at future B-factories. Moreover, $B_c \to AT$ decays can offer a scenario to study polarization of final state particles [2]. In the present work, we obtain the first estimates of branching ratios of exclusive $B_c \to AT$ decays. We calculate $B_c \to T$ form factors using the improved Isgur-Scora-Grinstein-Wise quark model framework (called ISGW II Model) [30,31]. The ISGW II model is one of the few successful models [30–33] which is used to calculate $B_c \to T$ transition form factors.

This paper is organized as follows: In Sec. II, we briefly discuss the effective weak Hamiltonian and decay width formula. We give input parameters like mixing scheme and decay constants for axial-vector mesons, and $B_c \to T$ transition form factors in Sec. III. Results and discussions are given in Sec. IV. In the final section, we present a summary and conclusions.

II. METHODOLOGY

A. Weak Hamiltonian

The QCD-modified weak Hamiltonian [34] generating the B_c decay involving $b \to c$ and $b \to u$ transitions in Cabibbo-Kobayashi-Maskawa (CKM) favored modes ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$; $\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) and CKM suppressed ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = -1$; $\Delta b = 1$, $\Delta C = 1$, $\Delta S = 1$; $\Delta b = 1$, $\Delta C = -1$, $\Delta S = -1$; $\Delta b = 1$, $\Delta C = -1$, $\Delta S = 0$) modes is expressed as follows:

*dhir.rohit@gmail.com
†Corresponding author.
cskim@yonsei.ac.kr

$$H_w^{(\Delta b=1)} = \frac{G_F}{\sqrt{2}} \sum_{Q(q)=u,c} \sum_{q'=d,s} V_{Qb}^* V_{qq'}(a_1(\mu) O_1^{qq'}(\mu) + a_2(\mu) O_2^{qq'}(\mu)) + \text{H.c.}, \tag{1}$$

where G_F is the Fermi constant and V_{ij} are the CKM matrix elements. a_1 and a_2 are the standard perturbative QCD coefficients, evaluated at renormalization scale $\mu \approx \mathcal{O}(m_b)$. Local tree-level operators $O_{1,2}$ involving $b \to q$ transition are given below:

$$\begin{aligned} O_1^{qd} &= (\bar{b}_\alpha q_\alpha)_{V-A} \cdot (\bar{q}_\beta d_\beta)_{V-A}, \\ O_2^{qd} &= (\bar{b}_\alpha q_\beta)_{V-A} \cdot (\bar{q}_\beta d_\alpha)_{V-A}, \\ O_1^{qs} &= (\bar{b}_\alpha q_\alpha)_{V-A} \cdot (\bar{q}_\beta s_\beta)_{V-A}, \\ O_2^{qs} &= (\bar{b}_\alpha q_\beta)_{V-A} \cdot (\bar{q}_\beta s_\alpha)_{V-A}, \end{aligned} \tag{2}$$

where $\bar{q}q' \equiv \bar{q}\gamma_\mu(1-\gamma_5)q'$, α and β are $SU(3)$ color indices. In addition, B_c meson can also decay via the bottom conserving modes, where the charm quark decays to an s or a d quark. However, such $B_c \to AT$ decays are kinematically forbidden.

By factorizing matrix elements of the four-quark operator contained in the effective weak Hamiltonian (1), one can divide these decays into three classes: Contribution from W-external emission diagram at tree level (color-favored diagram) is classified as class I type decays. In this case the decay amplitudes are proportional to a_1, where $a_1(\mu) = c_1(\mu) + \frac{1}{N_c} c_2(\mu)$, and N_c is the number of colors. Class II type decays consist of contribution from another W-internal emission diagrams (color-suppressed diagrams). The decay amplitude in this class is proportional to $a_2(\mu) = c_2(\mu) + \frac{1}{N_c} c_1(\mu)$. Class III transitions are caused by the interference of both color-favored and color-suppressed diagrams. Interestingly, we find that $B_c \to AT$ decays are independent of a_1 and a_2 interference.

For numerical calculations, we follow the B physics convention of taking $N_c = 3$ to fix the QCD coefficients a_1 and a_2, i.e.,

$$a_1(\mu) = 1.03, \qquad a_2(\mu) = 0.11,$$

where we use [34]

$$c_1(\mu) = 1.12, \qquad c_2(\mu) = -0.26 \quad \text{at } \mu \approx m_b^2.$$

We want to point out that N_c, the number of color degrees of freedom, is generally treated as a phenomenological parameter in weak meson decays to account for nonfactorizable contributions [34]. In the absence of any experimental and theoretical study of $B_c \to AT$, we also compare our results with those of large N_c limit. The obtained results would provide a reasonable estimate of range of branching ratio between $N_c = 3$ to $N_c \to \infty$.

B. Decay amplitudes

In the framework of generalized factorization the decay amplitudes $\langle AT|H_w|B_c\rangle$ are approximated as a product of the matrix elements of weak currents (up to the weak scale factor of $\frac{G_F}{\sqrt{2}} \times$ CKM elements $\times$ QCD factor) given by

$$\langle TA|H_w|B_c\rangle \sim \langle T|J^\mu|0\rangle\langle A|J_\mu|B_c\rangle + \langle A|J^\mu|0\rangle\langle T|J_\mu|B_c\rangle, \tag{3}$$

where $J^\mu = V^\mu - A^\mu$, V^μ and A^μ denote a vector and an axial-vector current, respectively.

In order to calculate the numerical values of decay amplitudes of $B_c \to AT$ decays, we have to obtain the above mentioned hadronic matrix elements. Notice that the polarization tensor $\epsilon_{\mu\nu}$ of the $^3P_2$ tensor meson satisfies the following relations [35]:

$$\begin{aligned} &\epsilon_{\mu\nu}(p_T,\lambda) = \epsilon_{\nu\mu}(p_T,\lambda), \\ &p_\mu \epsilon^{\mu\nu}(p_T,\lambda) = p_\nu \epsilon^{\mu\nu}(p_T,\lambda) = 0, \quad \epsilon^\mu_\mu(p_T,\lambda) = 0, \end{aligned} \tag{4}$$

where λ defines state of definite halicities. Consequently, the matrix element between the vacuum and final state tensor meson T is

$$\langle 0|(V-A)_\mu|T\rangle = a\epsilon_{\mu\nu}p^\nu(p_T,\lambda) + b\epsilon^\nu_\nu p_\mu(p_T,\lambda) = 0. \tag{5}$$

This relation, in general, follows from Lorentz covariance and parity consideration. Hence, the tensor meson cannot be produced from the $V-A$ current; i.e., the decay constant of the tensor meson vanishes. This fact further simplifies the decay amplitude $\langle AT|H_w|B_c\rangle$ expressed by relation (3), which yields

$$\langle AT|H_w|B_c\rangle \sim \langle A|J^\mu|0\rangle\langle T|J_\mu|B_c\rangle. \tag{6}$$

On the other hand, the matrix element between the vacuum and final state axial-vector meson is defined as

$$\langle A(p_A,\varepsilon)|A_\mu|0\rangle = \varepsilon^*_\mu m_A f_A, \tag{7}$$

where ε_μ denotes polarization of axial-vector meson and f_A is the corresponding decay constant.

Using Lorentz invariance, the hadronic transition matrix elements [30,31] for the relevant weak current between meson states can be parametrized as follows:

$$\begin{aligned} \langle T|V^\mu|B_c\rangle &= ih(q^2)\varepsilon^{\mu\nu\rho\sigma}\epsilon_{\nu\alpha}p^\alpha_{B_c}(p_{B_c}+p_T)_\rho(p_{B_c}-p_T)_\sigma, \\ \langle T|A^\mu|B_c\rangle &= k(q^2)\epsilon^{*\mu\nu}(p_{B_c})_\nu + \epsilon^*_{\alpha\beta}p^\alpha_{B_c}p^\beta_{B_c} \\ &\quad \times [b_+(q^2)(p_{B_c}+p_T)^\mu + b_-(q^2)(p_{B_c}-p_T)^\mu], \end{aligned} \tag{8}$$

where p_{B_c} and p_T denote the momentum of the B_c meson and the tensor meson T, respectively such that $q^\mu \equiv p^\mu_{B_c} - p^\mu_T$. $h(q^2)$, $k(q^2)$, $b_+(q^2)$, and $b_-(q^2)$ represent the relevant form factors for the $B_c \to T$ transition, $F^{B\to T}(m^2_{B_c})$, which have been calculated at $q^2 = q^2_{\rm max}$ using the ISGW II quark model [31].

Using definitions (7) and (8), the matrix element (6) takes the form

$$A(B_c \to AT) = \frac{G_F}{\sqrt{2}} \times (\text{CKM factors} \times \text{QCD factors}) \times f_A m_A \epsilon^{*\alpha\beta} F^{B_c\to T}_{\alpha\beta}(m_A^2), \quad (9)$$

where

$$F^{B_c\to T}_{\alpha\beta} = \epsilon^*_\mu (p_{B_c} + p_T)_\rho [ih\varepsilon^{\mu\nu\rho\sigma} g_{\alpha\nu}(p_A)_\beta (p_A)_\sigma + k\delta^\mu_\alpha \delta^\rho_\beta + b_+(p_A)_\alpha (p_A)_\beta g^{\mu\rho}]. \quad (10)$$

C. Decay widths

The simplified decay width formula, after summing over polarizations of the tensor meson T, for the $B_c \to AT$ decays is given as

$$\Gamma(B_c \to AT) = \frac{G_F^2}{6\pi m_T^4} m^2_{B_c} f_A^2 \times [\alpha(m_A^2)|\boldsymbol{p_A}|^7 + \beta(m_A^2)|\boldsymbol{p_A}|^5 + \gamma(m_A^2)|\boldsymbol{p_A}|^3], \quad (11)$$

where $|\boldsymbol{p_A}|(= |\boldsymbol{p_T}|)$ is the magnitude of 3 momentum of the final state particle in the rest frame of B_c meson. $|\boldsymbol{p_A}|$ can be expressed as $|\boldsymbol{p_A}| = \sqrt{\lambda}/2m_{B_c}$, where $\lambda \equiv \lambda(m_B^2, m_T^2, m_A^2) = (m^2_{B_c} + m_T^2 - m_A^2)^2 - 4m_A^2 m_T^2$ is the triangle function. The coefficients α, β, and γ are quadratic functions of form factors k, b_+ and h, evaluated at $q^2 = m_A^2$. These coefficients are given as follows:

$$\alpha(m_A^2) = 8b_+^2 m^2_{B_c}, \quad (12)$$

$$\beta(m_A^2) = \frac{1}{4}[k^2 + 6m_T^2 m_A^2 h^2 + 2(m_B^2 - m_T^2 - m_A^2)kb_+], \quad (13)$$

$$\gamma(m_A^2) = \frac{5k^2 m_A^2 m_T^2}{8m^2_{B_c}}. \quad (14)$$

III. INPUT PARAMETERS

A. Mixing angles for axial-vector mesons

Mixing scheme of the axial-vector mesons have thoroughly been discussed in the literatures [32,33,36–42]. Thus, we briefly mention the important facts. There are two types of axial-vector mesons, in spectroscopic notation $^{2s+1}L_J$, $^3P_1(J^{PC} = 1^{++})$ and $^1P_1$ $(J^{PC} = 1^{+-})$, respectively. These states can mix in 2 ways: first, states $^3P_1$ or $^1P_1$ can mix within themselves; secondly, mixing can occur between $^3P_1$ or $^1P_1$ states. Experimentally observed nonstrange and uncharmed $^3P_1$ meson sixteenplet consists of isovector $a_1(1.230)$ and four isoscalars $f_1(1.285)$, $f_1(1.420)/f_1'(1.512)$ and $\chi_{c1}(3.511)$. On the other hand, $^1P_1$ meson multiplet includes isovector $b_1(1.229)$ and 3 isoscalars $h_1(1.170)$, $h_1'(1.380)$, and $h_{c1}(3.526)$, where $h_{c1}(3.526)$ and $h_1'(1.380)$ are not well understood experimentally.[1]

The following mixing scheme has been proposed for the isoscalar (1^{++}) and (1^{+-}) mesons:

$$\begin{aligned} f_1(1.285) &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\phi_A + (s\bar{s})\sin\phi_A \\ f_1'(1.512) &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\phi_A - (s\bar{s})\cos\phi_A \\ \chi_{c1}(3.511) &= (c\bar{c}), \end{aligned} \quad (15)$$

and

$$\begin{aligned} h_1(1.170) &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\phi_{A'} + (s\bar{s})\sin\phi_{A'}, \\ h_1'(1.380) &= \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin\phi_{A'} - (s\bar{s})\cos\phi_{A'}, \\ h_{c1}(3.526) &= (c\bar{c}), \end{aligned} \quad (16)$$

respectively, with

$$\phi_{A(A')} = \theta(\text{ideal}) - \theta_{A(A')}(\text{physical}).$$

The observation that $f_1(1.285)\to 4\pi/\eta\pi\pi$, $f_1'(1.512)\to K\bar{K}\pi$, $h_1(1.170) \to \rho\pi$ and $h_1' \to K\bar{K}^*/\bar{K}K^*$ predominantly seems to favor the ideal mixing for these nonets, i.e., $\phi_A = \phi_{A'} = 0°$. The hidden-flavor neutral $a_1(1.230)$ and $b_1(1.229)$ states cannot mix due to opposite C-parity. Also, the opposite G-parities under isospin symmetry prevent mixing of charged $a_1(1.230)$ and $b_1(1.229)$ states. In contrast, states involving strange partners namely, K_{1A} and $K_{1A'}$ of $A(1^{++})$ and $A'(1^{+-})$ mesons, respectively, mix to generate the physical states in the following convention:

$$\begin{aligned} K_1(1.270) &= K_{1A}\sin\theta_{K_1} + K_{1A'}\cos\theta_{K_1}, \\ \underline{K}_1(1.400) &= K_{1A}\cos\theta_{K_1} - K_{1A'}\sin\theta_{K_1}. \end{aligned} \quad (17)$$

Several phenomenological analyses indicate that the mixing angle θ_{K_1} lies in the vicinity of $\sim 35°$ and $\sim 55°$, see for details [38]. Several studies based on the experimental information obtained twofold ambiguous solutions $\theta_{K_1} = \pm 37°$ and $\theta_{K_1} = \pm 58°$ [36,37]. We wish to point out that the study of $D \to K_1(1.270)\pi/K_1(1.400)\pi$ decays and experimental measurement of the ratio of $K_1\gamma$ production in B decays favor negative mixing angle solutions [32,33,37,41]. In a recent study [38], it has been argued that choice of angle for $f_1 - f_1'$ and $h_1 - h_1'$ mixing schemes is intimately related to choice of mixing angle θ_{K_1}. The mixing angle $\theta_{K_1} \sim 35°$ is preferred over $\sim 55°$ for nearly ideal mixing for $f_1 - f_1'$ and $h_1 - h_1'$. However,

[1]Here the quantities in brackets indicate their respective masses (in GeV).

we also give results at $\theta_{K_1} = -58°$ in our numerical calculations.

Following the Heavy Quark Spin scheme, the heavy axial-vector resonances (likely charmed and bottom axial-vector meson states) are generally taken to be the mixture of $P_1^{1/2}$ and $P_1^{3/2}$ states. In the heavy quark limit, the heavy quark spin S_Q and the total angular momentum of the light antiquark can be used as good quantum numbers, separately [37,40]. Therefore, in heavy axial vector resonances, the physical mass eigenstates $P_1^{3/2}$ and $P_1^{1/2}$ with $J^P = 1^+$ can be expressed as a mixture of $^3P_1$ and $^1P_1$ states, i.e.,

$$|P_1^{1/2}\rangle = -\sqrt{\frac{1}{3}}|^1P_1\rangle + \sqrt{\frac{2}{3}}|^3P_1\rangle,$$
$$|P_1^{3/2}\rangle = \sqrt{\frac{2}{3}}|^1P_1\rangle + \sqrt{\frac{1}{3}}|^3P_1\rangle. \tag{18}$$

In the heavy quark limit, the physical states $D_1(2.427)$ and $\underline{D}_1(2.422)$ can be identified as $P_1^{1/2}$ and $P_1^{3/2}$, respectively. However, beyond the heavy quark limit, there can be mixing between $P_1^{1/2}$ and $P_1^{3/2}$ given as

$$D_1(2.427) = D_1^{1/2}\cos\theta_{D_1} + D_1^{3/2}\sin\theta_{D_1},$$
$$\underline{D}_1(2.422) = -D_1^{1/2}\sin\theta_{D_1} + D_1^{3/2}\cos\theta_{D_1}. \tag{19}$$

Likewise, a mixing scheme for strange charmed axial-vector mesons is expressed as

$$D_{s1}(2.460) = D_{s1}^{1/2}\cos\theta_{D_{s1}} + D_{s1}^{3/2}\sin\theta_{D_{s1}},$$
$$\underline{D}_{s1}(2.535) = -D_{s1}^{1/2}\sin\theta_{D_{s1}3} + D_{s1}^{3/2}\cos\theta_{D_{s1}}. \tag{20}$$

Following the analysis given by the Belle [39], we use the mixing angle $\theta_{D_1} = (-5.7 \pm 2.4)°$, while the quark potential model analysis [40] yields $\theta_{D_{s1}} \approx 7°$.

B. Decay constants

The decay constants for tensor mesons vanish corresponding to the condition (5), while the decay constants for axial-vector mesons are defined by the reduced matrix elements expressed in relation (7) in the previous section. It is a well-established fact that the axial-vector meson $^3P_1$ and $^1P_1$ states transform under the charge conjugation [36] as

$$M_a^b(^3P_1) \to M_b^a(^3P_1), \qquad M_a^b(^1P_1) \to -M_b^a(^1P_1),$$
$$(a = 1, 2, 3). \tag{21}$$

Since the weak axial-vector current transfers as $(A_\mu)_a^b \to (A_\mu)_b^a$ under charge conjugation, the decay constants of the $^1P_1$ mesons state vanish in the flavor symmetry limit [36]. However, in the presence of symmetry breaking $^1P_1$ mesons may acquire nonzero values of the decay constants. In case of nonstrange axial-vector mesons, $f_{a1} = 0.203 \pm 0.018$ GeV is quoted in the analysis given by Bloch *et al.* [42]. The value $f_{a1} = 0.238 \pm 0.010$ GeV is obtained using the QCD sum rule method [43]. Nardulli and Pham [41] used $SU(3)$ symmetry to determine $f_{a_1} = 0.223(-0.215)$ GeV for $\theta_{K_1} = -58°(-32°)$ mixing angle for strange axial-vector mesons. Since, a_1 and f_1 lies in the same $SU(3)$ nonet we assume $f_{f_1} \approx f_{a_1}$. In isospin limit, owing to G-parity conservation decay constant $f_{b_1} = 0$. Experimental data on $\tau \to K_1(1270)\nu_\tau$ decay yields the decay constant $f_{K_1}(1270) = 0.175 \pm 0.019$ GeV [22,37], while decay constant for $\underline{K}_1(1.400)$ may be calculated by using relation $f_{\underline{K}_1}(1.400)/f_{K_1}(1.270) \approx \cot\theta_{K_1}$ i.e., $f_{\underline{K}_1}(1.400) = (-0.109 \pm 0.012)$ GeV, for $\theta_{K_1} = -58°$; $f_{\underline{K}_1}(1.400) = (-0.232 \pm 0.025)$ GeV, for $\theta_{K_1} = -37°$; used in the present work [37]. For decay constants of axial-vector charmed and strange charmed meson states, we have used $f_{D_{1A}} = -0.127$ GeV, $f_{D'_{1A}} = 0.045$ GeV, $f_{D_{s1A}} = -0.121$ GeV, $f_{D'_{s1A}} = 0.038$ GeV, and $f_{\chi_{c1}} \approx -0.160$ GeV for numerical evaluation [29,32,33,37]. It may be noted that the decay constants of $^3P_1$ states have opposite signs to that of $^1P_1$ as appear from (21).

C. Form factors

In the Isgur-Scora-Grinstein-Wise model [31], weak hadronic transition form factors are predicted using nonrelativistic quark model wave functions. The form factors obtained are assumed to be reliable at near the zero recoil point where q^2 reaches its maximum value $(m_B - m_X)^2$. The problem being that the form-factor q^2-dependence in the original ISGW model [30] is proportional to $e^{-(q_m^2-q^2)}$, as a result form factor decreases exponentially. Nevertheless, this has been improved in updated version of ISGW model (ISGW II) [30,31] where the form factors show a more realistic behavior at large $(q_m^2 - q^2)$. The exponential dependence of $B_c \to T$ form factors has been replaced by a polynomial term. In addition, ISGW II model incorporates heavy quark symmetry constraints, heavy-quark-symmetry-breaking color magnetic interaction, relativistic corrections, etc.

The simplified expressions for h, k, b_+ and b_- form factors in the ISGW II model for $B_c \to T$ transitions are given as [30,31]:

$$h = \frac{m_d}{2\sqrt{2\tilde{m}_{B_c}\beta_{B_c}}}\left(\frac{1}{m_q} - \frac{m_d\beta_{B_c}^2}{2\mu_-\tilde{m}_T\beta_{B_cT}^2}\right)F_5^{(h)}, \tag{22}$$

$$k = \frac{m_d}{\sqrt{2}\beta_{B_c}}(1+\tilde{\omega})F_5^{(k)}, \tag{23}$$

$$b_+ + b_- = \frac{m_d^2}{4\sqrt{2}m_q m_b \tilde{m}_{B_c}\beta_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2} \times\left(1 - \frac{m_d}{2\tilde{m}_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2}\right)F_5^{(b_+ + b_-)}, \tag{24}$$

$$b_+ - b_- = -\frac{m_d}{\sqrt{2}m_b\tilde{m}_T\beta_{B_c}}F_5^{(b_+-b_-)}\left(1 - \frac{m_d m_b}{2\mu_+\tilde{m}_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2} + \frac{m_d}{4m_q}\frac{\beta_T^2}{\beta_{B_cT}^2}\left(1 - \frac{m_d}{2\tilde{m}_{B_c}}\frac{\beta_T^2}{\beta_{B_cT}^2}\right)\right), \tag{25}$$

where

$$\mu_\pm = \left(\frac{1}{m_c} + \frac{1}{m_b}\right)^{-1}. \tag{26}$$

$t(\equiv q^2)$ dependence is given by

$$\tilde{\omega} - 1 = \frac{t_m - t}{2\bar{m}_{B_c}\bar{m}_A}, \tag{27}$$

where $t_m = (m_{B_c} - m_T)^2$ is the maximum momentum transfer, and

$$\begin{aligned} F_5^{(h)} &= F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-3/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{-1/2}, \\ F_5^{(k)} &= F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-1/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{1/2}, \\ F_5^{(b_++b_-)} &= F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-5/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{1/2}, \\ F_5^{(b_+-b_-)} &= F_5\left(\frac{\bar{m}_{B_c}}{\tilde{m}_{B_c}}\right)^{-3/2}\left(\frac{\bar{m}_T}{\tilde{m}_T}\right)^{-1/2}, \end{aligned} \tag{28}$$

where $\tilde{m}$ represents sum of the meson constituent quarks masses, $\bar{m}$ is the hyperfine averaged physical masses. The parameter β have different values for s-wave and p-wave mesons as shown in Table I [30,31].

The function F_5 is given by

$$F_5 = \left(\frac{\tilde{m}_T}{\tilde{m}_{B_c}}\right)^{1/2}\left(\frac{\beta_{B_c}\beta_T}{\beta_{B_cT}}\right)^{5/2}\left[1 + \frac{1}{18}\chi^2(t_m - t)\right]^{-3}, \tag{29}$$

where

$$\chi^2 = \frac{3}{4m_b m_c} + \frac{3m_c^2}{2\bar{m}_{B_c}\bar{m}_T\beta_{B_cT}^2} + \frac{1}{\bar{m}_{B_c}\bar{m}_T}\left(\frac{16}{33 - 2n_f}\right) \times \ln\left[\frac{\alpha_S(\mu_{QM})}{\alpha_S(m_c)}\right], \tag{30}$$

and

$$\beta_{B_cT}^2 = \frac{1}{2}(\beta_{B_c}^2 + \beta_T^2). \tag{31}$$

$n_f = 5$, the number of active flavors and μ_{QM} is the quark model scale. The following quark mass values (in GeV) have been used to evaluate $B_c \to T$ form factors:

TABLE I. The values of β parameter for s-wave and p-wave mesons in the ISGW II quark model.

Quark content	$u\bar{d}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{s}$	$u\bar{b}$	$s\bar{b}$	$c\bar{c}$	$b\bar{c}$
β_s (GeV)	0.41	0.44	0.53	0.45	0.56	0.43	0.54	0.88	0.92
β_p (GeV)	0.28	0.30	0.33	0.33	0.38	0.35	0.41	0.52	0.60

TABLE II. $B_c \to T$ transition form factors at $q^2_{\max}$ in the ISGW II quark model.

Modes	Transition	h	k	b_+	b_-
$\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$	$B_c \to D_2$	0.016	0.515	−0.008	0.010
	$B_c \to D_{s2}$	0.019	0.684	−0.010	0.013
$\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$	$B_c \to \chi_{c2}(c\bar{c})$	0.021	1.306	−0.015	0.018

$$m_u = m_d = 0.31, \quad m_s = 0.49, \quad m_c = 1.7, \quad \text{and} \quad m_b = 5.0.$$

Finally, the form factors thus obtained are given in Table II.

IV. RESULTS AND DISCUSSIONS

In this section, we present branching ratios of nonleptonic $B_c \to AT$ decays using ISGW II model framework for CKM favored and CKM suppressed modes. The branching ratios for B_c decaying to an axial-vector meson and a tensor in the final state for CKM enhanced and CKM suppressed modes are given in column 3 of Tables III, IV, and V, respectively. We also present our results at $N_c \to \infty$ (see column 4 of Tables III, IV, and V) for the sake of comparison, as no theoretical or experimental information is available now. We observe the following:

A. For CKM enhanced modes

(1) The most dominant decay channel in CKM-enhanced bottom changing and charm conserving ($\Delta S = -1$) mode is B_c meson decaying to p-wave charmonium χ_{c2} in the final state, i.e., $\mathrm{Br}(B_c^- \to a_1^-\chi_{c2}) = 4.69 \times 10^{-4}$. The branching ratio increases by 15% larger at large N_c limit i.e., 5.54×10^{-4}. However, our prediction is roughly half of the numerical value obtained by Chang *et al.* [23], which is based on the Bethe-Salpeter equation and QCD inspired potential approach at $N_c \to \infty$. The next order branching ratio are of $B_c^- \to D_1^0 D_2^-$ and $B_c^- \to \underline{D}_1^0 D_2^-$ decays. It may be noted that at large N_c limit, branching ratios for color favored modes increase roughly by 15%, while that of color suppressed modes increase by ~80% due to change in Wilson coefficients a_1 and a_2, respectively. However, our results remain unaffected from the interference of a_1 and a_2 terms as class III decays are not possible for $B_c \to AT$ decays.

(2) In CKM-enhanced bottom and charm changing ($\Delta S = 0$) mode, the highest order branching ratios is $\mathrm{Br}(B_c^- \to D_{s1}^-\chi_{c2}) = 3.34 \times 10^{-5}(3.95 \times 10^{-5})$. The next order values are $\mathrm{Br}(B_c^- \to \underline{D}_{s1}^-\chi_{c2}) = 7.49 \times 10^{-6}(8.85 \times 10^{-6})$ and $\mathrm{Br}(B_c^- \to \chi_{c1}D_{s2}^-) = 4.78 \times 10^{-7}(2.67 \times 10^{-6})$, where the numbers in the parenthesis are BRs at large N_c limit. The remaining decay modes are suppressed having branching ratios of $\mathcal{O}(10^{-8} \sim 10^{-10})$.

TABLE III. Branching ratios of $B_c \to AT$ decays for CKM-enhanced modes.

Mode	Decays	Branching ratios $N_c = 3$	$N_c \to \infty$
$\Delta S = -1$	$B_c^- \to a_1^- \chi_{c2}$	4.69×10^{-4}	5.54×10^{-4} (9.17×10^{-4}) [23]
	$B_c^- \to b_1^- \chi_{c2}$	3.28×10^{-9}	3.88×10^{-9}
	$B_c^- \to D_1^0 D_2^-$	4.23×10^{-7}	2.36×10^{-6}
	$B_c^- \to \underline{D}_1^0 D_2^-$	1.33×10^{-7}	7.43×10^{-7}
$\Delta S = 0$	$B_c^- \to D_{s1}^- \chi_{c2}$	3.34×10^{-5}	3.95×10^{-5}
	$B_c^- \to \underline{D}_{s1}^- \chi_{c2}$	7.49×10^{-6}	8.85×10^{-6}
	$B_c^- \to \chi_{c1} D_{s2}^-$	4.78×10^{-7}	2.67×10^{-6}
	$B_c^- \to K_1^- \bar{D}_2^0$	2.46×10^{-8} (2.53×10^{-8})[a]	2.90×10^{-8} (2.81×10^{-8})[a]
	$B_c^- \to \underline{K}_1^- \bar{D}_2^0$	3.66×10^{-8} (7.61×10^{-9})[a]	4.33×10^{-8} (8.85×10^{-9})[a]
	$B_c^- \to a_1^0 D_{s2}^-$	4.55×10^{-10}	2.54×10^{-9}
	$B_c^- \to f_1 D_{s2}^-$	4.54×10^{-10}	2.54×10^{-9}

[a]For $\theta_K = -58°$.

TABLE IV. Branching ratios of $B_c \to AT$ decays for CKM-suppressed modes.

Mode	Decays	Branching ratios $N_c = 3$	$N_c \to \infty$
$\Delta S = -1$	$B_c^- \to K_1^- \chi_{c2}$	1.66×10^{-5} (1.63×10^{-5})[a]	1.97×10^{-5} (1.90×10^{-5})[a]
	$B_c^- \to \underline{K}_1^- \chi_{c2}$	2.40×10^{-5} (4.76×10^{-5})[a]	2.84×10^{-5} (5.58×10^{-5})[a]
	$B_c^- \to D_1^0 D_{s2}^-$	4.45×10^{-8}	2.49×10^{-7}
	$B_c^- \to \underline{D}_1^0 D_{s2}^-$	1.40×10^{-8}	7.81×10^{-8}
$\Delta S = 0$	$B_c^- \to D_1^- \chi_{c2}$	2.32×10^{-6}	2.74×10^{-6}
	$B_c^- \to \underline{D}_1^- \chi_{c2}$	7.42×10^{-7}	8.77×10^{-7}
	$B_c^- \to \chi_{c1} D_2^-$	2.42×10^{-8}	1.35×10^{-7}
	$B_c^- \to a_1^- \bar{D}_2^0$	6.82×10^{-7}	8.07×10^{-7}
	$B_c^- \to f_1 D_2^-$	3.96×10^{-9}	2.21×10^{-8}
	$B_c^- \to a_1^0 D_2^-$	3.96×10^{-9}	2.21×10^{-8}
	$B_c^- \to b_1^- \bar{D}_2^0$	4.77×10^{-12}	5.65×10^{-12}

[a]For $\theta_K = -58°$.

TABLE V. Branching ratios of $B_c \to AT$ decays for CKM-doubly suppressed modes.

Mode	Decays	Branching ratios $N_c = 3$	$N_c \to \infty$
$\Delta S = -1$	$B_c^- \to D_{s1}^- \bar{D}_2^0$	2.53×10^{-7}	3.00×10^{-7}
	$B_c^- \to \underline{D}_{s1}^- \bar{D}_2^0$	8.43×10^{-8}	9.96×10^{-8}
	$B_c^- \to \bar{D}_1^0 D_{s2}^-$	6.76×10^{-9}	3.78×10^{-8}
	$B_c^- \to \bar{\underline{D}}_1^0 D_{s2}^-$	2.12×10^{-9}	1.19×10^{-8}
$\Delta S = 0$	$B_c^- \to D_1^- \bar{D}_2^0$	1.51×10^{-8}	1.79×10^{-8}
	$B_c^- \to \underline{D}_1^- \bar{D}_2^0$	4.75×10^{-9}	5.61×10^{-9}
	$B_c^- \to \bar{D}_1^0 D_2^-$	1.75×10^{-10}	9.76×10^{-10}
	$B_c^- \to \bar{\underline{D}}_1^0 D_2^-$	5.49×10^{-11}	3.07×10^{-10}

(3) We wish to point out that the branching ratios of $B_c \to A(^3P_1)T$ decays are larger than $B_c \to A(^1P_1)T$ decays due to larger values of respective decay constants. However, this trend is reversed in decays involving strange axial vector mesons for both angles, i.e., $\theta_{K_1} = -37°$ or $-58°$.

(4) It may also be noted that change in mixing angle θ_{K_1} from $-37°$ to $-58°$ does not affect the branching ratios of decays involving $K_1(^3P_1)$, whereas branching ratios of decays involving $\underline{K}_1(^1P_1)$ are almost doubled.

B. For CKM suppressed and doubly suppressed modes

(1) In CKM-suppressed bottom and charm changing mode ($\Delta S = -1$), branching ratios of dominant decays are $\mathrm{Br}(B_c^- \to \underline{K}_1^- \chi_{c2}) = 2.40 \times 10^{-5} (2.84 \times 10^{-5})$ and $\mathrm{Br}(B_c^- \to K_1^- \chi_{c2}) = 1.66 \times 10^{-5} (1.97 \times 10^{-5})$ for $\theta_{K_1} = -37°$; here also, numerical values in brackets indicate BRs at $N_c = \infty$. It is seen that branching ratio for $B_c^- \to \underline{K}_1^- \chi_{c2}$ decay increases by a factor of 2 for $\theta_{K_1} = -58°$. The branching ratios of the remaining decay modes are of $\mathcal{O}(10^{-7} \sim 10^{-8})$.

(2) As compared to ($\Delta C = 1$, $\Delta S = -1$), the dominant decay channels in ($\Delta C = \Delta S = 0$) have branching

ratios of $\mathcal{O}(10^{-6} \sim 10^{-7})$, i.e., $\mathrm{Br}(B_c^- \to D_1^- \chi_{c2}) = 2.32 \times 10^{-6} (2.74 \times 10^{-6})$; $\mathrm{Br}(B_c^- \to \underline{D}_1^- \chi_{c2}) = 7.42 \times 10^{-7} (8.77 \times 10^{-7})$ and $\mathrm{Br}(B_c^- \to \bar{D}_2^0 a_1^-) = 6.82 \times 10^{-7} (8.07 \times 10^{-7})$. However, rest of the decays have branching ratios of $\mathcal{O}(10^{-8} \sim 10^{-12})$.

(3) As expected, the branching ratios of CKM-suppressed decays are an order smaller than CKM-enhanced modes both for strangeness changing and strangeness conserving channels.

(4) Decay channels in Cabibbo doubly suppressed ($\Delta C = -1$, $\Delta S = -1$) and ($\Delta C = -1$, $\Delta S = 0$) modes remain highly suppressed except for $\mathrm{Br}(B_c^- \to \underline{D}_{s1}^- \bar{D}_2^0) = 2.53 \times 10^{-7} (3.00 \times 10^{-7})$. Other decays in these channels have branching ratios $\mathcal{O}(10^{-8} \sim 10^{-10})$.

(5) Here also, the branching ratios of decays involving $A(^3P_1)$ in the final state are larger in comparison to the decays involving $A(^1P_1)$ except for strange axial-vector mesons.

In addition, some decay channels are also possible through annihilation contributions but are ignored in our analysis, as these are suppressed due to helicity and color arguments.

As a test of factorization approximation, we propose to study the ratio of branching fractions of $B_c \to AT$ to $B_c \to VT$ decays for the same quark content, i.e.,

$$\frac{\mathrm{Br}(B_c \to AT)}{\mathrm{Br}(B_c \to VT)} = \left(\frac{f_A}{f_V}\right)^2 \left[\frac{\alpha(m_A^2)|\boldsymbol{p_A}|^7 + \beta(m_A^2)|\boldsymbol{p_A}|^5 + \gamma(m_A^2)|\boldsymbol{p_A}|^3}{\alpha(m_V^2)|\boldsymbol{p_V}|^7 + \beta(m_V^2)|\boldsymbol{p_V}|^5 + \gamma(m_V^2)|\boldsymbol{p_V}|^3}\right], \tag{32}$$

which is independent of QCD coefficients, kinematic constants, and CKM factors. We find this ratio for dominant modes in CKM-enhanced and CKM-suppressed modes as

$$\frac{\mathrm{Br}(B_c \to a_1^- \chi_{c2})}{\mathrm{Br}(B_c \to \rho^- \chi_{c2})} \sim 0.9;$$

$$\left.\frac{\mathrm{Br}(B_c \to \underline{K}_1^- \chi_{c2})}{\mathrm{Br}(B_c \to K^{*-} \chi_{c2})}\right|_{37^\circ} \sim 0.9,$$

and

$$\left.\frac{\mathrm{Br}(B_c \to K_1^- \chi_{c2})}{\mathrm{Br}(B_c \to K^{*-} \chi_{c2})}\right|_{37^\circ} \sim 0.6.$$

It is clear from the expression (32) that this ratio may be used as a test of the ISGW II model to determine q^2-dependence of the form factors provided that decay constants are known. Also, if the form factors are well known, relation (32) may be used to estimate the extent of mixing between K_{1A} and $K_{1A'}$ states and consequently to determine $f_{K_{1A}}$ and $f_{K_{1A'}}$ decay constants.

With remarkable technological improvements in experiments and availability of high precision instrumentation, branching ratios of the order of (10^{-6}) could be measured precisely [7] at the LHC, the LHC-b and Super-B factories in the near future, which could provide the necessary insight into the phenomenological studies related to B_c meson physics.

V. SUMMARY AND CONCLUSIONS

In the present work, we have investigated $B_c \to AT$ decays in the ISGW II model framework. We have obtained their branching ratios of $B_c \to T$ transition form factors in this model. We have presented results both at $N_c = 3$ and large N_c limit. We conclude the following:

(1) In CKM enhanced ($\Delta b = 1$, $\Delta C = 1$, $\Delta S = 0$) the only dominant decay $B_c^- \to a_1^- \chi_{c2}$ has branching ratio (2.32×10^{-4}). However, the dominant decays $B_c^- \to D_{s1}^- \chi_{c2}$ and $B_c^- \to \underline{D}_{s1}^- \chi_{c2}$ in ($\Delta b = 1$, $\Delta C = 0$, $\Delta S = -1$) have branching ratios around 10^{-5} and 10^{-6}, respectively. Branching ratios of all the decay modes range from ($10^{-4} \sim 10^{-12}$).

(2) In CKM suppressed modes, the branching ratios are further suppressed by an order of magnitude as compared to CKM enhanced modes. Branching ratios for the dominant decays $B_c^- \to K_1^- \chi_{c2}$, $B_c^- \to \underline{K}_1^- \chi_{c2}$ and $B_c^- \to D_1^- \chi_{c2}$ are of $\mathcal{O}$ ($10^{-5} \sim 10^{-6}$).

(3) In general, branching ratios of $B_c \to A(^3P_1)T$ decays are higher in comparison to the $B_c \to A(^1P_1)T$ decays due to larger decay constant of $A(^3P_1)$ mesons. However, this is reversed for decays involving K_1-meson in the final state.

(4) Change in mixing angle θ_{K_1} from -37° to -58° increases branching ratios of decays involving $\underline{K}_1(^1P_1)$ by a factor of 2, however, decays involving $K_1(^3P_1)$ remain almost unaffected.

(5) At large N_c limit, branching ratios of all the decay modes are enhanced due to increased values of a_1 and a_2. Also, our results are independent of sign of a_1 and a_2 as Class III type decays are not possible in $B_c \to AT$ mode.

Recently, several p-wave meson emitting decays have been reported in the Particle Data Group [2] whose branching ratios are $\mathcal{O}(10^{-6})$. Therefore, we hope that the predicted BRs will be measured soon as experiments like the LHC, the LHC-b, and the KEK-B are expected to accumulate more than 10^{10} B_c events per year.

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