Determination of exotic hadron structure by constituent-counting rule for hard exclusive processes

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We propose to use hard exclusive production of an exotic hadron for finding its internal quark-gluon configuration by the constituent-counting rule in perturbative QCD. In particular, the cross section for the exclusive process $\pi^- + p \to K^0 + \Lambda(1405)$ is estimated at the scattering angle $\theta = 90°$ in the center-of-mass frame by using current experimental data. In comparison, the cross section for the ground-state Λ production $\pi^- + p \to K^0 + \Lambda$ is also shown. We suggest that the internal quark configuration of $\Lambda(1405)$ should be determined by the asymptotic scaling behavior of the cross section. If it is an ordinary three-quark baryon, the scaling of the cross section is $s^8 d\sigma/dt =$ constant, whereas it is $s^{10} d\sigma/dt =$ constant if $\Lambda(1405)$ is a five-quark hadron, where s and t are Mandelstam variables. Such a measurement will be possible, for example, by using the high-momentum beam line at Japan Proton Accelerator Research Complex. In addition, another exclusive process $\gamma + p \to K^+ + \Lambda(1405)$ could be investigated at laser electron photon beam line at SPring-8 and JLab for finding the nature of $\Lambda(1405)$. We indicate that the constituent-counting rule could be used as a valuable quantity in determining internal structure of exotic hadrons by high-energy exclusive processes, where quark-gluon degrees of freedom explicitly appear. Furthermore, it is interesting to investigate the transition from hadron degrees of freedom to quark-gluon ones for exclusive exotic-hadron production processes.

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I. INTRODUCTION

A basic quark model indicates that baryons consist of three quarks (qqq) and mesons of a quark-antiquark pair ($q\bar{q}$). The family of the baryons and mesons is called hadrons, and a few hundred hadrons have been found experimentally [1]. However, an undoubted evidence has not been found for an exotic hadron, which has a different configuration from qqq and $q\bar{q}$, although the fundamental theory of strong interaction, QCD, does not prohibit the existence of such states like tetraquark ($qq\bar{q}\,\bar{q}$) and pentaquark ($qqqq\bar{q}$) hadrons [2].

It is, nevertheless, fortunate that the experimental situation changed in the last several years because there have been reports on exotic hadron candidates particularly from the Belle and *BABAR* collaborations [3]. Exotic hadrons were suggested in experimental measurements so far by looking at masses, spins, and decay widths, namely, global observables at low energies. For example, electromagnetic and strong decay widths could provide useful information on exotic hadrons [4]. However, at low energies, effective degrees of freedom are hadrons, and only integrated quantities are observed, so that it is not easy to judge whether or not a hadron has an exotic quark-gluon configuration. Therefore, it is appropriate to look for high-energy processes, where quark-gluon degrees of freedom appear explicitly. Keeping this idea in mind, we have been investigating possible high-energy processes for determining internal structure of exotic hadron candidates, for example, by fragmentation functions [5] and hadron-production processes in the e^+e^- annihilation [6].

In this article, we propose that the constituent-counting rule of perturbative QCD could be used for finding the internal quark configuration of exotic-hadron candidates in exclusive production processes. In the exclusive process $a + b \to c + d$ with large-momentum transfer, hard gluon exchange processes should occur to maintain the exclusive nature. Namely, quarks should share large momenta so that they should stick together to become a hadron by exchanging hard gluons. Then, considering hard quark and gluon propagators in the reaction, we obtain that the cross section of the $a + b \to c + d$ exclusive reaction should scale like $d\sigma/dt \sim s^{2-n} f(\theta_{\rm cm})$ with $n = n_a + n_b + n_c + n_d$, where s and t are Mandelstam variables, $\theta_{\rm cm}$ is the scattering angle in the c.m. system, and n_h is the number of constituents in the particle h. This asymptotic scaling relation is known as the constituent-counting rule [7–12]. Since the factor n_h clearly indicates the internal configuration in the hadron, this scaling relation can be used for finding internal configurations of exotic hadrons.

Here, we take an exotic hadron candidate $\Lambda(1405)$ as an example for proposing such an idea. The $\Lambda(1405)$ has been controversial for many years from 1960s. The $\Lambda(1405)$ is a baryon resonance with isospin 0, spin-parity $(1/2)^-$, strangeness -1, mass 1405.1 MeV, and width 50 MeV [1]. One of the remarkable properties for $\Lambda(1405)$ is its anomalously light mass. Namely, in the ground states which possess spin-parity $(1/2)^+$, Λ is heavier than the nucleon, $M_\Lambda - M_N \simeq +180$ MeV, due to the heavier strange quark in Λ. However, in the $(1/2)^-$ states, the lowest excitation states of Λ and nucleon are $\Lambda(1405)$ and $N(1535)$, respectively, and the puzzling reversal of the mass relation takes place as $M_{\Lambda(1405)} - M_{N(1535)} \simeq -130$ MeV, although $\Lambda(1405)$ should have the heavier strange quark. The mass of $\Lambda(1405)$ is found to be anomalously light also compared to the result of the SU(6) quark model, in which both $\Lambda(1405)$ and $N(1535)$ should be considered to be baryons in the 70-dimensional representation with p-wave excitation of a quark [13], but it is difficult to explain the lighter mass of $\Lambda(1405)$ than $N(1535)$ in the same representation. Therefore, it has been thought as an exotic hadron, beyond the naive three-quark (uds) configuration.

Instead of an uds three-quark system, the $\Lambda(1405)$ has been considered as a $\bar{K}N$ molecule for a long time [14] because it is slightly below the $\bar{K}N$ threshold, and the $\bar{K}N$ interaction is strongly attractive in the isospin 0 channel. There are recent theoretical progresses on $\Lambda(1405)$ as a dynamically generated resonance in meson-baryon scattering by the so-called chiral unitary model [15]. This model supports the meson-baryon molecule nature for $\Lambda(1405)$ by revealing, e.g., predominance of the meson-baryon component [16], its large-N_c scaling behavior [17], and its spatial size [18,19]. There is also a proposal that $\Lambda(1405)$ could be a strange hybrid baryon by the QCD sum rule [20]. For the last several years, there have been many articles on $\Lambda(1405)$, so that we suggest that the reader look at the reference section of the recent review article [21]. In the experimental side, precise measurement of the $\Lambda(1405)$ line shape has been recently performed in the photon induced $\Lambda(1405)$ production [22], which provides information on underlying dynamics and internal structure of $\Lambda(1405)$ [23]. In addition, hadron induced production experiments are currently in progress, e.g., by pp collision at 3.5 GeV by the HADES collaboration at Gesellschaft für Schwerionenforschung (GSI) [24] and the $K^- + d$ reaction planned by the E31 experiment at Japan Proton Accelerator Research Complex (J-PARC) [25].

In spite of theoretical studies on exotic hadrons for a long time, it is difficult to find a clear experimental evidence for the molecular or any other exotic configuration because global quantities such as masses and decay widths have been used. On the other hand, the quark-gluon degrees of freedom appear in high-energy reactions. For example, scaling behavior of exclusive cross sections is known as the constituent-counting rule. In addition, the transition from the hadron degrees of freedom to the quark-gluon ones seems to be clearly shown in the JLab measurements of $\gamma + p \to \pi^+ + n$ [26] by the differential cross section as the function of the c.m. energy $\sqrt{s}$.

In the same way, hard exclusive production processes of $\Lambda(1405)$ could be valuable for finding its internal quark configuration by looking at the scaling behavior of the cross section at high energies. Furthermore, it is interesting to investigate the transition phenomena from hadron degrees of freedom to the quark-gluon ones for exotic hadrons. Fortunately, the high-momentum beam line of the J-PARC will be built in a few years, and an unseparated hadron (essentially pion) beam with momentum up to 15–20 GeV will be available. Then, the exclusive reaction $\pi^- + p \to K^0 + \Lambda(1405)$ will become experimentally possible in principle. However, no theoretical study exists for estimating the cross section of $\pi^- + p \to K^0 + \Lambda(1405)$ at large-momentum transfer. As far as we are aware, even an idea does not exist for studying the internal quark configuration of exotic hadron candidates by the constituent-counting rule [6]. This article should be the first attempt to investigate such an idea by taking $\Lambda(1405)$ as an example. Since there is no prior study, we do not intend to show precise theoretical cross sections, which are not possible at this stage in any case. Instead, we try to provide an order of magnitude estimate of the cross sections in this work for future experimental proposal at the J-PARC or any other hadron facilities.

This article is organized in the following way. In Sec. II, the constituent-counting rule is explained for understanding cross section behavior at high energies. The cross sections are estimated in a high-momentum transfer region, where the counting rule could be applied, by using existing measurements for $\pi^- + p \to K^0 + \Lambda$ and $\pi^- + p \to K^0 + \Lambda(1405)$ in Sec. III. We summarize our studies in Sec. IV.

II. CONSTITUENT-COUNTING RULE IN HARD EXCLUSIVE REACTIONS

We introduce the constituent-counting rule especially for the readers who are not familiar with perturbative QCD. For a large-angle exclusive scattering $a+b\to c+d$, the reaction cross section is given by

$$\frac{d\sigma_{ab\to cd}}{dt} \simeq \frac{1}{16\pi s^2}\overline{\sum_{\rm pol}}|M_{ab\to cd}|^2, \tag{1}$$

where s and t are Mandelstam variables defined by

$$\begin{aligned} s &= (p_a + p_b)^2 \simeq 4|\vec{p}_{\rm cm}|^2, \\ t &= (p_a - p_c)^2 \simeq -2|\vec{p}_{\rm cm}|^2(1-\cos\theta_{\rm cm}), \end{aligned} \tag{2}$$

where the masses of hadrons are neglected by considering the kinematical condition s, $|t| \gg m_i^2$ ($i = a, b, c, d$), p_i is the momentum of the hadron i, and $p_{\rm cm}$ and $\theta_{\rm cm}$ are momentum and scattering angle in the c.m. frame, respectively. Since we are considering the large-angle scattering, the kinematical invariants s and $|t|$ are in the same order of magnitude. The summation of Eq. (1) indicates the average over the initial spins and the summation for the final spins.

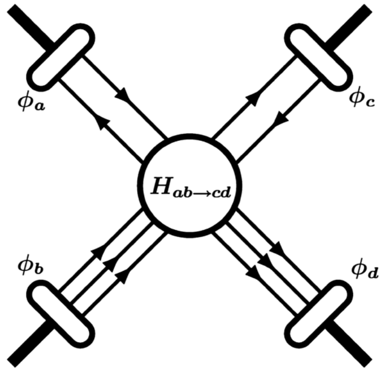

FIG. 1. Hard exclusive scattering $a + b \to c + d$.

The matrix element $M_{ab\to cd}$ is expressed in the factorized form at large momentum transfer [10,12,27]:

$$M_{ab\to cd} = \int [dx_a][dx_b][dx_c][dx_d]\phi_c([x_c])\phi_d([x_d]) \times H_{ab\to cd}([x_a],[x_b],[x_c],[x_d],Q^2)\phi_a([x_a])\phi_b([x_b]), \quad (3)$$

in terms of the partonic scattering amplitude $H_{ab\to cd}$ and the light-cone distribution amplitude of each hadron, ϕ_a, ϕ_b, ϕ_c, and ϕ_d, as illustrated in Fig. 1. Here, $[x]$ indicates a set of the light-cone momentum fractions of partons in a hadron: $x_i = p_i^+/p^+$ where p_i and p are ith parton and hadron momenta, respectively, and the light-cone component is defined as $p^+ = (p^0 + p^3)/\sqrt{2}$ by taking the third axis for the longitudinal direction.

For the nucleon, two independent variables are needed to describe the distribution amplitude by considering a constraint of the momentum conservation $x_1+x_2+x_3=1$. Namely, we have $[x] = x_1,\ x_2$ and $[dx] = dx_1 dx_2$ in Eq. (3). On the other hand, only one variable x is needed for mesons such as pions and kaons [see Eq. (7)]. As an example, the reaction is illustrated in Fig. 1 by taking the hadrons a and c as mesons and b and d as baryons. The variable Q^2 indicates a hard scale of the reaction, which is given by $Q^2 \simeq s$ for the large-angle elastic exclusive scattering. In Eq. (3), we have suppressed the renormalization and factorization scale dependencies: the former is controlled by the renormalization group equation for the coupling constant and the latter by the Efremov–Radyushkin–Brodsky–Lepage (ERBL) evolution equations for the distribution amplitudes [27]. Those scales are taken to be order of Q^2 to avoid large radiative corrections. The resulting Q^2 dependencies are logarithmic and do not largely affect the scaling behavior of the matrix element.

A. Constituent-counting rule by dimensional counting

Originally, the constituent-counting rule was suggested by dimensional counting [7,8]. Then, it was studied by considering hard scattering processes in perturbative QCD [8–10]. In this section, we explain derivation of the scaling rule by counting mass dimensions. Then, we outline how the counting rule is understood in perturbative QCD in Sec. II B.

Because the state vector of a hadron is normalized as $\langle h(p')|h(p)\rangle = 2p^0(2\pi)^3\delta^{(3)}(\vec{p} - \vec{p}')$, its mass dimension is $[1/M]$. If a hadron is made of n_h elementary constituents, its state vector could be written as

$$|h\rangle = \sqrt{N_h}|n_h\rangle, \qquad [\sqrt{N_h}] = [M^{n_h-1}], \quad (4)$$

where the second equation indicates that the normalization factor $\sqrt{N_h}$ has the mass dimension $[M^{n_h-1}]$ if the state vector of each constituent has the mass dimension $[1/M]$.

Here, we explain that the normalization factor N_h is free from the hard momentum scale. Let us take a pion state as an example. A pion state with momentum $p \simeq (p^+, 0^-, \vec{0}_T)$ in the c.m. system is expressed in terms of the Bethe–Salpeter (BS) wave function of the leading Fock state as

$$|\pi(p)\rangle = \int \frac{du}{\sqrt{u\bar{u}}}\frac{d\vec{k}_T}{16\pi^3}\Psi_{q\bar{q}/\pi}(u,\vec{k}_T)|q(k_q)\bar{q}(k_{\bar{q}})\rangle + \cdots, \quad (5)$$

where $\bar{u} = 1 - u$ and the ellipses denote the higher Fock states, whose contribution to the exclusive scattering amplitude is suppressed by some powers of s. The leading Fock state consists of a quark and antiquark with momenta $k_q \simeq (up^+, \vec{k}_T^2/(2up^+), \vec{k}_T)$ and $k_{\bar{q}} \simeq (\bar{u}p^+, \vec{k}_T^2/(2\bar{u}p^+), -\vec{k}_T)$, respectively. Ignoring the higher Fock states, we have the normalization of the BS wave function given by

$$\int_0^1 du \int \frac{d\vec{k}_T}{16\pi^3}|\Psi_{q\bar{q}/\pi}(u,\vec{k}_T)|^2 = 1. \quad (6)$$

Then, if one can assume that the wave function damps fast enough at large $|\vec{k}_T|$ such that it has nonzero values in the region $|\vec{k}_T| \lesssim Q_{\rm had}$, where $Q_{\rm had}$ is the hadronic scale, its magnitude is given by $\Psi_{q\bar{q}/\pi} \simeq \mathcal{O}(1/Q_{\rm had})$. This means that the normalization factor is given by $\sqrt{N_\pi} \sim \int d\vec{k}_T \Psi_{q\bar{q}/\pi} \simeq \mathcal{O}(Q_{\rm had})$.

Actually, in the perturbative calculation, the Feynman rule for the incoming pion, for example, is given from the following operator definition of the light-cone distribution amplitude as a matrix element of a bilocal operator between the pion and vacuum states [27,28]:

$$\langle 0|\bar{d}(0)_\alpha u(z)_\beta|\pi^+(p)\rangle = \frac{if_\pi}{4}\int_0^1 du e^{-iup^+z^-}(\gamma_5 \not{p})_{\beta\alpha}\phi_\pi(u,\mu), \quad (7)$$

where $z = (0, z^-, \vec{0}_T)$ is a lightlike vector and f_π is the pion decay constant defined as $\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(0)|\pi(p)\rangle = if_\pi p_\mu$, with the normalization $\int_0^1 du \phi_\pi(u) = 1$. A gauge link inserted between the two quark fields is understood on the lhs, so that the matrix element is gauge invariant. The variables u and μ are the longitudinal momentum fraction of a quark in the pion and the renormalization scale of the bilocal operator, respectively, where the latter dependence is governed by the ERBL evolution equation [27]. The relation between the light-cone distribution amplitude and the BS wave function is given by [29]

$$\int^{|\vec{k}_T|<\mu} \frac{d\vec{k}_T}{16\pi^3} \Psi_{q\bar{q}/\pi}(u, \vec{k}_T) \sim \frac{if_\pi}{4}\sqrt{\frac{2}{N_c}}\phi_\pi(u, \mu), \quad (8)$$

up to the scheme difference for subtracting the light-cone singularity in the bilocal operator. Here, N_c is the number of colors. From this expression, one can see the normalization factor is the order of magnitude of the pion decay constant: $\sqrt{N_\pi} \sim \mathcal{O}(f_\pi \simeq 0.13\ \text{GeV})$, which is the order of a typical hadron mass.

The same discussions also apply for the nucleon. By looking at its light-cone expression [30], one can explicitly see that the normalization factor is of the order of a soft mass scale squared. Actually, the normalization factor is always given by the corresponding "decay constant," which is free from the hard momentum scale.

Now, we consider the mass dimensions of the matrix element in Eq. (1) for obtaining the counting rule in the exclusive cross sections. The scattering matrix S is expressed by the transition matrix T as $S = \mathbf{1} + i(2\pi)^4 \times \delta^{(4)}(p_f - p_i)T$, so that the mass dimension of T is $[T] = [M^4]$. The matrix element $M_{ab\to cd}$ is given by T as

$$M_{ab\to cd} = \langle cd|T|ab\rangle = \sqrt{N_a N_b N_c N_d}\langle n_c n_d|T|n_a n_b\rangle. \quad (9)$$

Because the normalization factors N_i ($i = a, b, c, d$) are expressed by soft constants, we consider the matrix element $\hat{M}_{ab\to cd}$ by excluding them, and then the remaining hard part should be expressed in terms of two variables s and t:

$$\hat{M}_{ab\to cd} \equiv \frac{1}{\sqrt{N_a N_b N_c N_d}} M_{ab\to cd} = \langle n_c n_d|T|n_a n_b\rangle \equiv \hat{F}_{ab\to cd}(s, t). \quad (10)$$

From the dimensions $[T] = [M^4]$ and $[|n_i\rangle] = [1/M^{n_i}]$, the dimension of the matrix element is given as

$$[\hat{M}_{ab\to cd}] = [\langle n_c n_d|T|n_a n_b\rangle] = [M^{4-n}], \quad (11)$$

where $n \equiv n_a + n_b + n_c + n_d$. The variable s could be chosen as the only hard scale in the large-angle exclusive reaction, so that the matrix element is expressed, by considering the mass dimension, as

$$\hat{M}_{ab\to cd} = \hat{F}_{ab\to cd}(s, t) = s^{(4-n)/2} F_{ab\to cd}(t/s), \quad (12)$$

where $F_{ab\to cd}(t/s)$ is a dimensionless quantity and it is a function of scattering angle $-2t/s = 1 - \cos\theta_{\text{cm}}$ from Eq. (2). Using Eqs. (1), (10), and (12), we obtain the constituent-counting expression for the cross section:

$$\frac{d\sigma_{ab\to cd}}{dt} = \frac{1}{s^{n-2}} f_{ab\to cd}(t/s), \quad (13)$$

where $f(t/s)$ is the scattering-angle dependent part multiplied by the normalization factors. Because the mass dimensions of $f(t/s)$ are given by $[f(t/s)] = [N_a N_b N_c N_d |F_{ab\to cd}|^2] = [M^{2n-8}]$, the overall mass dimension of Eq. (13) is, of course, $[1/M^4]$. This is the derivation of the counting rule by considering the mass dimensions. Because it counts the number of constituents which actively participate in the reaction, this scaling behavior is called the "constituent-counting rule."

B. Constituent-counting rule in perturbative QCD

The argument by the dimensional counting described above is intuitively clear, but it does not provide a "proof" of the constituent-counting rule. For example, Eq. (13) is not valid for the contribution from disconnected so-called Landshoff diagrams [31]. Actually, each disconnected scattering amplitude is dimensionless, while the condition that the separately scattered partons form the hadrons in the final state requires that the c.m. momentum in each subdiagram must coincide up to Q^2_{had}/s. For example, in the elastic scattering of Fig. 1, $x_a = x_b + \mathcal{O}(Q^2_{\text{had}}/s)$ is imposed in the $[x]$-integral, which eventually yields some powers of Q^2_{had}/s [8,10]. Such a mechanism as the origin of the scaling power is not included in the naive dimensional counting in Sec. II A, where we treat $\sqrt{N_h}$ as a dimensionful constant and assume that the x integral does not affect the dimensional counting. Hence, in general, more rigorous arguments based on perturbative calculations are needed to correctly identify the scaling behavior [8–12]. In this subsection, we discuss how the counting rule emerges in QCD from rough estimation of Feynman diagrams and possible complications.

Before stepping into an exclusive hadron-hadron reaction, we explain a familiar elastic electron scattering from the proton, $e + p \to e' + p'$. Its cross section is described by elastic form factors of the proton:

$$\langle p'|J^\mu|p\rangle = \bar{u}(p')\left[\gamma^\mu F_1(Q^2) + i\frac{\kappa}{2m_N}\sigma^{\mu\nu}q_\nu F_2(Q^2)\right]u(p), \quad (14)$$

where $F_1(Q^2)$ and $F_2(Q^2)$ are Dirac and Pauli form factors, κ is the anomalous magnetic moment, m_N is the proton mass, and Q^2 is given by the momentum of the virtual photon q as $Q^2 = -q^2 \equiv \vec{q}^2 - (q^0)^2$. Then, the electric and magnetic form factors are defined by $G_E = F_1 - \frac{\kappa Q^2}{4m_N^2}F_2$ and $G_M = F_1 + \kappa F_2$. In the following discussions, we consider the magnetic form factor G_M which is dominant in the cross section at large Q^2.

At large Q^2, the elastic form factor is factorized into a hard-scattering part H_M and a soft part given by the proton distribution amplitude ϕ_p:

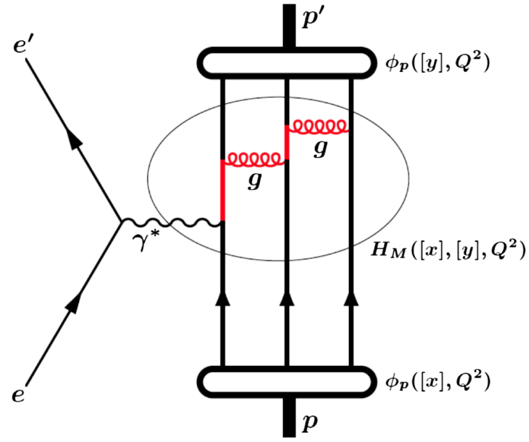

FIG. 2 (color online). A typical hard gluon-exchange process in elastic electron-proton scattering ($e + p \to e' + p'$). There are two hard quark propagators and two gluon ones which contribute to the counting rule in the elastic form factor.

$$G_M(Q^2) = \int [dx] \int [dy] \phi_p([y]) H_M([x], [y], Q^2) \phi_p([x]), \tag{15}$$

where we suppress the scale dependence of ϕ_p. The hard amplitude $H_M([x], [y], Q^2)$ should be evaluated in perturbative QCD. Because of the elastic scattering nature, the proton should not be broken up by the large momentum given by the virtual photon as shown in Fig. 2. The only way to sustain the identity of the proton for a given large momentum is to share the momentum among the constituents of the proton by exchanging hard gluons. Therefore, the leading contribution to the elastic ep cross section should be described by the hard gluon exchange processes typically shown in Fig. 2.

The amplitude H_M is controlled by the momentum scale Q, which is provided by the virtual photon, in the two quark propagators and two gluon ones in Fig. 2. If we consider a frame with large momentum for the proton, specifically, the Breit frame where the virtual photon 4-momentum is given by $q = (0, \vec{q})$, we have a relation $|\vec{p}| = |\vec{p}'| \equiv P \sim \mathcal{O}(Q) \gg m_N$. There are additional hard factors due to each quark external line $u \sim \sqrt{P}$. More precisely, the three quark lines are replaced by $(\not{p}\Gamma)_{\alpha\beta}(\Gamma' u(p))_\gamma \sim (\sqrt{P})^3$, where Γ and Γ' are appropriate γ matrices, multiplied by the proton's distribution amplitudes [30] for the incoming and outgoing proton. Anyway, there is a factor of $\sqrt{P} \sim \sqrt{Q}$ for each external quark line. Therefore, there are two quark propagators $\sim 1/Q^2$, two gluon propagators $\sim 1/(Q^2)^2$, and six external quark lines $\sim (\sqrt{Q})^6$, which give rise to the overall factor $1/(Q^2)^{3/2}$:

$$\langle p'|J^\mu|p\rangle \sim \frac{1}{Q^2} \frac{\alpha_s(Q^2)^2}{(Q^2)^2} (\sqrt{Q})^6 = \frac{\alpha_s(Q^2)^2}{(Q^2)^{3/2}}, \tag{16}$$

where α_s is the running coupling constant of QCD.

The proton distribution amplitude $\phi_p([x])$ is the amplitude for finding quarks with the momentum fractions x_1 and x_2 in the proton. This distribution amplitude also has a weak logarithmic Q^2 dependence [27] as we discussed in this section, which does not change the leading scaling behavior.

There is one more factor which needs to be considered due to the definition of the form factor in Eq. (14), so that there is another hard factor $\bar{u}\gamma^\mu u \sim P \sim (Q^2)^{1/2}$ in front of the definition of the form factor. Summarizing these results, we have

$$G_M(Q^2) \sim \frac{1}{(Q^2)^{1/2}} \langle p'|J^\mu|p\rangle \sim \frac{1}{Q^4} = \frac{1}{t^{n_N - 1}} \qquad (n_N = 3), \tag{17}$$

where t is the Mandelstam variable $t = -Q^2$ and $n_N = 3$ is the number of valence quarks in the proton. Actually, one can easily see that all factors of Q cancel with each other except the ones from the $n_h - 1$ gluon propagators. Therefore, the form factors generally scale as $1/t^{n_h - 1}$, which is consistent with the constituent-counting rule in Eq. (13) for the $e + h \to e + h$ scattering. Such a scaling has been experimentally observed in the form factors of the proton [32].

From these discussions, we understand a scaling rule for large-angle exclusive reactions in the following manner. First, for finding the scaling behavior, it is enough to consider a Feynman diagram with the simplest topology as shown in Fig. 3. For the time being, we forget the flavor contents of the hadrons. In order to become an exclusive reaction with large momentum transfer, a hard gluon should be exchanged between a quark in the hadron a and a quark in b. Then, the large momentum should be shared within the hadrons by exchanging hard gluons as shown in the figure. Denoting the hard momentum as P in an exclusive reaction, we have the following rule for calculating the scaling behavior of the cross section:

(i) Feynman diagram:
First, leading and connected Feynman diagrams are drawn for the exclusive process by connecting $n/2$ quark lines by gluons.

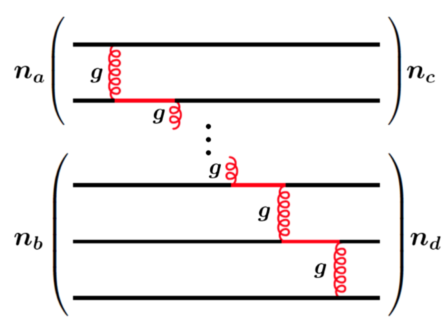

FIG. 3 (color online). Hard gluon exchange process for an exclusive hadron-hadron reaction $a + b \to c + d$ with large momentum transfer.

(ii) Gluon propagators:
The factor $1/P^2$ is assigned for each gluon propagator. Because there are $n/2 - 1$ gluon propagators in Fig. 3, the overall factor is $1/(P^2)^{n/2-1}$.

(iii) Quark propagators:
The factor $1/P$ is assigned for each quark propagator. There are $n/2 - 2$ quark propagators, so that the overall factor becomes $1/P^{n/2-2}$.

(iv) External quarks:
The factor $\sqrt{P}$ is assigned for each external quark. Because there are n quarks in the initial and final states in total, the overall factor is $(\sqrt{P})^n$.

Then, the matrix element $\hat{M}_{ab\to cd}$ has the mass dimension

$$[\hat{M}_{ab\to cd}] = \left[\frac{1}{(P^2)^{n/2-1}} \frac{1}{P^{n/2-2}} P^{n/2}\right] = \left[\frac{1}{s^{n/2-2}}\right]. \tag{18}$$

Because the hadron distribution amplitudes $\phi_{a,b,c,d}$ have the weak logarithmic scale dependence, the leading contribution should come from the hard matrix element. Then, the cross section is given by the constituent-counting expression of Eq. (13) by using Eqs. (1), (10), and (18). This is a diagrammatic explanation of the constituent-counting rule in perturbative QCD.

There are theoretical complications which need to be considered for the counting rule [8,10–12]. One is that the disconnected diagrams do not necessarily obey the counting rule as we explained before. Actually, they and some correction diagrams to them develop the "pinch singularity," which occurs when a denominator of a gluon propagator vanishes inside the interval of the x integration [31]. After regularizing the linear divergence of the infrared origin, the x integral around the pinch singularity gives a power of $\sqrt{s}/m$ in the matrix element, where m is a quark mass. For example, such diagrams in meson-meson scattering scale as s^{-5}, instead of s^{-6} by the counting rule [8]. However, the configuration at the pinch singularity is associated with the elastic scattering of colored particles and is subject to the Sudakov effects [33]. Actually, it has been shown that the Sudakov effects shift the scaling power of the hadron-hadron scattering amplitude significantly, and the resulting "effective" scaling power is close to the one by the counting rule [12,34].

Furthermore, the endpoint singularity at $x \to 0$ or 1 could also affect the scaling behavior. At the endpoints, the momentum transfer to one of the quarks becomes soft, and the rules (2)–(4) for calculating the scaling behavior do not apply. A typical endpoint singularity is given by an integral $\sim \int dx \alpha_s(xQ^2) \frac{\phi_h(x,\cdots)}{x}$, so that the validity of the perturbative QCD (pQCD) description in Eq. (3), let alone the counting rule, depends on the nonperturbative endpoint behavior of the distribution amplitude. According to the conformal symmetry of QCD, the distribution amplitudes are linear: $\phi_h(x) \sim x$ as $x \sim 0$ in the asymptotic limit [35], as is known for the pion distribution amplitude $\phi_\pi(x) = 6x(1-x)$. A numerical study for the pion form factor with the conventional collinear factorization like Eq. (3) suggests that the pQCD description is valid only at the very high energy [36]. On the other hand, a more elaborate study using the k_t factorization formalism tells that the effects of the Sudakov form factor provide a sufficient suppression of the contribution from the endpoint region above $Q \simeq 10\Lambda_{\mathrm{QCD}}$ [37]. Unfortunately, the precise experimental tests of pQCD for exclusive hard processes are still premature, but the recent *BABAR* and Belle data [38] for the photon-pion transition form factor are not far from the pQCD result [9,10,27].

Despite these theoretical complications, the constituent-counting rule seems to work well for hard exclusive reactions [39], so that the above mentioned contributions from the pinch/endpoint singularities are not expected to change the rule to a significant amount. Actually, it seems that the counting rule applies even at the energy which is lower than the region where the leading power QCD description is considered to be valid.

So far, we have ignored the hadron helicity in the exclusive processes. When the transverse momenta are integrated, only the S-wave states are projected, unless $x \sim 0$ or 1. Since the QCD interaction conserves the quark helicity up to the $\mathcal{O}(m^2/Q^2)$ effects, the total hadron helicity is also conserved to that accuracy: $\lambda_a + \lambda_b = \lambda_c + \lambda_d$ [10,40]. In other words, the helicity nonconserving processes are suppressed by a factor of m^2/Q^2 from the scaling behavior given by Eq. (13). We also note that the large-angle elastic scatterings, $\pi + p \to K + \Lambda, K + \Lambda(1405)$, which we discuss in this paper, are given by the quark exchange diagrams. Therefore, there appears no pinch singularity for these processes.

C. Internal structure of hadrons by counting rule

The scaling behavior of the exclusive cross section given by the constituent-counting rule has been confirmed by a number of experiments [39]. Another striking phenomenon, including the transition from hadron degrees of freedom to the quark degrees of freedom, was observed by the reaction $\gamma + p \to \pi^+ + n$ in Fig. 4. Here, the number of elementary constituents is $n = 1 + 3 + 2 + 3 = 9$ in this reaction, and the cross section is multiplied by the counting-rule factor s^{9-2} in the ordinate, and it is shown as the function of the c.m. energy $\sqrt{s}$. In the low-energy region $\sqrt{s} < 2.5$ GeV, the cross section is described by contributions from nucleon and delta resonances, whereas the scaling of $s^7 d\sigma/dt =$ constant seems to be obtained at higher energy $\sqrt{s} > 2.6$ GeV. Furthermore, the data suggest that the transition from the hadron degrees of freedom to the quark ones occurs at $\sqrt{s} \sim 2.5$ GeV, which is 1.6 GeV above the proton mass.

We intend to use the counting rule for probing the internal structure of exotic hadron candidates. For example, ordinary Λ should be counted as $n_\Lambda = 3$; however, it is expected to be $n_{\Lambda(1405)} = 5$ if the structure is a

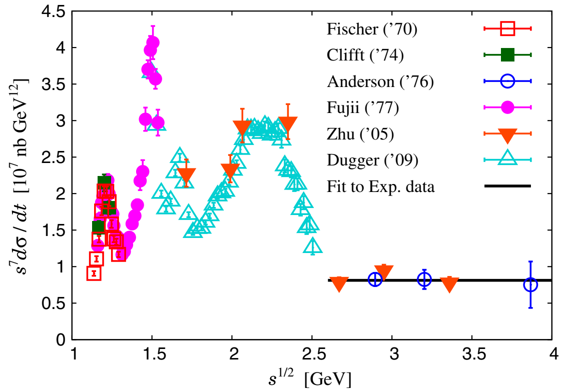

FIG. 4 (color online). $\gamma + p \to \pi^+ + n$ cross section from resonances to a large momentum-transfer region. The data are taken from Refs. [26,41]. The straight line is a fit to the data at $\sqrt{s} > 2.6$ GeV.

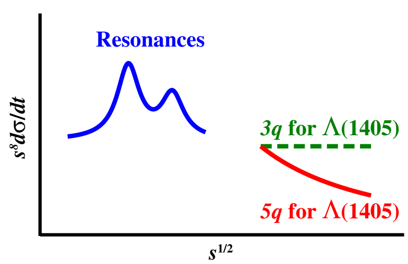

FIG. 5 (color online). Schematic figure of $\pi^- + p \to K^0 + \Lambda(1405)$ cross section $s^8 d\sigma/dt$ as the function of $\sqrt{s}$ from the resonance region to the scaling one. The scaling behavior at high energies indicates whether $\Lambda(1405)$ has an exotic five-quark configuration.

fivequark configuration including a $\bar{K}N$ molecule for $\Lambda(1405)$. It is schematically shown in Fig. 5 by the cross section $s^8 d\sigma/dt$ at high energies. If $\Lambda(1405)$ is a three-quark baryon, it scales like $s^8 d\sigma/dt =$ constant, whereas it should be $s^8 d\sigma/dt \sim 1/s^2$ if $\Lambda(1405)$ is a five-quark state.

III. RESULTS

Our research purpose is to estimate the order of magnitude of the exclusive cross section of $\pi^- + p \to K^0 + \Lambda(1405)$ for future experimental proposals by considering existing experimental data and theoretical estimates to extend them to the large-momentum transfer region, so that experimental measurements will be used for finding the internal structure of $\Lambda(1405)$ by the constituent-counting rule. As for the reference cross section, the order of magnitude of the ground-state Λ cross section is also estimated from the data at high energies from the available data. In addition, it is interesting to investigate the transition from the hadron degrees of freedom to the quark ones, as clearly shown in Fig. 4, particularly for exotic hadrons.

At this stage, a successful theoretical description has not been developed for estimating the magnitude of exclusive cross sections in the perturbative QCD region [42,43] although the scaling behavior $d\sigma/dt \sim 1/s^{n-2}$ is well known. The following points need to be done for the pQCD estimate. First, there are many combinations of gluon-exchange processes in addition to the typical example in Fig. 3. even for the ordinary three-quark Λ and especially if $\Lambda(1405)$ consists of five quarks. The number of diagrams is significantly large, and they should be systematically calculated. Second, the distribution amplitudes of hadrons have not been determined, and they are necessary for calculating the absolute cross section as obvious from Eq. (3). Even the distribution amplitude for the pion has not been established yet. In spite of these issues, the counting rule is a valid theoretical prediction in perturbative QCD, and it could be used for experimental studies on exotic hadrons. For experimental proposals and actual measurements, the order of magnitude of the $\Lambda(1405)$-production cross section is needed. Therefore, we intend to provide such information in this work.

A. Cross section for $\pi^- + p \to K^0 + \Lambda$

There are many available measurements on the cross section for $\pi^- + p \to K^0 + \Lambda$ [44,45] although the momentum transfer may not be sufficiently large. We could use these measurements together with the counting rule for calculating the cross section in the large momentum-transfer region. The cross-section measurements have been presented by $d\sigma/d\Omega$ as the function of the c.m. scattering angle θ_{cm}. From them, we calculate "experimental" cross sections of $\pi^- + p \to K^0 + \Lambda$ at $\theta_{cm} = 90°$ as shown in Fig. 4 as a function of the c.m. energy $\sqrt{s}$. Since the measured values are not necessarily provided at exactly 90°, we interpolate the data by smooth polynomials: $d\sigma/d\Omega = \sum_{n=0}^{n_{max}} a_n (\cos\theta_{cm})^n$. Then, the parameters a_n are determined from the χ^2 fit, and the value at $\theta_{cm} = 90°$ is given by $d\sigma/d\Omega|_{\theta_{cm}=90°} = a_0$. The results did not change significantly as long as n_{max} is taken as $n_{max} \sim 5$. In this work, only the statistical errors are

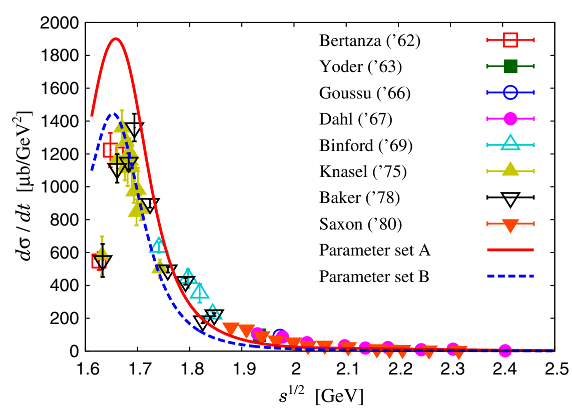

FIG. 6 (color online). Experimental data of $\pi^- + p \to K^0 + \Lambda$ cross section $d\sigma/dt$ at $\theta_{cm} = 90°$ are compared with theoretical cross sections calculated by the N^* contributions [46].

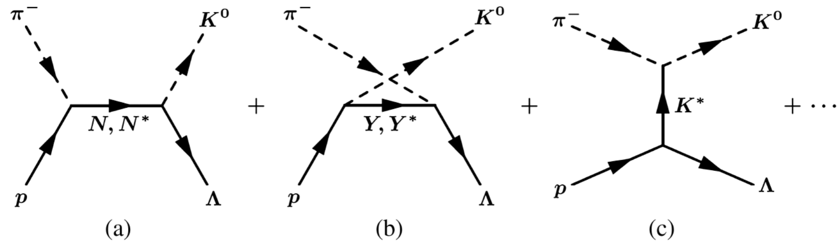

FIG. 7. Subprocess for $\pi^- + p \to K^0 + \Lambda$ at low energies. The figures indicate contributions from (a) s-channel N and N^* resonances, (b) Y and Y^*, and (c) t-channel K^*.

included. Then, the cross sections are converted to $d\sigma/dt$ by changing the variable to t. The obtained data are shown in Fig. 6 together with theoretical estimates with N^* resonances [46] in order to understand significant processes at low energies.

In Fig. 7, possible subprocesses are shown for the reaction $\pi^- + p \to K^0 + \Lambda$ at low energies by considering various intermediate resonances. There were some studies on Λ production processes [47], and complete studies of hyperon-production reactions became available recently by Rönchen *et al.* [46] and by Kamano *et al.* [48]. We do not step into the details of these reactions, and simply the contributions from s-channel N^* resonances in Fig. 7(a) are compared with the data in Fig. 6. As for the N^*, we took thirteen resonances: $N(1535)$, $N(1650)$, $N(1440)$, $N(1710)$, $N(1750)$, $N(1720)$, $N(1520)$, $N(1675)$, $N(1680)$, $N(1990)$, $N(2190)$, $N(2250)$, and $N(2220)$. Two possible parameter sets A and B are provided in Ref. [46] for these N^* resonances, and the two curves in Fig. 6 correspond to the two choices. At low energies, the experimental data agree with the curves, which indicates that the dominant subprocesses come from the intermediate N^* resonances. At higher energies at $\sqrt{s} > 1.8$ GeV, the curves deviate from the data. It is because other processes, namely, the crossed ones of (b) and t-channel resonances of (c), and the coupled-channel effects contribute to the cross section.

It is, however, not obvious to predict the cross section in the perturbative QCD region. The process should be described by Eq. (3) at large-momentum transfer, but it is not possible to obtain the accurate matrix element at this stage. The hard part $H_{ab\to cd}$ could be calculated in perturbative QCD in principle; however, there are too many processes to be evaluated easily by an analytical method. In addition, the distribution amplitudes $\phi_{a,b,c,d}$ are not determined for π, p, K, and Λ since there are still discussions whether the functional form should be the asymptotic form or the Chernyak–Zhitnitsky type even for the pion [27,28] at present experimental energies. In order to estimate the order of Λ production cross sections at high energies, we use a fit to the experimental data in Fig. 8.

In Fig. 8, the experimental data of $\pi^- + p \to K^0 + \Lambda$ are shown by the cross section multiplied by s^8, which is the factor predicted by the constituent-counting rule with the total number $n = 2 + 3 + 2 + 3 = 10$. Bumpy resonance like behavior is seen at low energies $\sqrt{s} < 1.9$ GeV, whereas the scaling appears at $\sqrt{s} > 2$ GeV. As explained in the last paragraph, it is not obvious what should be the high-energy region where the perturbative QCD can be applied. Therefore, we are not confident whether the constant cross section at $\sqrt{s} > 2$ GeV indicates the scaling by the counting rule. In the reaction $\gamma + p \to \pi^+ + n$ of Fig. 4, the scaling starts from the excitation energy $\sqrt{s} - m_p \simeq 2.5 - 0.9 = 1.6$ GeV. In Fig. 8, it starts at $\sqrt{s} - (m_K + m_\Lambda) \simeq 2.0 - (0.5 + 1.1) = 0.4$ GeV, which is rather small in comparison with the $\gamma + p \to \pi^+ + n$ case. However, the hadron distribution amplitudes $\phi_{\pi,p,K^0,\Lambda}$ together with the hard scattering amplitude $H_{\pi^- + p\to K^0+\Lambda}$ are not known, so that there could be no wonder even if the scaling starts from a lower energy. In any case, we fit the experimental cross sections at $\sqrt{s} > 2$ GeV in Fig. 8 by the straight line for an estimation in the scaling region. From the fit to the experimental data, we obtain

$$s^8 \frac{d\sigma}{dt} = (3.50 \pm 0.21) \times 10^6 \mu b\ \mathrm{GeV}^{14}. \qquad (19)$$

On the other hand, fitting the experimental data at $\sqrt{s} > 2$ GeV with the expression $d\sigma/dt = (\mathrm{constant}) \times s^{2-n}$, we obtain the scaling factor

$$n = 10.1 \pm 0.6, \qquad (20)$$

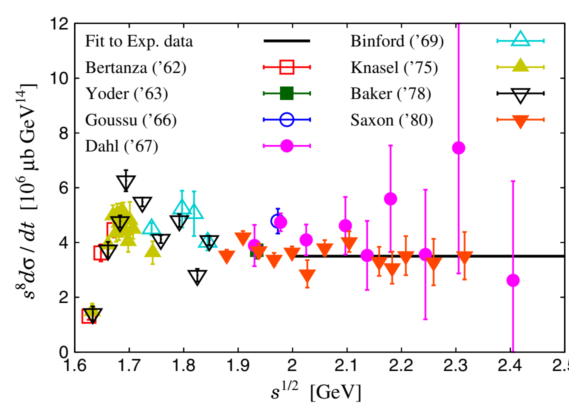

FIG. 8 (color online). Experimental data of $\pi^- + p \to K^0 + \Lambda$ cross section $s^8 d\sigma/dt$ as the function of $\sqrt{s}$ [44]. By considering the counting rule with $n = 10$, the cross section is multiplied by the factor s^{n-2}. The line is a fit to the data at $\sqrt{s} > 2$ GeV.

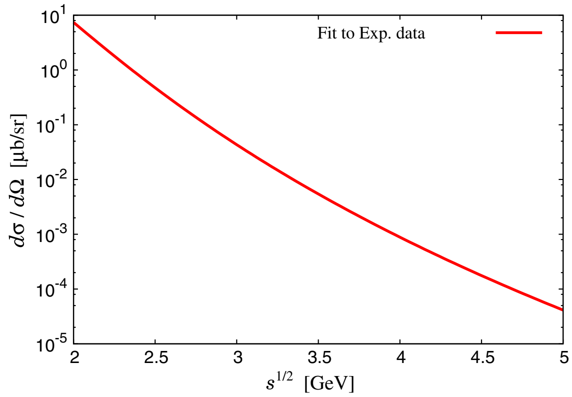

FIG. 9 (color online). The fitted cross section $d\sigma/dt$ of $\pi^- + p \to K^0 + \Lambda$ is extended to a high-energy region by assuming the constituent-counting rule and the fitted value of Eq. (19).

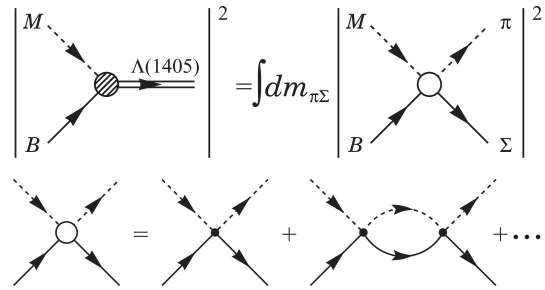

FIG. 11. The upper figures indicate that the cross section of $\pi^- + p \to K^0 + \Lambda(1405)$ is calculated by the cross section of $\pi^- + p \to K^0 + \pi + \Sigma$ integrated over the $\pi\Sigma$ invariant mass $m_{\pi\Sigma}$ in the $\Lambda(1405)$ region. The lower figures indicate that $\Lambda(1405)$ is generated in dynamical processes [50]. The intermediate meson M and baryon B indicate ten sets of meson-baryon systems explained in the main text.

which is consistent with the three-quark structure for Λ. It is an interesting and encouraging result for our studies.

Then, the cross section $d\sigma/d\Omega$ is shown in Fig. 9 for $\pi^- + p \to K^0 + \Lambda$ by extrapolating the constant cross section value in Fig. 8 to the higher-energy region up to $\sqrt{s} = 5$ GeV. The cross section is shown at $\theta_{\rm cm} = 90°$ in the c.m. system. Although it is a rough estimate, we show the cross section for planning future experimental measurements in comparison with the $\Lambda(1405)$ production in Sec. III B.

B. Cross section for $\pi^- + p \to K^0 + \Lambda(1405)$

We show the cross section of $\pi^- + p \to K^0 + \Lambda(1405)$ in the same way as the Λ production by using current information from theoretical and experimental studies for finding the internal structure of $\Lambda(1405)$ by the constituent-counting rule at high energies. However, both experimental and theoretical information is very limited even in the resonance region for $\Lambda(1405)$. Actually, there is only one experiment for the pion induced $\Lambda(1405)$ production [49], and only the chiral unitary model [50] is available for theoretical estimation.

In Ref. [50], the pion induced $\Lambda(1405)$ production at low energies is theoretically studied by taking into account the meson exchange contribution as well as the intermediate $N^*(1710)$ s-channel formation as shown in Fig. 10. First, the cross section $\pi^- + p \to K^0 + \pi + \Sigma$ is calculated, and then it is integrated over the invariant mass $m_{\pi\Sigma}$ of the final π and Σ in the $\Lambda(1405)$ energy region for obtaining the $\Lambda(1405)$-production cross section as shown in Fig. 11. The couplings of $\pi + N \to N^*(1710)$ and $N^*(1710) \to K^0 + M + B$ are calculated from the $N^*(1710)$ partial decay widths with the flavor SU(3) symmetry. Here, the intermediate MB states consist of the ten channels: $K^- p$, $\bar{K}^0 n$, $\pi^0\Lambda$, $\pi^0\Sigma^0$, $\eta\Lambda$, $\eta\Sigma^0$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$, $K^+\Xi^-$, and $K^0\Xi^0$.

If the $\Lambda(1405)$ is a five-quark state, the total number of interacting elementary fields is $n = 2 + 3 + 2 + 5 = 12$. The constituent-counting rule indicates the scaling $s^{10} d\sigma/dt =$ constant, so that the cross section multiplied by s^{10} is shown in Fig. 12 for $\pi^- + p \to K^0 + \Lambda(1405)$ as the function of $\sqrt{s}$, in which the data together with the available theoretical calculation are plotted. The experimental cross section at $\theta_{\rm cm} = 90°$ is extracted from the measurement [49] in the same with the Λ cross section, and its value

$$s^{10} \frac{d\sigma}{dt} = (1.89 \pm 0.36) \times 10^7 \mu b \ {\rm GeV}^{18}, \qquad (21)$$

at $\sqrt{s} = 2.02$ GeV is plotted in Fig. 12. On the other hand, the theoretical estimates roughly agree with the data, but they diverge at large energies at $\sqrt{s} > 2.1$ GeV simply because the strong energy dependence of s^{10} cannot be suppressed by the contributions from Fig. 10. In any case, other resonances and t channel contributions should be taken into account for a precise description of the cross section, and such hadronic models cannot be used at high energies. In this sense, we inevitably have to use

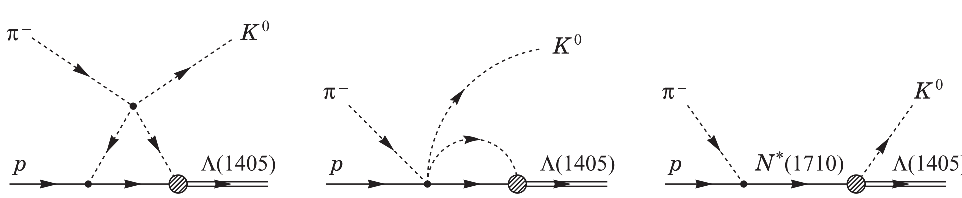

FIG. 10. At low energies, the cross section of $\pi^- + p \to K^0 + \Lambda(1405)$ is calculated by the meson induced processes and the intermediate s-channel $N^*(1710)$ [50].

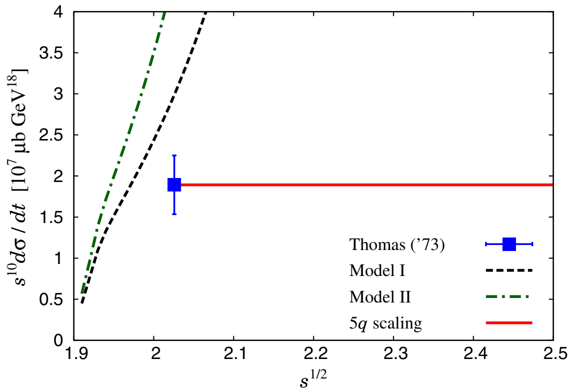

FIG. 12 (color online). Experimental data of $\pi^- + p \to K^0 + \Lambda(1405)$ cross section $s^{10}d\sigma/dt$ [49]. By considering the counting rule with $n = 12$, namely, a five-quark state for $\Lambda(1405)$, the cross section is multiplied by the factor s^{n-2}. The theoretical models I and II are from Ref. [50].

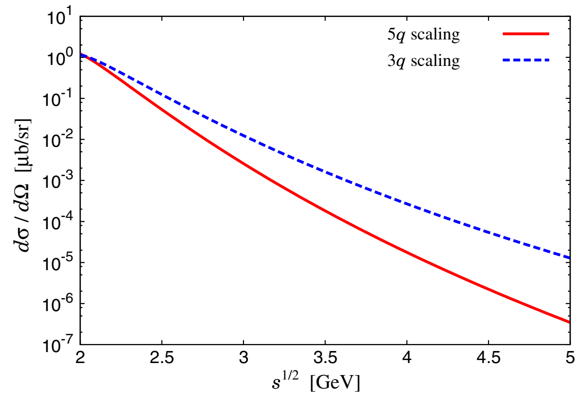

FIG. 13 (color online). The cross section $d\sigma/d\Omega$ of $\pi^- + p \to K^0 + \Lambda(1405)$ is extended to the high-energy region by assuming the constituent-counting rule and by using the experimental data in Eq. (21) at $\sqrt{s} = 2.02$ GeV [49].

experimental information for estimating the cross section in the scaling region. The straight line is drawn in Fig. 12 by assuming the scaling function for the five-quark type $\Lambda(1405)$.

At this stage, the theoretical and experimental information is very limited for estimating the order of the $\Lambda(1405)$ production cross section at high energy. On the other hand, even a rough estimate of the cross section is needed for proposing a future measurement at experimental facilities such as the J-PARC. For this purpose, we extended the cross section at $\sqrt{s} = 2.02$ GeV to high energies by assuming the scaling function with the five-quark $\Lambda(1405)$. Its cross section is shown in Fig. 13 by the solid curve with the condition $d\sigma/d\Omega = 1.09 \pm 0.21\,\mu b/sr$ at $\sqrt{s} = 2.02$ GeV. In comparison, the dashed curve is also shown for the scaling behavior to be observed if $\Lambda(1405)$ were an ordinary three-quark baryon by assuming $s^8 d\sigma/dt =$ constant and the same cross section at $\sqrt{s} = 2.02$ GeV. Because it is not clear where the perturbative QCD region starts, the cross sections should be considered as rough estimates. In any case, there is a distinct difference between the two functional forms if measurements will be done at high energies. In the scaling region, the quark-gluon degrees of freedom explicitly appear, which results in the constituent-counting rule, and the internal structure of $\Lambda(1405)$ could be clarified. If $\Lambda(1405)$ is a $\bar{K}N$ molecule, such investigations are similar to the scaling studies for the deuteron [51] in the sense that both are bound states of two hadrons. Therefore, in this case $\Lambda(1405)$ can be treated simply as a five-quark state for studying the scaling behavior.

C. Comments on experimental possibilities

As for the future experimental measurements, there are possibilities to measure $\pi^- + p \to K^0 + \Lambda(1405)$ at the J-PARC by using the high-momentum beam line [52,53], which will be ready in a few years. There is also a high-momentum pion beam in the COMPASS experiment, so that it could be possible. Furthermore, there is a plan at laser electron photon beam line at SPring-8 (LEPS) II to set up a detector for large-angle scattering measurements [54] in addition to the increase of photon energy. Currently, the reaction $\gamma + p \to K^+ + \Lambda(1405)$ is taken up to the c.m. energy $\sqrt{s} = 2.3$ GeV within a limited scattering angle at LEPS, and up to $\sqrt{s} = 2.85$ GeV at JLab. Then, the internal structure of $\Lambda(1405)$ could be also investigated by the exclusive reaction $\gamma + p \to K^+ + \Lambda(1405)$ as we explained in this article. According to the counting rule, if the $\Lambda(1405)$ were an ordinary three-quark baryon, the cross section should scale like $s^7 d\sigma/dt =$ constant as shown in Fig. 4; however, it is $s^9 d\sigma/dt =$ constant if $\Lambda(1405)$ is a five-quark state. Actually, there is an indication in Ref. [22] that the $\Lambda(1405)$ photoproduction cross section is suppressed at high energies in comparison with the $\Sigma(1385)$ one.

Here, we discussed only $\Lambda(1405)$; however, our idea can be used for investigating other exotic hadron candidates by using the counting rule for exclusive reactions. In addition to the J-PARC and LEPS, there are several hadron and lepton beam facilities in the world, such as the KEK-B, JLab, CERN-COMPASS, GSI, Fermilab, RHIC, LHC, etc. They could be used for such studies. The idea of the counting rule is quite different from ordinary approaches at low energies, and we hope that our proposal will shed light on a new direction of exotic-hadron studies at high energies, where quark-gluon degrees of freedom appear.

IV. SUMMARY

We proposed that the internal configuration of exotic hadron candidates should be investigated by the scaling behavior given by the constituent-counting rule for exclusive production processes. As an example, the cross section was estimated for $\Lambda(1405)$ production processes $\pi^- + p \to K^0 + \Lambda(1405)$ together with the ground-state Λ

production $\pi^- + p \to K^0 + \Lambda$. The production cross sections were shown at $\theta_{\rm cm} = 90°$ by using the existing experimental data, and they were compared with theoretical results in the resonance region. If the center-of-mass energy $\sqrt{s}$ becomes large enough, the cross sections should be described by perturbative QCD with light-cone wave functions of the hadrons. The cross sections of this scaling region were simply estimated by considering the counting rule in this work. Depending on the quark configuration whether $\Lambda(1405)$ is a five-quark state (including $\bar{K}N$ molecule) or an ordinary three-quark hadron, the scaling behavior is different. Measuring the exclusive cross sections at high energies, we should be able to learn about the internal structure of $\Lambda(1405)$. This method is completely different from other studies at low energies, and it provides a new approach for exotic-hadron studies by using high-energy processes. We hope that our idea will be materialized as future measurements at hadron facilities such as the J-PARC and other facilities such as LEPS and JLab.

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