

# $B \rightarrow DK_{0,2}^*$ decays and the $CP$ phase angle $\gamma$

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$B^\pm \rightarrow (D^0, \bar{D}^0, D_{CP})K_{0,2}^{*\pm}$  decays are helpful in determining the  $CP$  violation angle  $\gamma$ , and we analyze these decay processes within the perturbative QCD approach based on  $k_T$  factorization. We found that the branching ratio of  $B^- \rightarrow D^0 K_0^{*-}$  can reach the order of  $10^{-4}$ , due to the enhancement of nonfactorizable contributions in color-suppressed  $D^0$ -emission, while the branching ratio of  $B^- \rightarrow \bar{D}^0 K_0^{*-}$  is of the order  $10^{-5}$ . The ratio of decay amplitudes is about 3 times larger than the one in the channel  $B^\pm \rightarrow DK^\pm$ . Large branching ratios provide a good opportunity to observe  $B^\pm \rightarrow DK_2^{*\pm}$  on the ongoing and forthcoming experimental facilities and consequently these channels may be of valuable avail in reducing the errors in the  $CP$  violation phase angle  $\gamma$ . We also explore the possible time-dependent  $CP$  asymmetries of  $B_s$  decay into a scalar meson to determine the phase angle  $\gamma$ .

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## I. INTRODUCTION

The authentication of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix allows us to explore the standard model (SM) description of the  $CP$  violation and reveal new physics beyond the SM. Among the angles  $(\alpha, \beta, \gamma)$  of the so-called  $(bd)$  unitarity triangle derived from the  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ , satisfying the constraint  $\alpha + \beta + \gamma = 180^\circ$ , the angle  $\gamma$  is least constrained, with a precision of roughly  $10^\circ$ . This is one of the main sources of the current uncertainties in the apex of the unitary triangle [1,2].

One of the most efficient ways proposed in the literature to measure  $\gamma$  makes use of the two triangles formed by the six channels of  $B^\pm \rightarrow (D^0, \bar{D}^0, D_{CP})K^\pm$  [3–5]. The shape of the two triangles is controlled by two quantities,

$$r_B^{K_J} \equiv |A(B^- \rightarrow \bar{D}^0 K_J^-) / A(B^- \rightarrow D^0 K_J^-)|,$$

$$\delta_B^{K_J} \equiv \arg[e^{i\gamma} A(B^- \rightarrow \bar{D}^0 K_J^-) / A(B^- \rightarrow D^0 K_J^-)],$$

where  $K_J$  can be  $K$  or  $K_{0,2}^*$ . One of the most intriguing properties in this method is that it is independent of hadronic uncertainties, and moreover the  $CP$  violation from the  $D$  meson decays can also be incorporated [6]. Due to the fact that the  $B^- \rightarrow \bar{D}^0 K^-$  is both Cabibbo suppressed and color suppressed, the ratio  $r_B^K \sim |V_{ub}V_{cs}^* / (V_{cb}V_{us}^*)a_2/a_1| \sim 0.1$  is small and in particular the world averages for these parameters [7],

$$r_B^K = 0.107 \pm 0.010, \quad \delta_B^K = (112_{-13}^{+12})^\circ,$$

indicate that the two triangles formed by decay amplitudes are squashed. As a consequence, the measurement of  $\gamma$  requests a precise knowledge on the  $B^- \rightarrow \bar{D}^0 K^-$ .

In Ref. [8], we proposed a new method to determine the  $CP$  violation angle  $\gamma$  that uses the  $B^\pm \rightarrow DK_{0,2}^{*\pm}$  decays (see also Ref. [9]). Unlike the  $B \rightarrow DK^\pm$ , the color-allowed amplitudes in  $B^\pm \rightarrow DK_{0,2}^{*\pm}$  have vanishing/small decay constants and are comparable with the color-suppressed ones. Large interference between the two amplitudes is induced in the  $B \rightarrow D_{CP}K$  and the sensitivity to  $\gamma$  is greatly improved. Branching ratios of these channels are estimated to lie in the range from  $10^{-6}$  to  $10^{-5}$ , using a method of factorization in conjunction with experimental data [8]. The motif of this work is to adopt the QCD-based factorization method, more explicitly the perturbative QCD (PQCD) approach [10–12] (see Refs. [13,14] for the recent developments and applications of the PQCD approach), to calculate the branching ratios, strong phases, and  $CP$  asymmetries. The perturbative QCD approach is formulated on the basis of  $k_T$  factorization, and has been applied to  $B$  meson decays into charmed meson in a number of references and a global agreement of the results with the available data is found [15–22]. One of the most successful predictions is  $r_K = 0.092_{-0.003-0.003}^{+0.012+0.003}$  [21], in good agreement with the data [7]. To the end of this work, we show that the resulting branching ratios are enhanced by 1 order of magnitude higher than our previous estimates, due to the inclusion of the large nonfactorizable contribution in  $D^0$  emission diagram. Such large branching ratios provide a better opportunity for the measurement of these channels on the experimental facilities and constraining the  $\gamma$  angle.

The rest of this work is organized as follows: In Sec. II, we will calculate the  $B \rightarrow D^0(\bar{D}^0)K_{0,2}^*$  (1430) decay amplitudes and give the factorization formulas, while Sec. III contains the numerical analysis and discussions. The last section is our summary. We also relegate some of the calculation details to the Appendix.

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## II. PERTURBATIVE QCD CALCULATION

In the PQCD approach, the inclusion of the intrinsic transverse momentum of valence quarks smears the end point singularities appearing in the calculations under the collinear factorization context. In the  $m_b \rightarrow \infty$  limit, the decay amplitude is generically expressed as a convolution of wave functions and hard scattering kernel with both longitudinal momenta and transverse space coordinates,

$$\mathcal{M} = \int_0^1 dx_1 dx_2 dx_3 \int d^2\vec{b}_1 d^2\vec{b}_2 d^2\vec{b}_3 \phi_B(x_1, \vec{b}_1, t) \times T_H(x_1, x_2, \vec{b}_1, \vec{b}_2, t) \phi_2(x_2, \vec{b}_2, t) \phi_3(x_3, \vec{b}_3, t), \quad (1)$$

where the  $B$  in the indices represents a  $B$  meson and 2, 3 represent the two mesons in the final state. In the computation of higher order QCD corrections, the overlap of soft and collinear momentum results in double logarithm divergences. Resummation of them leads to the Sudakov factor which has the tendency to diminish the end point contributions and supports the hard-scattering picture used in this framework. For a review of this approach, see Ref. [23].

The wave functions, the most important entry in the perturbative QCD approach, are nonperturbative in nature and can only be acquired by some nonperturbative methods or with the aid from some simple but effective models. For the  $B$  meson which is a heavy-light system, we adopt the light cone matrix,

$$\Phi_B = \frac{i}{\sqrt{2N_c}} (\not{p}_B + m_B) \gamma_5 \phi_B(x_1, b_1), \quad (2)$$

in which we have neglected the numerically suppressed distribution amplitude [24]. Here  $x_1$  is the momentum fraction of the light spectator quark and  $N_c = 3$  is the color factor. As for the wave functions for the  $D$  meson, we use the form derived in Ref. [15]:

$$\Phi_D = \frac{i}{\sqrt{2N_c}} \gamma_5 (\not{p}_D + m_D) \phi_D(x_2, b_2). \quad (3)$$

The light-cone distribution amplitudes (LCDAs) for  $K_0^*$  are governed by the conformal spin symmetry of QCD and have the following definitions [25]:

$$\langle K_0^*(p_{K_0^*}) | q(0)_j \bar{q}(z)_l | 0 \rangle = \frac{-1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp_{K_0^*} \cdot z} \{ \not{p}_{K_0^*} \phi_{K_0^*}(x) + m_{K_0^*} \phi_{K_0^*}^s(x) + m_{K_0^*} (\not{n}\bar{n} - 1) \phi_{K_0^*}^T(x) \}_{jl},$$

in which  $\bar{n}$  is chosen as the flight direction of the  $K_0^*$  in the  $B$  meson rest frame and  $n$  is opposite to  $\bar{n}$ . These LCDAs, the twist-2  $\phi_{K_0^*}$  and the twist-3  $\phi_{K_0^*}^{s,T}$ , can be expanded in terms of Gegenbauer polynomials,

$$\phi_{K_0^*}(x) = \frac{\bar{f}_{K_0^*}}{2\sqrt{2N_c}} 6x(1-x) \sum_{m=0}^{\infty} B_m C_m^{3/2}(2x-1), \quad (4)$$

$$\phi_{K_0^*}^s(x) = \frac{\bar{f}_{K_0^*}}{2\sqrt{2N_c}}, \quad \phi_{K_0^*}^T(x) = \frac{\bar{f}_{K_0^*}}{2\sqrt{2N_c}} (1-2x),$$

with  $B_0 = (m_s - m_u)/m_{K_0^*}$ . The decay constant  $\bar{f}_{K_0^*}$  is defined by a scalar current,

$$\langle K_0^{*-}(1430) | \bar{s}u | 0 \rangle = \bar{f}_{K_0^*},$$

and is related to the vector decay constant by  $f_{K_0^*} = B_0 \bar{f}_{K_0^*}$ . We will leave out the higher Gegenbauer moments in twist-3 LCDAs [26,27] since their contributions are found to be typically small [28].

Similarly the LCDAs of a longitudinally polarized  $K_2^*$  state are defined as [29]

$$\langle K_2^*(p_{K_2^*}, \epsilon) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp_{K_2^*} \cdot z} \{ m_{K_2^*} \epsilon_{\bullet L}^* \phi_{K_2^*}(x) + \epsilon_{\bullet L}^* p_{K_2^*} \phi_{K_2^*}^t(x) + m_{K_2^*}^2 \frac{\epsilon_{\bullet L} \cdot v}{p_{K_0^*} \cdot v} \phi_{K_2^*}^s(x) \}_{\alpha\beta}, \quad (5)$$

with  $n^2 = v^2 = 0$  being lightlike unit vectors. The new vector  $\epsilon_{\bullet L}$  in Eq. (5) is related to the polarization tensor by  $\epsilon_{\bullet L\mu} \equiv \frac{\epsilon_{\mu\nu} v^\nu}{p_{K_2^*} \cdot v} m_{K_2^*}$  and can be simplified in terms of a polarization vector,

$$\epsilon_{\bullet L\mu} \equiv \frac{\epsilon_{\mu\nu} v^\nu}{p_{K_2^*} \cdot v} m_{K_2^*} \simeq \frac{\sqrt{2}}{\sqrt{3}} \epsilon_{L\mu}. \quad (6)$$

The above LCDAs have the asymptotic forms [29],

$$\begin{aligned} \phi_{K_2^*}(x) &= \frac{f_{K_2^*}}{2\sqrt{2N_c}} 30x(1-x)(2x-1), \\ \phi_{K_2^*}^t(x) &= \frac{f_{K_2^*}^T}{2\sqrt{2N_c}} \frac{15}{2} (2x-1)(1-6x+6x^2), \\ \phi_{K_2^*}^s(x) &= \frac{f_{K_2^*}^T}{4\sqrt{2N_c}} \frac{d}{dx} [15x(1-x)(2x-1)]. \end{aligned} \quad (7)$$

There are three types of diagrams contributing to the decay amplitudes which are depicted in Fig. 1: the color-allowed contributions in the process  $B^- \rightarrow D^0 K_{0(2)}^{*-}(1430)(a, b, c, d)$ , the color-suppressed one in the process  $B^- \rightarrow D^0 K_{0(2)}^{*-}(1430)(e, f, g, h)$ , and the annihilation one in the process  $B^- \rightarrow \bar{D}^0 K_{0(2)}^{*-}(1430)(i, j, k, l)$ . In the middle four diagrams, the exchange of  $c$  and  $u$  quark results in the corresponding diagrams for the process  $B^- \rightarrow \bar{D}^0 K_{0(2)}^{*-}(1430)$ .

Factorization formulas for the  $K_0^*$ -emission diagrams are given as

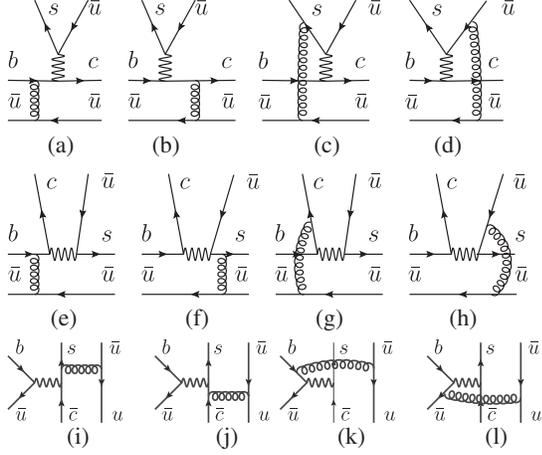


FIG. 1. Feynman diagrams for the color-allowed contributions in the process  $B^- \rightarrow D^0 K_{0(2)}^{*-}(1430)$ : (a), (b), (c), and (d); for the color-suppressed contributions in the process  $B^- \rightarrow D^0 K_{0(2)}^{*-}(1430)$ : (e), (f), (g), and (h); and for the annihilation contributions in process  $B^- \rightarrow \bar{D}^0 K_{0(2)}^{*-}(1430)$ : (i), (j), (k), and (l). In the middle four diagrams, the replacement of the  $c\bar{u}$  by  $u\bar{c}$  results in the corresponding diagrams for the process  $B^- \rightarrow \bar{D}^0 K_{0(2)}^{*-}(1430)$ .

$$\begin{aligned} \xi_{ex} = & N_1 f_{K_0^*} \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \\ & \times \phi_D(\bar{x}_3, b_3) [(2 - x_3 + r_D(2x_3 - 1))E_a(t_a)a_1(t_a)h_a \\ & + r_D(1 + r_D)E_b(t_b)a_1(t_b)h_b], \end{aligned} \quad (8)$$

with  $N_1 = 8\pi C_F m_B^4$ . The hard kernels  $E_i(t_i)$  and  $h_i$  in these formulas are determined by the virtualities of the intermediate quarks and gluons and they can be found in the Appendix. The nonfactorizable contributions, depicted by Figs. 1(c) and 1(d), have the formulas

$$\begin{aligned} \mathcal{M}_{ex} = & N_2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(\bar{x}_3, b_3) \\ & \times \phi_{K_0^*}(x_2) [(x_2 - r_D \bar{x}_3)E_c(t_c)h_c C_1(t_c) \\ & + (\bar{x}_3 r_D - \bar{x}_3 - \bar{x}_2)E_d(t_d)h_d C_1(t_d)], \end{aligned}$$

with  $\bar{x}_i = 1 - x_i$ ,  $N_2 = 32\pi m_B^4 C_F / \sqrt{2N_c}$ ,  $[dx] \equiv dx_1 dx_2 dx_3$ . In the collinear approximation, the amplitude is divergent when the momentum fraction of the light spectator in the final state goes to zero. The transverse momentum regulates this end point singularity and the threshold resummation function  $S_t$  will suppress the end point contribution further.

The factorizable color-suppressed diagrams, either the  $D^0$  or  $\bar{D}^0$  emission diagrams, have the same factorization formulas:

$$\begin{aligned} \xi_{in}^c = & \xi_{in}^l \\ = & N_1 f_D \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \\ & \times \{E_e(t_e)h_e a_2(t_e)[r_{K_0^*}(1 - 2x_3)(\phi_{K_0^*}^s(x_3) - \phi_{K_0^*}^T(x_3)) \\ & + (2 - x_3)\phi_{K_0^*}(x_3)] - 2r_{K_0^*}\phi_{K_0^*}^s(x_3)E_f(t_f)h_f a_2(t_f)\}, \end{aligned} \quad (9)$$

but the  $D^0$ -emission nonfactorizable diagram is

$$\begin{aligned} \mathcal{M}_{in} = & N_2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(\bar{x}_2, b_2) \\ & \times \{[x_2 \phi_{K_0^*}(x_3) + r_{K_0^*} \bar{x}_3 (\phi_{K_0^*}^s(x_3) + \phi_{K_0^*}^T(x_3))] \\ & \times h_g E_g(t_g)C_1(t_h) - h_h E_h(t_h)C_2(t_h) \\ & \times [(\bar{x}_2 + \bar{x}_3)\phi_{K_0^*}(x_3) + r_3 \bar{x}_3 (\phi_{K_0^*}^s(x_3) - \phi_{K_0^*}^T(x_3))]\}, \end{aligned}$$

while the  $\bar{D}^0$  emission is factorized as

$$\begin{aligned} \mathcal{M}_{in}^l = & N_2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(x_2, b_2) \\ & \times \{[x_2 \phi_{K_0^*}(x_3) + r_{K_0^*} \bar{x}_3 (\phi_{K_0^*}^s(x_3) \\ & + \phi_{K_0^*}^T(x_3))]h'_g E'_g(t'_g)C_2(t'_g) \\ & - h'_h E'_h(t'_h)C_2(t'_h)[(\bar{x}_2 + \bar{x}_3)\phi_{K_0^*}(x_3) \\ & + r_3 \bar{x}_3 (\phi_{K_0^*}^s(x_3) - \phi_{K_0^*}^T(x_3))]\}. \end{aligned} \quad (10)$$

For the  $B \rightarrow \bar{D}K_0^*$  decays, there are contributions from the annihilation diagrams, which are depicted in Figs. 1(i)–1(l). The amplitude of factorizable annihilation diagrams is given as

$$\begin{aligned} \xi_{exc} = & N_1 f_B \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_D(x_3, b_3) \\ & \times \{E_k(t_k)h_k a_1(t_k)[2(x_3 + 1)r_D r_{K_0^*} \phi_{K_0^*}^s(x_2) \\ & - x_3 \phi_{K_0^*}(x_2)] + E_l(t_b)h_l a_1(t_l) \\ & \times [r_D r_{K_0^*}((2x_2 - 3)\phi_{K_0^*}^s(x_2) \\ & - (2x_2 - 1)\phi_{K_0^*}^T(x_2)) - (x_2 - 1)\phi_{K_0^*}(x_2)]\}. \end{aligned} \quad (11)$$

The nonfactorizable annihilation amplitude is given by

$$\begin{aligned} \mathcal{M}_{exc} = & N_2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \phi_D(x_3, b_3) \\ & \times \{[r_D r_{K_0^*}((x_2 + x_3 - 1)\phi_{K_0^*}^T(x_2) \\ & - (x_2 - x_3 - 3)\phi_{K_0^*}^s(x_2)) \\ & + (x_2 - 1)\phi_{K_0^*}(x_2)]h_m E_m(t_m)C_1(t_m) \\ & + h_n E_n(t_n)C_1(t_n)[r_D r_{K_0^*}((x_2 - x_3 - 1)\phi_{K_0^*}^s(x_2) \\ & + (x_2 + x_3 - 1)\phi_{K_0^*}^T(x_2)) + x_3 \phi_{K_0^*}(x_2)]\}. \end{aligned}$$

The formulas for channels involving  $K_2^*$  are obtained by the replacement  $f_{K_0^*} \rightarrow 0$ ,  $\phi_{K_0^*} \rightarrow \phi_{K_2^*}$  and  $\phi_{K_0^*}^{s,T} \rightarrow \phi_{K_2^*}^{s,T}$ . Incorporating the CKM matrix elements, we have the total decay amplitudes,

$$\begin{aligned} A(B^- \rightarrow \bar{D}^0 K_{0,2}^{*-}) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (\xi_{\text{in}}^l + \mathcal{M}_{\text{in}}^l + \xi_{\text{exc}} + \mathcal{M}_{\text{exc}}), \\ A(B^- \rightarrow D^0 K_{0,2}^{*-}) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (\xi_{\text{ex}} + \mathcal{M}_{\text{ex}} + \xi_{\text{in}} + \mathcal{M}_{\text{in}}), \end{aligned} \quad (12)$$

where  $G_F$  is the Fermi constant. It should be pointed out that the color-allowed  $\xi_{\text{ex}}$  is zero in  $B^- \rightarrow D^0 K_2^{*-}$  due to the fact that the tensor meson cannot be generated by a local vector or axial-vector current.

### III. NUMERICAL RESULTS AND DISCUSSIONS

The expression for  $\phi_B(x, b)$  has been examined in various kinds of  $B$  decays and the currently accepted form in the PQCD approach is

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp \left[ -\frac{m_B^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right], \quad (13)$$

where the normalization factor  $N_B$  is related to the decay constant  $f_B$ . We adopt the ansatz that the  $B$  meson wave functions have a sharp peak at  $x \sim 0.1$ , in accordance with the most probable momentum fraction of the light quark. The best-fitted form for  $\phi_D$  from the  $B$  meson decays into a charmed meson derived in Refs. [16,20,21] is

$$\begin{aligned} \phi_D(x_2, b_2) &= \frac{f_D}{2\sqrt{2}N_c} 6x(1-x)[1 + C_D(1-2x)] \\ &\times \exp[-\omega_D^2 b_2^2/2]. \end{aligned} \quad (14)$$

Their numerical values (in GeV except  $C_D$ ) are used as

$$\begin{aligned} C_D &= (0.5 \pm 0.1), & \omega_b &= (0.40 \pm 0.05), \\ \omega_D &= 0.1, & f_B &= (0.1969 \pm 0.0089), \\ f_D &= (0.221 \pm 0.018), \end{aligned} \quad (15)$$

where  $f_B$  is from the recent lattice QCD simulation [30] and the  $f_D$  is extracted from  $D^- \rightarrow \mu \bar{\nu}_\mu$  [31].

For the LCDAs of the light scalar meson  $K_0^*$ , we adopt  $B_0 = (m_s - m_u)/m_{K_0^*} = 0.07$  [31] and the two different solutions in Ref. [25]:

$$\begin{aligned} S1: \bar{f}_{K_0^*} &= (-300 \pm 30) \text{ MeV}, & B_1 &= 0.58 \pm 0.07, \\ & & B_3 &= -1.20 \pm 0.08, \\ S2: \bar{f}_{K_0^*} &= (445 \pm 50) \text{ MeV}, & B_1 &= -0.57 \pm 0.13, \\ & & B_3 &= -0.42 \pm 0.22. \end{aligned} \quad (16)$$

The normalization constants in  $K_2^*$  LCDAs are [29]

$$f_{K_2^*} = (118 \pm 5) \text{ MeV}, \quad f_{K_2^*}^T = (77 \pm 14) \text{ MeV}. \quad (17)$$

These LCDAs have been used to calculate the form factors of  $B$  decays into a scalar/tensor meson in the same perturbative QCD approach [32–35].

For the CKM matrix elements, we use [31]

$$\begin{aligned} |V_{ub}| &= (3.89 \pm 0.44) \times 10^{-3}, & |V_{cs}| &= 0.97345, \\ |V_{us}| &= 0.2252, & |V_{cb}| &= (40.6 \pm 1.3) \times 10^{-3}, \end{aligned} \quad (18)$$

where the small uncertainties are not taken into account.

With the above inputs, we predict the branching ratios as

$$\begin{aligned} \mathcal{B}(B^- \rightarrow D^0 K_0^{*-}) &= (2.70_{-0.97-0.48}^{+1.09+0.25}) \times 10^{-4}, & S1 \\ \mathcal{B}(B^- \rightarrow \bar{D}^0 K_0^{*-}) &= (1.53_{-0.53-0.46}^{+0.82+0.62}) \times 10^{-5}, & S1 \\ \mathcal{B}(B^- \rightarrow D^0 K_0^{*-}) &= (1.16_{-0.41-0.22}^{+0.40+0.14}) \times 10^{-4}, & S2 \\ \mathcal{B}(B^- \rightarrow \bar{D}^0 K_0^{*-}) &= (3.38_{-1.18-0.59}^{+1.51+0.65}) \times 10^{-5}, & S2 \\ \mathcal{B}(B^- \rightarrow D^0 K_2^{*-}) &= (2.40_{-0.97-1.05}^{+1.30+0.72}) \times 10^{-5}, \\ \mathcal{B}(B^- \rightarrow \bar{D}^0 K_2^{*-}) &= (3.32_{-1.18-0.74}^{+1.90+1.07}) \times 10^{-6}, \end{aligned} \quad (19)$$

where the first uncertainties are from  $f_B$  and  $\omega_b$  in the  $B$  meson wave functions, the second errors are from  $\Lambda_{\text{QCD}}$  and the scales defined in the Appendix. (We vary the  $\sqrt{Q}$  and  $\sqrt{P}$  in the scales 25% for error estimation.) The results for the branching ratios made here are larger than our previous estimates in Ref. [8], obtained under the factorization approach. The main reason is due to the enhancement of nonfactorizable contributions in color-suppressed  $D^0$  emission. In Ref. [34], the authors also predict the branching ratios of  $B^- \rightarrow (D^0, \bar{D}^0) K_2^{*-}$  under the same approach, which are confirmed here. Our results of ratios and phases of the amplitudes are

$$\begin{aligned} r_{K_0^*} &= 0.24_{-0.01-0.04}^{+0.02+0.07}, & \delta_{K_0^*} &= (-125.65_{-0.00-17.42}^{+3.95+23.16})^\circ, & S1 \\ r_{K_0^*} &= 0.54_{-0.02-0.07}^{+0.03+0.11}, & \delta_{K_0^*} &= (-161.51_{-0.16-9.53}^{+0.91+12.01})^\circ, & S2 \\ r_{K_2^*} &= 0.37_{-0.00-0.09}^{+0.02+0.17}, & \delta_{K_2^*} &= (155.53_{-3.36-3.49}^{+0.00+2.98})^\circ. \end{aligned} \quad (20)$$

It should be pointed out that although the large uncertainties in many entries like decay constants will affect our predictions for branching ratios, the relative strength of decay amplitudes are almost unaffected.

Physical observables that are experimentally explored are defined as

$$\begin{aligned} R_{CP^\pm}^{K_J} &= 2 \frac{\mathcal{B}(B^- \rightarrow D_{CP^\pm} K_J^-) + \mathcal{B}(B^+ \rightarrow D_{CP^\pm} K_J^+)}{\mathcal{B}(B^- \rightarrow D^0 K_J^-) + \mathcal{B}(B^+ \rightarrow \bar{D}^0 K_J^+)} \\ &= 1 + (r_B^{K_J})^2 \pm 2r_B^{K_J} \cos \delta_B^{K_J} \cos \gamma, \\ A_{CP^\pm}^{K_J} &= \frac{\mathcal{B}(B^- \rightarrow D_{CP^\pm} K_J^-) - \mathcal{B}(B^+ \rightarrow D_{CP^\pm} K_J^+)}{\mathcal{B}(B^- \rightarrow D_{CP^\pm} K_J^-) + \mathcal{B}(B^+ \rightarrow D_{CP^\pm} K_J^+)} \\ &= \pm 2r_B^{K_J} \sin \delta_B^{K_J} \sin \gamma / R_{CP^\pm}^{K_J}. \end{aligned} \quad (21)$$

In the limit of  $r_B \rightarrow 0$ , the ratio  $R_{CP^\pm}^{K_J}$  is close to 1 while the  $CP$  asymmetries vanish. As we have pointed out, due to the suppression of the color-allowed decay amplitudes based

on the fact that the matrix element of a local vector or axial-vector current (at the lowest order in  $\alpha_s$ ) between the QCD vacuum and the  $K_0^*(K_2^*)$  state is small (identically zero), the low sensitivity to  $\gamma$  is improved and in particular large  $CP$  asymmetries are expected. The dependence of  $R_{CP}$  and  $A_{CP}$  on  $\gamma$  is shown in Fig. 2. Since the errors of  $r_{K_{0,2}^*}$  and  $\delta_{K_{0,2}^*}$  are not large, only their central values are

used. We investigate these observables in the region  $\gamma = (68_{-11}^{+10})^\circ$  which is from a combined analysis of  $B^\pm \rightarrow DK^\pm$  [7]. In this region we find that the observables of the  $B^- \rightarrow (\bar{D}^0, D^0)K_0^{*-}$  in  $S1$  have relative smaller variances because of the smaller  $r_{K_0^*}$ , most of which are around 10%. However, for the other cases the observables have large variances, and some of them even reach about 40%. Therefore these channels have the potential to improve the accuracy of  $\gamma$  extracted from the  $B^\pm \rightarrow DK^\pm$  decays.

It is also interesting to notice that due to the large value of  $r_{K_{0,2}^*}$ , the large impact arising from the direct  $CP$  violation of  $D^0$  decays into  $CP$  eigenstates  $K^+K^-/\pi^+\pi^-$ , of the order  $\mathcal{O}(A_{CP}^{\text{dir}}/r_{K_{0,2}^*})$  [6] are not important in  $B \rightarrow DK_{0,2}^*$ .

As discussed in Ref. [8], the time-dependent observables of  $B_s \rightarrow (D, \bar{D})f_0(980)$  and  $B_s \rightarrow (D, \bar{D})f_2'(1525)$  processes can be used to determine the  $\gamma$  as well. Therefore we will also predict their branching ratios in the perturbative QCD approach. In these channels only the color-suppressed diagrams depicted as Figs. 1(e)–1(h) contribute, in which the spectator quark  $\bar{u}$  needs to be replaced by  $\bar{s}$  and the  $c$  and  $\bar{u}$  in the emission meson should be exchanged for the  $B_s \rightarrow D(f_0, f_2')$  decays. The amplitudes are given by

$$A(\bar{B}_s^0 \rightarrow \bar{D}^0(f_0, f_2')) = \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (\xi_{\text{in}}^t + \mathcal{M}_{\text{in}}^t),$$

$$A(\bar{B}_s^0 \rightarrow D^0(f_0, f_2')) = \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (\xi_{\text{in}} + \mathcal{M}_{\text{in}}).$$
(22)

Our inputs for the  $\bar{B}_s^0 \rightarrow (\bar{D}, D)(f_0, f_2')$  decays are summarized as (decay constants in units of GeV) [25,29,30]

$$f_{B_s} = 0.2420 \pm 0.0095, \quad \bar{f}_{f_0} = 0.37 \pm 0.02,$$

$$B_1(f_0) = -0.78 \pm 0.08, \quad B_3(f_0) = 0.02 \pm 0.07,$$

$$f_{f_2'} = 0.126 \pm 0.004, \quad f_{f_2'}^T = 0.065 \pm 0.012,$$
(23)

with which the branching ratios are predicted as

$$\mathcal{B}(\bar{B}_s^0 \rightarrow D^0 f_0) = (3.50_{-1.15-0.77}^{+1.26+0.56}) \times 10^{-5},$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow \bar{D}^0 f_0) = (5.94_{-2.16-0.97}^{+3.13+1.47}) \times 10^{-6},$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow D^0 f_2') = (1.08_{-0.35-0.28}^{+0.37+0.26}) \times 10^{-5},$$

$$\mathcal{B}(\bar{B}_s^0 \rightarrow \bar{D}^0 f_2') = (2.85_{-0.95-0.43}^{+1.53+0.67}) \times 10^{-6}.$$
(24)

#### IV. SUMMARY

The determination of the CKM angles is crucial for the test of the CKM paradigm and also sheds light on the standard model description of the  $CP$  violation. To accomplish this goal, one of the most important efforts to be done in the next step is to reduce the uncertainties in these entries. What has been explored in Ref. [8] and this work

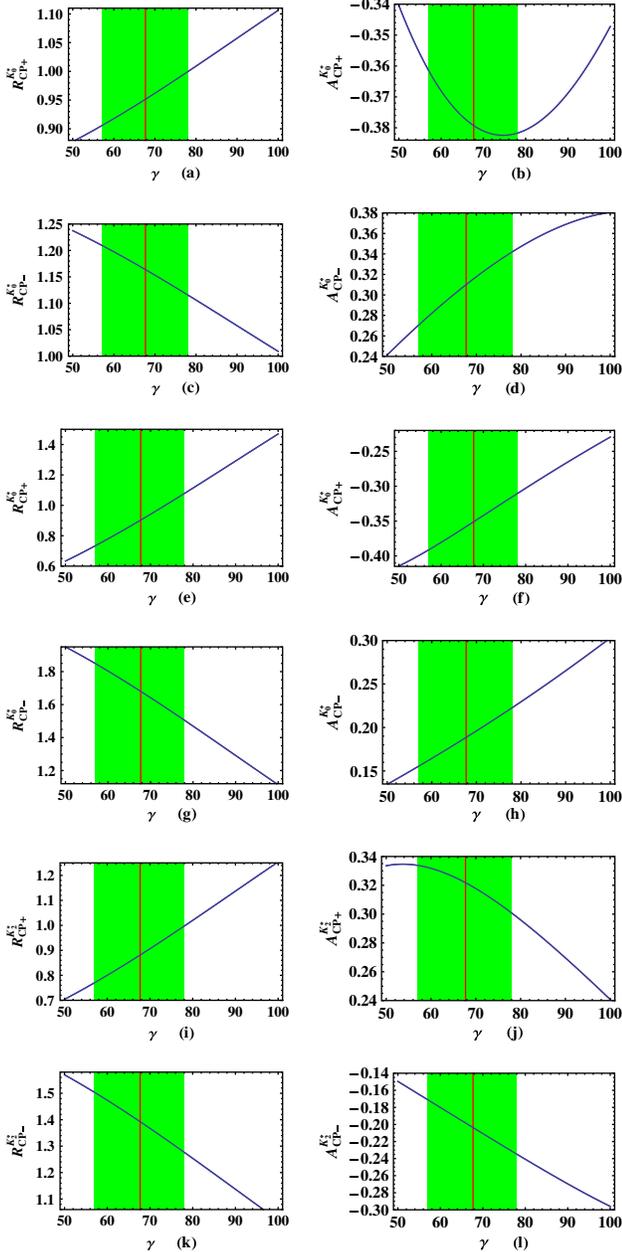


FIG. 2 (color online). The dependence of  $R_{CP}$  and  $A_{CP}$  on  $\gamma$ . Diagrams (a)–(d) show  $R_{CP}^{K_0^*}$  and  $A_{CP}^{K_0^*}$  in  $S1$ , (e)–(h) in  $S2$ , and diagrams (i)–(l) show  $R_{CP}^{K_2^*}$  and  $A_{CP}^{K_2^*}$ . The shadowed (green) region denotes the current bounds on  $\gamma = (68_{-11}^{+10})^\circ$  from a combined analysis of  $B^\pm \rightarrow DK^\pm$  [7], and the vertical (red) line represents the central value.

is to propose that the  $B \rightarrow DK_{0,2}^*$  is expedient to provide complementary information of the angle  $\gamma$ .

In this work we have calculated the branching ratios of  $B \rightarrow DK_{0,2}^*$  and the corresponding  $B_s$  relatives, by adopting the  $k_T$  factorization approach. We find that the BR of  $B \rightarrow DK_{0(2)}^*$  can reach  $10^{-4}(10^{-5})$  while the ratio of the magnitude is also significantly enhanced compared to the  $B \rightarrow DK$  mode. As a consequence, it seems promising for the LHCb experiment and the currently designed Super B factory to measure  $B_{u,d} \rightarrow DK_{0(2)}^*$  (1430) and time-dependent  $CP$  asymmetries in the  $B_s$  decays.

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### APPENDIX: HARD KERNELS IN THE PQCD CALCULATION

The offshellness of the intermediate gluon

$$Q_{a,b,c,d} = x_1 \bar{x}_3 m_B^2, \quad Q_{e,f,g,h,g',h'} = x_1 \bar{x}_3 (1 - r_D^2) m_B^2,$$

$$Q_{k,l,m,n} = x_3 \bar{x}_2 (1 - r_D^2) m_B^2,$$

and the quarks

$$P_a = \bar{x}_3 m_B^2, \quad P_b = x_1 m_B^2,$$

$$P_c = \bar{x}_3 [x_1 - x_2 (1 - r_D^2)] m_B^2,$$

$$P_d = \bar{x}_3 [x_1 - \bar{x}_2 (1 - r_D^2)] m_B^2,$$

$$P_e = \bar{x}_3 (1 - r_D^2) m_B^2, \quad P_f = x_1 (1 - r_D^2) m_B^2,$$

$$P_g = \{(x_1 - x_2) [(1 - r_D^2) \bar{x}_3 + r_D^2] + r_D^2\} m_B^2,$$

$$P_h = \{(x_1 - \bar{x}_2) (1 - r_D^2) \bar{x}_3\} m_B^2,$$

$$P'_g = \{(x_1 - x_2) (1 - r_D^2) \bar{x}_3\} m_B^2,$$

$$P_h = \{(x_1 - \bar{x}_2) [(1 - r_D^2) \bar{x}_3 + r_D^2] + r_D^2\} m_B^2,$$

$$P_k = x_3 (1 - r_D^2) m_B^2, \quad P_l = \bar{x}_2 (1 - r_D^2) m_B^2,$$

$$P_m = -\{\bar{x}_3 [1 - (1 - r_D^2) \bar{x}_2 - x_1] - 1\} m_B^2,$$

$$P_n = x_3 [x_1 - \bar{x}_2 (1 - r_D^2)] m_B^2,$$

result in the hard scales and kernels

$$t_i = \max\{\sqrt{Q_i}, \sqrt{P_i}, 1/b_1, 1/b_3\},$$

$$t_j = \max\{\sqrt{Q_j}, \sqrt{P_j}, 1/b_1, 1/b_2\},$$

$$t_x = \max\{\sqrt{Q_x}, \sqrt{P_x}, 1/b_2, 1/b_3\},$$

$$t_y = \max\{\sqrt{Q_y}, \sqrt{P_y}, 1/b_1, 1/b_2\},$$

$$h_{a,e} = H_e(P_{a,e}, Q_{a,e}, b_1, b_3) S_t(x_3),$$

$$h_{b,f} = H_e(P_{b,f}, Q_{b,f}, b_3, b_1) S_t(x_1),$$

$$h_j = H_{en}(Q_j, P_j, b_1, b_2),$$

$$h_k = H_{af}(P_k, Q_k, b_2, b_3) S_t(x_3),$$

$$h_l = H_{af}(P_l, Q_l, b_3, b_2) S_t(x_2),$$

$$h_{m,n} = H_{an},$$

for the factorizable diagram  $i = a, b, e, f$ , for the non-factorizable diagram  $j = c, d, g, h, g', h'$ , and for the annihilation diagrams  $x = k, l$  and  $y = m, n$ . For diagrams (c) and (d), we also keep the function  $S_t(x_3)$ . Here we use the definition of Bessel functions

$$H_e(\alpha, \beta, b_1, b_3) = K_0(\sqrt{\beta} b_1) [\theta(b_1 - b_3) I_0(\sqrt{\alpha} b_3) \\ \times K_0(\sqrt{\alpha} b_1) + (b_1 \leftrightarrow b_3)],$$

$$H_{en}(\alpha, \beta, b_1, b_2) = [\theta(b_2 - b_1) K_0(\sqrt{\alpha} b_2) \\ \times I_0(\sqrt{\alpha} b_1) + (b_1 \leftrightarrow b_2)] \\ \times \begin{cases} \frac{i\pi}{2} H_0^{(1)}(\sqrt{\beta} b_2), & \beta < 0 \\ K_0(\sqrt{\beta} b_2), & \beta > 0, \end{cases} \quad (A2)$$

$$H_{af}(\alpha, \beta, b_2, b_3) = \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{\beta} b_2) [\theta(b_2 - b_3) \\ \times H_0^{(1)}(\sqrt{\alpha} b_2) J_0(\sqrt{\alpha} b_3) \\ + \theta(b_3 - b_2) H_0^{(1)}(\sqrt{\alpha} b_3) J_0(\sqrt{\alpha} b_2)],$$

$$H_{an} = i \frac{\pi}{2} \left[ \theta(b_1 - b_2) H_0^{(1)}(\sqrt{Q_y} b_1) J_0(\sqrt{Q_y} b_2) \right. \\ \left. + \theta(b_2 - b_1) H_0^{(1)}(\sqrt{Q_y} b_2) J_0(\sqrt{Q_y} b_1) \right] \\ \times \begin{cases} K_0(\sqrt{|P_y|} b_1) & \text{for } P_y \leq 0 \\ \frac{i\pi}{2} H_0^{(1)}(\sqrt{|P_y|} b_1) & \text{for } P_y \geq 0 \end{cases}, \quad (A3)$$

where  $H_0^{(1)}(z) = J_0(z) + iY_0(z)$ . The  $S_t$  has been parameterized as

$$S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c, \quad (A4)$$

with  $c = 0.4 \pm 0.1$ .

The  $E_i$ s contain the Sudakov evolution factors and the strong coupling constants, and are given by

$$\begin{aligned}
 E_{a,b}(t) &= \alpha_s(t) \exp[-S_B(t) - S_D(t, \bar{x}_3, b_3)], \\
 E_{c,d}(t) &= \alpha_s(t) \exp[-S_B(t) - S_D(t, \bar{x}_3, b_1) - S_{K_0^*}(t, b_2)], \\
 E_{e,f}(t) &= \alpha_s(t) \exp[-S_B(t) - S_{K_0^*}(t, b_3)], \\
 E_{g,h}(t) &= \alpha_s(t) \exp[-S_B(t) - S_D(t, \bar{x}_2, b_2) - S_{K_0^*}(t, b_1)], \\
 E_{g',h'}(t) &= \alpha_s(t) \exp[-S_B(t) - S_D(t, x_2, b_2) - S_{K_0^*}(t, b_1)], \\
 E_{k,l}(t) &= \alpha_s(t) \exp[-S_D(t, x_3, b_3) - S_{K_0^*}(t, b_2)], \\
 E_{m,n}(t) &= \alpha_s(t) \exp[-S_B(t) - S_D(t, x_2, b_2) - S_{K_0^*}(t, b_2)].
 \end{aligned}$$

in which the Sudakov exponents are defined as

$$\begin{aligned}
 S_B(t) &= s\left(x_1 \frac{m_B}{\sqrt{2}}, b_1\right) + \frac{5}{3} \int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \\
 S_D(t, x_2, b_2) &= s\left(x_2 \frac{m_B}{\sqrt{2}}, b_2\right) + 2 \int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \\
 S_{K_0^*}(t, b_3) &= s\left(x_3 \frac{m_B}{\sqrt{2}}, b_3\right) + s\left((1-x_3) \frac{m_B}{\sqrt{2}}, b_3\right) \\
 &\quad + 2 \int_{1/b_3}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \tag{A5}
 \end{aligned}$$

with the quark anomalous dimension  $\gamma_q = -\alpha_s/\pi$ .

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