

Congeniality bounds on quark masses from nucleosynthesisM. Hossain Ali,^{1,*} M. Jakir Hossain,^{2,†} and Abdullah Shams Bin Tariq^{3,‡}¹*Department of Applied Mathematics, Rajshahi University, Rajshahi 6205, Bangladesh*²*Department of Mathematics, Rajshahi University, Rajshahi 6205, Bangladesh*³*Department of Physics, Rajshahi University, Rajshahi 6205, Bangladesh*

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The work of Jaffe, Jenkins and Kimchi [Phys. Rev. D **79**, 065014 (2009)] is revisited to see if indeed the region of congeniality found in their analysis survives further restrictions from nucleosynthesis. It is observed that much of their congenial region disappears when imposing conditions required to produce the correct and required abundances of the primordial elements as well as ensure that stars can continue to burn hydrogen nuclei to form helium as the first step in forming heavier elements in stellar nucleosynthesis. The remaining region is a very narrow slit reduced in width from around 29 MeV found by Jaffe *et al.* to only about 2.2 MeV in the difference of the nucleon/quark masses. Further bounds on $\delta m_q/m_q$ seem to reduce even this narrow slit to the physical point itself.

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A reasonably contemporary approach is to study, even without going into anthropic arguments, the nature of alternative universes as one changes the values of physical parameters. In the parameter space, one then looks for regions that could be similar to our Universe and may possibly be congenial to the creation and sustenance of intelligent life [1–8]. Bounds thus obtained may be referred to as congeniality bounds.

In a recent work, Jaffe, Jenkins and Kimchi [7] studied how sensitive our Universe would be to variations of quark masses. For this they chose to study the variations of masses of the three lightest quarks u , d and s , under the constraint that the sum of these masses, m_T , remained fixed. They also studied variations of m_T .

Their basic idea was to find the two lightest baryons for any quark mass combination and consider them to play the roles of the proton and neutron in forming nuclei. In this process they also considered Λ_{QCD} to be an adjusted free parameter that they tuned to keep the average nucleon mass at 940 MeV. They then studied the variation of nuclear stability and, in light of this, tried to obtain the regions of the parameter space where nuclear chemistry in a somewhat familiar form could be sustained.

The starting point is that the three light quark masses would be changed keeping their sum m_T fixed. This parameter space can be neatly shown in the form of an equilateral triangle (Fig. 1) where the distances of a point from the base and right and left sides are, respectively, the masses of the up, down and strange quarks.

In this manner they identified congenial regions in a triangle parametrized in terms of x_3 and x_8 (Fig. 2) defined as

$$x_3 = \frac{2m_3}{\sqrt{3}} \frac{100}{m_T^\oplus} = \frac{100(m_u - m_d)}{\sqrt{3}m_T^\oplus}, \quad (1)$$

$$x_8 = \frac{2m_8}{\sqrt{3}} \frac{100}{m_T^\oplus} = \frac{100(m_u + m_d - 2m_s)}{\sqrt{3}m_T^\oplus}. \quad (2)$$

Here m_T^\oplus is the sum of the light (u , d and s) quark masses in our Universe. It is obvious that x_3 basically gives the isospin splitting while x_8 is related to the breaking of $SU(3)_f$ due to the mass of the strange quark. Their results can be summarized in Fig. 3, where the congenial regions are indicated in green (lightly shaded). This triangle is for m_T^\oplus , i.e. with m_T as it is in the present Universe. They have also studied variations in m_T , but our work is limited to commenting on the case of m_T^\oplus , understanding that the same arguments qualitatively extend to other values of m_T . This is further justified in the discussions near the end of this report.

It is increasingly being understood that if there is complexity, fine-tuning is inevitable [9]. Even if one is not happy with anthropic arguments, we simply cannot get away from fine-tuning. With this in mind, the first impression that one has from Fig. 3 is that the congenial region seems to be surprisingly large—allowing around 1 order of magnitude variations in the quark masses. However, this already involves intricate compensating adjustments in Λ_{QCD} to keep the average “nucleon” mass fixed.

However, one should appreciate the difficulty in setting up a new framework in which a problem can be studied. From this perspective the authors of [7] should be commended for presenting, literally from scratch, a setup for studying the congeniality bounds on quark masses. This setup can be extended by removing some of the constraints used in any further work. Indeed, considering the significance of the work it was chosen first for a Viewpoint article in Physics [9] and then went into a cover story in Scientific American [10].

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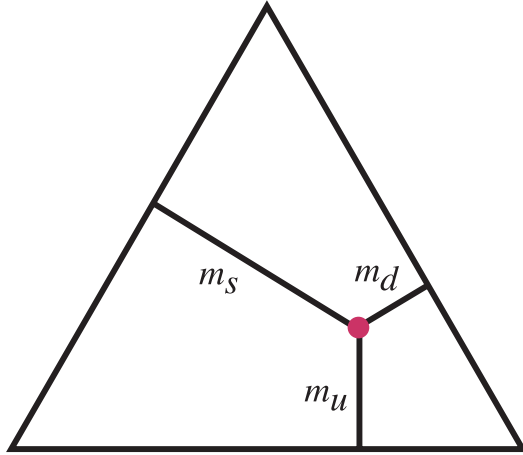


FIG. 1 (color online). The model space of light quark masses for a fixed m_T shown in the form of a triangle where the distances from the three sides give the three masses. The figure is reproduced from [7].

In our work, we remain within the provided setup, but extend the analysis to bounds provided by nucleosynthesis. It should be noted here that whereas, on the one hand, nuclear masses and stability expectedly vary comparatively slowly with quark masses, on the other hand, the observed abundances of the lightest nuclei hydrogen and helium provide much more stringent bounds on the variation of nucleon masses. We report below how the congeniality triangle of Fig. 3 is modified by the application of these constraints.

At the outset, the variations of the octet baryon masses as one traverses along the borders of the triangle (Figs. 10 and 11 of [7]) were reproduced to gain confidence in our code and our understanding of the framework. The fitted

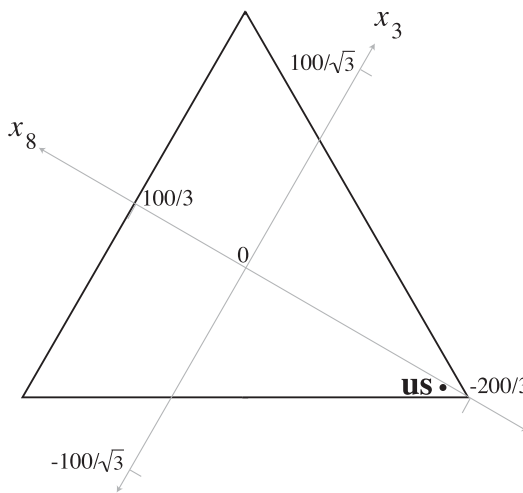


FIG. 2. The model space of light quark masses parametrized in terms of x_3 and x_8 reproduced from [7]. The point labeled “us” points to the physical value in our present Universe and therefore has coordinates $(x_3^{\oplus}, x_8^{\oplus})$.



FIG. 3 (color online). Figure reproduced from [7] identifying congenial regions in the quark mass triangle with green bands. The red and white regions are uncongenial and uncertain, respectively.

parameter values of c_T , c_3 and c_8 from Table III of Ref. [7] were used in the equation

$$M_B = C_0 + c_T x_T + c_8 x_8 + c_3 x_3 + \langle B | H_{EM} | B \rangle. \quad (3)$$

This leads to

$$M_p = C_0 + 3.68x_T + 3.53x_8 + 1.24x_3 + 0.63, \quad (4)$$

$$M_n = C_0 + 3.68x_T + 3.53x_8 - 1.24x_3 - 0.13. \quad (5)$$

The quantity occurring most in the analysis below being

$$M_n - M_p = -2.48x_3 - 0.76. \quad (6)$$

It may be noted here that using updated values of baryon masses from the Particle Data Book changes the parameters very slightly and this is neglected considering the qualitative nature of this work. After a clarification on the adjustment of C_0 (corresponding to an adjustment of Λ_{QCD}) from the authors [11] it was possible to reproduce the figures.

Then the issue of further bounds from nucleosynthesis was studied. It is well known that the observed abundances of the primordial nuclei hydrogen (protons), helium (alpha particles), etc. are sensitively tied to the masses of the nucleons [5,12]. The slight difference in the masses of the proton and neutron are responsible for the survival of protons with the observed abundance. The (un)congenial regions of the triangle are explored further under these constraints.

There are three cases that arise here.

Case I: $x_3 > x_3^{\oplus}$ —Let us concentrate on the region on the upper right of the triangle with x_3 values greater than at the point labeled “us” on the right-hand side of the triangle.

Of the two nucleons, the neutron is heavier by about 1.3 MeV. If it was just 0.8 MeV less, that would bring it below the electron capture threshold for protons; i.e. it

would become energetically favorable for protons to capture electrons and become neutrons. All the protons would have been converted to neutrons in the big bang. The Universe would be full of neutrons and nothing else. We would not be here. In the words of Barrow and Tipler [12],

Without electrostatic forces to support them, solid bodies would collapse rapidly into neutron stars or black holes. Thus, the coincidence that allows protons to partake in nuclear reactions in the early universe also prevents them decaying by weak interactions. It also, of course, prevents the 75% of the Universe which emerges from nucleosynthesis in the form of protons from simply decaying away into neutrons. If that were to happen no atoms would ever have formed and we would not be here to know it. (Ref. [12], p. 40)

The same issue is also discussed by Hogan [5]:

The u - d mass difference in particular attracts attention because the d is just heavier enough than u to overcome the electromagnetic energy difference to make the proton (uud) lighter than the neutron (udd) and therefore stable. On the other hand, if it were a little heavier still, the deuteron would be unstable and it would be difficult to assemble any nuclei heavier than hydrogen.

Therefore, it is necessary to have

$$M_n - M_p \geq 0.5 \text{ MeV.} \quad (7)$$

This reduces the congenial region on the upper right of the physical point to

$$x_3 \leq -0.51. \quad (8)$$

Case II: $x_3 < x_3^\oplus$ —Now let us move to the bottom-left side of the triangle (left of the x_3 axis) where x_3 values are smaller than that at the “us”-labeled point, again concentrating on the right-hand side of the triangle.

The key reaction by which hydrogen burns in stars such as the sun is

$$p + p \rightarrow d + e^+ + \nu + 0.42 \text{ MeV,} \quad (9)$$

$$e^+ + e^- \rightarrow 1 \text{ MeV.} \quad (10)$$

So the total amount of energy released in this reaction is 1.42 MeV.

If the neutron mass was 1.42 MeV (0.15%) more than it is, this reaction would not happen at all. It would need energy to make it go, rather than producing energy. Deuterons are a key step in burning hydrogen to helium. Without them, hydrogen would not burn, and there would be no long-lived stars and no stellar nucleosynthesis to produce the remaining elements.

Therefore, it is necessary that

$$M_n - M_p \leq 2.72 \text{ MeV.} \quad (11)$$

This reduces the congenial region on the lower left of the physical point to

$$x_3 \geq -1.4. \quad (12)$$

These two conditions, thus, significantly reduce the congenial corridor from

$$-12.9 \leq x_3 \leq 4.1 \quad (13)$$

to

$$-1.4 \leq x_3 \leq -0.5. \quad (14)$$

It may be noted here that the width of this region is of the same order as the uncertainty in x_3^\oplus itself due to uncertainties in the light quark masses given by $x_3^\oplus = -1.17 \pm 0.43$.

In fact it was a pleasant surprise to realize, rather late into our work, that Hogan [5] reached essentially similar conclusions which were expressed in terms of the up-down quark mass difference, δm_{d-u} , and Sec. 4 of his review [5] is a recommended read for anybody interested in this issue. The approximately $1.4 + 0.8 = 2.2$ MeV window of variation that we find is in agreement with the allowed region in Fig. 1 of the same paper [5].

Case III: Left half of the triangle—If we move to the left half of the triangle, we essentially replace the down quark with a strange quark. We know that the s quark is, in some ways, like a heavy d quark. In the left half of the triangle the s quark is light and the d quark is heavy, as if they simply interchanged positions. That is why Jaffe *et al.* [7] seem to find a symmetric congenial region in the left part of the triangle. The discussions for Cases I and II narrow it down, but they do not remove it.

However, let us now turn towards the coupling between u - d and u - s . The u - d coupling is much stronger, whereas the u - s coupling is suppressed. This is described by the well-known Cabibbo angle θ_C . The u - d coupling carries a factor $\cos\theta_C$ and the u - s coupling carries a factor of $\sin\theta_C$, the Cabibbo angle being about 13° .

This is like the present world with a much weaker weak interaction. This the case where the weak decay rate of neutrons is not strong enough to produce the primordial neutron-proton abundance ratio of 1:6. Without this we are left without enough protons, i.e. without enough hydrogen, which is key to both stellar burning and biological life itself. Therefore we are left with only a narrow region on the right [Fig. 4].

The only remaining question is probably regarding the length of this narrow region extending nearly up to the center of the triangle. As one moves up this narrow slit towards the center, away from the physical point, the up-down quarks become heavier, with the down quark slightly heavier than the up quark. Meanwhile, the strange quark becomes lighter to keep m_T fixed. The physics considered here is probably not very sensitive to the

strange quark mass. The increase in the up-down masses is offset by the compensating adjustment in Λ_{QCD} to keep the nucleon masses fixed. Therefore, the length of this region could probably be an artifact of the simultaneous and compensating tuning of quark masses and Λ_{QCD} .

This indeed was one of the conclusions in [7], as summarized more elegantly in [10], as well as [13] where, reviewing the alternative universe landscapes studied by [7,10,14–16], it was observed that if one is prepared to adjust another parameter in a compensating manner, it might be possible to find other regions in the parameter space that are also congenial. However, that does not remove the fine-tuning problem, as the alternative values are still finely tuned and this is inevitable to produce complexity as observed in our present Universe. Here most of the alternatives are removed and the narrow region remains as a result of the compensating adjustments of Λ_{QCD} .

Indeed along the narrow region the sum of the two lightest quarks varies with the strange quark mass going in the opposite direction to keep m_T fixed. If the effect of this could be quantified, it would probably be possible to restrict even the length of the narrow region (see the Appendix).

For example, as noted by Hogan [5],

... the sum of the (up and down) quark masses controls the pion mass, so changing them alters the range of the nuclear potential and significantly changes nuclear structure and energy levels. Even a small change radically alters the history of nuclear astrophysics, for example, by eliminating critical resonances of nucleosynthesis needed to produce abundant carbon [17].

Here it should be added that a more up-to-date view is that the strongest effect on the scalar scattering lengths and deuteron binding energy seems to be due to the sigma-resonance exchange (or correlated two-pion scalar-isoscalar exchange) dependence on m_π [18,19].

As mentioned at the outset, the analysis here has been limited to the case of $m_T = m_T^\oplus$. It has been noted by Jaffe *et al.* [7] that the widths of the two major congenial bands on the bottom left and bottom right of the triangle are independent of m_T . Therefore, naturally the further exclusions for $x_3 > x_3^\oplus$, $x_3 < x_3^\oplus$ reducing the width of the band should also apply to other values of m_T . The exclusion of the left half of the triangle should also extend to other values of m_T . Therefore, in summary, it can be expected that for all values of m_T , after applying constraints from nucleosynthesis, there will only remain a similar very narrow congenial band at the bottom right of the triangle.

An additional comment is due here on the possibilities of universes with deuterons, sigma-hydrogen, or delta-helium playing the roles of hydrogen as listed in [10] as a summary of [7]. The point made here does not contradict the fact that

these could be stable lightest elements. It is only pointed out that stability alone is not enough to produce and sustain nuclear chemistry in a manner familiar to us. Correct primordial abundances and conditions for sustained stellar burning provide constraints that are much more difficult to satisfy. This probably calls for a closer analysis of the other half in [10] related to possible universes without any weak interaction of [15], where they have claimed to find congenial universes even after a detailed discussion of these issues pertaining to nucleosynthesis. However, that would have to be another project, as this work is focused on [7].

In summary, it can be observed that primordial nuclear abundances and processes of stellar nucleosynthesis provide much more stringent constraints on quark masses than nuclear stability. Using these constraints it is possible to significantly reduce the congenial region in the space of light quark masses.

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APPENDIX

Our attention has been drawn through referee comments to studies of the bounds from nucleosynthesis [20,21], the latter appearing after the initial submission of this paper, on $\delta m_q/m_q$, where m_q is the average of the light (up and down) quark mass and δm_q is the change in m_q keeping m_u/m_d fixed. Coincidentally, along the length of the remaining narrow congenial region, m_u/m_d is approximately constant. The latest value is $|\delta m_q/m_q| < 0.009$ [21]. There are other values in the literature, but they are generally of the same order. Let us try to do a crude estimate of the effect of this constraint. For $m_q \approx 3.8$ MeV, $\delta m_q \approx 0.035$ MeV.

From Eqs. (8) and (9), we get

$$M_N = (M_n + M_p)/2 = C_0 + 3.68x_T + 3.53x_8 + 2.5, \quad (\text{A1})$$

which, given that x_T is kept fixed, leads to

$$\delta M_N = \delta C_0 + 3.53\delta x_8. \quad (\text{A2})$$

Now, x_8 as defined in Eq. (2) can be reexpressed in terms of m_q as

$$x_8 = \frac{200(m_q - m_s)}{\sqrt{3}m_T^\oplus}. \quad (\text{A3})$$

If m_T is kept fixed then $\delta m_s = -\delta m_q$, leading to

$$\delta x_8 = \frac{400(\delta m_q)}{\sqrt{3}m_T^\oplus} = 0.08, \quad (\text{A4})$$

where we have used $\delta m_q \approx 0.035$ MeV and $m_T^\oplus \approx 100$ MeV. The remaining congenial region is too small to show on a figure of this scale. In fact the region $x_8 = x_8^\oplus \pm 0.08$ is too small to show on a plot of this scale and is also very small compared to the uncertainty in the

value of x_8^\oplus itself, $x_8^\oplus = -59.5 \pm 1.1$, due to the uncertainties in the determination of the light quark masses. However, one should remember our estimate is rather crude without appropriate consideration of the uncertainties in m_q . Taking these into account will increase the region, but keep it within the same order as the uncertainty in x_8^\oplus itself. In short, there is practically no congenial region outside (x_3^\oplus, x_8^\oplus) .

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