Quark sector *CP* violation of the universal seesaw model

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We study the charge parity (*CP*) violation of the universal seesaw model, especially its quark sector. The model is based on $SU(2)_L \times SU(2)_R \times U(1)_{Y'}$. In order to count the number of parameters in the quark sector, we use the degree of freedom of the weak basis transformation. For the N(3)-generation model, the number of *CP* violating phases in the quark sector is identified as $3N^2 - 3N + 1$ (19). We also construct 19 *CP* violating weak basis invariants of Yukawa coupling matrices and $SU(2)$ singlet quark mass matrices in the three-generation universal seesaw model. The quark interaction terms induced by neutral currents are given as an exact formula. Both the charged current and the neutral current are expressed in terms of the mass basis by finding the transformations from the weak basis to the mass basis. Finally, we calculate the mixing matrix element approximately, assuming that the $SU(2)_R$ breaking scale v_R is much larger than the electroweak breaking scale v_L.

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I. INTRODUCTION

The universal seesaw mechanism [1–7] based on $SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{Y'}$ gauge symmetry is considered for fermion mass hierarchy with $SU(2)_R \times SU(2)_L$ isosinglet fermion masses. The ordinary fermion and the singlet fermion mix at the tree level after spontaneous symmetry breaking $SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{Y'} \to SU(3)_C \times U(1)_{EM}$. The universal seesaw mechanism provides us a clue for the mystery: why are ordinary fermions much lighter than the electroweak scale except for top quark [8,9]? When this mechanism works, all of the strength of the Yukawa couplings can be taken order of unity. The doublet quark and singlet quark are transformed by $SU(3)_C \times SU(2)_R \times SU(2)_L \times U(1)_{Y'}$ as follows:

$$q_L \sim \left(3, 1, 2, \frac{1}{6}\right), \qquad q_R \sim \left(3, 2, 1, \frac{1}{6}\right),$$
$$\mathcal{U} \sim \left(3, 1, 1, \frac{2}{3}\right), \qquad \mathcal{D} \sim \left(3, 1, 1, -\frac{1}{3}\right),$$

where $Q = T_R^3 + T_L^3 + Y'$.

A sophisticated discussion of *CP* violation using weak basis (WB) invariants is given by Jarlskog in Ref. [10] and by Bernabeu *et al.* in Ref. [11]. See also Ref. [12] for a review and Ref. [13] for WB invariants in the framework of the left-right symmetric model.

The gauge boson mass matrix in the universal seesaw model is identical to the left-right symmetric model studied in Ref. [14], except that the left-right symmetric model includes the $SU(2)_R \times SU(2)_L$ bidoublet Higgs. One can find the gauge boson mass matrix in the present model by taking the limit where the vacuum expectation value of the bidoublet Higgs vanishes. The possibility that the universal seesaw mechanism resolves the strong *CP* problem is explained by Babu and Mohapatra in Ref. [15]. Embedding the universal seesaw in the grand unified theory scenario is discussed by Cho in Ref. [16], Koide in Ref. [17], and Mohapatra in Ref. [18].

In this paper, we focus on the *CP* violation of the quark sector. Phenomenological aspects of the *CP* violation have been studied in Refs. [19,20]. In the literature [20], *CP* violation of the present model is studied with an additional assumption: left-right symmetry. We study the *CP* violation and the flavor mixing as general as possible so that one can study the phenomenology of the present model to the full extent. The recent study on mixings of the vectorlike quarks can be also found in Ref. [21].

Our paper is organized as follows. We count the number of the parameters in the quark sector in Sec. II. In Sec. III, we construct WB invariants of the quark sector. In Sec. IV, we propose a parametrization for the three-generation model by minimizing the numbers of the parameters with weak basis transformation (WBT). The relation between the WB invariant and *CP* violation parameters in the specific parametrization is discussed. The exact formulas for the mixing matrices are obtained in the mass basis in Sec. V. Finally, in Sec. VI, we carry out the diagonalization of 6×6 mass matrices with some approximation and write down the mixing matrix elements. Section VII is devoted to the summary.

II. COUNTING THE NUMBER OF REAL AND IMAGINARY PARAMETERS IN THE QUARK SECTOR OF THE UNIVERSAL SEESAW MODEL

In this section, by using the freedom of WBT, we minimize the number of real and imaginary parts of Yukawa couplings and singlet quark mass matrices. The number of imaginary parts which are left after WBT corresponds to the number of physical *CP* violating phases.

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We also verify the number of CP violating phases by counting the independent number of CP invariant conditions in a specific weak basis.

A. WBT of the universal seesaw model

We assume the singlet quark generation number is N, which is identical to an ordinary quark generation number. In this model, WBTs on singlet and doublet quarks are given by

$$\mathcal{U}'_R = V_{U_R}\mathcal{U}_R, \qquad \mathcal{U}'_L = V_{U_L}\mathcal{U}_L, \tag{1}$$

$$\mathcal{D}'_R = V_{D_R}\mathcal{D}_R, \qquad \mathcal{D}'_L = V_{D_L}\mathcal{D}_L, \tag{2}$$

$$q'_R = V_R q_R, \qquad q'_L = V_L q_L, \tag{3}$$

where $\mathcal{U}_{R(L)}$, $\mathcal{D}_{R(L)}$, and $q_{R(L)}$ denote the right-handed (left-handed) uptype singlet quark, downtype singlet quark, and ordinary doublet quark, respectively. Below, the matrices with superscript $'$ imply the matrices obtained by changing the WB. Yukawa matrices and mass matrices of the singlet quarks are transformed as

$$\begin{aligned} M'_{\mathcal{U}} &= V^\dagger_{U_L} M_{\mathcal{U}} V_{U_R}, \qquad M'_{\mathcal{D}} = V^\dagger_{D_L} M_{\mathcal{D}} V_{D_R}, \\ y'_{uL} &= V^\dagger_L y_{uL} V_{U_R}, \qquad y'_{uR} = V^\dagger_R y_{uR} V_{U_L}, \\ y'_{dL} &= V^\dagger_L y_{dL} V_{D_R}, \qquad y'_{dR} = V^\dagger_R y_{dR} V_{D_L}, \end{aligned} \tag{4}$$

where $M_{\mathcal{U}(\mathcal{D})}$ denotes the $N \times N$ uptype (downtype) mass matrix of a singlet quark, and y is the $N \times N$ Yukawa coupling constant matrix. One chooses the weak basis, and $M'_{\mathcal{U}(\mathcal{D})}$ is given by a real diagonal matrix by carrying out the suitable biunitary transformation as the WBT. In the basis, both of the uptype and downtype singlet mass matrices have N real parameters. Suppose that we find the biunitary transformation, which diagonalizes the mass matrices as

$$\tilde{V}^\dagger_{U_L} M_{\mathcal{U}} \tilde{V}_{U_R} = D_U, \tag{5}$$

$$\tilde{V}^\dagger_{D_L} M_{\mathcal{D}} \tilde{V}_{D_R} = D_D, \tag{6}$$

where D_U and D_D are real diagonal matrices. We note that real diagonal matrices are invariant under the similarity transformation P_U and P_D,

$$P^\dagger_U D_U P_U = D_U, \qquad P^\dagger_D D_D P_D = D_D, \tag{7}$$

where P_U and P_D are given by

$$\begin{aligned} P_U &= \begin{pmatrix} e^{ia_1} & & & \\ & e^{ia_2} & & \\ & & \ddots & \\ & & & e^{ia_N} \end{pmatrix}, \\ P_D &= \begin{pmatrix} e^{ib_1} & & & \\ & e^{ib_2} & & \\ & & \ddots & \\ & & & e^{ib_N} \end{pmatrix}. \end{aligned} \tag{8}$$

Unitary matrices which diagonalize the singlet quark mass matrices with biunitary transformation are not fixed uniquely. One can define the new unitary matrices,

$$\begin{aligned} V_{U_R} &= \tilde{V}_{U_R} P_U, \qquad V_{U_L} = \tilde{V}_{U_L} P_U, \\ V_{D_R} &= \tilde{V}_{D_R} P_D, \qquad V_{D_L} = \tilde{V}_{D_L} P_D. \end{aligned} \tag{9}$$

By using V_{U_R}, V_{U_L}, V_{D_R}, and V_{D_L} as WBT, one can also diagonalize the singlet quark mass matrices. Next we consider the weak basis transformation on Yukawa matrices,

$$V^\dagger_L y_{uL} V_{U_R} = P^\dagger_U (\tilde{V}^\dagger_L y_{uL} \tilde{V}_{U_R}) P_U, \tag{10}$$

$$V^\dagger_R y_{uR} V_{U_L} = P^\dagger_U (\tilde{V}^\dagger_R y_{uR} \tilde{V}_{U_L}) P_U, \tag{11}$$

$$V^\dagger_L y_{dL} V_{D_R} = P^\dagger_U (\tilde{V}^\dagger_L y_{dL} \tilde{V}_{D_R}) P_D, \tag{12}$$

$$V^\dagger_R y_{dR} V_{D_L} = P^\dagger_U (\tilde{V}^\dagger_R y_{dR} \tilde{V}_{D_L}) P_D. \tag{13}$$

In Eqs. (10)–(13), we extract the diagonal phase matrix P_U from V_L and V_R,

$$V_L = \tilde{V}_L P_U, \qquad V_R = \tilde{V}_R P_U. \tag{14}$$

We can choose unitary matrix $\tilde{V}_L$ so that $y'_{\Delta_{uL}} = \tilde{V}^\dagger_L y_{uL} \tilde{V}_{U_R}$ is a lower triangular matrix with real diagonal elements. One can also choose $\tilde{V}_R$ so that $y'_{\Delta_{uR}} = \tilde{V}^\dagger_R y_{uR} \tilde{V}_{U_L}$ is a lower triangular matrix with real diagonal elements. Therefore, Eqs. (10) and (11) are rewritten as

$$\begin{aligned} V^\dagger_L y_{uL} V_{U_R} &= P^\dagger_U y'_{\Delta_{uL}} P_U = y_{\Delta_{uL}}, \\ V^\dagger_R y_{uR} V_{U_L} &= P^\dagger_U y'_{\Delta_{uR}} P_U = y_{\Delta_{uR}}. \end{aligned} \tag{15}$$

In the triangular form of the Yukawa couplings $y'_{\Delta_{uL(R)}}$, one reduces $\frac{1}{2}N(N-1)$ real parameters and $\frac{1}{2}N(N+1)$ imaginary parameters from $N \times N$ complex Yukawa matrices y_{uL} and y_{uR}, respectively. Therefore, each triangular matrix includes $\frac{1}{2}N(N+1)$ real parts and $\frac{1}{2}N(N-1)$ imaginary parts. With P_U, one can remove the $N-1$ imaginary parts in $y'_{\Delta_{uL}}$. Therefore, with the WBT in Eq. (15), $y_{\Delta_{uL}}$ includes $\frac{1}{2}N(N+1)$ real parts and $\frac{1}{2}(N-1)(N-2)$ imaginary parts, while $y_{\Delta_{uR}}$ includes $\frac{1}{2}N(N+1)$ real parts and $\frac{1}{2}N(N-1)$ imaginary parts.

TABLE I. The number of parameters included in quark sector matrices for the N generations universal seesaw model in a specific WB.

	M_U	M_D	$y_{\Delta uL}$	$y_{\Delta uR}$	y_{dL}	y_{dR}	Sum.
Re.	N	N	$\frac{1}{2}N(N+1)$	$\frac{1}{2}N(N+1)$	N^2	N^2	$3N(N+1)$
Im.	0	0	$\frac{1}{2}(N-1)(N-2)$	$\frac{1}{2}N(N-1)$	$N(N-1)$	N^2	$3N^2-3N+1$

Next we count the number of the parameters in y_{dL} and y_{dR}. We can use the similarity transformation P_D. Then one removes N imaginary parts in y_{dL}. Therefore, y_{dL} includes N^2 real parts and $N^2 - N$ imaginary parts. Since we have already used all the freedom of WBT, N^2 real parts and N^2 imaginary parts are left in y_{dR}.

We summarize the number of degrees of freedom in the quark sector of the universal seesaw model for N generations. Table I shows the number of real and imaginary parameters in the matrices obtained by the WBT. Table II shows the number of real and imaginary parameters for specific generation numbers $N = 1$–4.

B. *CP* invariant condition

Let us prove the previous derivation of the number of *CP* violating phases with an alternative argument. To count the numbers of nontrivial *CP* violating phases, one can study the numbers of independent *CP* invariant conditions. The *CP* invariant conditions are then

$$M'_{\mathcal{U}} = M^*_{\mathcal{U}}, \qquad M'_{\mathcal{D}} = M^*_{\mathcal{D}}, \tag{16}$$

$$y'_{uL} = y^*_{uL}, \qquad y'_{uR} = y^*_{uR}, \tag{17}$$

$$y'_{dL} = y^*_{dL}, \qquad y'_{dR} = y^*_{dR}. \tag{18}$$

We consider these conditions in a specific weak basis. In the basis, the singlet quark mass matrices are given by real diagonal matrices D_U and D_D. Yukawa coupling matrices y_{uL} and y_{uR} are given by the lower triangular matrices $y'_{\Delta uL}$ and $y'_{\Delta uR}$. Note that the diagonal elements of the triangular matrix are real. In this basis, *CP* invariant conditions for singlet quark mass matrices are written as

$$V^\dagger_{U_L} D_U V_{U_R} = D_U, \qquad V^\dagger_{D_L} D_D V_{D_R} = D_D. \tag{19}$$

To satisfy the conditions given above, Vs are determined as

$$V_{U_L} = V_{U_R} = P_U, \qquad V_{D_L} = V_{D_R} = P_D. \tag{20}$$

TABLE II. The number of parameters for the specific generation number N.

	$N = 1$	$N = 2$	$N = 3$	$N = 4$
Re.	6	18	36	60
Im.	1	7	19	37

The *CP* invariant conditions for Yukawa matrices are then

$$P^\dagger_U {y'}_{\Delta_{uL}} P_U = {y'}^*_{\Delta_{uL}}, \tag{21}$$

$$P^\dagger_U {y'}_{\Delta_{uR}} P_U = {y'}^*_{\Delta_{uR}}, \tag{22}$$

$$P^\dagger_U y_{dL} P_D = y^*_{dL}, \tag{23}$$

$$P^\dagger_U y_{dR} P_D = y^*_{dR}. \tag{24}$$

These four relations are also written in terms of the argument of their matrix element,

$$\arg({y'}_{\Delta_{uL}ij}) = \arg({y'}_{\Delta_{uR}ij}) = \frac{a_i - a_j}{2}, \tag{25}$$

$$\arg(y_{dLij}) = \arg(y_{dRij}) = \frac{a_i - b_j}{2}. \tag{26}$$

We count the nontrivial *CP* invariant conditions which cannot be satisfied by adjusting the phases in P_U and P_D. Since one can choose the $N-1$ phase difference, $a_i - a_1$ ($i = 1 - N$) as $\arg(y_{\Delta uLi1}) = \frac{a_i - a_1}{2}$, the $N-1$ *CP* invariant conditions are automatically satisfied. Therefore, the number of the nontrivial conditions in Eq. (25) is $(N-1)^2 = 2 \times \frac{N(N-1)}{2} - (N-1)$. As for the conditions in Eq. (26), b_i is chosen as $b_i = a_i - 2\arg(y_{dLii})$ so that the N condition of Eq. (26) is satisfied. Therefore, there are $2N^2 - N$ nontrivial conditions. Then, in total, we find $3N^2 - 3N + 1$ *CP* invariant conditions, which are identical to the number of *CP* violating phases. It also agrees with the number of the imaginary parts in the Yukawa matrices obtained with the WBT (see Table I).

III. *CP* VIOLATING WEAK BASIS INVARIANTS IN THE THREE-GENERATION MODEL

In this section, we derive the *CP* violating WB invariants for a three-generation model. The use of the WB invariants including $SU(2)$ singlet quarks within the standard model gauge group is discussed in Ref. [22]. We define the following Hermitian matrices in order to write down the WB invariants for *CP* violation in the universal seesaw model:

$$
\begin{aligned}
H_U &= M_{\mathcal{U}} M_{\mathcal{U}}^\dagger, & H_D &= M_{\mathcal{D}} M_{\mathcal{D}}^\dagger, & H_{uL} &= y_{uL} y_{uL}^\dagger, \\
H_{uR} &= y_{uR} y_{uR}^\dagger, & H_{dL} &= y_{dL} y_{dL}^\dagger, & H_{dR} &= y_{dR} y_{dR}^\dagger, \\
h_U &= M_{\mathcal{U}}^\dagger M_{\mathcal{U}}, & h_D &= M_{\mathcal{D}}^\dagger M_{\mathcal{D}}, & h_{uL} &= y_{uL}^\dagger y_{uL}, \\
h_{uR} &= y_{uR}^\dagger y_{uR}, & h_{dL} &= y_{dL}^\dagger y_{dL}, & h_{dR} &= y_{dR}^\dagger y_{dR}.
\end{aligned} \tag{27}
$$

In the case that the singlet quark generation number is 3, identical to the ordinary quark generation number, the 19 CP violating WB invariants in the quark sector of the universal seesaw model are then

$$I_1 = \mathrm{Im\,tr}[h_{uL}, h_U]^3, \tag{28}$$

$$I_2 = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U h_{uL} M_{\mathcal{U}}^\dagger h_{uR}), \tag{29}$$

$$I_3 = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U^2 h_{uL} M_{\mathcal{U}}^\dagger h_{uR}), \tag{30}$$

$$I_4 = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U^2 h_{uL} H_U M_{\mathcal{U}}^\dagger h_{uR}), \tag{31}$$

$$I_5 = \mathrm{Im\,tr}[h_{dL}, h_D]^3, \tag{32}$$

$$I_6 = \mathrm{Im\,tr}(M_{\mathcal{D}} h_D h_{dL} M_{\mathcal{D}}^\dagger h_{dR}), \tag{33}$$

$$I_7 = \mathrm{Im\,tr}(M_{\mathcal{D}} h_D^2 h_{dL} M_{\mathcal{D}}^\dagger h_{dR}), \tag{34}$$

$$I_8 = \mathrm{Im\,tr}(M_{\mathcal{D}} h_D^2 h_{dL} H_D M_{\mathcal{D}}^\dagger h_{dR}), \tag{35}$$

$$I_9 = \mathrm{Im\,tr}[H_{uL}, H_{dL}]^3, \tag{36}$$

$$I_{10} = \mathrm{Im\,tr}[H_{uR}, H_{dR}]^3, \tag{37}$$

$$I_{11} = \mathrm{Im\,tr}(M_{\mathcal{U}} y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger y_{dR}^\dagger y_{uR}), \tag{38}$$

$$I_{12} = \mathrm{Im\,tr}(M_{\mathcal{U}} y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger H_D y_{dR}^\dagger y_{uR}), \tag{39}$$

$$I_{13} = \mathrm{Im\,tr}(M_{\mathcal{U}} y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger H_D^2 y_{dR}^\dagger y_{uR}), \tag{40}$$

$$I_{14} = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger y_{dR}^\dagger y_{uR}), \tag{41}$$

$$I_{15} = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger H_D y_{dR}^\dagger y_{uR}), \tag{42}$$

$$I_{16} = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger H_D^2 y_{dR}^\dagger y_{uR}), \tag{43}$$

$$I_{17} = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger y_{dR}^\dagger y_{uR}), \tag{44}$$

$$I_{18} = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger H_D y_{dR}^\dagger y_{uR}), \tag{45}$$

$$I_{19} = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U y_{uL}^\dagger y_{dL} M_{\mathcal{D}}^\dagger H_D^2 y_{dR}^\dagger y_{uR}). \tag{46}$$

We briefly explain how to construct the CP violating WB invariants in Eqs. (28)–(46). First, we can construct the WB invariance which does not vanish trivially by considering the trace of the cube of the commutator,

$$I_1 = \mathrm{Im\,tr}[h_{uL}, h_U]^3. \tag{47}$$

Note that the real part of the trace of the cube of the commutator does vanish. The nonzero value of the trace of the cubic commutator signals CP violation, and the proof follows in the same way as the Jarlskog invariant [10] and the CP violating WB invariant [11] for the Kobayashi-Maskawa model [23]. Next we consider the WB invariant with the form,

$$\mathrm{tr}\,(M_{\mathcal{U}} h_U h_{uL} M_{\mathcal{U}}^\dagger h_{uR}). \tag{48}$$

When CP is conserved, the imaginary part of Eq. (48) vanishes,

$$\mathrm{tr}\,(M_{\mathcal{U}}^* h_{\mathcal{U}}^* h_{uL}^* M_{\mathcal{U}}^T h_{uR}^*) = [\mathrm{tr}(M_{\mathcal{U}} h_U h_{uL} M_{\mathcal{U}}^\dagger h_{uR})]^*. \tag{49}$$

Therefore, the imaginary $I_2 = \mathrm{Im\,tr}(M_{\mathcal{U}} h_U h_{uL} M_{\mathcal{U}}^\dagger h_{uR})$ is a CP violating WB invariant. By inserting some Hermitian matrices, we can also construct the other CP violating WB invariants.

IV. A PARAMETRIZATION OF THE YUKAWA SECTOR IN THE THREE-GENERATION MODEL

In Sec. II, we introduced a specific WB; i.e., the uptype Yukawa matrices are given by the triangular matrices and the singlet quark matrices are real diagonal. This WB is obtained by fully utilizing the freedom of the WBT. Then the number of the real parts and imaginary parts included in the parameters of the Yukawa sector is minimized and should be equal to the number of independent physical parameters. In this section, we introduce a parametrization of the Yukawa sector for the three-generation model which is associated with the WB in Table I. The parametrization includes the same number of the real and imaginary parameters with that of the WB for $N = 3$. The Yukawa terms for the quarks in the WB are given by the following Lagrangian:

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} = {} & y_{\Delta uLij} \overline{q_L^i} \tilde{\phi}_L \mathcal{U}_R^j + y_{\Delta uRji}^* \overline{\mathcal{U}_L^i} \tilde{\phi}_R^\dagger q_R^j \\
& + \overline{\mathcal{U}_L^i} D_U^i \mathcal{U}_R^i + \text{H.c.} + y_{dLij} \overline{q_L^i} \phi_L \mathcal{D}_R^j \\
& + y_{dRji}^* \overline{\mathcal{D}_L^i} \phi_R^\dagger q_R^j + \overline{\mathcal{D}_L^i} D_D^i \mathcal{D}_R^i + \text{H.c.},
\end{aligned} \tag{50}
$$

where $i, j = 1$–3. After the symmetry breaking of $SU(2)_L$ and $SU(2)_R$, the doublet Higgses ϕ_L and ϕ_R acquire the vacuum expectation values v_L and v_R, respectively. Then the mass matrix for six up (down) quarks are generated as

$$
\begin{pmatrix} \overline{u_L} & \overline{\mathcal{U}_L} \end{pmatrix} \mathcal{M}_{\mathcal{U}} \begin{pmatrix} u_R \\ \mathcal{U}_R \end{pmatrix}, \qquad \begin{pmatrix} \overline{d_L^0} & \overline{\mathcal{D}_L} \end{pmatrix} \mathcal{M}_{\mathcal{D}}{}^0 \begin{pmatrix} d_R^0 \\ \mathcal{D}_R \end{pmatrix}, \tag{51}
$$

where, $\mathcal{M}_{\mathcal{U}}$ and $\mathcal{M}_{\mathcal{D}}^0$ are given as

$$\mathcal{M}_{\mathcal{U}} = \begin{pmatrix} 0 & y_{\Delta uL} v_L \\ y_{\Delta uR}^\dagger v_R & D_U \end{pmatrix}, \tag{52}$$

$$\mathcal{M}_{\mathcal{D}}{}^0 = \begin{pmatrix} 0 & y_{dL} v_L \\ y_{dR}^\dagger v_R & D_D \end{pmatrix}. \tag{53}$$

D_U and D_D are singlet quark mass matrices which are real diagonal,

$$D_U = \begin{pmatrix} M_U & 0 & 0 \\ 0 & M_C & 0 \\ 0 & 0 & M_T \end{pmatrix}, \qquad D_D = \begin{pmatrix} M_D & 0 & 0 \\ 0 & M_S & 0 \\ 0 & 0 & M_B \end{pmatrix}, \tag{54}$$

where the diagonal elements satisfy the following order, $M_U > M_C > M_T$ and $M_D > M_S > M_B$, in order to acquire the light quark mass spectrum $m_u < m_c < m_t$ and $m_d < m_s < m_b$. Applying the result in Table I to the three-generation model, the uptype Yukawa matrices $y_{\Delta uL}$ and $y_{\Delta uR}$ are given as triangular matrices,

$$y_{\Delta uL} = \begin{pmatrix} y_{uL1} & 0 & 0 \\ y_{uL21} & y_{uL2} & 0 \\ y_{uL31} & y_{uL32} & y_{uL3} \end{pmatrix}, \quad y_{\Delta uR} = \begin{pmatrix} y_{uR1} & 0 & 0 \\ y_{uR21} & y_{uR2} & 0 \\ y_{uR31} & y_{uR32} & y_{uR3} \end{pmatrix}, \tag{55}$$

where y_{uL32} and $y_{uRij}(i > j)$ are complex and the other elements are real. Two phases of y_{uL21} and y_{uL31} are removed by using the freedom of the similarity transformation P_U in Eq. (10). The downtype Yukawa couplings are given by 3×3 matrices. According to Table I, y_{dL} includes nine real parts and six imaginary parts. They can be parametrized as

$$y_{dL} = U_L y_{\Delta dL}, \tag{56}$$

where $y_{\Delta dL}$ is a lower triangular matrix [$(y_{\Delta dL})_{ij} = 0$, for $(i < j)$] which includes six real parts and only one imaginary part in $(y_{\Delta dL})_{32}$. It is parametrized exactly the same as that of $y_{\Delta uL}$,

$$y_{\Delta dL} = \begin{pmatrix} y_{dL1} & 0 & 0 \\ y_{dL21} & y_{dL2} & 0 \\ y_{dL31} & y_{dL32} & y_{dL3} \end{pmatrix}. \tag{57}$$

U_L includes three angles and five phases as

$$U_L = P(\alpha_{L1}, \alpha_{L2}, 0) V(\theta_{L1}, \theta_{L2}, \theta_{L3}, \delta_L) P(\beta_{L1}, \beta_{L2}, 0), \tag{58}$$

$$P(\phi_1, \phi_2, \phi_3) = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{i\phi_3} \end{pmatrix}. \tag{59}$$

In U_L, V denotes the Kobayashi-Maskawa-type parametrization of the unitary matrix which includes three mixing angles θ_{Li} $(i = 1\text{–}3)$ and a single *CP* violating phase δ_L [see Eq. (C4) in Appendix C for the explicit form for V]. There are four more *CP* violating phases, α_{Li}, β_{Li} $(i = 1, 2)$, which are parametrized in the diagonal phase matrix in $P(\alpha_{1L}, \alpha_{2L}, 0)$ and $P(\beta_{L1}, \beta_{L2}, 0)$ in Eq. (59). Next we parametrize the down-quark Yukawa coupling y_{dR}. Since y_{dR} is a completely general 3×3 complex matrix, it has three more *CP* violating phases compared with y_{dL}. Therefore, one can parametrize it as the product of a unitary matrix and triangular matrix as

$$y_{dR} = U_R y_{\Delta dR}. \tag{60}$$

In the parametrization given in Eq. (60), the unitary matrix U_R includes six phases [see Eq. (C3)],

$$U_R = P(\alpha_{1R}, \alpha_{2R}, \alpha_{3R}) V(\theta_{1R}, \theta_{2R}, \theta_{3R}, \delta_R) P(\beta_{R1}, \beta_{R2}, 0). \tag{61}$$

$y_{\Delta dR}$ has the same form as that of $y_{\Delta uR}$,

$$y_{\Delta dR} = \begin{pmatrix} y_{dR1} & 0 & 0 \\ y_{dR21} & y_{dR2} & 0 \\ y_{dR31} & y_{dR32} & y_{dR3} \end{pmatrix}, \tag{62}$$

where $y_{dRij}(i > j)$ are complex and $y_{dRi}(i = 1, 2, 3)$ are real.

We show how the 19 *CP* violating WB invariants I_1–I_{19} in Eqs. (28)–(46) can be written in the specific WB in which the singlet quark mass matrices are real diagonal and the Yukawa couplings are parametrized by Eqs. (55), (56), and (60). Then one can relate the *CP* violating WB invariants to the *CP* violating parameters defined by the specific WB. We first show that the first eight WB invariants I_1–I_8 can be written in terms of the *CP* violating phases of the Yukawa couplings of the triangular matrices. Note that there are also eight *CP* violating phases in the triangular matrices of the Yukawa couplings. By taking the real diagonal mass matrices for the singlet quarks, one can show I_1 is written in terms of a combination of the Yukawa coupling $y_{\Delta uL}$,

$$I_1 \ni \mathrm{Im}[h_{uL12} h_{uL23} h_{uL31}], \tag{63}$$

where $h_{uL} = y_{\Delta uL}^\dagger y_{\Delta uL}$. Because $\mathrm{Im}(y_{uL32})$ is the only *CP* violating phase in $y_{\Delta uL}$, I_1 corresponds to the *CP* violating phase $\mathrm{Im}(y_{uL32})$. One can also show that I_2, I_3, and I_4 are written by linear combinations of the following quantities:

$$\chi_u^{ij} = \mathrm{Im}(h_{uLij} h_{uRji}), \ (i, j) = (1, 2), (2, 3), (3, 1), \tag{64}$$

where $h_{uR} = y_{\Delta uR}^\dagger y_{\Delta uR}$. I_2, I_3, and I_4 depend on $\mathrm{Im}(y_{uRij})$ $(i > j)$ and $\mathrm{Im}(y_{uL32})$. All the four *CP* violating phases in uptype Yukawa couplings $y_{\Delta uL}$ and $y_{\Delta uR}$ can be found in the WB invariants I_1–I_4. Similarly, the four WB invariants I_5–I_8 are related to the four *CP* violating phases in the triangular matrices of the down quark sector. I_5 is related to $\mathrm{Im}(y_{dL32})$ since I_5 is proportional to

$$I_5 \ni \mathrm{Im}(h_{dL12}h_{dL23}h_{dL31}), \tag{65}$$

where $h_{dL} = y^\dagger_{\Delta d}y_{\Delta d}$. I_6–I_8 are written in terms of three combinations of Yukawa couplings χ_d^{12}, χ_d^{23}, and χ_d^{31}. They are defined by

$$\chi_d^{ij} = \mathrm{Im}(h_{dLij}h_{dRji}), \qquad (i,j) = (1,2), (2,3), (3,1), \tag{66}$$

where $h_{dRji} = y^\dagger_{\Delta dR}y_{\Delta dR}$. They are related to $\mathrm{Im}(y_{\Delta dRij})$ $(i > j)$ and $\mathrm{Im}(y_{dL32})$.

So far, all the *CP* violating phases in the triangular matrices in the Yukawa couplings are identified in the WB invariants I_1–I_8. Next, we show how the other 11 WB invariants are related to the rest of the *CP* violating phases in U_L and U_R. Although I_9–I_{19} depend on the *CP* violating phases of the triangular matrices, we focus on their dependence on the *CP* violation of unitary matrices U_L and U_R. I_9 depends on U_L and I_{10} depends on U_R. There are still four *CP* violating phases in U_L and five *CP* violating phases which are not identified yet in the WB invariants. One can easily see I_{11}–I_{19} can be written in terms of

$$\mathrm{Im}\,(y^\dagger_{\Delta uL}U_L y_{\Delta dR})_{ij}(y^\dagger_{\Delta dR}U_R^\dagger y_{\Delta uR})_{ji}. \tag{67}$$

They depend on the 11 *CP* violating phases in U_L and U_R.

Now we carry out the following unitary transformations on the downtype quarks d_L^0 and d_R^0,

$$d_L^0 = U_L d_L, \tag{68}$$

$$d_R^0 = U_R d_R. \tag{69}$$

With the new basis given in Eqs. (68) and (69), only the form of charged currents changes as

$$\overline{u_L}\gamma_\mu d_L^0 = \overline{u_L}\gamma_\mu U_L d_L, \qquad \overline{u_R}\gamma_\mu d_R^0 = \overline{u_R}\gamma_\mu U_R d_R. \tag{70}$$

The neutral currents keep their diagonal form as

$$\overline{u_L}\gamma_\mu u_L, \qquad \overline{u_R}\gamma_\mu u_R, \qquad \overline{d_L}\gamma_\mu d_L, \qquad \overline{d_R}\gamma_\mu d_R. \tag{71}$$

In terms of the new basis, the downtype mass matrix ${\mathcal{M}_{\mathcal{D}}}^0$ is changed into

$$\mathcal{M}_{\mathcal{D}} = \begin{pmatrix} U_L^\dagger & 0 \\ 0 & 1 \end{pmatrix} {\mathcal{M}_{\mathcal{D}}}^0 \begin{pmatrix} U_R & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & y_{\Delta dL}v_L \\ y^\dagger_{\Delta dR}v_R & D_D \end{pmatrix}. \tag{72}$$

Note that in the new basis, the downtype Yukawa matrices are given by the triangular matrices. To summarize, at this stage the mass terms of the quarks are

$$\begin{pmatrix} \overline{u_L} & \overline{\mathcal{U}_L} \end{pmatrix} \mathcal{M}_{\mathcal{U}} \begin{pmatrix} u_R \\ \mathcal{U}_R \end{pmatrix}, \quad \begin{pmatrix} \overline{d_L} & \overline{\mathcal{D}_L} \end{pmatrix} \mathcal{M}_{\mathcal{D}} \begin{pmatrix} d_R \\ \mathcal{D}_R \end{pmatrix}. \tag{73}$$

V. DIAGONALIZATION OF THE MASS MATRICES

In the previous section, we performed the unitary transformation on $SU(2)$ doublet fields. In this section, we carry out the diagonalization of the 6×6 mass matrices $\mathcal{M}_{\mathcal{U}}$ and $\mathcal{M}_{\mathcal{D}}$. Therefore, by the unitary transformation, doublet and singlet quarks are mixed in the mass eigenstates. Now we diagonalize the mass matrices given in Eqs. (52) and (72),

$$V^\dagger_{uL}\mathcal{M}_{\mathcal{U}}V_{uR} = \begin{pmatrix} d_u & 0 \\ 0 & \tilde{D}_U \end{pmatrix}. \tag{74}$$

Note that d_u is a diagonal mass matrix for light uptype quarks and $\tilde{D}_U$ denotes that for heavy quarks. The downtype mass matrix is diagonalized as

$$V^\dagger_{dL}\mathcal{M}_{\mathcal{D}}V_{dR} = \begin{pmatrix} d_d & 0 \\ 0 & \tilde{D}_D \end{pmatrix}, \tag{75}$$

where d_d is a diagonal mass matrix for light downtype quarks and $\tilde{D}_D$ is that for heavy quarks. In terms of the mass eigenstates, charged currents and neutral currents are written as

$$\overline{u_{Li}}\gamma_\mu U_{Lij}d_{Lj} = \mathcal{V}_{L\alpha\beta}\overline{u^m_{L\alpha}}\gamma_\mu d^m_{L\beta}, \tag{76}$$

$$\overline{u_{Ri}}\gamma_\mu U_{Rij}d_{Rj} = \mathcal{V}_{R\alpha\beta}\overline{u^m_{R\alpha}}\gamma_\mu d^m_{R\beta}, \tag{77}$$

$$\overline{u_{Li}}\gamma_\mu u_{Li} = Z_{uL\alpha\beta}\overline{u^m_{L\alpha}}\gamma_\mu u^m_{L\beta}, \tag{78}$$

$$\overline{u_{Ri}}\gamma_\mu u_{Ri} = Z_{uR\alpha\beta}\overline{u^m_{R\alpha}}\gamma_\mu u^m_{R\beta}, \tag{79}$$

$$\overline{d_{Li}}\gamma_\mu d_{Li} = Z_{dL\alpha\beta}\overline{d^m_{L\alpha}}\gamma_\mu d^m_{L\beta}, \tag{80}$$

$$\overline{d_{Ri}}\gamma_\mu d_{Ri} = Z_{dR\alpha\beta}\overline{d^m_{R\alpha}}\gamma_\mu d^m_{R\beta}, \tag{81}$$

where u^m_α and $d^m_\alpha (\alpha = 1, \ldots, 6)$ denote the mass eigenstates.

We parametrize V_{qL} and V_{qR} $(q = u,\, d)$ with 3×3 submatrices as

$$V_{qL} = \begin{pmatrix} K_{qL} & R_{qL} \\ S_{qL} & T_{QL} \end{pmatrix}, \qquad V_{qR} = \begin{pmatrix} K_{qR} & R_{qR} \\ S_{qR} & T_{QR} \end{pmatrix}. \tag{82}$$

The 6×6 mixing matrices $\mathcal{V}_L$ and $\mathcal{V}_R$ for the charged currents in Eqs. (76) and (77) are written as

$$\mathcal{V}_L = \begin{pmatrix} K^\dagger_{uL}U_L K_{dL} & K^\dagger_{uL}U_L R_{dL} \\ R^\dagger_{uL}U_L K_{dL} & R^\dagger_{uL}U_L R_{dL} \end{pmatrix}, \tag{83}$$

$$\mathcal{V}_R = \begin{pmatrix} K^\dagger_{uR}U_R K_{dR} & K^\dagger_{uR}U_R R_{dR} \\ R^\dagger_{uR}U_R K_{dR} & R^\dagger_{uR}U_R R_{dR} \end{pmatrix}. \tag{84}$$

The mixing matrices for the neutral currents in Eqs. (78)–(81) are given by

$$Z_{uL} = \begin{pmatrix} K_{uL}^\dagger K_{uL} & K_{uL}^\dagger R_{uL} \\ R_{uL}^\dagger K_{uL} & R_{uL}^\dagger R_{uL} \end{pmatrix},\quad Z_{uR} = \begin{pmatrix} K_{uR}^\dagger K_{uR} & K_{uR}^\dagger R_{uR} \\ R_{uR}^\dagger K_{uR} & R_{uR}^\dagger R_{uR} \end{pmatrix}, \tag{85}$$

$$Z_{dL} = \begin{pmatrix} K_{dL}^\dagger K_{dL} & K_{dL}^\dagger R_{dL} \\ R_{dL}^\dagger K_{dL} & R_{dL}^\dagger R_{dL} \end{pmatrix},\quad Z_{dR} = \begin{pmatrix} K_{dR}^\dagger K_{dR} & K_{dR}^\dagger R_{dR} \\ R_{dR}^\dagger K_{dR} & R_{dR}^\dagger R_{dR} \end{pmatrix}. \tag{86}$$

The quark interaction terms induced by neutral currents are written in terms of mass eigenstate quark fields and mass eigenstate gauge fields as follows:

$$\begin{aligned}
-\mathcal{L}_{\rm NC} =& +\frac{2}{3}e\bar{u}^m_\alpha \not{A} u^m_\alpha - \frac{1}{3}e\bar{d}^m_\alpha \not{A} d^m_\alpha - \frac{2}{3}g_1(c_R s_W c_\xi + s_R s_\xi)\bar{u}^m_\alpha \not{Z} u^m_\alpha + \frac{1}{3}g_1(c_R s_W c_\xi + s_R s_\xi)\bar{d}^m_\alpha \not{Z} d^m_\alpha \\
& + \frac{2}{3}g_1(s_W c_R s_\xi - s_R c_\xi)\bar{u}^m_\alpha \not{Z}' u^m_\alpha - \frac{1}{3}g_1(s_W c_R s_\xi - s_R c_\xi)\bar{d}^m_\alpha \not{Z}' d^m_\alpha \\
& + \left[\frac{1}{2}Z_{uL\alpha\beta}(g_L c_W c_\xi + g_1(c_R s_W c_\xi + s_R s_\xi))\right]\bar{u}^m_{L\alpha}\not{Z} u^m_{L\beta} + \left[-\frac{1}{2}Z_{dL\alpha\beta}(g_L c_W c_\xi + g_1(c_R s_W c_\xi + s_R s_\xi))\right]\bar{d}^m_{L\alpha}\not{Z} d^m_{L\beta} \\
& + \left[\frac{1}{2}Z_{uR\alpha\beta}(g_R(c_R s_\xi - s_R s_W c_\xi) + g_1(c_R s_W c_\xi + s_R s_\xi))\right]\bar{u}^m_{R\alpha}\not{Z} u^m_{R\beta} + \left[-\frac{1}{2}Z_{dR\alpha\beta}(g_R(c_R s_\xi - s_R s_W c_\xi) \right. \\
& \left. + g_1(c_R s_W c_\xi + s_R s_\xi))\right]\bar{d}^m_{R\alpha}\not{Z} d^m_{R\beta} + \left[\frac{1}{2}Z_{uL\alpha\beta}(-g_L c_W s_\xi + g_1(s_R c_\xi - c_R s_W s_\xi))\right]\bar{u}^m_{L\alpha}\not{Z}' u^m_{L\beta} \\
& + \left[-\frac{1}{2}Z_{dL\alpha\beta}(-g_L c_W s_\xi + g_1(s_R c_\xi - c_R s_W s_\xi))\right]\bar{d}^m_{L\alpha}\not{Z}' d^m_{L\beta} + \left[\frac{1}{2}Z_{uR\alpha\beta}(g_R(c_R c_\xi + s_R s_W s_\xi) \right. \\
& \left. + g_1(s_R c_\xi - c_R s_W s_\xi))\right]\bar{u}^m_{R\alpha}\not{Z}' u^m_{R\beta} + \left[-\frac{1}{2}Z_{dR\alpha\beta}(g_R(c_R c_\xi + s_R s_W s_\xi) + g_1(s_R c_\xi - c_R s_W s_\xi))\right]\bar{d}^m_{R\alpha}\not{Z}' d^m_{R\beta}.
\end{aligned} \tag{87}$$

In Eq. (87), we used the notation of the mass eigenstates of neutral gauge fields,

$$\begin{pmatrix} A \\ Z \\ Z' \end{pmatrix} = \begin{pmatrix} c_W c_R & s_W & c_W s_R \\ -(s_W c_R c_\xi + s_R s_\xi) & c_W c_\xi & c_R s_\xi - s_W s_R c_\xi \\ s_W c_R s_\xi - s_R c_\xi & -c_W s_\xi & c_R c_\xi + s_W s_R s_\xi \end{pmatrix} \begin{pmatrix} B \\ W_L^3 \\ W_R^3 \end{pmatrix}, \tag{88}$$

where the mixing angles of the neutral gauge bosons satisfy the following equations:

$$g_L t_W = g_1 c_R, \qquad g_R t_R = g_1,$$
$$\tan 2\xi = -\frac{s_R^2 \sin 2\theta_R v_L^2}{s_W[v_R^2 + (s_R^4 - \frac{\sin^2 2\theta_R}{\sin^2 2\theta_W})v_L^2]}. \tag{89}$$

They are derived by taking the vacuum expectation value of the bidoublet Higgs field zero in the formulas of Ref. [14]. In order to acquire the derivation of the relation (88) and the definition of mixing parameters s_W, s_R, s_ξ, and e, see Ref. [14].

VI. THE APPROXIMATE FORMULAS FOR THE MIXING MATRICES

So far, we derive the exact formulas for the mixing matrices. In this section, we carry out the diagonalization of the mass matrices and determine the unitary matrices for the diagonalization. In Appendix B, we show the procedure of the diagonalization and the approximation. We have determined the submatrices of the unitary matrices V_{qL} and V_{qR} in Eqs. (74), (75), and (82). The approximate formulas on K_{uL} in Eq. (B25) and R_{uL} in Eq. (B6) are given as

$$K_{uL} = \begin{pmatrix} 1 & \frac{M_C y_{uL1} y^*_{uR21}}{M_U y_{uL2} y_{uR2}} & \frac{M_T y_{uL1} y^*_{uR31}}{M_U y_{uL3} y_{uR3}} \\ -\frac{M_C y_{uL1} y_{uR21}}{M_U y_{uL2} y_{uR2}} & 1 & \frac{M_T y_{uL2} y^*_{uR32}}{M_C y_{uL3} y_{uR3}} \\ \frac{M_T}{M_U} \frac{y_{uL1}(y_{uR32} y_{uR21} - y_{uR2} y_{uR31})}{y_{uR2} y_{uL3} y_{uR3}} & -\frac{M_T y_{uL2} y_{uR32}}{M_C y_{uL3} y_{uR3}} & 1 \end{pmatrix}, \tag{90}$$

$$R_{uL} = \begin{pmatrix} \frac{v_L}{M_U} y_{uL1} & 0 & 0 \\ \frac{v_L}{M_U} y_{uL21} & \frac{v_L}{M_C} y_{uL2} & 0 \\ \frac{v_L}{M_U} y_{uL31} & \frac{v_L}{M_C} y_{uL32} & \frac{v_L M_T}{D_T^2} y_{uL3} \end{pmatrix}, \tag{91}$$

where D_T denotes the mass eigenvalue of the lightest state of the heavy uptype quarks and the definition can be found in Eq. (B4). Similarly, the downtype mixing matrices K_{dL}, R_{dL} have following forms:

$$K_{dL} = \begin{pmatrix} 1 & \frac{M_S y_{dL1} y^*_{dR21}}{M_D y_{dL2} y_{dR2}} & \frac{M_B y_{dL1} y^*_{dR31}}{M_D y_{dL3} y_{dR3}} \\ -\frac{M_S y_{dL1} y_{dR21}}{M_D y_{dL2} y_{dR2}} & 1 & \frac{M_B y_{dL2} y^*_{dR32}}{M_S y_{dL3} y_{dR3}} \\ \frac{M_B}{M_D} \frac{y_{dL1}(y_{dR32} y_{dR21} - y_{dR2} y_{dR31})}{y_{dR2} y_{dL3} y_{dR3}} & -\frac{M_B y_{dL2} y_{dR32}}{M_S y_{dL3} y_{dR3}} & 1 \end{pmatrix}, \tag{92}$$

$$R_{dL} = \begin{pmatrix} \frac{v_L}{M_D} y_{dL1} & 0 & 0 \\ \frac{v_L}{M_D} y_{dL21} & \frac{v_L}{M_S} y_{dL2} & 0 \\ \frac{v_L}{M_D} y_{dL31} & \frac{v_L}{M_S} y_{dL32} & \frac{v_L}{M_B} y_{dL3} \end{pmatrix}. \tag{93}$$

The approximate forms for K_{uR}, R_{uR}, K_{dR}, and R_{dR} are also derived using the formulas

$$K_{qR} = y_{qR} v_R S_{qL} / d_q, \qquad R_{qR} = y_{qR} v_R T_{QL} / \tilde{D}_Q, \tag{94}$$

where Eq. (94) is derived using Eq. (A1). By substituting the approximate formulas for S_{qL} and T_{QL} given in Eqs. (B7) and (B5), K_{qR} and R_{qR} are given as

$$\begin{aligned} K_{uR} &= -y_{\Delta uR} \frac{D_U}{D_{0U}^2} y^\dagger_{\Delta uL} K_{uL} \frac{v_L v_R}{d_u} \\ &= -\begin{pmatrix} 1 & \frac{M_C}{M_U} \frac{y_{uR1} y^*_{uL21}}{y_{uL2} y_{uR2}} & \frac{D_T}{M_U} \frac{y_{uR1} y^*_{uL31}}{y_{uL3} y_{uR3}} \\ \frac{M_T}{M_C} \frac{y^*_{uL32}(y_{uR21} y_{uR32} - y_{uR2} y_{uR31})}{y_{uR1} y_{uL3} y_{uR3}} - \frac{M_C}{M_U} \frac{y_{uR21} y_{uR21} y^*_{uL21}}{y_{uR1} y_{uL2} y_{uR2}} & 1 & \frac{D_T}{M_C} \frac{y_{uR2} y^*_{uL32}}{y_{uL3} y_{uR3}} \\ \left(1 - \frac{M_T^2}{D_T^2}\right) \frac{y_{uR2} y_{uR31} - y_{uR21} y_{uR32}}{y_{uR1} y_{uR2}} & \left(1 - \frac{M_T^2}{D_T^2}\right) \frac{y_{uR32}}{y_{uR2}} & \frac{M_T}{D_T} + \frac{D_T}{M_C} \frac{y^*_{uL32} y_{uR32}}{y_{uL3} y_{uR3}} \end{pmatrix}, \end{aligned} \tag{95}$$

$$R_{uR} = y_{\Delta uR} \frac{v_R}{D_{0U}} = \begin{pmatrix} \frac{v_R}{M_U} y_{uR1} & 0 & 0 \\ \frac{v_R}{M_U} y_{uR21} & \frac{v_R}{M_C} y_{uR2} & 0 \\ \frac{v_R}{M_U} y_{uR31} & \frac{v_R}{M_C} y_{uR32} & \frac{v_R}{D_T} y_{uR3} \end{pmatrix}, \tag{96}$$

$$K_{dR} = -y_{\Delta dR} \frac{1}{D_D} y^\dagger_{\Delta dL} K_{dL} \frac{v_L v_R}{d_d} = -\begin{pmatrix} 1 & \frac{M_S}{M_D} \frac{y_{dR1} y^*_{dL21}}{y_{L2} y_{dR2}} & \frac{M_B}{M_D} \frac{y_{dR1} y^*_{dL31}}{y_{dL3} y_{dR3}} \\ -K_{dR21} & 1 & \frac{M_B}{M_S} \frac{y_{dR2} y^*_{dL32}}{y_{dL3} y_{dR3}} \\ -K_{dR31} & \frac{M_S}{M_D} \frac{y^*_{dL21} y_{dR31}}{y_{dL2} y_{dR2}} - \frac{M_B}{M_S} \frac{y^*_{dL32} y_{dR32} y_{dR32}}{y_{dR2} y_{dL3} y_{dR3}} & 1 \end{pmatrix}, \tag{97}$$

$$K_{dR21} = -\frac{M_B}{M_S}\frac{y^*_{dL32}(y_{dR21}y_{dR32} - y_{dR2}y_{dR31})}{y_{dR1}y_{dL3}y_{dR3}} + \frac{M_S}{M_D}\frac{y^*_{dL21}y_{dR21}y_{dR21}}{y_{dR1}y_{dL2}y_{dR2}},$$
$$K_{dR31} = -\frac{M_B}{M_S}\frac{y^*_{dL32}(y_{dR21}y_{dR32}y_{dR32} - y_{dR2}y_{dR31}y_{dR32})}{y_{dR1}y_{dR2}y_{dL3}y_{dR3}} + \frac{M_S}{M_D}\frac{y^*_{dL21}y_{dR21}y_{dR31}}{y_{dR1}y_{dL2}y_{dR2}},$$
$$R_{dR} = y_{\Delta dR}\frac{v_R}{D_D} = \begin{pmatrix} \frac{v_R}{M_D}y_{dR1} & 0 & 0 \\ \frac{v_R}{M_D}y_{dR21} & \frac{v_R}{M_S}y_{dR2} & 0 \\ \frac{v_R}{M_D}y_{dR31} & \frac{v_R}{M_S}y_{dR32} & \frac{v_R}{M_B}y_{dR3} \end{pmatrix}, \tag{98}$$

where the definition of D_{0U} can be found in Eq. (B3).

We summarize the results of the mixing matrices. The left-handed charged current $\mathcal{V}_L$ is determined in a good approximation as follows:

$$\mathcal{V}_L \simeq \begin{pmatrix} U_L & U_L R_{uL} \\ R^\dagger_{dL}U_L & R^\dagger_{dL}U_L R_{uL} \end{pmatrix}, \tag{99}$$

where we ignore the corrections suppressed by heavy quark masses by setting $K_{uL} \simeq K_{dL} \simeq K_{dR} \simeq 1$. The 3×3 submatrix, which corresponds to light quark mixings, is mostly determined by the 3×3 unitary matrix U_L. In our parametrization, U_L includes five *CP* violating phases α_{L1}, α_{L1}, δ_L, β_{L1}, β_{L2}. The mixing between the light quark and heavy quark is suppressed by a factor of $\frac{v_L}{D_{0Uii}} \ll 1$ or $\frac{v_L}{D_{Di}} \ll 1$. The mixing among heavy quarks is suppressed by a factor of the product $\frac{v_L^2}{D_{0Uii}D_{Dj}}$. One finds that the mixing of the heavy uptype quarks and the light downtype quarks corresponding to $\mathcal{V}_{R6i}$ ($i = 1$–3) is large. The large mixing occurs because the component of R_{uR33} is not suppressed. This phenomenon is related to the enhancement mechanism of the top quark mass as shown in Refs. [8,9].

The flavor changing neutral current (FCNC) for up quarks is determined by Z_{uL}, Z_{uR} in Eq. (85). We first show the approximate formulas for the FCNC among the light uptype quarks, Z_{uLij} ($i, j = 1$–3). They are derived using the relation, $Z_{uLij} = (K^\dagger_{uL}K_{uL})_{ij} \simeq \delta_{ij} - (S^\dagger_{uL}S_{uL})_{ij}$,

$$Z_{uL11} \simeq 1 - \left(\frac{m_u}{v_R}\right)^2\left[\sum_{k=1}^{2}(y^{-1}_{\Delta uR})^*_{k1}(y^{-1}_{\Delta uR})_{k1} + \left(\frac{M_T}{D_T}\right)^4|(y^{-1}_{\Delta uR})_{31}|^2\right],$$
$$Z_{uL12} \simeq -\frac{m_u m_c}{v_R^2}\left[(y^{-1}_{\Delta uR})_{22}(y^{-1}_{\Delta uR})^*_{21} + \left(\frac{M_T}{D_T}\right)^4(y^{-1}_{\Delta uR})_{31}(y^{-1}_{\Delta uR})_{32}\right],$$
$$Z_{uL13} \simeq \frac{m_u m_c}{v_R^2}\left(\frac{1}{y_{uL2}}(y^{-1}_{\Delta uR})^*_{21}(y^{-1}_{\Delta uR})_{32}(y^{-1}_{\Delta uR})^*_{33}\right) - \frac{m_u m_t}{v_R^2}\left(\frac{M_T}{D_T}\right)^3(y^{-1}_{\Delta uR})_{31}(y^{-1}_{\Delta uR})_{33},$$
$$Z_{uL22} \simeq 1 - \left(\frac{m_c}{v_R}\right)^2\left[(y^{-1}_{\Delta uR})^2_{22} + \left(\frac{M_T}{D_T}\right)^4|(y^{-1}_{\Delta uR})_{31}|^2\right],$$
$$Z_{uL23} \simeq -\left(\frac{m_c}{v_R}\right)^2\frac{y^*_{uL32}}{y^2_{uR2}y_{uL2}} - \frac{m_c m_t}{v_R^2}\left(\frac{M_T}{D_T}\right)^3(y^{-1}_{\Delta uR})^*_{32}(y^{-1}_{\Delta uR})^{-1}_{33},$$
$$Z_{uL33} \simeq 1 - \left(\frac{m_c}{v_R}\right)^2\frac{|y_{uL32}|^2}{y^2_{uL2}y^2_{uR2}} - \left(\frac{m_t}{v_R}\right)^2\left(\frac{M_T}{D_T}\right)(y^{-1}_{\Delta uR})^2_{33}. \tag{100}$$

We note that the *CP* violation of the tree-level FCNC for the left-handed current is determined by the *CP* violating phases in the right-handed Yukawa couplings $y_{uRij}(i > j)$ and left-handed Yukawa coupling y_{uL32}. The strength of the FCNC is naturally suppressed by the $SU(2)_R$ breaking scale. The FCNC of the left-handed current between the light uptype quarks and the heavy ones can be written as

$$K^\dagger_{uL}R_{uL} \simeq \begin{pmatrix} \frac{m_u}{v_R}\frac{1}{y_{uR1}} & -\frac{m_u}{v_R}\frac{y^*_{uR21}}{y_{uR1}y_{uR2}} & \left(\frac{M_T}{D_T}\right)^2\frac{m_u}{v_R}\frac{y^*_{uR21}y^*_{uR32}-y_{uR2}y^*_{uR31}}{y_{uR1}y_{uR2}y_{uR3}} \\ \frac{m_u}{v_R}\frac{y_{uL21}}{y_{uR1}y_{uL1}} & \frac{m_c}{v_R}\frac{1}{y_{uR2}} & -\left(\frac{M_T}{D_T}\right)^2\frac{m_c}{v_R}\frac{y^*_{uR32}}{y_{uR2}y_{uR3}} \\ \frac{m_u}{v_R}\frac{y_{uL31}}{y_{uL1}y_{uR1}} & \frac{m_c}{v_R}\frac{y_{uL32}}{y_{uL2}y_{uR2}} & \left(\frac{M_T}{D_T}\right)\frac{m_t}{v_R}\frac{1}{y_{uR3}} \end{pmatrix}. \tag{101}$$

The strength of the FCNC between the heavy left-handed uptype quark and the light uptype quark is suppressed by the $SU(2)_R$ breaking scale. We also note that CP violation is determined by the phases of y_{L32} and y_{Rij}, $(i > j)$. The FCNC of the left-handed current among the heavy uptype quarks is given as

$$R_{uL}^{\dagger} R_{uL} \simeq \begin{pmatrix} \left(\frac{m_u}{v_R}\right)^2 \frac{y_{uL1}^2 + |y_{uL21}|^2 + |y_{uL31}|^2}{y_{uL1}^2 y_{uR1}^2} & \frac{m_u m_c}{v_R^2} \frac{y_{uL2} y_{uL21}^* + y_{uL31}^* y_{uL32}}{y_{uL1} y_{uL2} y_{uR1} y_{uR2}} & \frac{M_T}{D_T} \frac{m_u m_t}{v_R^2} \frac{y_{uL31}^*}{y_{uL1} y_{uR1} y_{uR3}} \\ \frac{m_u m_c}{v_R^2} \frac{y_{uL2} y_{uL21} + y_{uL31} y_{uL32}^*}{y_{uL1} y_{uL2} y_{uR1} y_{uR2}} & \left(\frac{m_c}{v_R}\right)^2 \frac{y_{uL2}^2 + |y_{uL32}|^2}{y_{uL2}^2 y_{uR2}^2} & \frac{M_T}{D_T} \frac{m_c m_t}{v_R^2} \frac{y_{uL32}^*}{y_{uL2} y_{uR2} y_{uR3}} \\ \frac{M_T}{D_T} \frac{m_u m_t}{v_R^2} \frac{y_{uL31}}{y_{uL1} y_{uR1} y_{uR3}} & \frac{M_T}{D_T} \frac{m_c m_t}{v_R^2} \frac{y_{uL32}}{y_{uL2} y_{uR2} y_{uR3}} & \left(\frac{M_T}{D_T}\right)^2 \frac{m_t^2}{v_R^2} \frac{1}{y_{uR3}^2} \end{pmatrix}. \tag{102}$$

All the components are suppressed by a factor of $\frac{1}{v_R^2}$. The CP violation of the FCNC is determined by the left-handed Yukawa coupling y_{uL32}. Similarly, the FCNCs for the light right-handed uptype quarks are given as

$$Z_{uRij} = (K_{uR}^{\dagger} K_{uR})_{ij} \simeq \delta_{ij} - (S_{uR}^{\dagger} S_{uR})_{ij} \simeq \delta_{ij} - \frac{m_u^i m_u^j}{v_L^2} \sum_{k \geq i,j} (y_{\Delta uL}^{-1})_{ki}^* (y_{\Delta uL}^{-1})_{kj}, \quad (\text{for } i, j = 1, 2, 3). \tag{103}$$

Note that the flavor diagonal coupling Z_{uR33} of the right-handed top quark current $\overline{t_R^m} \gamma_\mu t_R^m$ is suppressed,

$$Z_{uR33} \simeq 1 - \frac{m_t^2}{y_{uL3}^2 v_L^2}. \tag{104}$$

The suppression of the FCNC for the right-handed current is weaker than that of the left-handed one. It is suppressed by a factor of $\frac{1}{v_L^2}$. The CP violation of the FCNC in Eq. (103) is determined by a phase of y_{uL32}. We note that the same phase appears in the FCNC of the left-handed current among the heavy uptype quarks. Below we show all the components of the FCNC couplings for the right-handed currents between the light uptype quark and the heavy uptype quark:

$$\begin{aligned}
Z_{uR14} &= -\frac{m_u}{v_L} \frac{1}{y_{uL1} y_{uR1}^2 y_{uR2}} \left[y_{uR1}^2 y_{uR2} + \left(1 - \left(\frac{m_t M_T}{v_L v_R}\right)^2 \frac{1}{y_{uL3}^2 y_{uR3}^2}\right) (y_{uR2} |y_{uR31}|^2 - y_{uR21}^* y_{uR31} y_{uR32}^*) \right], \\
Z_{uR15} &= \frac{m_c}{v_L} \left[1 - \left(\frac{m_t M_T}{v_L v_R}\right)^2 \frac{1}{y_{uL3}^2 y_{uR3}^2} \right] \frac{y_{uR32} (y_{uR21}^* y_{uR32}^* - y_{uR2} y_{uR31}^*)}{y_{uL2} y_{uR1} y_{uR2}^2}, \\
Z_{uR16} &= \frac{m_t}{v_L} \left[1 - \left(\frac{m_t M_T}{v_L v_R}\right)^2 \frac{1}{y_{uL3}^2 y_{uR3}^2} \right] \frac{y_{uR21}^* y_{uR32}^* - y_{uR2} y_{uR31}^*}{y_{uR1} y_{uR2} y_{uL3}}, \\
Z_{uR24} &= -\frac{m_u}{v_L} \frac{1}{y_{uL1} y_{uR1} y_{uR2}} \left[y_{uR2} y_{uR21} + \left(1 - \left(\frac{m_t M_T}{v_L v_R}\right)^2 \frac{1}{y_{uL3}^2 y_{uR3}^2}\right) y_{uR31} y_{uR32}^* \right], \\
Z_{uR25} &= -\frac{m_c}{v_L} \frac{1}{y_{uL2} y_{uR2}^2} \left[y_{uR2}^2 + \left(1 - \left(\frac{m_t M_T}{v_L v_R}\right)^2 \frac{1}{y_{uL3}^2 y_{uR3}^2}\right) |y_{uR32}|^2 \right], \\
Z_{uR26} &= -\frac{m_t}{v_L} \frac{y_{uR32}^*}{y_{uR2} y_{uL3}} \left[1 - \left(\frac{m_t M_T}{v_L v_R}\right)^2 \frac{1}{y_{uL3}^2 y_{uR3}^2} \right], \\
Z_{uR34} &= -\frac{m_u}{v_L} \frac{1}{y_{uL1} y_{uR1}} \left[\frac{m_c}{m_t} \frac{y_{uL32} (y_{uR2} y_{uR21} + y_{R31} y_{R32}^*)}{y_{uL2} y_{uR2}} + \frac{m_t M_T}{v_L v_R} \frac{y_{uR31}}{y_{uL3} y_{uR3}} \right], \\
Z_{uR35} &= -\frac{m_c}{v_L} \frac{1}{y_{uL2} y_{uR2}} \left[\frac{m_c}{m_t} \frac{y_{uL32} (y_{uR2}^2 + |y_{uR32}|^2)}{y_{uL2} y_{uR2}} + \frac{m_t M_T}{v_L v_R} \frac{y_{uR32}}{y_{uL3} y_{uR3}} \right], \\
Z_{uR36} &= -\frac{m_t}{v_L} \frac{1}{y_{uL3}} \left[\frac{m_c}{m_t} \frac{y_{uL32} y_{uR32}^*}{y_{uL2} y_{uR2}} + \frac{m_t M_T}{v_L v_R} \frac{1}{y_{uL3} y_{uR3}} \right].
\end{aligned} \tag{105}$$

We observe that CP violation is determined by the four phases, $\mathrm{Im}(y_{uRij})$ $(i > j)$ and $\mathrm{Im}(y_{uL32})$. The FCNC among the heavy right-handed uptype quarks is given as

$$Z_{uRi+3,j+3} \simeq (R^\dagger_{uR} R_{uR})_{ij} \simeq \frac{m_u^i m_u^j}{v_L^2} \frac{1}{y_{uLi} y_{uRi} y_{uLj} y_{uRj}} \sum_{k \geq i,j}^{3} y^*_{uRki} y_{uRkj}. \tag{106}$$

Note that the flavor diagonal coupling $Z_{uR66} \simeq \frac{m_t^2}{y_{uL3}^2 v_L^2}$ is not suppressed, which is in contrast to the coupling Z_{uR33} in Eq. (104).

Next we show the FCNC of the down-quark sector in Eq. (86). The approximate formulas for the FCNC for the left-handed currents is given by

$$\begin{aligned}
Z_{dLij} &= \delta_{ij} - \frac{m_d^i m_d^j}{v_R^2} \sum_{k \geq i,j}^{3} (y^{-1}_{\Delta dR})^*_{ki} (y^{-1}_{\Delta dR})_{kj}, \qquad (\text{for } i, j = 1, 2, 3), \\
K^\dagger_{dL} R_{dL} &\simeq \begin{pmatrix} \frac{m_d}{v_R} \frac{1}{y_{dR1}} & -\frac{m_d}{v_R} \frac{y^*_{dR21}}{y_{dR1} y_{dR2}} & \frac{m_d}{v_R} \frac{y^*_{dR21} y^*_{dR32} - y_{dR2} y^*_{dR31}}{y_{dR1} y_{dR2} y_{dR3}} \\ \frac{m_d}{v_R} \frac{y_{dL21}}{y_{dL1} y_{dR1}} & \frac{m_s}{v_R} \frac{1}{y_{dR2}} & -\frac{m_s}{v_R} \frac{y^*_{dR32}}{y_{dR2} y_{dR3}} \\ \frac{m_d}{v_R} \frac{y_{dL31}}{y_{dL1} y_{dR1}} & \frac{m_s}{v_R} \frac{y_{dL32}}{y_{dL2} y_{dR2}} & \frac{m_b}{v_R} \frac{1}{y_{dR3}} \end{pmatrix}, \\
(R^\dagger_{dL} R_{dL})_{ij} &\simeq \frac{m_d^i m_d^j}{v_R^2} \frac{1}{y_{dLi} y_{dRi} y_{dLj} y_{dRj}} \sum_{k \geq i,j}^{3} y^*_{dLki} y_{dLkj}, \qquad (\text{for } i, j = 1, 2, 3).
\end{aligned} \tag{107}$$

As we can easily see from Eq. (108), the FCNC of the down-quark sector is much simpler than that of the up-quark sector. The FCNC for the left-handed current among the light downtype quarks is suppressed by a factor of $\frac{1}{v_R^2}$. The same suppression occurs in the FCNC among heavy quarks. The FCNC between the heavy quark and light quark is suppressed by a factor of $\frac{1}{v_R}$. For the right-handed current, the FCNC couplings are given as

$$\begin{aligned}
Z_{dRij} &= (K^\dagger_{dR} K_{dR})_{ij} \simeq \delta_{ij} - (S^\dagger_{dR} S_{dR})_{ij} \simeq \delta_{ij} - \frac{m_d^i m_d^j}{v_L^2} \sum_{k \geq i,j}^{3} (y^{-1}_{\Delta dL})^*_{ki} (y^{-1}_{\Delta dL})_{kj}, \quad (\text{for } i, j = 1, 2, 3), \\
Z_{dR14} &= -\frac{m_d}{v_L} \frac{1}{y_{dL1}}, \\
Z_{dR15} &= \frac{m_d}{v_L} \frac{y_{dL21} y^*_{dR21} (y_{dR2} y^*_{dR21} + y^*_{dR31} y_{dR32})}{y_{dL1} y^2_{dR1} y_{dL2} y_{dR2}} \\
&\quad + \frac{m_s^2}{v_L m_b} \frac{y_{dL32} [y^2_{dR2} (y_{dR2} y^*_{dR31} - y^*_{dR21} y^*_{dR32}) + |y_{dR32}|^2 (y_{dR2} y^*_{dR31} - y^*_{dR21} y^*_{dR32})]}{y_{dR1} y^2_{dL2} y^3_{dR2}}, \\
Z_{dR16} &= \frac{m_s}{v_L} \frac{y_{dL32} y^*_{dR32} (y_{dR2} y^*_{dR31} - y^*_{dR21} y^*_{dR32})}{y_{dR1} y_{dL2} y^2_{dR2} y_{dL3}} + \frac{m_d m_b}{m_s v_L} \frac{y_{dL21} y^*_{dR21} y^*_{dR31}}{y_{dL1} y^2_{dR1} y_{dL3}}, \\
Z_{dR24} &= -\frac{m_d}{v_L} \frac{y_{dR21}}{y_{dL1} y_{dR1}}, \\
Z_{dR25} &= -\frac{m_s}{v_L} \frac{1}{y_{dL2}}, \\
Z_{dR26} &= \frac{m_s}{v_L} \frac{y_{dL32} y^*_{dR32} y^*_{dR32}}{y_{dL2} y^2_{dR2} y_{dL3}} - \frac{m_d m_b}{m_s v_L} \frac{y_{dL21} y^*_{dR31}}{y_{dL1} y_{dR1} y_{dL3}}, \\
Z_{dR34} &= -\frac{m_d}{v_L} \frac{y_{dR31}}{y_{dL1} y_{dR1}}, \\
Z_{dR35} &= -\frac{m_s}{v_L} \frac{y_{dR32}}{y_{dL2} y_{dR2}}, \\
Z_{dR36} &= -\frac{m_b}{v_L} \frac{1}{y_{dL3}}, \\
(R^\dagger_{dR} R_{dR})_{ij} &\simeq \frac{m_d^i m_d^j}{v_L^2} \frac{1}{y_{dLi} y_{dRi} y_{dLj} y_{dRj}} \sum_{k \geq i,j}^{3} y^*_{dRki} y_{dRkj}.
\end{aligned} \tag{108}$$

The FCNC among the light downtype quarks is suppressed by a factor of $\frac{m_{di}m_{dj}}{v_L^2}$. Since the downtype quarks' masses are smaller than v_L, the FCNC for the down-quark sector is naturally suppressed. We observe that the suppression of the FCNC among the heavy quarks is similar to the light-quark case. The FCNC from heavy quarks to light quarks is also suppressed by a factor of $\frac{1}{v_L}$, which is weaker than that for the left-handed current. However, it is much suppressed compared with that of the corresponding up-quark case. Concerning *CP* violation, we observe that the *CP* violation of the tree-level FCNCs are determined by the imaginary parts of the triangular matrices of the Yukawa couplings y_Δ.

VII. CONCLUSION

We study *CP* violation and flavor mixings in the quark sector of the universal seesaw model. We find the number of independent parameters in a specific weak basis. The basis is obtained using all the freedom of the WBT. There is no redundancy due to WBT in the parameters left. Therefore, the number of the parameters in such weak basis corresponds to the number of independent parameters of the model. The results of the number of parameters (real parts and imaginary parts) are summarized in Table I and in Table II for the case where the singlet quark generation number is identical to the doublet quark generation number. The number of *CP* violating parameters is also obtained by counting the number of *CP* invariant conditions that are nontrivially satisfied, which agrees with the one in the specific weak basis.

For the three-generation model, the number of *CP* violating phases is 19. The corresponding *CP* violating WB invariants are constructed in terms of the Yukawa matrices and singlet quark matrices. To identify the *CP* violation and mixings in mass eigenstates of quarks, we study the unitary matrices V_{qL}, $V_{qR} (q = u, d)$ which are used to diagonalize the 6×6 mass matrices for the up-quark sector and down-quark sector. These unitary matrices are related to the mixing matrices for the charged currents and neutral currents so that the 3×6 submatrix of the unitary matrices in V_{qL}, V_{qR} enters into both charged currents $\mathcal{V}_L$, $\mathcal{V}_R$ and the neutral currents $\mathcal{Z}_{uL}$, $\mathcal{Z}_{uR}$, $\mathcal{Z}_{dL}$, and $\mathcal{Z}_{dR}$. The *CP* violation of the tree-level FCNC is determined by the imaginary parts of the triangular matrices. Therefore, we conclude that the FCNC is determined by the WB invariants I_1–I_8. The mixing matrices for the charged currents also depend on the 3×3 unitary matrices U_L and U_R defined by Eqs. (68) and (69).

We obtain the mixing matrix elements by carrying out the approximate diagonalization so that we have some insight on the mixings and *CP* violation in terms of the mass eigenstates. As discussed in Sec. VI, we identify all 19 *CP* violating phases for the three-generation model in the couplings of charged currents and the neutral currents in terms of the mass eigenstates.

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APPENDIX A: EXACT FORMULAS OF MATRICES FOR THE DIAGONALIZATION

In this appendix, we give the derivation of the formulas for Eq. (94). We also collect the formulas that the submatrices of V_{qL} satisfy [Eqs. (A4)–(A12)]. The proof of the formulas is given below. One starts with Eqs. (74) and (75) for the diagonalization of the mass matrix $\mathcal{M}_Q$, which leads to the following relation:

$$V_{qR} = \mathcal{M}_Q^\dagger V_{qL} \begin{pmatrix} \frac{1}{d_q} & 0 \\ 0 & \frac{1}{\tilde{D}_Q} \end{pmatrix} = \begin{pmatrix} y_{\Delta qR} v_R S_{qL}/d_q & y_{\Delta qR} v_R T_{QL}/\tilde{D}_Q \\ (y_{\Delta qL}^\dagger v_L K_{qL} + D_Q S_{qL})/d_q & (y_{\Delta qL}^\dagger v_L R_{qL} + D_Q T_{QL})/\tilde{D}_Q \end{pmatrix}, \tag{A1}$$

where $(q, Q, \mathcal{Q})$ denotes $(u, U, \mathcal{U})$ or $(d, D, \mathcal{D})$. Equation (A1) leads to the formulas in Eq. (94). Since V_{qL} satisfies the eigenvalue equation,

$$V_{qL}^\dagger \mathcal{M}_Q \mathcal{M}_Q^\dagger V_{qL} = \begin{pmatrix} d_q^2 & 0 \\ 0 & \tilde{D}_Q^2 \end{pmatrix}, \tag{A2}$$

where

$$\mathcal{M}_Q \mathcal{M}_Q^\dagger = \begin{pmatrix} y_{\Delta qL} y_{\Delta qL}^\dagger v_L^2 & y_{\Delta qL} v_L D_Q \\ D_Q v_L y_{\Delta qL}^\dagger & y_{\Delta qR}^\dagger y_{\Delta qR} v_R^2 + D_Q^2 \end{pmatrix}, \tag{A3}$$

the submatrices of K_{qL}, S_{qL}, R_{qL}, and T_{QL} satisfy

$$y_{\Delta qL} y_{\Delta qL}^\dagger v_L^2 K_{qL} + y_{\Delta qL} v_L D_Q S_{qL} = K_{qL} d_q^2, \tag{A4}$$

$$D_Q y^\dagger_{\Delta qL} v_L K_{qL} + (y^\dagger_{\Delta qR} y_{\Delta qR} v_R^2 + D_Q^2) S_{qL} = S_{qL} d_q^2, \tag{A5}$$

$$y_{\Delta qL} y^\dagger_{\Delta qL} v_L^2 R_{qL} + y_{\Delta qL} v_L D_Q T_{QL} = R_{qL} \tilde{D}_Q^2, \tag{A6}$$

$$D_Q y^\dagger_{\Delta qL} v_L R_{qL} + (y^\dagger_{\Delta qR} y_{\Delta qR} v_R^2 + D_Q^2) T_{QL} = T_{QL} \tilde{D}_Q^2. \tag{A7}$$

They also satisfy the unitarity conditions. $V^\dagger_{qL} V_{qL} = 1$ leads to

$$K^\dagger_{qL} K_{qL} + S^\dagger_{qL} S_{qL} = 1, \tag{A8}$$

$$R^\dagger_{qL} R_{qL} + T^\dagger_{QL} T_{QL} = 1, \tag{A9}$$

$$K^\dagger_{qL} R_{qL} + S^\dagger_{qL} T_{QL} = 0. \tag{A10}$$

$V_{qL} V^\dagger_{qL} = 1$ leads to

$$K_{qL} K^\dagger_{qL} + R_{qL} R^\dagger_{qL} = 1, \tag{A11}$$

$$S_{qL} S^\dagger_{qL} + T_{QL} T^\dagger_{QL} = 1, \qquad K_{qL} S^\dagger_{qL} + R_{qL} T^\dagger_{QL} = 0. \tag{A12}$$

Using the equations above, one can rewrite V_{qR} as

$$V_{qR} = \begin{pmatrix} y_{\Delta qR} v_R S_{qL}/d_q & y_{\Delta qR} v_R T_{QL}/\tilde{D}_Q \\ \frac{1}{y_{\Delta qL} v_L} K_{qL} d_q & \frac{1}{y_{\Delta qL} v_{qL}} R_{qL} \tilde{D}_Q \end{pmatrix}. \tag{A13}$$

APPENDIX B: DERIVATION OF THE APPROXIMATE FORMULAS

In this appendix, we show the derivation for the approximate formulas Eqs. (90)–(98) for V_{qL} and V_{qR}. The approximate diagonalization of the mass matrix of the universal seesaw model has been carried out in the previous works [9,20]. Compared to the previous works, we relax the condition imposed on the singlet mass parameter M_T. In this work, we do not assume that the parameter is very small compared to the $SU(2)_R$ breaking scale. We also keep all the *CP* violating parameters in the approximation so that we can keep track of the *CP* violating phases in the mixing matrices.

We show the derivation for the uptype quark case. The derivation for the down-quark sector follows in the same way as that of the up-quark sector. The submatrices of K_{uL}, S_{uL}, R_{uL}, and T_{uL} satisfy Eqs. (A4)–(A7). One also notes that S_{uL} and R_{uL} are smaller than K_{uL} and T_{uL}. Let us start with the Hermitian matrix,

$$H_{\mathcal{U}} = y^\dagger_{\Delta uR} y_{\Delta uR} v_R^2 + D_U^2. \tag{B1}$$

By neglecting the small contribution proportional to R_{uL}, Eq. (A7) is rewritten as

$$T^{0\dagger}_{UL} H_{\mathcal{U}} T^0_{UL} = \tilde{D}_U^2, \tag{B2}$$

where we denote the leading form for T_{uL} as T^0_{uL} and we use $T^{0\dagger}_{uL} T^0_{uL} = 1$. The dominant term of $H_{\mathcal{U}}$ is

$$H_{\mathcal{U}} \sim D_{0U}^2 \equiv \begin{pmatrix} M_U^2 & 0 & \\ 0 & M_C^2 & 0 \\ 0 & 0 & D_T^2 \end{pmatrix}, \tag{B3}$$

$$D_T = \sqrt{y^2_{uR3} v_R^2 + M_T^2}. \tag{B4}$$

Therefore,

$$T^0_{uL} \simeq 1, \qquad \tilde{D}_U \simeq D_{0U}. \tag{B5}$$

Then one can solve Eq. (A6) for R_{uL},

$$R_{uL} \simeq y_{\Delta uL} v_L \frac{D_U}{D_{0U}^2}. \tag{B6}$$

In Eq. (A5), by neglecting the term proportional to d^2_{uL}, one can solve S_{uL} as

$$S_{uL} \simeq -\frac{1}{H_{\mathcal{U}}} D_U y^\dagger_{\Delta uL} v_L K_{uL}. \tag{B7}$$

Then, V_{uL} is approximately given as

$$V_{uL} = \begin{pmatrix} K_{uL} & y_{\Delta uL} v_L \frac{D_U}{D_{0U}^2} \\ -\frac{D_U v_L}{D_{0U}^2} y^\dagger_{\Delta uL} K_{uL} & 1 \end{pmatrix}, \tag{B8}$$

where we use the approximation $\mathcal{H}_{\mathcal{U}} \simeq D_{0U}^2$. One can also substitute S_{uL} in Eq. (B7) and R_{uL} in Eq. (B6) into Eq. (A13) and obtain V_{uR},

$$V_{uR} = \begin{pmatrix} -y_{\Delta uR} v_R \frac{D_U}{D_{0U}^2} y^\dagger_{\Delta uL} K_{uL} \frac{v_L}{d_u} & y_{\Delta uR} \frac{v_R}{D_{0U}} \\ \frac{1}{y_{\Delta uL} v_L} K_{uL} d_u & \frac{D_U}{D_{0U}} \end{pmatrix}. \tag{B9}$$

Both V_{uL} and V_{uR} can be determined once the submatrix K_{uL} is fixed. The equation which determines K_{uL} is obtained as

$$v_L^2 \mathcal{H} K_{uL} = K_{uL} d_u^2, \tag{B10}$$

where $\mathcal{H}$ is defined as

$$\mathcal{H} = y_{\Delta uL}\left(1 - D_U \frac{1}{H_{\mathcal{U}}} D_U\right) y^\dagger_{\Delta uL}. \tag{B11}$$

When deriving Eq. (B10), Eq. (B7) is substituted into Eq. (A4). Using the approximation $\det H_{\mathcal{U}} \simeq H_{\mathcal{U}11} H_{\mathcal{U}22} H_{\mathcal{U}33} \simeq M_U^2 M_C^2 H_{\mathcal{U}33}$, $H_{\mathcal{U}11} \simeq M_U^2$, $H_{\mathcal{U}22} \simeq M_C^2$, and by keeping the leading term in each matrix element, one obtains

$$\mathcal{H}/v_R^2 = \begin{pmatrix} \frac{y_{uL1}^2}{M_U^2}\left(y_{uR1}^2 + |y_{uR21}|^2 + \frac{|M_T|^2|y_{uR31}|^2}{D_T^2}\right) & \frac{y_{uL1}y_{uL2}}{M_U M_C}\left(y_{uR21}^* y_{uR2} + \frac{y_{uR31}^* y_{uR32} M_T^2}{D_T^2}\right) & \frac{M_T}{M_U}\frac{y_{uL1}y_{uL3}y_{uR31}^* y_{uR3}}{D_T^2} \\ \frac{y_{uL1}y_{uL2}}{M_U M_C}\left(y_{uR21}y_{uR2} + \frac{y_{uR32}^* y_{uR31} M_T^2}{D_T^2}\right) & \frac{y_{uL2}^2}{M_C^2}\left(y_{uR2}^2 + \frac{|y_{uR32}|^2 M_T^2}{D_T^2}\right) & \frac{M_T}{M_C}\frac{y_{uL2}y_{uL3}y_{uR32}^* y_{uR3}}{D_T^2} \\ \frac{M_T}{M_U}\frac{y_{uL1}y_{uL3}y_{uR3}y_{uR31}}{D_T^2} & \frac{M_T}{M_C}\frac{y_{uL2}y_{uL3}y_{uR3}y_{uR32}}{D_T^2} & \frac{y_{uL3}^2 y_{uR3}^2}{D_T^2} \end{pmatrix}. \tag{B12}$$

Now we solve the eigenvalue equation for K_{uL}. The eigenvalues of $\mathcal{H}$ are related to the up, charm, and top quark masses squared. We write the eigenvalue equation as

$$\mathcal{H}\begin{pmatrix} \mathbf{u} \\ u_3 \end{pmatrix} = \frac{m_i^2}{v_L^2}\begin{pmatrix} \mathbf{u} \\ u_3 \end{pmatrix}, \tag{B13}$$

where $\mathbf{u}^T = (u_1, u_2)$ and $i = u, c, t$. We can rewrite Eq. (B13) as

$$\begin{pmatrix} \mathcal{H}_{11} & \mathcal{H}_{12} \\ \mathcal{H}_{12}^* & \mathcal{H}_{22} \end{pmatrix}\mathbf{u} + \begin{pmatrix} \mathcal{H}_{13} \\ \mathcal{H}_{23} \end{pmatrix}u_3 = \frac{m_i^2}{v_L^2}\mathbf{u}, \tag{B14}$$

$$(\mathcal{H}_{13}^* \quad \mathcal{H}_{23}^*)\cdot\mathbf{u} + \mathcal{H}_{33}u_3 = \frac{m_i^2}{v_L^2}u_3. \tag{B15}$$

We first determine the eigenvalue and eigenvector for the top quark. Because $\frac{m_t^2}{v_L^2} \gg \mathcal{H}_{ij}(i, j = 1, 2)$, one can solve Eq. (B14),

$$\mathbf{u} = \frac{v_L^2}{m_t^2}\begin{pmatrix} \mathcal{H}_{13} \\ \mathcal{H}_{23} \end{pmatrix}u_3. \tag{B16}$$

Since $\mathbf{u} \ll u_3$, the top quark mass is approximately given as

$$m_t = v_L\sqrt{\mathcal{H}_{33}} = y_{uL3}v_L\frac{y_{uR3}v_R}{D_T}. \tag{B17}$$

The corresponding eigenvector for the top quark is given as

$$\mathbf{v}_t = \frac{1}{\sqrt{1 + |\frac{\mathcal{H}_{13}}{\mathcal{H}_{33}}|^2 + |\frac{\mathcal{H}_{23}}{\mathcal{H}_{33}}|^2}}\begin{pmatrix} \frac{\mathcal{H}_{13}}{\mathcal{H}_{33}} \\ \frac{\mathcal{H}_{23}}{\mathcal{H}_{33}} \\ 1 \end{pmatrix}. \tag{B18}$$

We ignore the correction in the following analysis since the normalization factor of Eq. (B18) is close to 1. The other two eigenvectors $\mathbf{v}_u$ and $\mathbf{v}_c$ correspond to the eigenvalues m_u^2/v_L^2 and m_c^2/v_L^2. For the small eigenvalues, Eq. (B15) can be solved as

$$u_3 = -\frac{(\mathcal{H}_{13}^* \quad \mathcal{H}_{23}^*)\cdot\mathbf{u}}{\mathcal{H}_{33}}. \tag{B19}$$

Substituting the relation into Eq. (B14), one obtains the following equation for the up and charm quarks:

$$\begin{pmatrix} h_{11} & h_{12} \\ h_{12}^* & h_{22} \end{pmatrix}\mathbf{u} = \frac{m_i^2}{v_L^2}\mathbf{u}, \tag{B20}$$

where $i = u, c$ and h_{ij} $(i, j = 1, 2)$ is defined as

$$h_{ij} = \mathcal{H}_{ij} - \mathcal{H}_{i3}\frac{1}{\mathcal{H}_{33}}\mathcal{H}_{3j}. \tag{B21}$$

The components of h are written explicitly,

$$\begin{aligned} h_{11} &= \frac{y_{uL1}^2 v_R^2(y_{uR1}^2 + |y_{uR21}|^2)}{M_U^2}, \\ h_{12} &= \frac{y_{uL1}y_{uL2}v_R^2(y_{uR21}^* y_{uR2})}{M_U M_C}, \\ h_{22} &= \frac{y_{uL2}^2 v_R^2 y_{uR2}^2}{M_C^2}. \end{aligned} \tag{B22}$$

Then for the up quark, the eigenvalue and the eigenvector are given as

$$m_u = y_{uL1}y_{uR1}\frac{v_L v_R}{M_U}, \qquad \mathbf{v}_u = \begin{pmatrix} 1 \\ -\frac{h_{12}^*}{h_{22}} \\ -\frac{\mathcal{H}_{13}^* - \frac{h_{12}^*}{h_{22}}\mathcal{H}_{23}^*}{\mathcal{H}_{33}} \end{pmatrix}, \tag{B23}$$

and for charm quark, they are given as

$$\begin{aligned} m_c &= y_{uL2}y_{uR2}\frac{v_L v_R}{M_C}, \\ \mathbf{v}_c &= \begin{pmatrix} \frac{h_{12}}{h_{22}} \\ 1 \\ -\frac{\mathcal{H}_{13}^*\frac{h_{12}}{h_{22}} + \mathcal{H}_{23}^*}{\mathcal{H}_{33}} \end{pmatrix} \simeq \begin{pmatrix} \frac{h_{12}}{h_{22}} \\ 1 \\ -\frac{\mathcal{H}_{23}^*}{\mathcal{H}_{33}} \end{pmatrix}. \end{aligned} \tag{B24}$$

K_{uL} is written in terms of the eigenvectors,

$$K_{uL} = (\mathbf{v_u} \quad \mathbf{v_c} \quad \mathbf{v_t}.). \tag{B25}$$

Similarly, the downtype mixing matrices K_{dL} and R_{dL} are obtained. The eigenvalues for the quark masses agree with the ones obtained in Ref. [20].

APPENDIX C: PARAMETRIZATION OF THE YUKAWA MATRIX IN TERMS OF A PRODUCT OF THE UNITARY MATRIX AND TRIANGULAR MATRIX

In this appendix, we give proof of the parametrization of the general 3×3 Yukawa matrices in terms of the product of the unitary matrices and triangular matrices.

The decomposition and the parametrization are used in Eqs. (56) and (60). The general 3×3 complex matrix of the Yukawa coupling Y with $\det Y \neq 0$ is written in terms of three independent complex vectors in C^3 $\mathbf{y^0}_i$, ($i = 1$–3) as follows:

$$Y = (\mathbf{y}_1^0 \quad \mathbf{y}_2^0 \quad \mathbf{y}_3^0) = P(\alpha_1, \alpha_2, \alpha_3)(\mathbf{y}_1 \quad \mathbf{y}_2 \quad \mathbf{y}_3), \tag{C1}$$

$$\alpha_i = \arg(\mathbf{y}_{3i}^0), \tag{C2}$$

where $P(\alpha_1, \alpha_2, \alpha_3)$ is a diagonal unitary matrix defined in Eq. (59), and $\mathbf{y}_3$ is a real vector in R^3. Then, we show Y can be parametrized as

$$Y = P(\alpha_1, \alpha_2, \alpha_3)V(\theta_1, \theta_2, \theta_3, \delta)P(\alpha, \beta, 0)Y_\Delta, \tag{C3}$$

where $V(\theta_1, \theta_2, \theta_3, \delta)$ is a Kobayashi-Maskawa-type parametrization for the unitary matrix, and Y_Δ is a lower triangular matrix with real diagonal elements,

$$V(\theta_1, \theta_2, \theta_3, \delta) = \begin{pmatrix} \cos\theta_3\cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1 e^{i\delta} & \cos\theta_3\cos\theta_2\sin\theta_1 - \sin\theta_2\cos\theta_1 e^{i\delta} & \sin\theta_3\cos\theta_2 \\ \cos\theta_3\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1 e^{i\delta} & \cos\theta_3\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\theta_1 e^{i\delta} & \sin\theta_3\sin\theta_2 \\ -\sin\theta_3\cos\theta_1 & -\sin\theta_3\sin\theta_1 & \cos\theta_3 \end{pmatrix}, \tag{C4}$$

$$Y_\Delta = \begin{pmatrix} y_{\Delta 11} & 0 & 0 \\ y_{\Delta 21} & y_{\Delta 22} & 0 \\ y_{\Delta 31} & y_{\Delta 32} & y_{\Delta 33} \end{pmatrix} = \begin{pmatrix} \cos\theta_{21}|\mathbf{y}_1| & 0 & 0 \\ \sin\theta_{21}\cos\theta_{31}e^{i\phi_{21}}|\mathbf{y}_1| & \cos\theta_{32}|\mathbf{y}_2| & 0 \\ \sin\theta_{21}\sin\theta_{31}e^{i\phi_{31}}|\mathbf{y}_1| & \sin\theta_{32}e^{i\phi_{32}}|\mathbf{y}_2| & |\mathbf{y}_3| \end{pmatrix}. \tag{C5}$$

Equation (C3) shows a well-known result [9]; i.e., the matrix Y is written as the product of the unitary matrix and the triangular matrix. Here we show that a particular form of the parametrization including some phases, angles, etc., shown in Eq. (C3) is indeed a generic parametrization. In this parametrization, there are nine real parts constructed by six angles,

$$\theta_1, \theta_2, \theta_3, \theta_{21}, \theta_{32}, \theta_{31}, \tag{C6}$$

and three norms of the complex vectors $|\mathbf{y}_\mathbf{i}^0| = |\mathbf{y}_\mathbf{i}|$, ($i = 1$, 2, 3). The nine phases are given by

$$\alpha_1, \alpha_2, \alpha_3, \alpha, \beta, \delta, \phi_{21}, \phi_{32}, \phi_{31}. \tag{C7}$$

Now we prove the parametrization is completely general. One can start with

$$P(-\alpha_1, -\alpha_2, -\alpha_3)Y = (\mathbf{y}_1 \quad \mathbf{y}_2 \quad \mathbf{y}_3), \tag{C8}$$

where $\mathbf{y}_3$ is a real vector in R^3. Further, one can take out the norm of $\mathbf{y}_\mathbf{i}$ as

$$(\mathbf{y}_1 \ \mathbf{y}_2 \ \mathbf{y}_3) = (\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3)\begin{pmatrix} |\mathbf{y}_1| & 0 & \\ 0 & |\mathbf{y}_2| & 0 \\ 0 & 0 & |\mathbf{y}_3| \end{pmatrix}. \tag{C9}$$

Note that $\mathbf{v}_i$ ($i = 1$–3) are normalized as $\mathbf{v}_\mathbf{i}^\dagger \cdot \mathbf{v}_\mathbf{i} = 1$ but are not necessarily orthogonal. $\mathbf{v}_3$ is a real normalized vector, which implies

$$\mathbf{v}_3 = \mathbf{e}_3 = \begin{pmatrix} \sin\theta_3\cos\theta_2 \\ \sin\theta_3\sin\theta_2 \\ \cos\theta_3 \end{pmatrix}. \tag{C10}$$

We first show the general parametrization for orthonormal basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, which are orthogonal to $\mathbf{e}_3$ in complex C^3 satisfying $\mathbf{e}_\mathbf{i}^\dagger \cdot \mathbf{e}_\mathbf{j} = \delta_{ij}$. Since $\mathbf{e}_\mathbf{i}(i = 1, 2)$ are orthogonal to $\mathbf{e}_3$, both real part and imaginary parts of $\mathbf{e}_\mathbf{i}$ ($i = 1$, 2) are orthogonal to $\mathbf{e}_3$. Therefore, they are unitary superpositions of the two real orthogonal vectors $\mathbf{e}_1^0$ and $\mathbf{e}_2^0$,

$$(\mathbf{e}_1 \quad \mathbf{e}_2) = (\mathbf{e}_1^0 \quad \mathbf{e}_2^0)U, \quad \mathbf{e}_1^0 = \begin{pmatrix} \cos\theta_3\cos\theta_2 \\ \cos\theta_3\sin\theta_2 \\ -\sin\theta_3 \end{pmatrix}, \quad \mathbf{e}_2^0 = \begin{pmatrix} -\sin\theta_2 \\ \cos\theta_2 \\ 0 \end{pmatrix}, \tag{C11}$$

where the two-by-two unitary matrix denoted by U can be parametrized as

$$U \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}\begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{i\beta} \end{pmatrix}. \tag{C12}$$

Then, one can write $\mathbf{e}_2$ and $\mathbf{e}_1$ as

$$\mathbf{e}_2 = \begin{pmatrix} \cos\theta_3\cos\theta_2\sin\theta_1 - \sin\theta_2\cos\theta_1 e^{i\delta} \\ \cos\theta_3\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\theta_1 e^{i\delta} \\ -\sin\theta_3\sin\theta_1 \end{pmatrix} e^{i\beta}, \quad \mathbf{e}_1 = \begin{pmatrix} \cos\theta_3\cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1 e^{i\delta} \\ \cos\theta_3\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1 e^{i\delta} \\ -\sin\theta_3\cos\theta_1 \end{pmatrix} e^{i\alpha}. \tag{C13}$$

From $\mathbf{v_2}$ one can form the vector that is orthogonal to $\mathbf{e_3}$. This vector can be identified with $\mathbf{e_2}$,

$$\frac{\mathbf{v_2} - \mathbf{e_3^T} \cdot \mathbf{v_2}\mathbf{e_3}}{\sqrt{1 - |\mathbf{e_3^T} \cdot \mathbf{v_2}|^2}} = \mathbf{e_2}. \tag{C14}$$

Therefore, one can write $\mathbf{v_2}$ with the superposition,

$$\begin{aligned}\mathbf{v_2} &= \mathbf{e_3^T} \cdot \mathbf{v_2}\mathbf{e_3} + \sqrt{1 - |\mathbf{e_3^T} \cdot \mathbf{v_2}|^2}\mathbf{e_2}, \\ &= \sin\theta_{32} e^{i\phi_{32}}\mathbf{e_3} + \cos\theta_{32}\mathbf{e_2},\end{aligned} \tag{C15}$$

where we set $\mathbf{e_3^T} \cdot \mathbf{v_2} = \sin\theta_{32} e^{i\phi_{32}}$. Next, from $\mathbf{v_1}$, one can form the vector that is orthogonal to $\mathbf{e_3}$ and $\mathbf{e_2}$. This can be identified as $\mathbf{e_1}$,

$$\begin{aligned}&\frac{\mathbf{v_1} - \mathbf{e_3^T} \cdot \mathbf{v_1}\mathbf{e_3} - \mathbf{e_2^\dagger} \cdot \mathbf{v_1}\mathbf{e_2}}{\sqrt{1 - |\mathbf{e_3^T} \cdot \mathbf{v_1}|^2 - |\mathbf{e_2^\dagger} \cdot \mathbf{v_1}|^2}} = \mathbf{e_1}, \\ &\mathbf{v_1} = \cos\theta_{21}\mathbf{e_1} + \sin\theta_{21}\cos\theta_{31} e^{i\phi_{21}}\mathbf{e_2} \\ &\qquad + \sin\theta_{21}\sin\theta_{31} e^{i\phi_{31}}\mathbf{e_3},\end{aligned} \tag{C16}$$

where one sets $\mathbf{e_3^T} \cdot \mathbf{v_1} = \sin\theta_{21}\sin\theta_{31} e^{i\phi_{31}}$ and $\mathbf{e_2^\dagger} \cdot \mathbf{v_1} = \sin\theta_{21}\cos\theta_{31} e^{i\phi_{21}}$. We summarize the relation $(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3})$ with $(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})$ using Eqs. (C15) and (C16),

$$(\mathbf{v_1} \quad \mathbf{v_2} \quad \mathbf{v_3}) = (\mathbf{e_1} \quad \mathbf{e_2} \quad \mathbf{e_3}) \begin{pmatrix} \cos\theta_{21} & 0 & 0 \\ \sin\theta_{21}\cos\theta_{31} e^{i\phi_{21}} & \cos\theta_{32} & 0 \\ \sin\theta_{21}\sin\theta_{31} e^{i\phi_{31}} & \sin\theta_{32} e^{i\phi_{32}} & 1 \end{pmatrix}. \tag{C17}$$

Note that the unitary matrix $(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3})$ is written in terms of three angles and three phases as

$$(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}) = V(\theta_1, \theta_2, \theta_3, \delta)P(\alpha, \beta, 0). \tag{C18}$$

We substitute the relation Eq. (C18) into Eq. (C17). Then one obtains

$$(\mathbf{v_1} \quad \mathbf{v_2} \quad \mathbf{v_3}) = V(\theta_1, \theta_2, \theta_3, \delta)P(\alpha, \beta, 0) \begin{pmatrix} \cos\theta_{21} & 0 & 0 \\ \sin\theta_{21}\cos\theta_{31} e^{i\phi_{21}} & \cos\theta_{32} & 0 \\ \sin\theta_{21}\sin\theta_{31} e^{i\phi_{31}} & \sin\theta_{32} e^{i\phi_{32}} & 1 \end{pmatrix}, \tag{C19}$$

which implies

$$P(-\alpha_1, -\alpha_2, -\alpha_3)Y = V(\theta_1, \theta_2, \theta_3, \delta)P(\alpha, \beta, 0)Y_\Delta. \tag{C20}$$

One can easily derive Eq. (C3) from Eq. (C20).

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