

Double neutrino production and detection in neutrino detectors

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Large, high-energy ($E > 100$ GeV) cosmic neutrino telescopes are now quite mature. IceCube, for example, observes about 50 000 well-reconstructed single atmospheric neutrino events/year, with energies above 100 GeV. Although the neutrino detection probability is small, current detectors are large enough so that it is possible to detect two neutrinos from the same cosmic-ray interaction. In this paper, we calculate the expected rate of double-neutrino interactions from a single cosmic-ray air shower. The rate is small, about 0.07 events/year for a 1 km³ detector like IceCube, with only a small dependence on the assumed cosmic-ray composition and hadronic interaction model. For a larger detector, like the proposed KM3Net, the rate is about 0.8 events/year, high enough to be easily observable. These double neutrino interactions are the major irreducible background to searches for pairs of supersymmetric particles produced in neutrino or cosmic-ray air-shower interactions. Other standard model backgrounds are considered, and found to be small.

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I. INTRODUCTION

High-energy astrophysical neutrinos are being actively studied, with a view to using them to find the acceleration sites for high-energy cosmic rays [1,2]. Three large detectors, the 1 km³ IceCube [3], the 10⁷ m³ Baikal detector [4], and the 1.5×10^7 m³ ANTARES detector [5] are taking data, and the 5–6 km³ KM3NeT detector [6,7] has been proposed. These detectors are optimized for neutrinos with energies in the TeV to PeV range, where the atmospheric neutrino flux is substantial; the completed IceCube detector, for example, observes about 50 000 well-reconstructed single atmospheric neutrino events/year [8]. Most of these events are from ν_μ (or $\bar{\nu}_\mu$; we do not distinguish between neutrinos and antineutrinos here), which interact and produce muons which travel upward through the detector.

Although these detectors are focused on the detection of single-neutrino interactions, they also look for more complex topologies. One signature of great interest consists of two parallel tracks going upward through the detector. This signature could be a sign of some type of “new physics,” such as supersymmetry (SUSY) or Kaluza-Klein models. In supersymmetry, parallel tracks can be created when a neutrino (or cosmic ray) interacts in the Earth below a detector, producing a pair of SUSY particles [9,10]. These supersymmetric particles decay, eventually producing a pair of next-to-lightest SUSY particles. If SUSY has a high mass scale, then these particles have a relatively long lifetime, of order μ s. They live long enough to travel long distances (≈ 1000 km) through the Earth. As they propagate, they will slowly separate, and will appear in a neutrino detector as a pair of upward-going parallel tracks, with a typical separation of order a few hundred

meters [11]. Since these particles are typically quite heavy, they lose energy via specific ionization (dE/dx) at a rate only slightly higher than minimum ionizing particles. Kaluza-Klein particles are produced via a different mechanism, but have similar observational consequences [12].

Previous studies have considered the standard model backgrounds to these processes; the major background is from charm production, where both of the charmed particles decay semileptonically [9]. This produces a pair of muons. These muons have a rather short range (even a 1 PeV muon has a range of less than 10 km in rock). With a typical maximum transverse momentum of a few GeV/c, the muons will not separate significantly before they lose their energy.

Two muons from a pair of neutrino interactions, from the same cosmic-ray air shower, are the only background that is likely to mimic the signatures described above. If the neutrinos are produced in the same cosmic-ray air shower, then they will be nearly parallel, but with a large enough opening angle to separate by a few hundred meters as they pass through the Earth. If the neutrinos have an energy below a few TeV, their ionization is within a few times of being minimum ionizing, similar to the energy deposition expected for supersymmetric or Kaluza-Klein particles [13].

In this paper, we calculate the rate of double-neutrino events expected to be observable in IceCube and KM3NeT, and discuss the expected characteristics of the events [14].

II. AIR SHOWERS, PRODUCTION MODEL AND NEUTRINO INTERACTIONS

The calculation was done in two parts. In the first part, cosmic-ray air showers were generated, and the neutrino data was retained for analysis in the second part. All the neutrinos in each event were paired with all of the other neutrinos in that event, and the separation distance

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computed. The neutrino-pair flux was weighted with the probability of detection. The detection probability has three components: 1) and 2) the energy-dependent probabilities of the two neutrinos interacting with the resulting muon being observed (assumed independent for each neutrino), and 3) the probability (based on their separation) of both muons passing within the detector active volume. The lateral separation distance $S = \theta D$ is a critical parameter. It depends on the distance D between the shower and the detector, and on the opening angle θ between the two neutrinos. θ depends on the neutrino energy and transverse momentum, p_T (relative to the shower core); the neutrino p_T depends on the p_T of the pion/kaon progenitor when it decays.

A small (length scale of a few km or less) detector can see only a fraction of the pairs, namely those with the smallest S . These neutrinos come mostly from the decays of pions and kaons [15] with a very small relative p_T —generally pairs where both muons have a very small p_T with respect to the shower core. Unlike studies of high- p_T muons [16], the p_T spectra depend on the soft hadronic physics. The progenitor pion and kaons also bend in the Earth’s magnetic field before they decay, with non-negligible effect.

We only consider neutrinos with energies above 100 GeV. Because of their small interaction cross section and large angular spread, lower-energy neutrinos do not contribute significantly. We assume that the two neutrinos come from the decays of different pions/kaons, rather than from a single decay chain like $\pi \rightarrow \nu\mu$, followed by $\mu \rightarrow e\nu_e\nu_\mu$.

Pair generation was simulated using cosmic-ray air showers that were generated using CORSIKA version 6.980 [17] to model the cosmic-ray air showers. Two different hadronic interaction models, QGSJET v01c [18] and DPMJET v2.55 [19], were used. The cosmic-ray spectrum was approximated by the Hörandel spectrum [20], with the low-energy end of the cosmic-ray spectrum following an $E^{-2.7}$ slope, and the high-energy end following E^{-3} ,

$$\Phi(E_p) = \begin{cases} 1.8 \times 10^4 E_p^{-2.7} & E_p < 10^6 \text{ GeV}, \\ 1.1 \times 10^6 E_p^{-3.0} & E_p > 10^6 \text{ GeV}, \end{cases} \quad (1)$$

with E_p the energy of the primary particle in units of GeV and the flux, $\Phi(E_p)$ in $1/(\text{s sr m}^2 \text{ GeV})$.

CORSIKA includes the bending effect of the Earth’s magnetic field and multiple scattering in the Earth’s atmosphere. Multiple scattering is negligible, but the magnetic bending of the π^\pm and K^\pm that decay into ν_μ affects the calculation. Like-sign pion/kaon pairs are bent in the same direction (but, depending on their energies, by a different amount), so their separation is less affected than unlike-sign pairs, which are bent in different directions. For a 5×10^{-4} T field (typical in the atmosphere above Antarctica) perpendicular to the

pion direction of motion, a pion is bent by an angle $\theta_B = qBc\tau_\pi/m_\pi \approx 117 \text{ keV}/m_\pi \approx 8.4 \times 10^{-4}$, where m_π and τ_π are the π^\pm mass and lifetime, respectively. The actual angle between the pion direction and the magnetic field is usually less than 90° , so the magnetic bending will be smaller, typically by a factor of order $1/\sqrt{2} \approx 0.7$.

The bending due to the pion/kaon transverse momentum p_T with respect to the cosmic-ray direction is $\theta_p = p_T/E_\pi$. For a typical scale $\Lambda_{\text{QCD}} = 300 \text{ MeV}$, the magnetic bending is larger than the p_T -induced bending for pion energies above 500 GeV. For kaons at the same energy, the bending is a factor of 8 smaller. The typical neutrino energy is around 1 TeV, so magnetic bending cannot be neglected.

Both QGSJET and DPMJET generate low- p_T particles using phenomenological, Pomeron-based models. Both of them reproduce accelerator data quite well, and so have similar p_T spectra in this region. In the low- p_T region, both models predict thermal spectra that are in agreement with experimental data obtained in accelerator experiments. Collider experiments are not sensitive to very low p_T , so there is some uncertainty here, but we can use data on high-energy cosmic-ray muon separations to check the models. MACRO [21] and IceCube [16] have studied muon separation spectra at small and large separations, respectively. The observed separation spectra and overall rates are in reasonable agreement with Monte Carlo expectations, although the zenith angle distributions do not agree well.

CORSIKA generates downward-going showers; this analysis uses the transformation shown in Fig. 1 to convert the downward-going neutrinos into upward-going neutrinos. The transformation maps the zenith angle, θ_Z into $-\theta_Z$. The two neutrinos are propagated through the Earth, separating as they go. In the relevant energy range (100 GeV to 10 TeV), neither neutrino oscillation nor absorption in the Earth is significant.

One weakness of this approach is that it uses both the magnetic field and ground elevation at the South Pole for all showers. Most of the relevant cosmic-ray air showers occur within $D < 1000 \text{ km}$ of the detector. Estimates of the inaccuracy due to the simplification need only consider field variations over this distance scale. For IceCube, we consider the region south of latitude -75° . Although the magnetic field strength does not vary significantly over this region, its direction does. The dip angle (angle between the magnetic field lines and vertical) ranges from -65° to -78° there. For a given longitude, the maximum change is 6° as the latitude varies from -75° to -90° [22]. The field declination varies more, up to 30° . These variations can alter the magnetic bending by up to 50%, but, after averaging over all possible angles of incidence, the net effect will be much smaller; the overall change in rate should be less than 25%.

The neutrino detection probability depends on both the neutrino interaction probability and the probability of

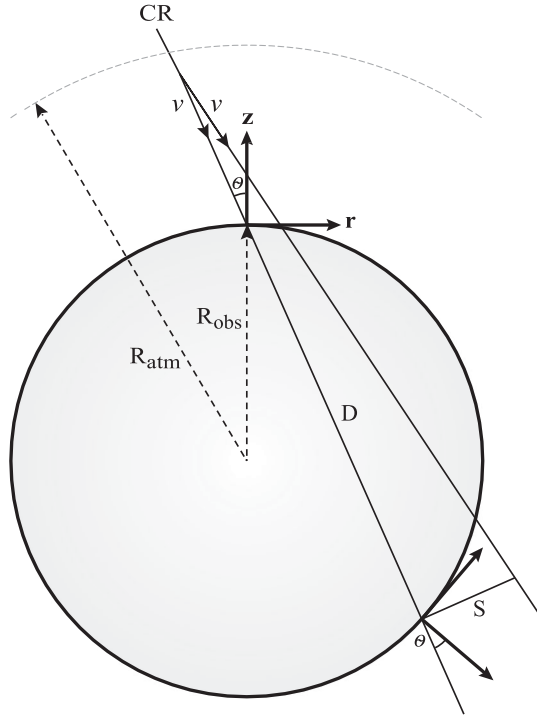


FIG. 1. The geometry used in the calculation for an incident cosmic ray at zenith angle θ . The parameters used are R_{obs} for the radius from the center of the Earth to the observation height, at which the particles are saved, R_{atm} for the radius from the center of the Earth to the top of the atmosphere, D for the distance between the cosmic-ray source and the detector, and S for the neutrino-neutrino separation in the detector.

observing the produced muon. We use a simple model which includes both factors [2],

$$P(\text{detection}|E_{\nu i}) = \begin{cases} 1.3 \cdot 10^{-6} E_{\nu}^{2.2} & \text{if } E_{\nu} \leq 1 \text{ TeV,} \\ 1.3 \cdot 10^{-6} E_{\nu}^{0.8} & \text{if } E_{\nu} > 1 \text{ TeV,} \\ 0 & \text{if } S > S_{\text{max}}, \end{cases} \quad (2)$$

with E_{ν} the neutrino energy in units of TeV and S_{max} the maximum separation where both neutrinos can be visible in the detector.

These probabilities are based on the model of a detector as a thin plate, sensitive to muons (from ν_{μ} and $\bar{\nu}_{\mu}$ interactions). Neutrinos are detected if they interact close enough to the plate that the muons they produce are energetic enough to reach the plate.

Parallel muon tracks that are too close together will not be resolvable as separate tracks; the minimum separation to be resolvable is S_{min} . The reconstruction of two parallel tracks is more challenging than for single tracks, with additional degrees of freedom [23]. The IceCube collaboration found that downward-going isolated muons were separable from muon bundles at separations larger than 135 m [16]. For two single muons, the minimum observable separation could be somewhat lower, especially for

near-horizontal muon pairs. The fraction of missed tracks with $S < S_{\text{min}}$ is $(S_{\text{min}}/S_{\text{max}})^2$. For IceCube $S_{\text{min}} = 135$ m, and $S_{\text{max}} = 1$ km, so less than 2% of the muon pairs within IceCube are not resolvable. For KM3NeT, the fraction of lost pairs should be even smaller.

Both IceCube and KM3NeT are three-dimensional, so that neutrinos that interact anywhere in the detector volume may be observed, in addition to neutrinos that interact outside the detector, but whose muons reach the detector volume. Both detectors contain holes—regions where a low-energy (minimum ionizing) muon may pass through undetected. As the neutrino energy rises, the muon range [24] and energy loss both rise, and both effects become less important. Over the relevant neutrino energies, these effects are both less than a factor of two. Fortunately, they work in opposite directions, and we will assume that they will cancel out. A more accurate calculation would require a detailed dedicated detector Monte Carlo to account for the correlated detection probability, event reconstruction software, and a well-defined set of event selection criteria.

III. RESULTS

Figure 2 shows the zenith-angle distribution of accepted pairs. As expected, most of the detected pairs are just below the horizon, where D is the smallest. The near-horizontal neutrino pairs come from near-horizontal cosmic rays which interact high enough in the atmosphere, typically above 40 km [16], so that D never gets too small. The dominant horizontal sensitivity means that the expected rates are somewhat sensitive to the detector shape; detectors with a larger horizontal frontal area should see more neutrino pairs.

Figure 3 shows the primary energies of the cosmic-ray progenitors of the pairs that would be detected. The distribution is peaked for primaries around 30 TeV, well below the knee of the cosmic-ray spectrum, where the cosmic-ray

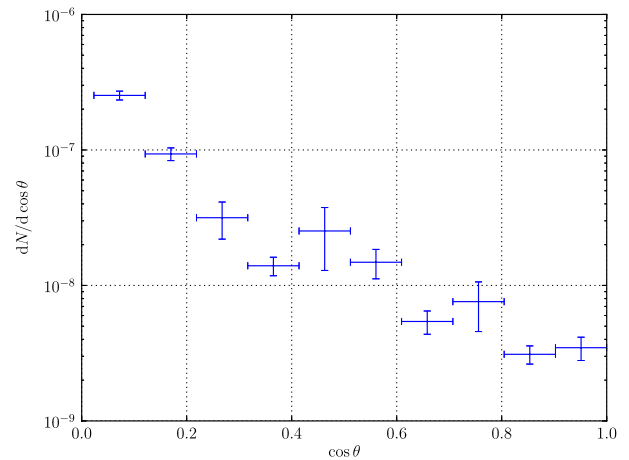


FIG. 2 (color online). The zenith-angle distribution of pairs that would be detected in the IceCube-model detector. Most of the pairs come from just below the horizon.

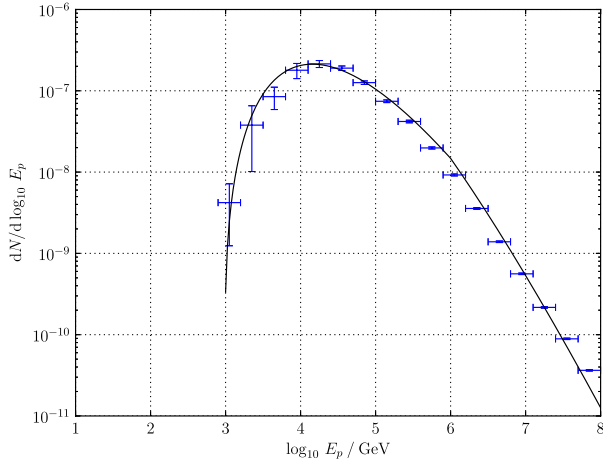


FIG. 3 (color online). The primary energy of the cosmic-ray progenitor of the pairs that would be detected in the IceCube-model detector. The solid black line shows the result of a fit to the detection probability distribution, weighted with the flux $\Phi(E_p)$.

composition is dominated by protons. This peak reflects several factors: the decrease in the cosmic-ray flux with increasing energy, the increasing neutrino production and detection cross sections, and the decrease in the average opening angle with increasing neutrino energy. With the rapid falloff with increasing energy, uncertainties in the cosmic-ray flux at high energies, above the knee, are unimportant.

Figure 4 shows the energy of the observed neutrinos (with two entries/pair), with a 1 km maximum separation. This distribution is peaked around 1 TeV, about 3% of the peak of the primary energy distribution. The maximum reflects the competition between the rapid decrease in the atmospheric neutrino flux with increasing energy, the increasing interaction probability and the decreasing opening angle (p_T/E_ν) with increasing neutrino energy. Events near the minimum energy cutoff, 100 GeV, do not significantly contribute to the rate. Prompt neutrinos become a significant contribution to

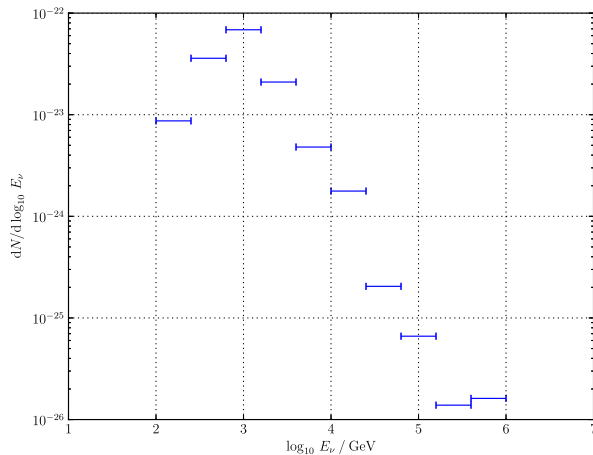


FIG. 4 (color online). The neutrino energies of the pairs that would be detected in the IceCube-model detector (two entries/pair).

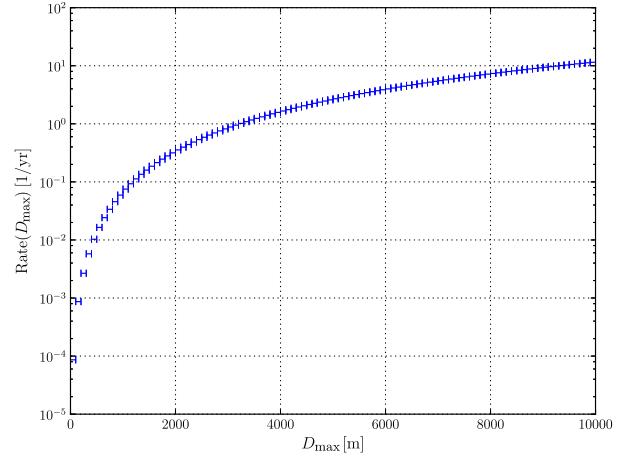


FIG. 5 (color online). The predicted pair-detection rate as a function of detector diameter, D_{\max} . This is for an assumed roughly spherical detector. IceCube fits this model fairly well. KM3NeT is likely to be wider than it is high, so the KM3NeT rates determined here may be slight overestimates.

the ν_μ flux at energies far above 100 TeV, and so do not contribute significantly to the pair rate.

The correlation between the energies of the two neutrinos is small. This is expected, since the separation distance is determined largely by the p_T and energy of the lowest-energy neutrino; as long as one neutrino has an energy substantially above the other one, the energy of the second is largely irrelevant.

Figure 5 shows the predicted detection rate as a function of detector diameter. For small detectors, the naive rate should scale as roughly the square of the surface area of the detector, or as the effective volume to the 4/3 power. For larger detectors, the rate of increase is slower because of the drop in neutrino flux at large transverse momentum.

IV. SIGNAL AND BACKGROUND RATES

The overall neutrino rates for a 1 km³ detector are shown in Table I, for both QGSJET and DPMJET, for both an all-proton and all-iron assumed cosmic-ray composition. The rates are all in quite good agreement, with the composition making at most a 21% difference. At the relevant energies—a few hundred TeV—cosmic rays are expected to be mostly protons and lighter nuclei.

TABLE I. The calculated event rates in IceCube for two interaction models and assumed all-proton or all-iron cosmic-ray composition. The rates do not vary very much for the four choices. Most of the double-neutrino events have cosmic-ray progenitors with energies of a few hundred TeV, where cosmic rays are expected to be mostly protons and light nuclei.

	QGSJET [1/yr]	DPMJET [1/yr]
Protons	0.068	0.070
Iron	0.065	0.056

For KM3NeT, using a 6 km³ effective volume [7], the rate is about 11 times higher, or about 0.8 events/year. KM3NeT is likely to be wider than it is high, so Fig. 5 may slightly overestimate its rate. These rates do not capture the details of either detector construction, but the IceCube rate should be accurate within 50%. More detailed calculations would require a complete simulation and an analysis chain.

The flux of double neutrino events is large enough that a signal should be visible in the proposed KM3NeT detector, and an event might be seen in IceCube. Once events are seen, then it is necessary to try to classify them as double neutrinos or as due to new physics. The observed specific energy loss (dE/dx) [25] and zenith-angle distributions may help in separating the two classes of events. As Fig. 4 shows, about 20% of the muons have energies above ~ 2 TeV, with an average dE/dx more than 10 times minimum ionizing; this is a larger energy loss than is expected from the considerably heavier supersymmetric or Kaluza-Klein particles. More importantly, most of the neutrino pairs come from near the horizon, whereas neutrino-induced supersymmetry interactions are more evenly spread over the upward-going hemisphere. However, the angular distribution is similar to what one would expect from supersymmetric or Kaluza-Klein particles that are produced directly in cosmic-ray air showers. Although these identifying criteria may be inadequate to classify a single event, a small event sample should allow clear conclusions to be drawn.

One potential background to these events (and to searches for supersymmetric and Kaluza-Klein particles) is from muon pairs that are produced in neutrino interactions (or in cosmic-ray air showers) from decays of charmed particles or Drell-Yan pairs. This background has been discussed previously [12]. However, the constraint that the two tracks appear parallel is a powerful constraint to eliminate background. For long muon tracks, IceCube has an angular resolution that is better than 1° [26]; KM3NeT is expected to be a few times better. Here, we use a maximum opening angle $\theta_o = 1^\circ$. Track pairs that diverge by more than twice that, or 2° , can be eliminated.

Two tracks from a single vertex can only be nearly parallel if the vertex is far from the detector. For the 2° parallelism requirement, and a minimum resolvable separation of 135 m, the vertex must be at least 2900 m from the detector. If we require that the tracks traverse through 1 km of IceCube, this gives a minimum track length of 3800 m. For a muon with average energy loss (dE/dx) in the ice, the 3.8 km range requires that the muon energy at the vertex must be at least 6 TeV. If the muon travels mostly through denser material (such as the rock below IceCube or KM3NeT), the minimum energy at the vertex would be three times higher.

The opening angle required for the muon pair to separate depends on the muon transverse momenta. As with the neutrinos, $S = \theta_o D_\mu$, where D_μ is the distance between the muon creation vertex and the middle of the detector.

If the muons both have $\theta_o = 1^\circ$, then, for the 6 TeV initial muon energy, $p_T = E_\mu \cdot \sin(\theta) = 105$ GeV/c. If the p_T is smaller, than the muons opening angle will be smaller, and their lateral separation will be too small for them to be resolvable. Such a large p_T is extremely rare; for comparison the IceCube studies of down-going muons covered the range of a few GeV/c.

The stringent p_T requirement remains under different conditions. For vertices farther from the detector, the opening angle is smaller, but the muon energy rises more quickly, increasing the minimum p_T . The inclusion of multiple scattering will alter these numbers slightly, but should not change the overall conclusion that the background rate due to neutrino interactions is negligible.

Angular misreconstruction does not affect the conclusions very much. As the allowed actual opening angle rises, the vertex can be closer to the detector and the minimum muon energy drops. However, the opening angle rises in concert, so the required p_T remains large. For example, for $\theta_o = 5^\circ$, $D_\mu \geq 1100$ m, but the required p_T is still 87 GeV. If the distance between the two tracks were misreconstructed, effectively reducing the two-track separation requirement, a larger background could be found.

Similar arguments apply for dimuons coming from cosmic-ray air showers. At a depth of 1500 m, the horizontal distance to the surface is 138 km. Muons cannot travel this far through rock or ice, but there may be a small background from downward-going dimuons where both muons are misreconstructed as upward-going.

A third background is from neutrino pairs where the two neutrinos are produced by different cosmic-ray interactions. The rate for this background depends on the detector angular and temporal resolution; it can be estimated with Poisson statistics. IceCube observes about 50 000 muons from high-energy neutrino interactions per year, spread over 2π steradians and 3×10^7 s. For a time-difference resolution of 450 ns [16], the number of temporal overlaps is 0.0008 per year. Including the 2° parallelism requirement reduces the rate by another factor of 6000. This calculation ignores nonuniformities in the acceptance, but these are not large effects. KM3NeT is larger, but its better angular resolution should lead to a similar background rate.

V. CONCLUSIONS

In conclusion, the expected rate for a 1 km³ detector like IceCube to observe two upward-going neutrinos from the same cosmic-ray air shower is about one every 14 years. Future, larger detectors, like a 6 km³ KM3NeT will have a substantially larger rate, i.e. 0.8 per year, and so should observe a signal. These double-neutrino events are an irreducible background to searches for pairs of upward-going particles produced by beyond-the-standard-model processes. The other standard model backgrounds to these processes appear to be very small.

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