

**Partial wave analysis of  $\psi(2S) \rightarrow p\bar{p}\eta$** 

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Using a sample of  $1.06 \times 10^8$   $\psi(2S)$  events collected with the BESIII detector at BEPCII, the decay  $\psi(2S) \rightarrow p\bar{p}\eta$  is studied. A partial wave analysis determines that the intermediate state  $N(1535)$  with a mass of  $1524 \pm 5_{-4}^{+10}$  MeV/ $c^2$  and a width of  $130_{-24}^{+27+57}_{-10}$  MeV/ $c^2$  is dominant in the decay; the product branching fraction is determined to be  $B(\psi(2S) \rightarrow N(1535)\bar{p}) \times B(N(1535) \rightarrow p\eta) + \text{c.c.} = (5.2 \pm 0.3_{-1.2}^{+3.2}) \times 10^{-5}$ . Furthermore, the branching fraction of  $\psi(2S) \rightarrow p\bar{p}\eta$  is measured to be  $(6.4 \pm 0.2 \pm 0.6) \times 10^{-5}$ . Here, the first error is statistical and the second is systematic.

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## I. INTRODUCTION

Baryon spectroscopy is an important field to understand the internal structure of hadrons. Within the static quark model, the baryon octet and decuplet are well described. About half a century after the introduction of the quark model, however, a substantial number of light baryons

predicted by the quark model have not been observed experimentally; this is known as the “missing baryon problem” [1,2]. One possibility could be that the missing states simply do not exist, which has led to the development of new phenomenological models, e.g., the diquark model [3]. Alternatively, the coupling of the unobserved states through conventional production channels could be small, which makes their observation more difficult.

In addition to fixed target experiments [4–11], charmonium decays produced in  $e^+e^-$  collisions open a window to hunt for the missing baryons [12]. The Beijing Spectrometer (BES) [13] experiment started a baryon program about a decade ago with the study of  $N(1535)$  and  $N(1650)$  in  $J/\psi \rightarrow p\bar{p}\eta$  by partial wave analysis (PWA) [14] using a sample of  $7.8 \times 10^6$   $J/\psi$  events. Using  $58 \times 10^6$   $J/\psi$  events collected at the BESII detector, a new excited nucleon  $N(2065)$  [15,16] was observed in  $J/\psi \rightarrow p\bar{n}\pi^-$  [17] and subsequently confirmed in  $J/\psi \rightarrow p\bar{p}\pi^0$  [18]. BESII also studied  $\psi(2S) \rightarrow p\bar{p}\gamma\gamma$ , where both  $p\bar{p}\pi^0$  and  $p\bar{p}\eta$  were observed, and  $\psi(2S) \rightarrow p\bar{p}\eta$  for the first time with a branching fraction of  $(5.8 \pm 1.1 \pm 0.7) \times 10^{-5}$ . In both decays, there was weak evidence for a  $p\bar{p}$  threshold mass enhancement but no PWA was performed [19]. Most recently, BESIII reported PWA results of  $\psi(2S) \rightarrow p\bar{p}\pi^0$  [20], and two new broad excited nucleons  $N(2300)$  and  $N(2570)$  were observed. However, no clear evidence for  $N(2065)$  was found. Using  $24.5 \times 10^6$   $\psi(2S)$  events, CLEO-c [21] reported the analysis of  $\psi(2S) \rightarrow \gamma p\bar{p}$ ,  $p\bar{p}\pi^0$ , and  $p\bar{p}\eta$  without considering interference effects, in which  $N(1535)$  and a  $p\bar{p}$  enhancement [ $R_1(2100)$ ] were investigated in  $\psi(2S)$  decay to  $p\bar{p}\eta$ . These results show that  $J/\psi$  and  $\psi(2S)$  decays offer a unique place to study baryon spectroscopy.

In this paper, using the  $1.06 \times 10^8$   $\psi(2S)$  events taken at the BESIII detector, a full PWA of the decay  $\psi(2S) \rightarrow p\bar{p}\eta$  is performed.

## II. BESIII DETECTOR AND MONTE CARLO SIMULATION

BEPCII [22] is a double-ring  $e^+e^-$  collider designed to provide a peak luminosity of  $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  at a beam current of 0.93 A. The BESIII [22] detector has a geometrical acceptance of 93% of  $4\pi$  and consists of four main components: (1) A small-cell, helium-based (40% He, 60%  $\text{C}_3\text{H}_8$ ) main drift chamber (MDC) with 43 layers providing an average single-hit resolution of  $135 \mu\text{m}$ , charged-particle momentum resolution in a 1 T magnetic field of 0.5% at  $1 \text{ GeV}/c^2$ , and a  $dE/dx$  resolution, which is better than 6%; (2) a time-of-flight system (TOF) constructed of 5-cm-thick plastic scintillators, with 176 detectors of 2.4 m length in two layers in the barrel and 96 fan-shaped detectors in the end caps [the barrel (end cap) time resolution of 80 ps (110 ps) provides  $2\sigma K/\pi$  separation for momenta up to  $\sim 1.0 \text{ GeV}/c^2$ ]; (3) an electromagnetic calorimeter

(EMC) consisting of 6240 CsI(Tl) crystals in a cylindrical structure (barrel) and two end caps [the energy resolution at 1.0 GeV is 2.5% (5%) in the barrel (end caps), and the position resolution in the barrel (end caps) is 6 mm (9 mm)]; (4) the muon counter consists of 1000  $\text{m}^2$  of resistive plate chambers in nine barrel and eight end cap layers and provides 2 cm position resolution.

A GEANT4-based simulation software BOOST [23] includes the geometric and material description of the BESIII detectors, the detector response, and digitization models, as well as the tracking of the detector running conditions and performance. The production of the  $\psi(2S)$  resonance is simulated by the Monte Carlo (MC) event generator KKMC [24], while the decays are generated by EvtGen [25] for known decay modes with branching ratios being set to the Particle Data Group [26] world average values, and by Lundcharm [27] for the remaining unknown decays. The analysis is performed in the framework of the BESIII Offline Software System, which takes care of the detector calibration, event reconstruction, and data storage.

## III. EVENT SELECTION

For  $\psi(2S) \rightarrow p\bar{p}\eta(\eta \rightarrow \gamma\gamma)$ , the topology is quite simple,  $p\bar{p}\gamma\gamma$ . Each candidate event is required to have two good charged tracks reconstructed from the MDC with total charge zero. The point of closest approach to the beam line of each charged track is required to be within  $\pm 20$  cm in the beam direction and 2 cm in the plane perpendicular to the beam. Both tracks must have the polar angle  $\theta$  in the range of  $|\cos\theta| < 0.93$ . The TOF and the specific energy loss  $dE/dx$  of a particle measured in the MDC are combined to calculate particle identification probabilities for pion, kaon, and proton hypotheses. The particle type with the highest probability is assigned to each track. In this analysis, one charged track is required to be identified as a proton and the other as an antiproton.

Photon candidates are reconstructed by clustering EMC crystal energies. For each photon, the minimum energy is 25 MeV for barrel showers ( $|\cos\theta| < 0.80$ ) and 50 MeV for end cap showers ( $0.86 < |\cos\theta| < 0.92$ ). To exclude showers from charged particles, the angle between the nearest proton track and the shower must be greater than  $10^\circ$ , while for the antiproton, the angle has to be greater than  $30^\circ$ . Timing requirements are used to suppress electronic noise and energy deposits in the EMC unrelated to the event. At least two good photons are required.

For the candidates remaining, a four-constraint kinematic fit imposing energy-momentum conservation is made under the  $p\bar{p}\gamma\gamma$  hypothesis. If the number of selected photons is greater than two, the fit is repeated using all combinations of photons. The two photon combination with the minimum fit  $\chi^2_{p\bar{p}\gamma\gamma}$  is selected, and  $\chi^2_{p\bar{p}\gamma\gamma}$  is required to be less than 20. Because the particle

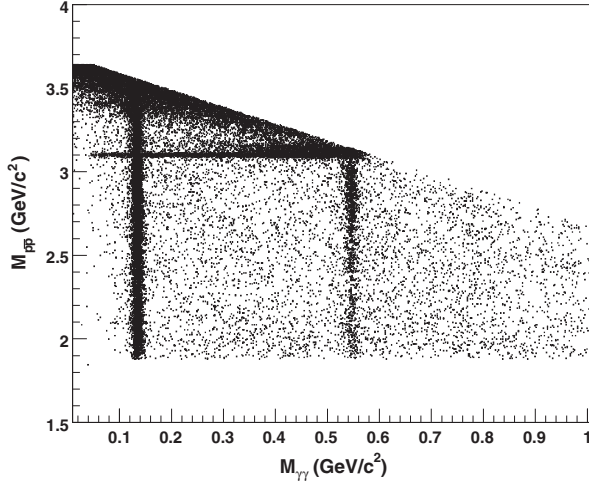


FIG. 1. Scatter plots of  $p\bar{p}$  invariant mass versus  $\gamma\gamma$  invariant mass.

identification efficiency between the data and MC simulation do not agree well in the low momentum region, the momenta of the proton and antiproton are required to be greater than 300 MeV/c. Figure 1 shows the scatter plot of  $M_{p\bar{p}}$  versus  $M_{\gamma\gamma}$  for events satisfying the above requirements, where the two vertical bands correspond to the decays  $\psi(2S) \rightarrow p\bar{p}\pi^0$  and  $\psi(2S) \rightarrow p\bar{p}\eta$ , and the horizontal band corresponds to the decay  $\psi(2S) \rightarrow X + J/\psi (J/\psi \rightarrow p\bar{p})$ . To remove the background events from  $\psi(2S) \rightarrow \eta J/\psi (J/\psi \rightarrow p\bar{p}, \eta \rightarrow \gamma\gamma)$  and  $\psi(2S) \rightarrow \gamma\chi_{cJ} (\chi_{cJ} \rightarrow p\bar{p}\gamma\gamma)$ ,  $M_{p\bar{p}} < 3.067 \text{ GeV}/c^2$  and  $M_{p\bar{p}} < (3.4 \text{ GeV}/c^2 - 0.75 \times M_{\gamma\gamma})$  are required. To select a clean sample,  $M_{\gamma\gamma}$  is required to be in the  $\eta$  mass region,  $|M_{\gamma\gamma} - M_{\eta}| < 21 \text{ MeV}/c^2$ .

After the above event selection, 745 candidate events are selected. The Dalitz plot of  $M_{p\eta}$  versus  $M_{\bar{p}\eta}$  is shown in Fig. 2(a), where two clusters corresponding to the  $p\eta$  mass threshold enhancement displayed in Figs. 2(b) and 2(c) are visible. Both the mass spectra and the Dalitz plot display an asymmetry for  $p\eta$  and  $\bar{p}\eta$ , which is mainly caused by

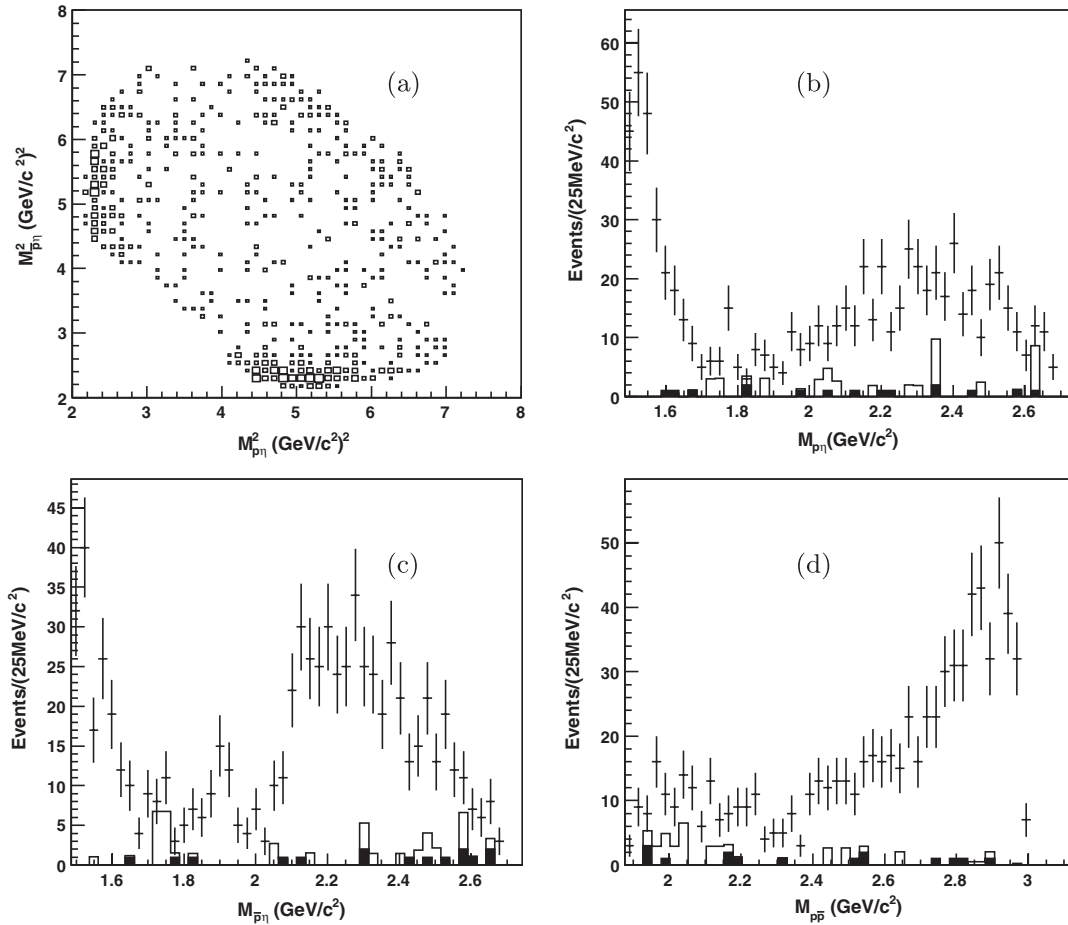


FIG. 2. (a) The Dalitz plot of  $\psi(2S) \rightarrow p\bar{p}\eta$  and distributions for (b)  $M_{p\eta}$ , (c)  $M_{\bar{p}\eta}$ , and (d)  $M_{p\bar{p}}$ . The crosses represent the data, the white histograms show background events from continuum data, and the black histograms represent the background events from  $\eta$  sidebands. The background events have been normalized to the data.



different detection efficiencies for the proton and antiproton.

To investigate possible background events, the same analysis is performed on the MC sample of  $1.0 \times 10^8$  inclusive  $\psi(2S)$  events, and 11 background events are found from the channels,  $\psi(2S) \rightarrow \gamma \chi_{cJ} (\chi_{cJ} \rightarrow p \bar{p} \pi^0, \pi^0 \rightarrow \gamma \gamma)$ ,  $\psi(2S) \rightarrow \gamma \chi_{cJ} (\chi_{cJ} \rightarrow \gamma J/\psi, J/\psi \rightarrow \gamma p \bar{p})$ , and  $\psi(2S) \rightarrow \gamma \chi_{c0} (\chi_{c0} \rightarrow \bar{p} \Delta^+, \Delta^+ \rightarrow p \pi^0)$ , which is compatible with the number of background events, 15, estimated with  $\eta$  sidebands ( $|M_{\gamma\gamma} - 0.43| < 50 \text{ MeV}/c^2$  and  $|M_{\gamma\gamma} - 0.65| < 50 \text{ MeV}/c^2$ ). Additionally,  $42.6 \text{ pb}^{-1}$  of continuum data taken at  $3.65 \text{ GeV}/c^2$  are used for an estimation of the background from QED processes, and 51 background events are obtained after normalization with the luminosity of the continuum data and  $\psi(2S)$  data. The background events from  $\eta$  sidebands and the continuum data will be considered in the PWA of  $\psi(2S) \rightarrow p \bar{p} \eta$ .

#### IV. PARTIAL WAVE ANALYSIS

The two-body decay amplitudes in the sequential decay process  $\psi(2S) \rightarrow N^* \bar{p}$ ,  $N^* \rightarrow p \eta$  (the charge-conjugate reaction is always implied unless explicitly mentioned) are constructed using the relativistic covariant tensor amplitude formalism [28], and the maximum likelihood method is used in the PWA [18]. In  $\psi(2S) \rightarrow N_X \bar{p}$ ,  $N_X \rightarrow p \eta$ ,  $A_j$  is the  $j$ th partial wave amplitude, which is described as

$$A_j = A_{\text{prod}-X}^j (BW)_X A_{\text{decay}-X}, \quad (1)$$

where  $A_{\text{prod}-X}^j$  is the amplitude describing the production of the intermediate resonance  $N_X$ ,  $BW_X$  is the Breit-Weigner propagator of  $N_X$ , and  $A_{\text{decay}-X}$  is the decay amplitude of  $N_X$ . The total differential cross section  $\frac{d\sigma}{d\Phi}$  is

$$\frac{d\sigma}{d\Phi} = \left| \sum_j c_j A_j + F_{\text{phsp}} \right|^2, \quad (2)$$

where  $F_{\text{phsp}}$  denotes the nonresonant contribution described by an interfering phase space term. The probability to observe the event characterized by the measurement  $\xi$  is

$$P(\xi) = \frac{\omega(\xi) \epsilon(\xi)}{\int d\xi \omega(\xi) \epsilon(\xi)}, \quad (3)$$

where  $\omega(\xi) \equiv \frac{d\sigma}{d\Phi}$  and  $\epsilon(\xi)$  is the detection efficiency.  $\int d\xi \omega(\xi) \epsilon(\xi)$  is the normalization integral calculated from the exclusive Monte Carlo sample. The joint probability density for observing  $n$  events in the data sample is

$$\mathcal{L} = P(\xi_1, \xi_2, \dots, \xi_n) = \prod_{i=1}^n P(\xi_i) = \prod_{i=1}^n \frac{\omega(\xi_i) \epsilon(\xi_i)}{\int d\xi \omega(\xi) \epsilon(\xi)}. \quad (4)$$

Rather than maximizing the likelihood function  $\ln \mathcal{L}$ ,  $S = -\ln \mathcal{L}$  is minimized to obtain  $c_j$  parameters, as well as the masses and widths of the resonances

$$-\ln \mathcal{L} = -\sum_{i=1}^n \ln \left( \frac{\omega(\xi_i)}{\int d\xi \omega(\xi) \epsilon(\xi)} \right) - \sum_{i=1}^n \ln \epsilon(\xi_i). \quad (5)$$

For a given data set, the second term is a constant and has no impact on the determination of the parameters of the amplitudes or on the relative changes of  $S$  values. So, for the fitting,  $-\ln \mathcal{L}$  is defined as

$$-\ln \mathcal{L} \equiv -\sum_{i=1}^n \ln \left( \frac{\omega(\xi_i)}{\int d\xi \omega(\xi) \epsilon(\xi)} \right). \quad (6)$$

The contribution of non- $\eta$  events and QED processes can be estimated with  $\eta$  sidebands and continuum data. In the log-likelihood calculation, the likelihood value of  $\eta$  sidebands and continuum data events are given negative weights and are removed from the data since the log-likelihood value of the data is the sum of the log-likelihood values of the signal and background events

$$S = -[(-\ln \mathcal{L})_{\text{data}} - (-\ln)_{\text{bg}}]. \quad (7)$$

The free parameters are optimized by FUMILI [29]. In the minimization procedure, a change in log-likelihood of 0.5 represents one standard deviation for each parameter.

In the analysis, the following two Breit-Wigner formulas are used to describe the resonance. One form has a width that is independent of the energy of the intermediate state, which is a default description and always implied

$$BW(s) = \frac{1}{M_{N^*}^2 - s - iM_{N^*} \Gamma_{N^*}}, \quad (8)$$

where  $s$  is the invariant mass squared. For  $N(1535)$  with its mass close to the threshold of its dominant decay channel  $N\eta$ , the approximation of a constant width is not very good. Thus, a phase-space-dependent width for  $N(1535)$  is also used

$$BW(s) = \frac{1}{M_{N^*}^2 - s - iM_{N^*} \Gamma_{N^*}(s)}. \quad (9)$$

The phase-space-dependent widths can be written as [30]

$$\Gamma_{N^*}(s) = \Gamma_{N^*}^0 \left( 0.5 \frac{\rho_{N\pi}(s)}{\rho_{N\pi}(M_{N^*}^2)} + 0.5 \frac{\rho_{N\eta}(s)}{\rho_{N\eta}(M_{N^*}^2)} \right), \quad (10)$$

where  $\rho_{N\pi}$  and  $\rho_{N\eta}$  are the phase space factors for the  $N\pi$  and  $N\eta$  final states, respectively,

$$\rho_{NX}(s) = \frac{2q_{NX}(s)}{\sqrt{s}} = \frac{\sqrt{(s - (M_N + M_X)^2)(s - (M_N - M_X)^2)}}{s}, \quad (11)$$

where  $X$  is  $\pi$  or  $\eta$ , and  $q_{NX}(s)$  is the momentum of  $X$  in the center-of-mass system of  $NX$ .

The best solution indicates that  $N(1535)$  combined with an interfering phase space is sufficient to describe the data.

## V. SYSTEMATIC ERRORS

The systematic error sources and their corresponding contributions to the measurement of mass, width, and branching fractions are discussed below.

- (i) To investigate the impact on the PWA results from other possible components, the analysis is also performed including other possible  $N^*$  states [e.g.,  $N(1520)$ ,  $N(1650)$ ,  $N(1700)$ ,  $N(1710)$ ,  $N(1720)$ ,  $N(1895)$ , and  $N(1900)$ , only one  $N^*$  is added in the fit each time]. The changes of the mass, width, and observed number of  $N(1535) \rightarrow p\eta$  events are taken as the systematic errors by summing them in quadrature.
- (ii) In the analysis, the background level is quite low, and the events from the  $\eta$  sidebands and continuum data are considered in the PWA. To estimate the uncertainty, the background events from  $\eta$  sidebands are varied by  $\pm 50\%$ , and the bigger change of the results is assigned as the systematic error.
- (iii) In Eq. (10), the weight of the phase space factors for both  $N\eta$  and  $N\pi$  is set to 0.5. The change of the results due to the variation of the weights in the range of 0–1 is taken as the systematic error.
- (iv) The MDC tracking efficiency is studied with the clean sample of  $J/\psi \rightarrow p\bar{p}\pi^+\pi^-$  events, as described in Ref. [31]. The difference between the data and the MC simulation is less than 2% per charged track. Here, 4% is taken as the systematic error for the proton and antiproton.
- (v) According to the particle identification efficiency study in Ref. [31], the difference of the particle identification efficiencies between the MC simulation and the data is around 2% for each charged track. In this study, the two charged tracks are required to be identified as  $p$  and  $\bar{p}$ , so 4% is taken as its systematic error from this source.
- (vi) The systematic error from the photon detection efficiency is studied using  $J/\psi \rightarrow \rho^0\pi^0$  events in Ref. [32]. The result indicates that the difference between the data and the MC simulation is about 1% for each photon. For the decay mode analyzed in this paper, 2% is taken as the systematic error from two photons in the final states.
- (vii) In order to estimate the systematic error of the kinematic fit, a clean sample of  $J/\psi \rightarrow p\bar{p}\pi^0$  is

TABLE I. Summary of general systematical errors.

Source	$\Delta B(\%)$
MDC tracking	$\pm 4$
Photon detection	$\pm 2$
Particle ID	$\pm 4$
Kinematic fit	$\pm 7$
The number of $\psi(2S)$ events	$\pm 0.82$
Total	$\pm 9$

TABLE II. Summary of the systematic errors on  $N(1535)$ .

Source	$\Delta M(\text{MeV}/c^2)$	$\Delta \Gamma(\text{MeV}/c^2)$	$\Delta B(\%)$
Systematic error from Table I	...	...	$\pm 9$
Additional resonances	+2	+56	+59
	-4	-0	-6
Different BW formula	...	...	+17
			-21
Background uncertainty	+10	+10	+8
	+10	-10	+63
Total	-4	-10	-23

selected. The difference of the efficiency between the data and the MC simulation with and without using the four-constraint kinematic fit, 7%, is taken as the systematic error.

- (viii) The number of  $\psi(2S)$  events  $(1.06 \pm 0.86) \times 10^8$  [33] is determined from a inclusive hadronic process, and the systematic uncertainty is 0.82%.

There are two categories of systematic errors here. The first includes the systematic errors listed in Table I, which applies to all branching fraction measurements. The total is the sum of them in quadrature. The second is from the fitting procedure, which applies to the mass, width, and branching fraction measurements of  $N(1535)$ . Table II summarizes the systematic errors from both categories on  $N(1535)$ .

## VI. RESULTS

The PWA results including the invariant mass spectra of  $p\bar{p}$ ,  $p\eta$ ,  $\bar{p}\eta$ , and angular distributions are shown as histograms in Fig. 3 and are consistent with the data. We observe  $527 \pm 27 N(1535) \rightarrow p\eta$  events with a mass  $M = (1524 \pm 5^{+10}_{-4}) \text{ MeV}/c^2$ , a width  $\Gamma = 130^{+27+57}_{-24-10} \text{ MeV}/c^2$ , and a statistical significance larger than  $10\sigma$ . Here, the first error is statistical and the second is systematic. The contributions of  $N(1535)$  and phase space are 70.8% and 61.0%. To determine the detection efficiency of  $\psi(2S) \rightarrow N(1535)\bar{p}$ , the MC events are generated in accordance with the PWA amplitudes for  $\psi(2S) \rightarrow N(1535)\bar{p}$ . With the detection efficiency of 24.1%, the product branching fraction of  $\psi(2S) \rightarrow N(1535)\bar{p}(N(1535) \rightarrow p\eta)$  is calculated to be

$$B(\psi(2S) \rightarrow N(1535)\bar{p}) \times B(N(1535) \rightarrow p\eta) = \frac{N_{\text{obs}}}{\varepsilon \cdot N_{\psi(2S)} \cdot B(\eta \rightarrow \gamma\gamma)} = (5.2 \pm 0.3^{+3.2}_{-1.2}) \times 10^{-5}, \quad (12)$$

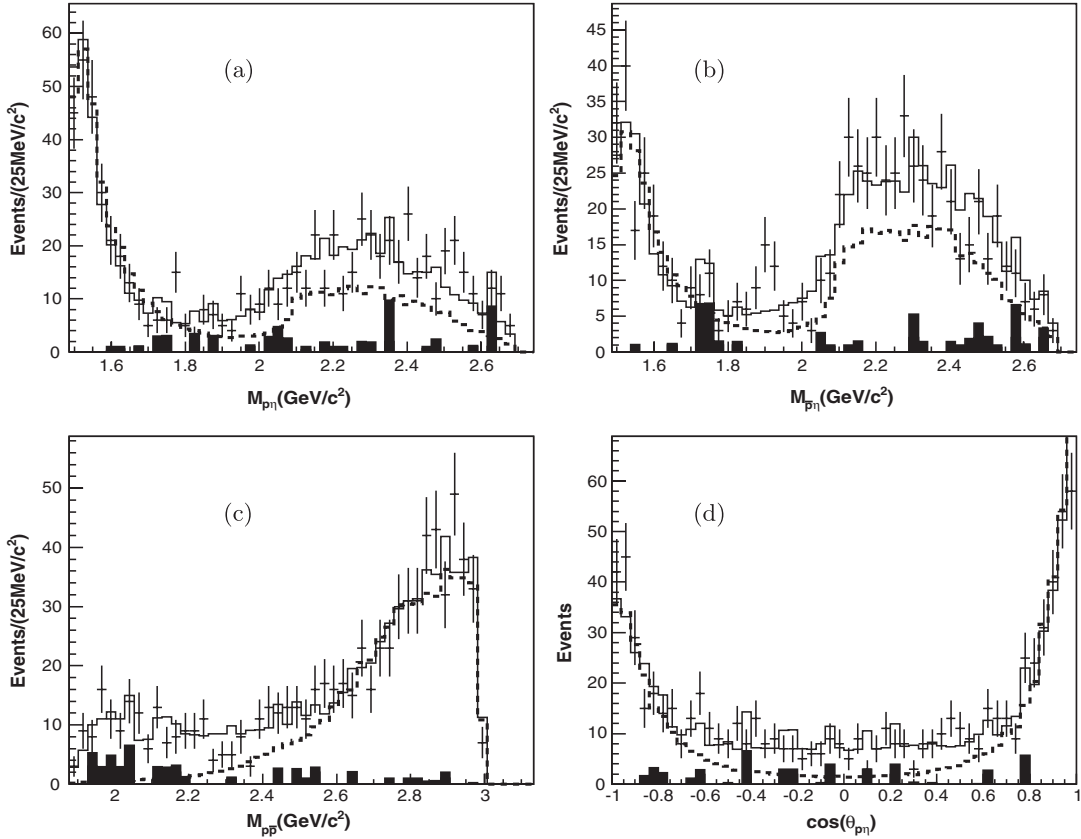


FIG. 3. Distributions of (a)  $M_{p\eta}$ , (b)  $M_{\bar{p}\eta}$ , (c)  $M_{p\bar{p}}$ , and (d) the angle between  $p\eta$  in the  $p\bar{p}$  system. The crosses are for data, the white histograms for PWA projections, the dashed lines for the contribution of  $N(1535)$ , and the black histograms for the background events from  $\eta$  sidebands and continuum data. The background events have been normalized to the data.

where the number of  $\psi(2S)$  events  $N_{\psi(2S)}$  is  $(1.06 \pm 0.86) \times 10^8$  determined from  $\psi(2S)$  inclusive decays [33];  $B(\eta \rightarrow \gamma\gamma)$  is the world average value [34], and the first error is statistical and the second systematic.

To investigate the  $p\bar{p}$  mass enhancement observed at BESII [19] and CLEO-c [21], which did not use a PWA, a scan for an additional  $1^{--}$  resonance described by a Breit-Wigner function is performed. The widths used are 50 MeV/c<sup>2</sup>, 100 MeV/c<sup>2</sup>, 200 MeV/c<sup>2</sup>, 300 MeV/c<sup>2</sup>, 400 MeV/c<sup>2</sup>, 500 MeV/c<sup>2</sup>, and 600 MeV/c<sup>2</sup>. The mass is allowed to vary from 1900 MeV/c<sup>2</sup> to 3000 MeV/c<sup>2</sup> with steps of 2 MeV/c<sup>2</sup>. There is no evidence for a  $p\bar{p}$  resonance in this region, indicating that the threshold enhancement can be explained by interference between the  $N(1535)$  and phase space.

Subtracting the 51 and 15 background events from QED processes and from  $\eta$  sidebands, respectively, the number of  $\psi(2S) \rightarrow p\bar{p}\eta$  events is calculated to be  $679 \pm 26$ . In addition to the contribution from  $N(1535)$ , the contribution from the phase space events is taken into account in the determination of the detection efficiency according to the PWA results. With the detection efficiency of 25.6%, the branching fraction of  $\psi(2S) \rightarrow p\bar{p}\eta$  is measured to be

$$B(\psi(2S) \rightarrow p\bar{p}\eta) = (6.4 \pm 0.2 \pm 0.6) \times 10^{-5}. \quad (13)$$

## VII. SUMMARY

Based on  $1.06 \times 10^8$   $\psi(2S)$  events collected with the BESIII detector, a full PWA on the 745  $\psi(2S) \rightarrow p\bar{p}\eta$  candidates is performed, and the results indicate that the dominant contribution is from  $\psi(2S) \rightarrow N(1535)\bar{p}$ . The mass and width of  $N(1535)$  are determined to be  $1524 \pm 5_{-4}^{+10}$  MeV/c<sup>2</sup> and  $130_{-24}^{+27+57}_{-10}$  MeV/c<sup>2</sup>, respectively, which are consistent with those from previous measurements listed in the Particle Data Group [34]. The product of the branching fractions is calculated to be  $B(\psi(2S) \rightarrow N(1535)\bar{p}) \times B(N(1535) \rightarrow p\eta) + c.c. = (5.2 \pm 0.3_{-1.2}^{+3.2}) \times 10^{-5}$ . The  $p\bar{p}$  mass enhancement observed by BESII is investigated, and the statistical significance of an additional  $p\bar{p}$  resonance is less than  $3\sigma$ .

The branching fraction of  $\psi(2S) \rightarrow p\bar{p}\eta$  is determined to be  $(6.4 \pm 0.2 \pm 0.6) \times 10^{-5}$ , where the detection efficiency is determined from MC simulation events generated based on the PWA results. Compared with the branching fraction of  $J/\psi \rightarrow p\bar{p}\eta$  [34],

$$Q_{p\bar{p}\eta} = \frac{B(\psi(2S) \rightarrow p\bar{p}\eta)}{B(J/\psi \rightarrow p\bar{p}\eta)} = (3.2 \pm 0.4)\%, \quad (14)$$

which improves the BESII measurement [19] of  $(2.8 \pm 0.7)\%$ , and indicates that the decay  $\psi(2S) \rightarrow p\bar{p}\eta$  is suppressed compared with the “12% rule” [35–37].

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- [1] S. Capstick and W. Roberts, *Phys. Rev. D* **47**, 1994 (1993).  
 [2] N. Isgur and G. Karl, *Phys. Rev. D* **19**, 2653 (1979).  
 [3] M. Oettel, G. Hellstern, R. Alkofer, and H. Reinhardt, *Phys. Rev. C* **58**, 2459 (1998).  
 [4] M. Dugger *et al.* (CLAS Collaboration), *Phys. Rev. C* **76**, 025211 (2007).  
 [5] V. Kubarovsky *et al.* (CLAS Collaboration), *Phys. Rev. Lett.* **92**, 032001 (2004).  
 [6] S. Stepanyan *et al.* (CLAS Collaboration), *Phys. Rev. Lett.* **91**, 252001 (2003).  
 [7] M. Williams *et al.* (CLAS Collaboration), *Phys. Rev. C* **80**, 045213 (2009).  
 [8] M. Dugger *et al.* (CLAS Collaboration), *Phys. Rev. Lett.* **89**, 222002 (2002).  
 [9] I. Horn *et al.* (CB-ELSA Collaboration), *Phys. Rev. Lett.* **101**, 202002 (2008).  
 [10] O. Bartholomy *et al.* (CB-ELSA Collaboration), *Phys. Rev. Lett.* **94**, 012003 (2005).  
 [11] E. F. McNicoll *et al.* (Crystal Ball Collaboration at MAMI), *Phys. Rev. C* **82**, 035208 (2010).  
 [12] H. B. Li, H. C. Chiang, G. X. Peng, and B. S. Zou (BES Collaboration), *Nucl. Phys.*, **A675** 189 (2000).  
 [13] D. M. Asner, T. Barnes, J. M. Bian, I. I. Bigi, N. Brambilla, I. R. Boyko, V. Bytev, K. T. Chao *et al.*, *Int. J. Mod. Phys. A* **24**, 499 (2009).  
 [14] J. Z. Bai *et al.* (BES Collaboration), *Phys. Lett. B* **510**, 75 (2001).  
 [15] R. Koniuk and N. Isgur, *Phys. Rev. D* **21**, 1868 (1980).  
 [16] S. Capstick and W. Roberts, *Phys. Rev. D* **49**, 4570 (1994).  
 [17] M. Ablikim *et al.* (BES Collaboration), *Phys. Rev. Lett.* **97**, 062001 (2006).  
 [18] M. Ablikim *et al.* (BES Collaboration), *Phys. Rev. D* **80**, 052004 (2009).  
 [19] M. Ablikim *et al.* (BES Collaboration), *Phys. Rev. D* **71**, 072006 (2005).  
 [20] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. Lett.* **110**, 022001 (2013).  
 [21] J. P. Alexander *et al.* (CLEO Collaboration), *Phys. Rev. D* **82**, 092002 (2010).  
 [22] M. Ablikim *et al.* (BES Collaboration), *Nucl. Instrum. Methods Phys. Res., Sect. A* **614**, 345 (2010).  
 [23] Z. Y. Deng *et al.*, *Chinese Phys. C* **30**, 371 (2006).  
 [24] S. Jadach, B. F. L. Ward, and Z. Was, *Phys. Chem. Comm.* **130**, 260 (2000); *Phys. Rev. D* **63**, 113009 (2001).  
 [25] R. G. Ping *et al.*, *Chinese Phys. C* **32**, 599 (2008).  
 [26] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).  
 [27] J. C. Chen, G. S. Huang, X. R. Qi, D. H. Zhang, and Y. S. Zhu, *Phys. Rev. D* **62**, 034003 (2000).  
 [28] J. X. Wang, *Nucl. Instrum. Methods Phys. Res., Sect. A* **534**, 241 (2004).  
 [29] S. N. Dymov, V. S. Kurbatov, I. N. Silin, and S. V. Yaschenko, *Nucl. Instrum. Methods Phys. Res., Sect. A* **440**, 431 (2000).  
 [30] T. P. Vrana, S. A. Dytman, and T. S. H. Lee, *Phys. Rep.* **328**, 181 (2000).  
 [31] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **83**, 112009 (2011).  
 [32] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **83**, 112005 (2011).  
 [33] M. Ablikim *et al.* (BESIII Collaboration), *Chinese Phys. C* **37**, 063001 (2013).  
 [34] J. Beringer *et al.* (Particle Data Group), *Phys. Rev. D* **86**, 010001 (2012).  
 [35] A. Duncan and A. H. Mueller, *Phys. Lett.* **93B**, 119 (1980).  
 [36] S. J. Brodsky and G. P. Lepage, *Phys. Rev. D* **24**, 2848 (1981).  
 [37] V. L. Chernyak and A. R. Zhitnitsky, *Nucl. Phys.* **B201**, 492 (1982).