Conservation laws in theories with universal gravity/matter coupling

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In this paper, we investigate the conservation laws of different types of particles in theories with a universal gravity/matter coupling. The result brings new insight about previous studies on universal gravity/matter theories. Especially, the paper demonstrates that, for perfect fluids, there is an equivalence between the assumption $\mathcal{L}_m = -\epsilon$, where ϵ is the total energy density, and the assumption that the matter fluid current is conserved [$\nabla_\sigma(\rho u^\sigma) = 0$, where ρ is the rest mass density]. However, the main result is given in the general case in which one does not make any assumption on the conservation of the matter fluid current.

I. INTRODUCTION

Recently, many phenomenological theories of gravitation have been proposed with a universal gravity/matter coupling. The coupling appears either as a universal nonminimal coupling between the Ricci scalar curvature and the nongravitational fields [1–6] or as a universal nonminimal coupling between a scalar field and the nongravitational fields [7–10]. In order to study both types of theories, we define universal gravity/matter coupling theories by the following action:

$$S = \frac{1}{c} \int [f(R, \phi, (\nabla\phi)^2) + h(R, \phi, (\nabla\phi)^2)\mathcal{L}_m]\sqrt{-g}\, d^4x, \tag{1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, $f(R, \phi, (\nabla\phi)^2)$ and $h(R, \phi, (\nabla\phi)^2)$ are arbitrary analytical functions of the Ricci scalar R, a scalar field ϕ and the gradients constructed from the scalar field. $\mathcal{L}_m$ is the Lagrangian of the nongravitational sector—which we assume here to be the usual standard model of the particle Lagrangian.[1] We dub the coupling *universal* because the coupling is the same for the whole material sector; while a more general set of theories would require different coupling for each material field [11–13]. It is worth stressing that such an action encompasses Brans-Dicke-like scalar-tensor theories (with [7–10] and without [14–17] universal scalar/matter coupling), $f(R)$ theories (with [2–4] and without [18] universal R/matter coupling), some low-energy (tree-level) string-inspired 4-dimensional models in the string frame [19–21], several Kaluza-Klein theories reduced to four dimensions [22–24], and most of the models present in $f(R, \mathcal{L}_m, \phi, (\nabla\phi)^2)$ theories [25]. However, we consider only theories in which ϕ is a gravitational field—which requires some sort of coupling

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[1]This assumption can be relaxed to include potential dark matter fields (see the discussion in the conclusion).

between ϕ and R, as in Brans-Dicke-like scalar-tensor theories, for instance [26,27]. Moreover, the main feature of the action in which we are interested is due to the nonconstant term $h(R, \phi, (\nabla\phi)^2)$. This last fact explains the chosen name for this kind of nonminimal coupling: *gravity/matter coupling*.

This type of universal gravity/matter coupling has been proposed in order to explain the phenomena usually associated to *dark matter* (without modifications of the standard model of particles) [2,28–32], to mimic the cosmological constant [33], to give a reheating scenario in the context of Starobinsky inflation [34], to explain the acceleration of the expansion of the Universe [1,6,7,21,35–37], to predict (essentially) stable weakly interacting massive partices [20], or to explain the smallness of the gravitational constant G compared to the proton mass scale [38]. Moreover, it has been shown that, under specific assumptions, such theories can be compatible with current tight constraints on the equivalence principle(s) (see, for instance, Refs. [12,39–41]). In the end, the goal of such a phenomenological action is to tend to an explanation of the whole observed phenomena, without the need of separate theories for inflation, dark matter, or dark energy.

As first noticed in Refs. [2–4] in the case of $f(R)$ with universal gravity/matter coupling theories, the field equations of theories with a gravity/matter coupling depend explicitly on the *on-shell* Lagrangian $\mathcal{L}_m$. In particular, whether one chooses $\mathcal{L}_m = \epsilon$—in which ϵ is the total energy density [42]—or $\mathcal{L}_m = P$—in which P is the pressure of the perfect fluid (or any normalized linear combination of the two)—explicitly modifies the field equations; while the difference between the different on-shell Lagrangians should reduce to surface integrals that do not contribute to the field equations—just as in the case of general relativity in which one can freely choose between $\mathcal{L}_m = \epsilon$ and $\mathcal{L}_m = P$ without modifying the field equations [43]. Therefore, it was first thought that there was a degeneracy of supposedly equivalent actions that lead to different field equations [3]—which is not

satisfactory with respect to the monist view of modern physics, which requires a unique mathematical description of the natural phenomena. However, Ref. [42] showed that some assumptions on the nature of the matter fluid can kill the degeneracy, hence showing that different Lagrangians actually correspond to matter fluids with different fundamental characteristics. To be specific, Ref. [42] showed that, when the matter fluid current is conserved, the Lagrangian of a barotropic perfect fluid is $\mathcal{L}_m = -\epsilon$— and not anything else. In the following we discuss this issue in more details and generalize the result found in Ref. [42] to general perfect fluids for which the matter fluid current is not conserved.

In Sec. II, we give the field equations corresponding to the Eq. (1). After, in Sec. III, we study the equation of motion of interacting particles forming an arbitrary perfect fluid. Then, in Sec. IV, we derive the equation of motion of noninteracting point particles. Finally, in Sec. V, we derive the equation of motion of light rays in the geometric optic approximation before concluding in Sec. VI.

II. FIELD EQUATIONS OF THE UNIVERSAL GRAVITY/MATTER COUPLING MODEL

We use the usual definition of the stress-energy tensor:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \tag{2}$$

By assuming that the matter Lagrangian $\mathcal{L}_m$ depends only on the metric tensor components $g_{\mu\nu}$, and not on its derivatives, we obtain the stress-energy tensor as

$$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\delta\mathcal{L}_m}{\delta g^{\mu\nu}}. \tag{3}$$

The extremization of the action (1) then reduces to the following field equations [25]:

$$[f_R + h_R\mathcal{L}_m]R_{\mu\nu} + (g_{\mu\nu}\nabla_\lambda\nabla^\lambda - \nabla_\mu\nabla_\nu)$$
$$\times [f_R + h_R\mathcal{L}_m] - \frac{1}{2}fg_{\mu\nu}$$
$$= \frac{1}{2}hT_{\mu\nu} - (f_{(\nabla\phi)^2} + h_{(\nabla\phi)^2}\mathcal{L}_m)\nabla_\mu\phi\nabla_\nu\phi, \tag{4}$$

where the subscript of f or h denotes a partial derivative with respect to the arguments, i.e., $f_R = \partial f/\partial R$, $f_{(\nabla\phi)^2} = \partial f/\partial(\nabla\phi)^2$,

$$\Box_{(\nabla\phi)^2}\phi = \frac{1}{2}(f_\phi + h_\phi\mathcal{L}_m), \tag{5}$$

where $f_\phi = \partial f/\partial\phi$ and

$$\Box_{(\nabla\phi)^2} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left[(f_{(\nabla\phi)^2} + h_{(\nabla\phi)^2}\mathcal{L}_m)\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right]. \tag{6}$$

The (non)conservation of the stress-energy tensor therefore reduces to

$$\nabla_\sigma T^{\mu\sigma} = (\mathcal{L}_m g^{\mu\sigma} - T^{\mu\sigma})\partial_\sigma \ln h. \tag{7}$$

III. PERFECT FLUID PARTICLES

The general Lagrangian that leads to the stress-energy tensor of a perfect fluid is

$$\mathcal{L}_m = [-(1-\alpha)\epsilon + \beta P]\gamma^{-1}, \tag{8}$$

where α and β are constants, γ is a normalization constant such that $\gamma = 1 - \alpha + \beta$, and where P and ϵ are the pressure and the so-called total energy density of the fluid [44]. Indeed, following the development made in Refs. [3,43], one can show that $\mathcal{L}_m = [-(1-\alpha)\epsilon + \beta P]\gamma^{-1}$ in addition with Eq. (3) induces $T^{\alpha\beta} = (\epsilon + P)U^\alpha U^\beta + Pg^{\alpha\beta}$—see the appendix of Ref. [42] for an example. The normalization procedure is necessary in order to conserve energy through the change of parametrization. Indeed, adding surface integral terms, one can transform the on-shell Lagrangian from $-\epsilon$ to P (and vice versa). Therefore, the most general Lagrangian has to be a normalized sum of the two possible on-shell Lagrangians. For instance, for $P = 0$, the normalization induces that $\mathcal{L}_m = -c^2\rho$ for any α; while $\mathcal{L}_m$ could be any times the rest mass density if not normalized. On the other side, one should note that the normalization procedure discards $\mathcal{L}_m = T$ as a possible on-shell Lagrangian for fluids with $P \neq 0$. The corresponding normalized on-shell Lagrangian would write $\mathcal{L}_m = T/4$ instead. However, from the trace of Eq. (3), one gets that $\mathcal{L}_m = T/4$ implies that $\delta\mathcal{L}_m/\delta g^{\mu\nu}$ has to be trace free. Therefore, $\mathcal{L}_m = T/4$ is not suitable any kind of effective particles.

Also, as shown in the appendix of Ref. [42], the matter current is generally not conserved when considering $\mathcal{L}_m = P$. Therefore, we deduce that the matter current is generally not conserved when considering the Lagrangian (8). Thus, let us write $\nabla_\sigma(\rho U^\sigma) = D$, where D is any scalar to be determined. One deduces

$$\nabla_\sigma(\epsilon U^\sigma) = \left(\frac{\epsilon + P}{\rho}\right)D - P\nabla_\sigma U^\sigma, \tag{9}$$

where [42]

$$\epsilon = \rho\left(c^2 - \frac{P}{\rho} + \int\frac{dP}{\rho}\right). \tag{10}$$

Therefore, taking the divergence of $T^{\alpha\beta} = (\epsilon + P)U^\alpha U^\beta + Pg^{\alpha\beta}$, one gets

$$\nabla_\sigma T^{\mu\sigma} = (\epsilon + P)U^\sigma\nabla_\sigma U^\mu + (g^{\mu\sigma} + U^\mu U^\sigma)\nabla_\sigma P$$
$$+ U^\mu\left(\frac{\epsilon + P}{\rho}\right)D. \tag{11}$$

On the other side, Eq. (7) implies

$$\nabla_\sigma T^{\mu\sigma} = -(\epsilon + P)[g^{\mu\sigma} + U^\mu U^\sigma]\partial_\sigma \ln h$$
$$+ \left[\left(1 - \frac{1-\alpha}{\gamma}\right)\epsilon + \frac{\beta}{\gamma}P\right]g^{\mu\sigma}\partial_\sigma \ln h. \quad (12)$$

Multiplying Eqs. (11) and (12) by U_μ and equating them, one gets

$$\nabla_\sigma(\rho U^\sigma) = -\frac{\rho}{\epsilon + P}\left[\left(1 - \frac{1-\alpha}{\gamma}\right)\epsilon + \frac{\beta}{\gamma}P\right]U^\sigma \partial_\sigma \ln h. \quad (13)$$

One notes that $\alpha = \beta = 0$ induces $\nabla_\sigma(\rho u^\sigma) = 0$—independently of the value of $\partial_\alpha \ln h$. Thus, it proves that $\mathcal{L}_m = -\epsilon \Rightarrow \nabla_\sigma(\rho u^\sigma) = 0$ for all perfect fluids. Since Ref. [42] proves $\nabla_\sigma(\rho u^\sigma) = 0 \Rightarrow \mathcal{L}_m = -\epsilon$ for barotropic perfect fluids, one has $\mathcal{L}_m = -\epsilon \Leftrightarrow \nabla_\sigma(\rho u^\sigma) = 0$ (at least) for barotropic perfect fluids. Now, injecting Eq. (13) in Eq. (7) gives

$$[\epsilon + P]U^\sigma\nabla_\sigma U^\alpha$$
$$= (g^{\alpha\sigma} + U^\alpha U^\sigma)\left[\partial_\sigma P + \left(-\frac{1-\alpha}{\gamma}\epsilon + \frac{\beta}{\gamma}P\right)\partial_\sigma \ln h\right]. \quad (14)$$

This equation generalizes the result found in Ref. [45] for noninteracting monopoles to interacting particles in a perfect fluid.

IV. MONOPOLE TEST PARTICLES

In this section, we derive the equation of motion of point particles directly from the material sector of the action—while Ref. [45] starts from the (non)conservation equation of the stress-energy tensor. Eventually, we shall show the equivalence of the two approaches. The material part of the action writes

$$S_m = \frac{1}{c}\int h(R, \phi, (\nabla\phi)^2)\mathcal{L}_m\sqrt{-g}d^4x. \quad (15)$$

For a noninteracting point particle, one has $\mathcal{L}_m = -c^2\rho$, with $\rho = \mu\delta(\vec{x})$ (μ being a mass), such that

$$S_m = -c\int_W \sqrt{-g}\mu hcdt = -c\int_W \sqrt{-g}\mu u^0 hds, \quad (16)$$

where the integral is taken on the world line W of the particle and where $u^\alpha = dx^\alpha/ds$, s being an affine parameter of the world line. According to the last section, since noninteracting point particles are the simplest form of perfect fluids ($P = 0$), the Lagrangian $\mathcal{L}_m = -c^2\rho$ induces the conservation of the matter fluid current $\nabla_\sigma(\rho u^\sigma) = 0$. Therefore, one shows that $\mathcal{L}_m = -c^2\rho$ induces the Newtonian conservation of the so-called *conserved density* $\rho^* = \sqrt{-g}\rho u^0$ [46],

$$\partial_0(\rho^*) + \partial_i(\rho^* v^i) = 0, \quad (17)$$

where $v^i = u^i/u^0$. Hence, one deduces the conservation of the mass m $(dm/dx^0 = (u^0)^{-1}dm/ds = 0)$ defined as $m = \sqrt{-g}\mu u^0$. One has

$$\nabla_\sigma(\rho u^\sigma) = 0 \Leftrightarrow \frac{dm}{ds} = 0. \quad (18)$$

Therefore, one can take m out of the integral in Eq. (16), and one gets the modified point particle action:

$$S_m = -mc\int_W hds. \quad (19)$$

Parametrizing by the proper time τ, one gets $S_m = -mc^2\int_W Ld\tau$, with $L = h\sqrt{g_{\alpha\beta}U^\alpha U^\beta}$, with $U^\alpha = c^{-1}dx^\alpha/d\tau$, such that one can use the Euler-Lagrange equation:

$$\frac{\partial L}{\partial x^\gamma} - \frac{d}{cd\tau}\frac{\partial L}{\partial U^\gamma} = 0. \quad (20)$$

Computing the terms

$$\frac{\partial L}{\partial x^\gamma} = \frac{ds}{cd\tau}\partial_\gamma h + \frac{1}{2}\frac{cd\tau}{ds}h\partial_\gamma g_{\alpha\beta}U^\alpha U^\beta \quad (21)$$

and

$$\frac{d}{cd\tau}\frac{\partial L}{\partial U^\gamma}$$
$$= \frac{cd\tau}{ds}\times\left(g_{\alpha\gamma}\partial_\beta hU^\alpha U^\beta + h\partial_\beta g_{\alpha\gamma}U^\alpha U^\beta + hg_{\alpha\gamma}\frac{dU^\alpha}{cd\tau}\right), \quad (22)$$

one gets the modified equation of motion of a point particle,

$$\frac{DU^\alpha}{cd\tau} = -(g^{\alpha\sigma} + U^\alpha U^\sigma)\partial_\sigma \ln h, \quad (23)$$

where $D/d\tau$ stands for the usual covariant derivative ($D/d\tau \equiv cU^\sigma\nabla_\sigma$). One can check that this equation is in accordance with the result found in Ref. [45] and that, for $P = 0$, Eq. (14) reduces to Eq. (23). It shows that, as expected, deriving the particle equation of motion directly from the Lagrangian is equivalent to deriving it from the (non)conservation equation of the stress-energy tensor.

V. ELECTROMAGNETIC WAVES IN THE GEOMETRIC OPTIC REGIME

In the following, the electromagnetic field is not considered as a significant source of curvature (i.e., photons are considered as test particles). Then, from Eq. (1) the electromagnetic equation writes

$$\nabla_\sigma(h(R, \phi, (\nabla\phi)^2)F^{\mu\sigma}) = 0. \quad (24)$$

Using the Lorenz gauge ($\nabla_\sigma A^\sigma = 0$), it reduces to

$$-\Box A^\mu + g^{\mu\epsilon}R_{\gamma\epsilon}A^\gamma + (\nabla^\mu A^\sigma - \nabla^\sigma A^\mu)\partial_\sigma \ln h = 0. \quad (25)$$

Following the analysis made in Ref. [47], we expand the 4-vector potential as follows:

$$A^\mu = \Re\{(a^\mu + \epsilon b^\mu + O(\epsilon^2))\exp^{i\theta/\epsilon}\}. \quad (26)$$

The two first leading orders of Eq. (25), respectively, give

$$k_\sigma k^\sigma = 0, \quad (27)$$

where $k_\sigma \equiv \partial_\sigma \theta$, and

$$a^\mu \nabla_\sigma k^\sigma + 2k^\sigma \nabla_\sigma a^\mu = (k^\mu a^\sigma - k^\sigma a^\mu)\partial_\sigma \ln h. \quad (28)$$

Remembering that the Lorenz gauge condition gives $k_\sigma a^\sigma = 0$ at the leading order, one gets

$$k^\sigma \nabla_\sigma k^\mu = 0. \quad (29)$$

This equation is the usual null-geodesic equation, showing that the presence of nonminimal gravity/matter coupling will not affect light ray trajectories at the geometric optic approximation. However, defining $a^\mu = af^\mu$, the propagation equation for the scalar amplitude (a) as well as the propagation equation for the polarization vector (f^μ) are modified:

$$k^\sigma \nabla_\sigma a = -\frac{a}{2}\nabla_\sigma k^\sigma + \frac{1}{2}ak^\sigma \partial_\sigma \ln h, \quad (30)$$

$$k^\sigma \nabla_\sigma f^\mu = \frac{1}{2}k^\mu f^\sigma \partial_\sigma \ln h. \quad (31)$$

From there follows that the conservation law of the "photon number" (i.e., intensity) is modified:

$$\nabla_\sigma(k^\sigma a^2) = -a^2 k^\sigma \partial_\sigma \ln h. \quad (32)$$

One notes that the last three equations may give alternative ways to put constraints on those theories.

For instance, at the classical level, Eq. (32) leads to a modification of the distance luminosity d_L because of the energy transfer between the gravitational fields and the electromagnetic field. As an example, for a flat Friedmann-Lemaitre-Robertson-Walker metric, the new equation reads[2]

$$d_{L,k=0} = (1 + z)\sqrt{\frac{h|_{z=0}}{h|_z}}\int_0^z \frac{dz}{H(z)}, \quad (33)$$

where $H(z)$ is the Hubble parameter as it would be measured by an observer at redshift z. This has implications in observational cosmology since the accelerated expansion of the Universe is historically deduced from the observation of high redshift supernovae while assuming the usual equation for the distance luminosity [48]:

$$d_{L,k=0-usual} = (1 + z)\int_0^z \frac{dz}{H(z)}. \quad (34)$$

Therefore, in models represented by the action (1), one not only has to check whether or not the Universe can be accelerated (such as in Refs. [6,7,35,36]), but one also

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[2]This equation has been obtained in a parallel work, in collaboration with Aurelien Hees (unpublished at the present time).

has to pay attention on how the distance luminosity equation is affected by the evolution of the Universe when considering Eq. (33). As far as we know, no study took that point into account so far.

VI. CONCLUSION

In the present paper, we studied the motion of noninteracting point particles, interacting particles composing a general perfect fluid, and the motion of light rays in theories that exhibit a universal gravity/matter coupling. We saw that the transfer of energy between the gravity fields and the material fields—which is due to the universal gravity/matter coupling—depends on the nature of the material fields. Indeed, such a transfer can take two possible forms: either it modifies the matter fluid current conservation, or it modifies the equation of motion (or both at the same time). As an example, it seems that for massive noninteracting point particles, the energy transfer is totally incorporated into the modification of the equation of motion; while for light rays, the energy transfer is totally incorporated into the modification of the conservation of the intensity. Otherwise, we saw that, for a very general set of perfect fluids, the energy transfer modifies both the matter current fluid conservation and the equation of motion. However, all the perfect fluid Lagrangians that give a nonconservation of the rest mass seem to be in physical disagreement with the noninteracting point particle case. Indeed, using the relation $\mathcal{L}_m = -\epsilon \Rightarrow \nabla_\sigma(\rho U^\sigma) = 0$ demonstrated for perfect fluids in Sec. III, the noninteracting point particle case necessarily induces the conservation of the rest mass (see Sec. IV). This seems to be in accordance with the assumption that $\mathcal{L}_m$ is the usual material Lagrangian. Therefore, we argue that perfect fluid cases in which $\mathcal{L}_m = [-(1-\alpha)\epsilon + \beta P]\gamma^{-1}$, with $\alpha \neq 0$ and $\beta \neq 0$, might not have any physical significance—unless one modifies the material sector in such a specific way that the nonconservation of matter precisely reduces to Eq. (13), with $\alpha \neq 0$ and $\beta \neq 0$. Hence, since the last case seems very unlikely (unless maybe for some exotic material fields), we believe that theorists should be careful when using Lagrangians that are such that $\mathcal{L}_m = [-(1-\alpha)\epsilon + \beta P]\gamma^{-1}$, with $\alpha \neq 0$ and $\beta \neq 0$. This point is very important if one considers the numerous publications using various values for the parameters α and β without justifications (see in the Appendix a sample of cases found in the literature). However, this issue would get a definitive answer if, instead of using a phenomenological perfect fluid description of matter, one works with the actual fundamental fields of the material sector (for instance, with $\mathcal{L}_m$ describing the standard model of particle fields). Work in that direction is in progress.

Also, although Eq. (1) generally leads to a violation of the equivalence principle(s), specific cases can still be in accordance with the present tight experimental constraints (see, for instance, Refs. [12,39–41]). However, a dedicated

study of the equivalence principle issue in each specific case is mandatory.

APPENDIX: PERFECT FLUID MODELS IN THE LITERATURE

Sample of cases considered in the literature			
Bertolami *et al.* [31]	$\alpha = 0$, $\beta = 0$, $\gamma = 1$		$dm/ds = 0$
Minazzoli and Harko [42]	$\alpha = 0$, $\beta = 0$, $\gamma = 1$		$dm/ds = 0$
Harko *et al.* [25]	$\alpha = 0$, $\beta = 0$, $\gamma = 1$		$dm/ds = 0$
Sotiriou and Faraoni [4]	$\alpha = 1$, $\beta = 1$, $\gamma = 1$		$dm/ds \neq 0$
Bisabr [6,36]	$\alpha = 1$, $\beta = 1$, $\gamma = 1$		$dm/ds \neq 0$
Farajollahi and Salehi [49]	$\alpha = 0$, $\beta = 3$, $\gamma = 4$		$dm/ds \neq 0$
Jamil *et al.* [50]	$\alpha = 0$, $\beta = 3$, $\gamma = 4$		$dm/ds \neq 0$
Sheykhi and Jamil [51]	$\alpha = 0$, $\beta = 3$, $\gamma = 4$		$dm/ds \neq 0$
Saaidi *et al.* [52]	$\alpha = 0$, $\beta = 3$, $\gamma = 4$		$dm/ds \neq 0$
Sharif and Waheed [53]	$\alpha = 0$, $\beta = 3$, $\gamma = 4$		$dm/ds \neq 0$

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