Sakai-Sugimoto model in D0-D4 background

Chao Wu,[1,*] Zhiguang Xiao,[1,2,†] and Da Zhou[1,‡]

[1] *Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026, China*

[2] *State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China*

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We add smeared D0 charges to the D4 background and discuss the Sakai-Sugimoto model under this background. The corresponding gauge theory is in a state with an expectation value of $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$. The D8-branes go less deep than in the original S-S model and massless Goldstones are still found in the spectrum. The effects of this condensate on the meson spectra, pion decay constant, and couplings of the vector mesons and Goldstones in this background state are then investigated.

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I. INTRODUCTION

Confinement as a nonperturbative phenomenon of QCD attracts lots of attention from theoretical physicists. There are many mechanisms proposed as the possible cause of the confinement. See [1] for a review. Among these mechanisms, some classical or semi-classical gauge field configurations could play an important role, such as some topologically nontrivial solutions, including monopoles, instantons, etc. There could also be solutions with constant field strength for the classical equation of motion (EOM). Self-dual field strength is studied in [2–5], and was proposed to be a mechanism for the confinement [6]. So there could be states with nonzero $\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$ background where $F_{\mu\nu}$ is the field strength and $\tilde{F}^{\mu\nu}$ its dual, and they may play a role in the confinement.

Such states may also be produced in heavy ion collisions. There were some proposals that the P-or CP-odd bubble may be created during the collisions [7–9]. A metastable state with nonzero QCD vacuum θ angle or $\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$ could be produced in some space-time region in the hot and dense condition when deconfinement happens. Then, with the rapid expansion of the bubble, it cools down and the metastable state freezes inside the bubble [8]. Then a P- or CP-odd bubble may form. It will soon decay into the true vacuum.

As nonperturbative phenomena in QCD, the effects of the states with nonzero $\mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$ must be studied using nonpertubative methods. String-gauge duality provides a way to study this kind of phenomena. To add $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$ condensate in $N = 4$ SUSY YM corresponds to adding smeared D(-1) charges into D3-brane configurations. Supersymmetric (SUSY) D(-1)-D3 background was studied in [10,11] and was proposed to correspond to gauge field theory with a self-dual background field strength [10]. Non-SUSY D(-1)-D3 was studied in [11,12] and corresponds to adding a temperature to the corresponding gauge theory. By introducing D7 probe branes into the background geometry, according to the proposal of Karch and Katz [13], flavors can also be introduced into these backgrounds, and then quark condensates and meson spectra can be studied [12,14–18]. Also by introducing baryonic D5-branes, studies can be carried out on baryon properties in the glue condensates [19–23].

Another holographic construction of the QCD-like theory is to use the D4 background initiated by Witten [24]. By compactifying the D4-brane on a circle, four-dimensional Yang-Mills theory can be obtained from the five-dimensional Yang-Mills theory, and by imposing the antiperiodic boundary condition on the fermions, supersymmetry is broken. Flavors can be added into the Yang-Mills by introducing flavor D6- [25] or D8-branes [26]. In particular, Sakai-Sugimoto(S-S) [26] proposed a model with $\mathrm{D8}-\overline{\mathrm{D8}}$ probe branes, where the spontaneous breaking of chiral symmetry is geometrically realized as the joining of N_f D8-branes and N_f anti-D8-branes into N_f D8-branes at the tip. Massless Goldstones with the right quantum numbers can be found in the spectrum. Meson spectra and interactions then can be studied along these lines [27]. Baryons can also be easily realized as instantons in this model such that the nucleon interactions can also be modeled [28–31]. As in the D(-1)-D3 background, adding condensate $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$ in the gauge theory corresponds to adding smeared $D0$ charges into the $D4$ background. The gauge theory in this background is studied in [32,33]. Putting the Sakai-Sugimoto model (S-S model) into this background allows us to study the hadron phenomena in the nonzero $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$ background. In the present paper, as a first step, we study the meson spectra and the interactions of the lowest-lying vector mesons and Goldstones in this background. To keep the $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$ dependence in the large N_c, we require it to be of $\mathcal{O}(N_c)$ as in [10], $\tilde{\kappa} \sim \langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle/N_c$. There are still massless Goldstone modes indicating the massless nature of the flavor quarks. We analyze the lowest-lying scalar and vector meson spectra in this model and the three point couplings for the lowest-lying vector mesons and Goldstones and find out that $\tilde{\kappa}$ really enters the formulas

*wuchao86@mail.ustc.edu.cn
†xiaozg@ustc.edu.cn
‡zhouda@mail.ustc.edu.cn

for these quantities. The detailed results are presented in Secs. IV, V, and VI.

This paper is organized as follows: In Sec. II, we review the D0-D4 background and its relation to the gauge field theory. In Sec. III we put D8 probe branes into this background and study the stability of the configuration. In Secs. IV and V, we study the scalar and vector meson spectra with one flavor, respectively. In Sec. VI, we extend our discussion to the multiflavor case, and the interactions of vector mesons and Goldstones are studied. Section VII is the conclusion and discussion.

II. THE D0-D4 BACKGROUND

Some of the results in this section are already presented in [32]. The solution of D4- branes with smeared D0 charges in Type IIA supergravity in the Einstein frame is [32,33]

$$ds^2 = H_4^{-\frac{3}{8}}(-H_0^{-\frac{7}{8}}f(U)d\tau^2 + H_0^{\frac{1}{8}}((dx^0)^2 + (dx^1)^2 + \cdots + (dx^3)^2)) + H_4^{\frac{5}{8}}H_0^{\frac{1}{8}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \tag{1}$$

$$e^{-(\Phi-\Phi_0)} = (H_4/H_0^3)^{\frac{1}{4}}, \tag{2}$$

$$f_2 = \frac{A}{U^4}\frac{1}{H_0^2}dU \wedge d\tau, \tag{3}$$

$$f_4 = B\epsilon_4, \tag{4}$$

where

$$A = \frac{(2\pi\ell_s)^7 g_s N_0}{\omega_4 V_4}, \qquad B = \frac{(2\pi\ell_s)^3 N_c g_s}{\omega_4}, \tag{5}$$

$$H_4 = 1 + \frac{U_{Q4}^3}{U^3}, \qquad H_0 = 1 + \frac{U_{Q0}^3}{U^3}, \qquad f(U) = 1 - \frac{U_{\mathrm{KK}}^3}{U^3}. \tag{6}$$

$d\Omega_4$, ϵ_4, and $\omega_4 = 8\pi^2/3$ are the line element, the volume form, and the volume of a unit S^4. U_{KK} is the coordinate radius of the horizon, and V_4 the volume of the D4-brane. N_0 and N_c are the numbers of D0 and D4 branes, respectively. D0 branes are smeared in the $x^0, \ldots, x^3$ directions.

In string frame the metric reads

$$ds^2 = H_4^{-\frac{1}{2}}(-H_0^{-\frac{1}{2}}f(U)d\tau^2 + H_0^{\frac{1}{2}}dx^2) + H_4^{\frac{1}{2}}H_0^{\frac{1}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \tag{7}$$

where $dx^2 = (dx^0)^2 + (dx^1)^2 + \cdots + (dx^3)^2$ is used. The EOM requires

$$A^2 = 9U_{Q0}^3(U_{Q0}^3 + U_{\mathrm{KK}}^3), \qquad B^2 = 9U_{Q4}^3(U_{Q4}^3 + U_{\mathrm{KK}}^3), \tag{8}$$

which can be solved,

$$U_{Q0}^3 = \frac{1}{2}\left(-U_{\mathrm{KK}}^3 + \sqrt{U_{\mathrm{KK}}^6 + \frac{4}{9}A^2}\right), \tag{9}$$

$$U_{Q4}^3 = \frac{1}{2}\left(-U_{\mathrm{KK}}^3 + \sqrt{U_{\mathrm{KK}}^6 + \frac{4}{9}B^2}\right). \tag{10}$$

We have required U_{KK} to be the horizon and no bare singularity, and then $U_{Q0}^3 > 0$, $U_{Q4}^3 > 0$ are chosen. To use this solution in the Sakai-Sugimoto model, we make a double wick rotation in τ and x^0 directions and the metric becomes

$$ds^2 = H_4^{-\frac{1}{2}}(H_0^{-\frac{1}{2}}f(U)d\tau^2 + H_0^{\frac{1}{2}}dx^2) + H_4^{\frac{1}{2}}H_0^{\frac{1}{2}}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \tag{11}$$

where $dx^2 = -(dx^0)^2 + (dx^1)^2 + \cdots + (dx^3)^2$ now. In fact, the metric is a bubble geometry and the space-time ends at $U = U_{\mathrm{KK}}$.

In order not to have the conical singularity, the period of τ should be

$$\beta = \frac{4\pi}{3}U_{\mathrm{KK}}H_0^{1/2}(U_{\mathrm{KK}})H_4^{1/2}(U_{\mathrm{KK}}). \tag{12}$$

We can then define a Kaluza-Klein mass scale $M_{\mathrm{KK}} = 2\pi/\beta$, which indicates the UV cutoff of the gauge theory. The D4-brane tension can be related to the five-dimensional Yang-Mills coupling constant,

$$\frac{1}{g_5^2} = \frac{(2\pi\alpha')^2}{(2\pi)^4\ell_s^5 g_s} = \frac{1}{(2\pi)^2\ell_s g_s}. \tag{13}$$

Then, by dimensional reduction to four dimensions, the four-dimensional Yang-Mills coupling constant can be expressed as

$$\frac{1}{g_{\mathrm{YM}}^2} = \frac{\beta}{g_5^2} = \frac{\beta}{4\pi^2 g_s\ell_s}. \tag{14}$$

In another way, the string coupling constant can be expressed using gauge theory parameters,

$$g_s = \frac{g_{\mathrm{YM}}^2}{2\pi M_{\mathrm{KK}}\ell_s} = \frac{\lambda}{2\pi M_{\mathrm{KK}} N_c \ell_s}, \tag{15}$$

where $\lambda = g_{\mathrm{YM}}^2 N_c$ is the 't Hooft coupling. Substituting this into (9), we have

$$H_0(U_{\mathrm{KK}}) = \frac{1}{2}(1 + (1 + C\beta^2)^{1/2}), \qquad C \equiv (2\pi\ell_s^2)^6\lambda^2\tilde{\kappa}^2/U_{\mathrm{KK}}^6. \tag{16}$$

In order to keep the backreaction of the D0-brane, we require N_0 to be of order N_c as in [10] and define $\tilde{\kappa} = N_0/(N_c V_4)$. It is easy to see that $H_0(U) \geq 1$.

Going to the near horizon limit by taking U/α' and $U_{\rm KK}/\alpha'$ finite, we have

$$U_{Q4}^3 \to \pi\alpha'^{3/2} g_s N_c = \frac{\beta g_{\rm YM}^2 N_c \ell_s^2}{4\pi} \equiv R^3, \tag{17}$$

$$H_4(U_{\rm KK}) \to \frac{R^3}{U_{KK}^3}, \tag{18}$$

$$\beta \to \frac{4\pi}{3} U_{\rm KK}^{-1/2} R^{3/2} H_0^{1/2}(U_{\rm KK}), \tag{19}$$

$$M_{\rm KK} \to \frac{3}{2} U_{\rm KK}^{1/2} R^{-3/2} H_0^{-1/2}(U_{\rm KK}). \tag{20}$$

The metric in string frame then becomes

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} (H_0^{1/2}(U)\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{-1/2}(U)f(U)d\tau^2) + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2}\left(\frac{1}{f(U)}dU^2 + U^2 d\Omega_4^2\right), \tag{21}$$

and the dilaton,

$$e^\Phi = g_s\left(\frac{U}{R}\right)^{3/4} H_0^{3/4}. \tag{22}$$

From (17) and (19) we have

$$\beta^{1/2} = \frac{2}{3}\pi^{1/2} U_{\rm KK}^{-1/2} \lambda^{1/2} \ell_s H_0^{1/2}(U_{\rm KK}), \tag{23}$$

or

$$\beta = \frac{4\pi\lambda\ell_s^2}{9U_{\rm KK}} H_0(U_{\rm KK}), \qquad M_{\rm KK} = \frac{9}{2}\frac{U_{\rm KK}}{\lambda\ell_s^2 H_0(U_{\rm KK})}. \tag{24}$$

Since $H_0(U_{\rm KK}) \geq 1$, $U_{\rm KK} \geq 2\lambda\ell_s^2 M_{\rm KK}/9$.

From this equation, β can be solved, and comparing with (24) we have

$$\beta = \frac{4\pi\lambda\ell_s^2}{9U_{\rm KK}} \frac{1}{1 - \frac{(2\pi\ell_s^2)^8}{81U_{\rm KK}^8}\lambda^4\tilde{\kappa}^2}, \qquad H_0(U_{\rm KK}) = \frac{1}{1 - \frac{(2\pi\ell_s^2)^8}{81U_{\rm KK}^8}\lambda^4\tilde{\kappa}^2}. \tag{25}$$

If we define $D = \frac{2}{9}\pi\lambda\ell_s^2/U_{\rm KK}$ and use the definition of C in (16), β then can be expressed as $\beta = 2D/(1 - CD^2)$ and $H_0(U_{\rm KK}) = 1/(1 - CD^2)$. Since $H_0 > 0$ and $CD^2 \leq 1$, this gives a constraint for $\tilde{\kappa}$,

$$|\tilde{\kappa}| \leq \frac{9U_{\rm KK}^4}{(2\pi\ell_s)^4\lambda^2} = \frac{\lambda^2 M_{\rm KK}^4 H_0^4(U_{\rm KK})}{9^3\pi^4}. \tag{26}$$

If we fix β, λ, from (24), $U_{\rm KK}$ goes the same as $H_0(U_{\rm KK})$. And together with (25), $H_0(U_{\rm KK})$ and $\tilde{\kappa}$ can be related,

$$H_0^8(U_{\rm KK}) - H_0^7(U_{\rm KK}) = \frac{9^6\pi^8\tilde{\kappa}^2}{\lambda^4 M_{\rm KK}^8} = 9^6\pi^8\xi^2. \tag{27}$$

For future convenience, we have defined a dimensionless quantity ξ,

$$\xi \equiv \frac{|\tilde{\kappa}|}{\lambda^2 M_{\rm KK}^4}. \tag{28}$$

Since we fix λ and $M_{\rm KK}$, changing $\tilde{\kappa}$ is equivalent to changing ξ. The left-hand side of (27) is a monotonic function increasing from zero for $H_0(U_{\rm KK}) \geq 1$. So for each $\tilde{\kappa}$, there is only one solution of $H_0(U_{\rm KK})$, going up as $\tilde{\kappa}$ increases (see Fig. 1), and $U_{\rm KK}$ is similar. Since we are interested in the region with $\lambda \gg 1$, if we choose $\lambda \sim 10$ and $|\tilde{\kappa}| < M_{\rm KK}^4$, ξ should be within $0 < \xi < 0.01$. And the corresponding $H_0(U_{\rm KK})$ falls in $1 < H_0(U_{\rm KK}) < 5.3$. So in future numerical analysis we constrain ourselves in this region.

This background actually introduces another free parameter $\tilde{\kappa}$ in the Sakai-Sugimoto model. This string theory background is not dual to the vacuum state of the gauge theory. The dual state may describe some excited state with some constant homogeneous field strength background, which gives the expectation value of ${\rm tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$. On the supergravity side, $\tilde{\kappa}N_c$ is the flux or charge of f_2. Since C_1 is coupled to ${\rm tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$ in the Euclidean Chern-Simons action,

$$S_{\rm CS} = i\frac{\mu_4}{2}(2\pi\alpha')^2 \int d\tau C_\tau \wedge {\rm tr}(F \wedge F). \tag{29}$$

$\tilde{\kappa}$ characterizes the expectation value of the Euclidean ${\rm tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$. Just as in [10], by rotating to the Euclidean space and naively using the classical EOM of C_1, we have the real Euclidean condensate,

$$\langle {\rm tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle = 8\pi^2 N_c\tilde{\kappa}. \tag{30}$$

We suppose this is a stochastic average over the background fields in all directions so that the $\langle F\rangle$ is still zero and the four-dimensional space-time translation invariance and proper Lorentz invariance are preserved, which is manifest in the string background solution. Obviously the P and CP invariances are violated, which is similar to the situation in [10]. Self-dual constant homogeneous backgrounds in the gauge theory are studied in [2–5] and may be related to the confinement. However, the field strength may not be self-dual in the present paper since the gravity background is

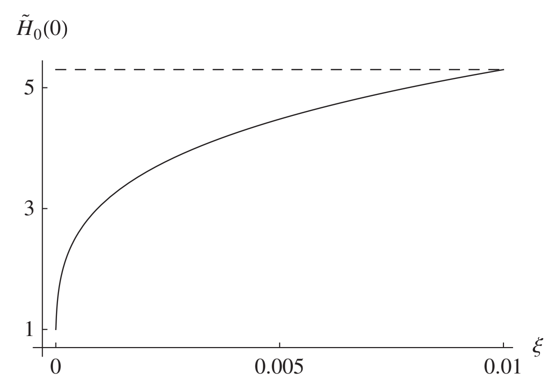

FIG. 1. The relation between $\tilde{H}_0(0)$ and parameter ξ.

nonsupersymmetric, and so we will not take it as a necessary assumption. Whether the background field strength is self-dual or not is beyond the scope of this paper. Our interest is to put the S-S model in this background to study the $\tilde{\kappa}$ dependence of the meson spectra and couplings.

Now we have the following independent parameters on the gravity side: R^3, U_{Q0}^3, $U_{\rm KK}$, and g_s, and ℓ_s will be canceled out in the final physical results. We also have the following parameters on the gauge theory side: N_c, $M_{\rm KK}$, λ, and $\tilde{\kappa}$. We have seen that $\tilde{\kappa}$ can be related to $H_0(U_{\rm KK})$, and we can use $H_0(U_{\rm KK})$ to represent $\tilde{\kappa}$. The final results on the gauge theory side can be expressed using N_c, $M_{\rm KK}$, λ, and $H_0(U_{\rm KK})$. We collect the relations here,

$$R^3 = \frac{\lambda \ell_s^2}{2M_{\rm KK}}, \qquad g_s = \frac{\lambda}{2\pi M_{\rm KK} N_c \ell_s}, \qquad U_{\rm KK} = \frac{2}{9} M_{\rm KK} \lambda \ell_s^2 H_0(U_{\rm KK}). \tag{31}$$

We fix the gauge theory parameters $M_{\rm KK}$, N_c, and λ, and then change $\tilde{\kappa}$. This corresponds to fixing the following parameters on the gravity side: R^3, g_s, $H_0(U_{\rm KK})/U_{KK}$, and changing $H_0(U_{\rm KK})$ or $U_{\rm KK}$.

Similar to the discussion of the D4-soliton background [25] in the S-S model, we can discuss the reliability of the background. First we require the curvature near the horizon to be small compared to the string scale $1/|(\mathbf{R}\ell_s^2)| \gg 1$. The curvature at $U_{\rm KK}$ is

$$\mathbf{R}(U_{\rm KK}) \sim \frac{9}{R^{3/2} U_{\rm KK}^{1/2} H_0^{1/2}(U_{KK})} \left(2 - \frac{3}{H_0(U_{\rm KK})}\right). \tag{32}$$

We have used $U_{\rm KK}^3/R^3 \sim \ell_s^6/\ell_s^2 \to 0$. Then using (31), we have

$$1 \ll \left|\frac{1}{\mathbf{R}\ell_s^2}\right| \sim \left|\frac{R^{3/2} U_{\rm KK}^{1/2} H_0^{1/2}(U_{\rm KK})}{9\ell_s^2(2 - 3/H_0(U_{\rm KK}))}\right| \sim \left|\frac{g_{\rm YM}^2 N_c H_0(U_{\rm KK})}{27(2 - 3/H_0(U_{\rm KK}))}\right|. \tag{33}$$

Since the factor $|H_0(U_{\rm KK})/(27(2 - 3/H_0(U_{\rm KK}))| \geq 1/27$ is bounded from below for $H_0(U_{\rm KK}) > 1$, $g_{\rm YM}^2 N_c \gg 1$ satisfies this inequality. However, the denominator $(2 - 3/H_0(U_{\rm KK}))$ could be zero for H_0 near 3/2. This may indicate that the gravity may not correspond to the strong coupling region. Nevertheless, by analyzing the scalar $R_{\mu\nu}R^{\mu\nu}\ell_s^4 \ll 1$, we can conclude that near $H_0 = 3/2$, the corresponding gauge theory is really in the strong 't Hooft coupling region,

$$1 \ll 1/|R_{\mu\nu}R^{\mu\nu}\ell_s^4| \simeq \frac{\lambda^2 H_0^4(U_{\rm KK})}{729(H_0^2 - H_0 + 1)}. \tag{34}$$

However, H_0 cannot be arbitrarily large. We require the factor $H_0^4/(729(H_0^2 - H_0 + 1)$ to be of $\mathcal{O}(1)$, which corresponds to $H_0(U_{\rm KK}) \sim 30$. Notice that previously $|\tilde{\kappa}| < M_{\rm KK}^4$ and large λ requires $1 \leq H_0(U_{\rm KK}) < 5.3$, which falls in this region. So the smallness of the curvature corresponds to the large 't Hooft coupling in the gauge theory in $1 \leq H_0 < 5.3$.

Next we require $e^\Phi \ll 1$ to suppress the string loop effect. From $e^\Phi = g_s H_0^{3/4} H_4^{-1/4}$, we have

$$U H_0(U) \ll g_s^{-4/3} R. \tag{35}$$

Since $1 \leq H_0 \sim \mathcal{O}(1)$, this means $U \ll g_s^{-4/3} R \equiv U_{\rm crit}$. This introduces no new information to the D4 soliton results. To repeat, the critical radius can be expressed as $U_{\rm crit} \simeq (2\pi^{4/3}\ell_s^2 N_c^{1/3} M_{\rm KK})/g_{\rm YM}^2$, and we require $U_{\rm crit} \gg U_{\rm KK}$. So, we have

$$g_{\rm YM}^4 \ll \frac{1}{g_{\rm YM}^2 N_c} \ll 1. \tag{36}$$

This just suggests that the supergravity solution is a valid dual description of the strong coupling region of the four-dimensional gauge theory in the 't Hooft limit.

Before ending this section, let us discuss a little about the meaning of changing $\tilde{\kappa}$ or the D0 charge density on the gauge theory side. We have related the D0 charge density to the condensate in (30). In fact, it can also be related to the θ angle in the gauge theory as discussed in [32]. So the background is dual to a state with nonzero $\langle {\rm tr}(F\tilde{F})\rangle$ expectation value in the gauge theory with nonzero θ angle, and these two quantites are not independent. Similar to the situation in Liu *et al.*'s paper, this is not the vacuum state, since in the true vacuum state, θ should be zero and there is no $\langle {\rm tr}(F\tilde{F})\rangle$ condensate (as an abuse of terminology, we use "condensate" to denote the expectation value of ${\rm tr}(F\tilde{F})$ not only in the vacuum state but also in the excited state). Thus, changing $\tilde{\kappa}$ can also be viewed as changing the θ angle and hence changing the background state. We assume that there could exist such excited states in the corresponding gauge theory, and we are interested in the hadron properties in these states. These states may have some possibilities of being created in the heavy ion collisions as we stated in the Introduction. In the next few sections, we will turn on the massless probe flavors and study the $\tilde{\kappa}$ dependence of the hadron physics. As is well known, one can make a phase rotation to eliminate the theta dependence in the gauge theory. So changing θ does not make sense anymore with massless flavors turned on. However, this does not mean that one can rotate away the condensate of the background state. $\langle {\rm tr}(F\tilde{F})\rangle$ is a physical observable, which should not be rotated away by an unobservable phase rotation. The back-reaction of the probe flavors on the background is ignored and hence the condensate in the backgound state should not be affected. In fact, in this situation, neither θ nor the expectation value of η' in this state (we use η' to denote its expectation value from now on) has a physical meaning, but the combination $\theta + \eta'\sqrt{2N_f}/f_\pi$ does. Only this combination could appear in the physical observables. By almost the same reasoning as in Sec. 5.8 of [26], taking the field strength of C_1 corresponding to the D0 charge in our paper as gauge invariant, one can show that $\theta + \eta'\sqrt{2N_f}/f_\pi$ is really related to $\tilde{\kappa}$, which is an observable in our paper. Since $\tilde{\kappa}$ appears also in the metric, the dependence on $\theta + \eta'\sqrt{2N_f}/f_\pi$ in the free energy comes not only from the kinetic part of C_1 as in [26], but also from the other parts of the action. We will not go further into the analysis of the free energy which is beyond the scope of our paper. As a further support to our argument, in [34], Witten argued that the θ dependence of the low-energy effective action of glueballs in the pure gauge theory should be changed to the dependence on $\theta + \eta'\sqrt{2N_f}/f_\pi$ after the switch-on of the flavor quarks. Suppose there is a state in the pure gauge sector corresponding to the D0-D4 background and this state has a nonzero value of θ and $\langle \mathrm{tr}(F\tilde{F})\rangle$ condensate. When the probe flavors are turned on, the θ angle is replaced by that combined quantity. So it is this combined quantity which takes the value that the original θ takes in the original pure gauge theory and the physical observable $\langle \mathrm{tr}(F\tilde{F})\rangle$ should not be changed. In this situation, changing the condensate can effectively be viewed as changing that combination of θ and η', which is a parameter of the theory.

III. SAKAI-SUGIMOTO MODEL IN D0-D4 BACKGROUND

Now we embed the D8-brane into the background with $U = U(\tau)$. The metric then becomes

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} H_0(U)^{-1/2}\left(f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0(U)}{f(U)} U'^2\right) d\tau^2 + \left(\frac{U}{R}\right)^{3/2} H_0^{1/2}(U)\eta_{\mu\nu}dx^\mu dx^\nu + H_0^{1/2}\left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2, \tag{37}$$

where $U' = dU/d\tau$. Substitute this into the D8-brane action, and we have

$$S_{D8} \sim \frac{1}{g_s}\int d^4x d\tau H_0(U)U^4\left(f(U) + \frac{H_0(U)}{f(U)}\left(\frac{R}{U}\right)^3 U'^2\right)^{1/2}, \tag{38}$$

from which the equation of motion can be obtained,

$$\frac{d}{d\tau}\left(\frac{H_0(U)U^4 f(U)}{[f(U) + \frac{H_0(U)}{f(U)}(\frac{R}{U})^3 U'^2]^{1/2}}\right) = 0, \tag{39}$$

which is just the conservation of the energy. With initial conditions $U(0) = U_0$ and $U'(0) = 0$ at $\tau = 0$, $\tau(U)$ can be solved,

$$\tau(U) = E(U_0)\int_{U_0}^{U} dU \frac{H_0^{1/2}(U)(\frac{R}{U})^{3/2}}{f(U)(H_0^2(U)U^8 f(U) - E^2(U_0))^{1/2}}, \tag{40}$$

where $E(U_0) = H_0(U_0)U_0^4 f^{1/2}(U_0)$.

The difference between the present background and the D4-soliton background is the $H_0(U)$ factor in all the equations. If we set $H_0(U) \to 1$, all the results degenerate to the original S-S model. For the antipodal case the profile is the same as the original S-S model, with $\tau(U) = \beta/4$. As the D8-$\overline{\mathrm{D8}}$ moves away from the antipodes, the profile goes less deep than in the original S-S model (Fig. 2). In this paper, as a first step, we constrain ourselves to the antipodal case to see the effects of the condensate $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$.

As in the S-S model, we introduce the new coordinate (r, θ) or (y, z),

$$y = r\cos\theta, \qquad z = r\sin\theta, \qquad U^3 = U_{\mathrm{KK}}^3 + U_{\mathrm{KK}}r^2,$$
$$\theta = \frac{2\pi}{\beta}\tau = \frac{3}{2}\frac{U_{\mathrm{KK}}^{1/2}}{R^{3/2}H_0^{1/2}(U_{\mathrm{KK}})}\tau, \tag{41}$$

and then the metric in the (y, z) plane becomes

$$ds_{\tau,U}^2 = \frac{4}{9}\frac{R^{3/2}}{U^{3/2}}\frac{H_0(U_{\mathrm{KK}})}{H_0^{1/2}(U)}[(1 - h(r)y^2)dy^2 + (1 - h(r)z^2)dz^2 - 2yzh(r)dydz], \tag{42}$$

where

$$h(r) = \frac{1}{r^2}\left[1 - \frac{U_{\mathrm{KK}}H_0(U)}{UH_0(U_{\mathrm{KK}})}\right]. \tag{43}$$

In the antipodal case, the D8-brane is put along x^0, x^1, x^2, x^3 and z direction at $y = 0$, wrapping the S^4. We can also study the fluctuations of the D8-brane in the y direction to examine the stability of this configuration. Then y is considered as a function of x and z, $y(x, z)$. The induced metric then reads

$$ds^2 = ds_{5\mathrm{d}}^2 + H_0^{1/2}(U)R^{3/2}U^{1/2}d\Omega_4^2 \tag{44}$$

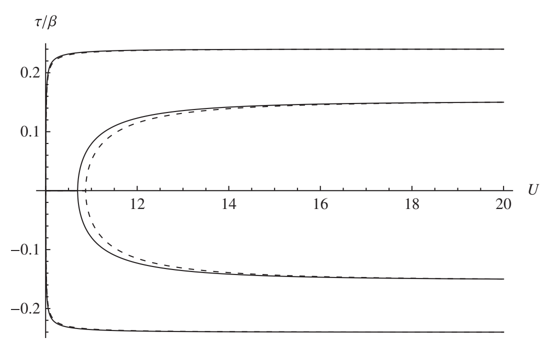

FIG. 2. The profile of the D8-brane. The dashed lines denote the profile with $\tilde{\kappa} \neq 0$ and the solid lines, with zero $\tilde{\kappa}$. With $\tilde{\kappa} \neq 0$, the D8-brane goes less deep.

$$ds_{5\mathrm{d}}^2 = H_0^{1/2}(U)\left(\frac{U}{R}\right)^{\frac{3}{2}}\left[\eta_{\mu\nu} + \frac{4}{9}\left(\frac{R}{U}\right)^3 \frac{H_0(U_{\mathrm{KK}})}{H_0(U)}\partial_\mu y \partial_\nu y\right]dx^\mu dx^\nu$$
$$+ \frac{4}{9}\left(\frac{R}{U}\right)^{\frac{3}{2}} \frac{H_0(U_{\mathrm{KK}})}{H_0^{1/2}(U)}\left[\frac{U_{\mathrm{KK}} H_0(U)}{U H_0(U_{\mathrm{KK}})} + \dot{y}^2 + h(z)(y^2 - 2zy\dot{y})\right]dz^2 + \frac{8}{9}\left(\frac{R}{U}\right)^{\frac{3}{2}} \frac{H_0(U_{\mathrm{KK}})}{H_0^{1/2}(U)}\partial_\mu y[\dot{y} - zyh(z)]dx^\mu dz + \mathcal{O}(y^4). \tag{45}$$

And the Dirac-Born-Infeld (DBI) action of the D8-brane turns out to be

$$S_{D8} = -\tilde{T} H_0^{3/2}(U_{\mathrm{KK}}) \int d^4x dz \left[\frac{H_0^{3/2}(U_z)}{H_0^{3/2}(U_{\mathrm{KK}})} U_z^2 + \frac{H_0^{1/2}(U_z)}{H_0^{1/2}(U_{\mathrm{KK}})}\left(\frac{2}{9}\frac{R^3}{U_z}\eta^{\mu\nu}\partial_\mu y \partial_\nu y + \frac{U^3}{2U_{\mathrm{KK}}}\dot{y}^2 + \frac{1}{2}\left(1 + \frac{1}{H_0(U_z)}\right)y^2\right)\right] + \mathcal{O}(y^4), \tag{46}$$

where we have defined $U_z = U_{\mathrm{KK}}(1 + z^2/U_{\mathrm{KK}}^2)^{1/3}$, $\tilde{T} = \frac{2}{3g_s} T_8 \Omega_4 U_{\mathrm{KK}}^{1/2} R^{3/2}$, with $T_8 = ((2\pi)^8 \ell_s^9)^{-1}$ the tension of the D8-brane. Then the energy density of the fluctuations in the y direction can be read off,

$$\mathcal{E} \simeq \tilde{T} H_0^{3/2}(U_{\mathrm{KK}}) \int dz \frac{H_0^{1/2}(U_z)}{H_0^{1/2}(U_{\mathrm{KK}})}\left(\frac{2}{9}\frac{R^3}{U_z}\sum_{i=0}^{3}(\partial_i y)^2 + \frac{U^3}{2U_{\mathrm{KK}}}\dot{y}^2 + \frac{1}{2}\left(1 + \frac{1}{H_0(U_z)}\right)y^2\right) \geq 0. \tag{47}$$

So adding the D0 flux does not affect the stability of the D8-brane probe configuration with respect to small fluctuations.

IV. SCALAR MESON SPECTRUM

Using the results of the previous section, we are ready to discuss the scalar spectrum for the one -flavor case. The fluctuations of y can be expanded in terms of some orthogonal basis $\rho_n(z)$,

$$y(x^\mu, z) = \sum_{n=1}^{\infty} \mathcal{U}^{(n)}(x^\mu)\rho_n(z). \tag{48}$$

We now define the dimensionless $Z = z/U_{\mathrm{KK}}$, $K = 1 + Z^2 = (U_z/U_{\mathrm{KK}})^3$, $U_z^3 = U_{\mathrm{KK}}^3(1 + Z^2)$, and $\tilde{H}_0(Z) = H_0(U_z)$. The orthogonal condition for ρ_m reads

$$\frac{4}{9}\tilde{T} R^3 \tilde{H}_0(0) \int dZ \tilde{H}_0^{1/2}(Z) K^{-1/3}(Z)\rho_m \rho_n = \delta_{mn}, \tag{49}$$

and $\rho_m (m \geq 1)$ are eigenfunctions of equation

$$K^{1/3}(Z)\left[-\tilde{H}_0^{-1/2}(Z)\partial_Z(\tilde{H}_0^{1/2}(Z)K(Z)\partial_Z \rho_n(Z)) + \left(1 + \frac{1}{\tilde{H}_0(Z)}\right)\rho_n(Z)\right] = \lambda_n \rho_n(Z). \tag{50}$$

Then the D8 action can be written as

$$S_{D8} = -\int d^4x \frac{1}{2}\sum_{n=1}^{\infty} \partial_\mu \mathcal{U}^{(n)} \partial^\mu \mathcal{U}^{(n)} + \frac{1}{2} M_{KK}^2 \tilde{H}_0(0) \sum_{n=1}^{\infty} \lambda_n (\mathcal{U}^{(n)})^2, \tag{51}$$

from which we can read off the mass for scalar mesons,

$$m_n^2 = M_{KK}^2 \tilde{H}_0(0)\lambda_n. \tag{52}$$

We see that the $\tilde{\kappa}$ dependence of the mass is through the $\tilde{H}_0(0)$ factor and is also hidden in λ_n as a result of the eigenvalue equation (50).

Now, we proceed to solve the eigenvalue equation. Similar to the method in Sakai and Sugimoto's original paper, from (50) we first find out the asymptotic behavior of $\rho_n(Z)$ as Z goes to infinity,

$$\rho_n \sim \frac{1}{Z^2}. \tag{53}$$

Then we can define

$$Z \equiv e^\eta, \qquad \tilde{\rho}_n(\eta) \equiv e^{2\eta}\rho_n(e^\eta), \tag{54}$$

such that $\tilde{\rho}_n$ is of $\mathcal{O}(Z^0)$. So the equation for $\tilde{\rho}_n$ reads

$$\frac{d^2\tilde{\rho}_n}{d\eta^2} + G\frac{d\tilde{\rho}_n}{d\eta} + F\tilde{\rho}_n = 0, \tag{55}$$

where

$$F = 6 - \frac{3}{1 + e^{-2\eta}} - \frac{3}{1 + \tilde{H}_0(0)e^{-2\eta}} + \frac{\lambda_n e^{-2\eta/3}}{(1 + e^{-2\eta})^{4/3}} \equiv \sum_{k=0}^{\infty} F_k e^{-2k\eta/3},$$
$$G = -5 + \frac{1}{1 + e^{-2\eta}} + \frac{1}{1 + \tilde{H}_0(0)e^{-2\eta}} \equiv \sum_{k=0}^{\infty} G_k e^{-2k\eta/3}. \tag{56}$$

The first few nonvanishing coefficients are listed below:

$$F_1 = \lambda_n, \qquad F_3 = 3 + 3\tilde{H}_0(0), \qquad F_4 = -\frac{4}{3}\lambda_n, \ldots$$
$$G_0 = -3, \qquad G_3 = -1 - \tilde{H}_0(0), \ldots. \tag{57}$$

Next we expand $\tilde{\rho}_n$ as

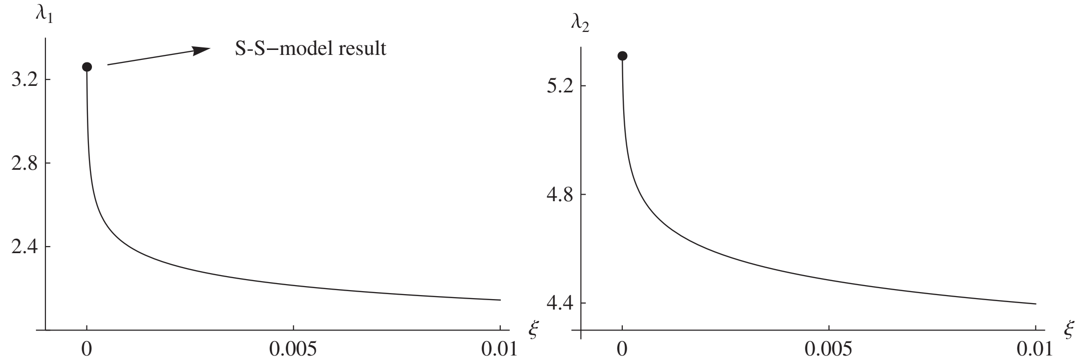

FIG. 3. The ξ dependence of λ_1 and λ_2.

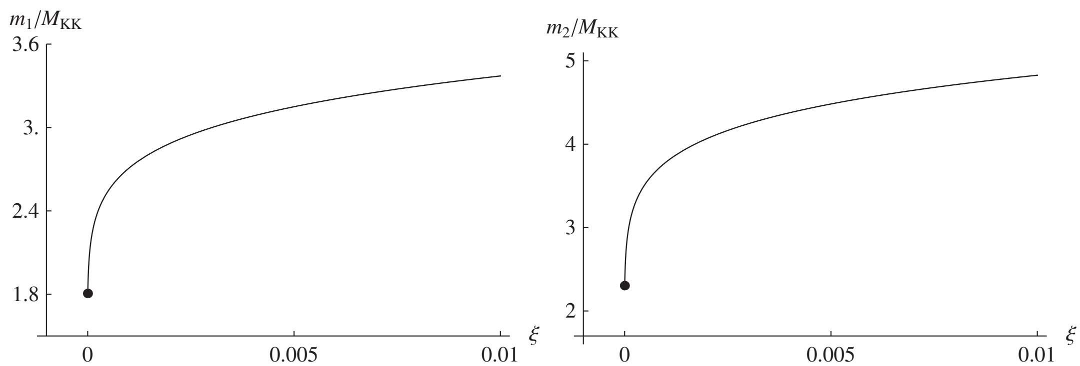

FIG. 4. The ξ dependence of m_1 and m_2.

$$\tilde{\rho}_n \sim 1 + \sum_{k=1}^{\infty} \beta_k e^{-2k\eta/3}. \tag{58}$$

And it is easy to verify that

$$\beta_1 = -\frac{9}{22}\lambda_n, \qquad \beta_2 = \frac{81}{1144}\lambda_n^2,$$
$$\beta_3 = -\frac{3}{10} - \frac{3}{10}\tilde{H}_0(0) - \frac{81}{11440}\lambda_n^3, \ldots. \tag{59}$$

We then solve the eigenvalue equation using the "shooting" method with ξ running from 0 to 0.01. As in S-S's original paper [26], we choose the eigenfunction to be even or odd for $n \geq 1$,

$$\text{Even}: \partial_Z \rho_n(0) = 0, \qquad \text{Odd}: \rho_n(0) = 0. \tag{60}$$

As a result, the eigenfunctions are even for odd n, and odd for even n. The charge conjugate C and parity properties are the same as in the S-S model. Then the lightest scalar meson has $CP = ++$ and the next level, $CP = --$. The ξ dependence of the lowest two λ_n and masses are shown in Figs. 3 and 4, respectively.

From these figures, we can see that even though the first two eigenvalues go down as ξ increases, the contributions from $\tilde{H}_0(0)$ overcome the eigenvalue contributions and make the mass grow with ξ. This is different from the results using the D(−1)-D3 background in [14]. In their model, the Liu-Tseytlin [10] background is used, which is supersymmetric, and in the corresponding gauge theory, the condensate is claimed to be self-dual $\langle F_{\mu\nu}F^{\mu\nu}\rangle = \langle F_{\mu\nu}\tilde{F}^{\mu\nu}\rangle \sim q$. Also, the current quark mass is nonzero. In our model, the background is not supersymmetric and it is highly possible that the field strength is not self-dual. And since the Goldstone is massless, the current quark mass is also zero. So the difference is not surprising. In their model, the meson mass is only determined by the eigenvalue of the fluctuation, which is going down with increasing q. In our model, though the eigenvalues have the same tendency as theirs, the masses are also proportional to $\tilde{H}_0(0)$, which is increasing and dominates in the contributions.

V. GAUGE FIELD FLUCTUATIONS AND VECTOR MESON SPECTRA

Now we consider the gauge field excitations on the D8-brane in this background. As in the S-S model, we are only interested in the SO(5) singlets, A_μ ($\mu = 0$, 1, 2, 3) and A_z, which are independent of the angular coordinates of the S^4. We consider only one flavor in this section. The DBI action can be cast into

$$S_{D8} = -\tilde{T}(2\pi\alpha')^2 \int d^4x dz H_0^{1/2}(U) \times \left[\frac{1}{4}\frac{R^3}{U}F_{\mu\nu}F^{\mu\nu} + \frac{9}{8}\frac{U^3}{U_{\rm KK}}F_{\mu z}F^{\mu z}\right]. \tag{61}$$

As in the S-S model, we expand the gauge field $A_\mu(\mu = 0, 1, 2, 3)$ and A_z in terms of some orthogonal basis,

$$A_\mu(x, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x)\psi_n(z),$$
$$A_z(x, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x)\phi_n(z), \tag{62}$$

and the orthogonal conditions are

$$\tilde{T}(2\pi\alpha')^2 R^3 \int dZ \frac{\tilde{H}_0^{1/2}(Z)}{K^{1/3}(Z)} \psi_m \psi_n = \delta_{mn}, \tag{63}$$

$$\tilde{T}(2\pi\alpha')^2 R^3 M_{\mathrm{KK}}^2 \tilde{H}_0(0) U_{\mathrm{KK}}^2 \int dZ \tilde{H}_0^{1/2}(Z) K(Z) \phi_m \phi_n = \delta_{mn}. \tag{64}$$

The eigenvalue equation for ψ_m is

$$-\tilde{H}_0^{-1/2}(Z) K^{1/3}(Z) \partial_Z(\tilde{H}_0^{1/2}(Z) K(Z) \partial_Z \psi_m) = \Lambda_m \psi_m, \tag{65}$$

with Λ_n the eigenvalue. The eigenfunction $\phi_n(z)$ can be chosen as

$$\phi_n = \frac{1}{M_n U_{\mathrm{KK}}} \partial_Z \psi_n = \frac{1}{M_n} \dot{\psi}_n(z),$$
$$M_n = \Lambda_n^{1/2} M_{\mathrm{KK}} \tilde{H}_0^{1/2}(0), \tag{66}$$

for $n \neq 0$, and for $n = 0$,

$$\phi_0 = \frac{c}{\tilde{H}_0^{1/2}(Z) K(Z)},$$
$$c = \left(\tilde{T}(2\pi\alpha')^2 R^3 M_{\mathrm{KK}}^2 \tilde{H}_0(0) U_{\mathrm{KK}}^2 \times \int dZ \tilde{H}_0^{-1/2}(Z) K^{-1}(Z)\right)^{-1/2}$$
$$= \frac{9(3\pi)^{3/2}}{\sqrt{2\lambda N_c}\lambda M_{\mathrm{KK}}^2 \ell_s^2 \tilde{H}_0^{3/2}(0)} \frac{1}{\sqrt{\mathcal{F}_0}}, \tag{67}$$

with

$$\mathcal{F}_0 \equiv \mathcal{F}\left(\frac{\pi}{2}, \sqrt{1 - \tilde{H}_0^{-1}(0)}\right),$$
$$\mathcal{F}(\phi, k) \equiv \int_0^\phi \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}},$$

the elliptic integrals of the first kind. The DBI action of the D8-brane can be recast into

$$S_{D8} = -\int d^4x \left[\sum_{n=1}^{\infty}\left(\frac{1}{4} F_{\mu\nu}^{(n)} F^{(n)\mu\nu} + \frac{1}{2} M_n^2 B_\mu^{(n)} B^{\mu(n)} - M_n \partial_\mu \varphi^{(n)} B^{\mu(n)}\right) + \sum_{n=0}^{\infty} \frac{1}{2} \partial_\mu \varphi^{(n)} \partial^\mu \varphi^{(n)}\right]. \tag{68}$$

By replacing $B_\mu^{(n)} \to B_\mu^{(n)} + M_n^{-1} \partial_\mu \varphi^{(n)}$ through a gauge transformation, the action becomes

$$S_{D8} = -\int d^4x \left[\sum_{n=1}^{\infty}\left(\frac{1}{4} F_{\mu\nu}^{(n)} F^{(n)\mu\nu} + \frac{1}{2} M_n^2 B_\mu^{(n)} B^{\mu(n)}\right) + \frac{1}{2} \partial_\mu \varphi^{(0)} \partial^\mu \varphi^{(0)}\right]. \tag{69}$$

We see that the masses for the massive vector bosons are just M_n and there is a massless pseudoscalar $\varphi^{(0)}$, which is just the Nambu-Goldstone boson. For the $U(1)$ case here, this Goldstone boson is just like the η' in the real world. Due to the $U(1)_A$ anomaly, its mass is related to the topological susceptibility of the pure Yang-Mills theory. This has already been discussed in [32] and met some difficulties in obtaining the analytic results. So we will not go deep in this direction.

We can now analyze the $\tilde{\kappa}$ dependence of the mass spectrum of the vector mesons by performing the same procedure as in the previous section. First we find out the asymptotic behavior when Z approaches infinity,

$$\psi_n \sim \frac{1}{Z}, \tag{70}$$

and define a new function,

$$\tilde{\psi}_n(\eta) = e^\eta \psi_n(e^\eta), \tag{71}$$

which satisfies the equation

$$\frac{d^2\tilde{\psi}_n}{d\eta^2} + G' \frac{d\tilde{\psi}_n}{d\eta} + F' \tilde{\psi}_n = 0, \tag{72}$$

where

$$F' = 2 - \frac{1}{1 + e^{-2\eta}} - \frac{1}{1 + \tilde{H}_0(0) e^{-2\eta}} + \frac{\Lambda_n e^{-2\eta/3}}{(1 + e^{-2\eta})^{4/3}} \equiv \sum_{k=0}^{\infty} F_k' e^{-2k\eta/3},$$
$$G' = -3 + \frac{1}{1 + e^{-2\eta}} + \frac{1}{1 + \tilde{H}_0(0) e^{-2\eta}} \equiv \sum_{k=0}^{\infty} G_k' e^{-2k\eta/3}, \tag{73}$$

in which the first few nonvanishing components are

$$F_1' = \Lambda_n, \qquad F_3' = 1 + \tilde{H}_0(0), \qquad F_4' = -\frac{4}{3}\Lambda_n, \ldots$$
$$G_0' = -1, \qquad G_3' = -1 - \tilde{H}_0(0), \ldots. \tag{74}$$

With these coefficients we can work out the expansion of ψ_n,

$$\psi_n(Z) \sim \frac{1}{Z} + \frac{\beta_1'}{Z^{5/3}} + \frac{\beta_2'}{Z^{7/3}} + \frac{\beta_3'}{Z^2} + \cdots, \tag{75}$$

where

$$\beta'_1 = -\frac{9}{10}\Lambda_n, \qquad \beta'_2 = \frac{81}{280}\Lambda_n^2,$$
$$\beta'_3 = -\frac{1+\tilde{H}_0(0)}{6} - \frac{27}{560}\Lambda_n^3, \ldots. \tag{76}$$

Using the shooting method to solve this two-point boundary value problem, we obtain the evolutions of the first two eigenvalues with respect to ξ, which are shown in Fig. 5, and the corresponding masses are shown in Fig. 6.

Similar to the scalar meson cases, the contributions from the eigenvalues are not comparable to the one from the $H_0(U_{\rm KK})$ factor: the trend of the eigenvalues for large ξ is downward, while the trend for the final masses is upward. This result is also different from the D(−1)-D3 case [14] in which the vector mass is independent of q.

As Sakai and Sugimoto did in their original paper [26], we could also consider the mass ratios,

$$\frac{M_2^2}{M_1^2} = \frac{\Lambda_2}{\Lambda_1}, \qquad \frac{m_1^2}{M_1^2} = \frac{\lambda_1}{\Lambda_1}. \tag{77}$$

With the lowest two vector mesons assigned to $\rho(770)$ and $a_1(1260)$, and the lowest-lying scalar assigned to isospin one $a_0(1450)$, these two ratios can be estimated to be 2.51 and 3.61 [26], respectively. Our results for these ratios are plotted in Fig. 7. It is interesting to see that the first estimated ratio can be reached by tuning the $\tilde{\kappa}$ parameter

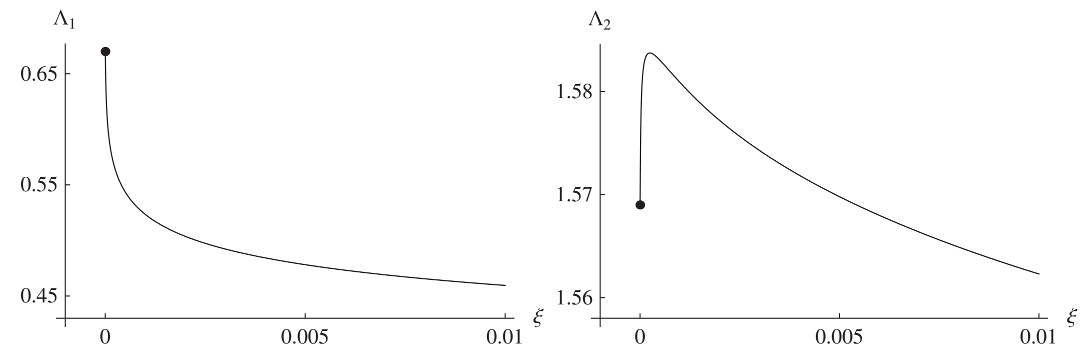

FIG. 5. The ξ dependence of Λ_1 and Λ_2.

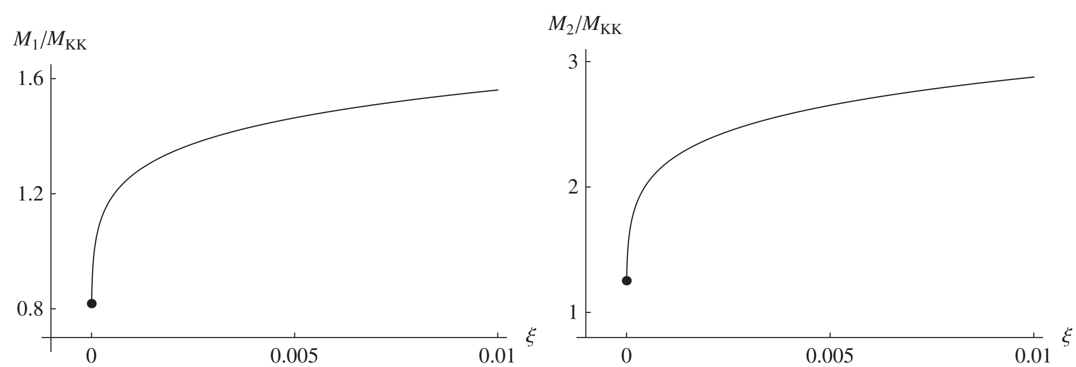

FIG. 6. The ξ dependence of M_1 and M_2.

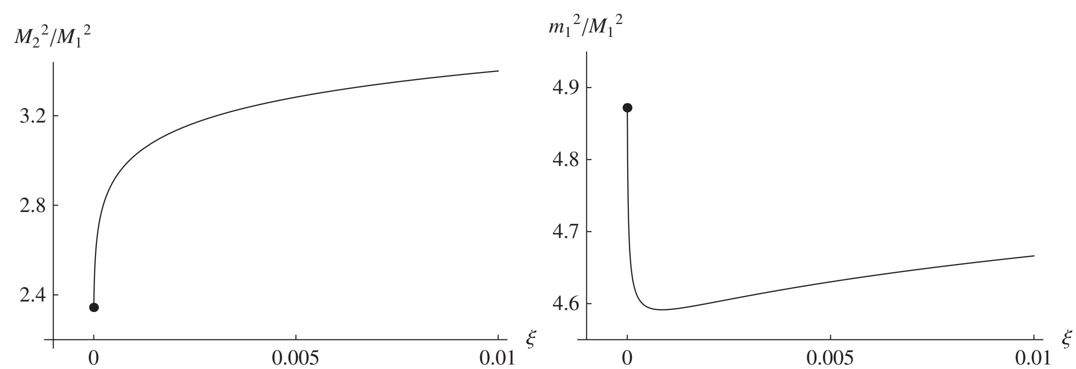

FIG. 7. The ξ dependence of M_2^2/M_1^2 and m_1^2/M_1^2.

to some certain value, and the second ratio in our result is closer to the experimental value with $\tilde{\kappa}$ turned on. However, this should not be taken seriously, since the experimental value is in the true vacuum in which the condensate $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$ may be almost zero.

VI. MULTIFLAVOR CASE

As in the S-S model, we can extend the previous discussion to the multiflavor case, i.e., $N_f > 1$. We will see that the mass formulas for vector mesons are the same as in the one-flavor case, and there is no new information for the vector meson mass spectrum. However, we can study the $\tilde{\kappa}$ dependence of f_π and the couplings of vectors and Goldstones. Since we closely follow S-S's original paper in the deduction, we will be brief and refer the readers to S-S's paper [26].

In the multiflavor case, the gauge fluctuations on N_f-flavor D8-branes are non-Abelian, and the DBI action becomes

$$S_{\mathrm{D8}} = -\hat{T}(2\pi\alpha')^2 \int d^4x dz 2H_0^{1/2}(U_z)\mathrm{Tr} \times \left[\frac{1}{4}\frac{R^3}{U_z}F_{\mu\nu}F^{\mu\nu} + \frac{9}{8}\frac{U_z^3}{U_{\mathrm{KK}}}F_{z\mu}F^{z\mu}\right], \quad (78)$$

where

$$\hat{T} \equiv \frac{\tilde{T}}{2} = \frac{1}{3g_s}T_8\Omega_4 U_{\mathrm{KK}}^{1/2}R^{3/2} = \frac{M_{\mathrm{KK}}N_c\tilde{H}_0^{1/2}(0)}{432\pi^5\ell_s^6}, \quad (79)$$

with field strength $F_{MN} = \partial_M A_N - \partial_N A_M + [A_M, A_N]$ for $U(N)$ gauge field A_M on the D8-branes. The contractions for μ and ν are done by using $\eta^{\mu\nu}$.

The $U(x^\mu) \equiv \exp\{2i\pi/f_\pi\}$ field in the usual chiral Lagrangian is realized as

$$U(x^\mu) = P\exp\left\{-\int_{-\infty}^{\infty} dz' A_z(x^\mu, z')\right\} = \xi_+^{-1}(x^\mu)\xi_-(x^\mu), \quad (80)$$

where $\xi_\pm^{-1}(x^\mu) \equiv P\exp\{-\int_0^{\pm\infty} dz' A_z(x^\mu, z')\}$ is defined for convenience.

A. Pion Lagrangian

In $\xi_-(x^\mu) = 1$ gauge, one can expand the non-Abelian gauge field as

$$A_\mu(x^\mu, z) = U^{-1}(x^\mu)\partial_\mu U(x^\mu)\frac{1+\hat{\psi}_0(z)}{2} + \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z), \quad (81)$$

where

$$\hat{\psi}_0(z) = \frac{\int_0^Z dZ\tilde{H}_0^{-1/2}(Z)K^{-1}(Z)}{\int_0^\infty dZ\tilde{H}_0^{-1/2}(Z)K^{-1}(Z)} = \frac{1}{\mathcal{F}_0}\mathcal{F}\left(\arctan\frac{z}{U_{\mathrm{KK}}}, \sqrt{1-\tilde{H}_0^{-1}(0)}\right), \quad (82)$$

with $\mathcal{F}$ and $\mathcal{F}_0$ the elliptic integrals defined in (68). Now since we are only interested in the pion field, all the excited vector modes $B_\mu^{(n)}$ $(n \geq 1)$ can be omitted, and the field strength can be written as

$$F_{\mu\nu} = [U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]\frac{\hat{\psi}_0^2 - 1}{4}, \quad F_{z\mu} = U^{-1}\partial_\mu U\frac{\partial_z\hat{\psi}_0}{2}. \quad (83)$$

Substituting (83) into (78), we obtain the effective action for pion

$$S_{\mathrm{D8}} = -\hat{T}(2\pi\alpha')^2 \int d^4x\, \mathrm{Tr}(A(U^{-1}\partial_\mu U)^2 + B[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2), \quad (84)$$

with

$$A \equiv \int dz \frac{9}{4}\frac{U_z^3}{U_{\mathrm{KK}}}H_0^{1/2}(U_z)\left(\frac{\partial_z\hat{\psi}}{2}\right)^2 = \frac{9U_{\mathrm{KK}}\tilde{H}_0^{1/2}(0)}{8\mathcal{F}_0},$$
$$B \equiv \int dz \frac{R^3}{2U_z}H_0^{1/2}(U_z)\left(\frac{\hat{\psi}_0^2 - 1}{4}\right)^2 = \frac{R^3}{32\mathcal{F}_0^4}b(\tilde{H}_0(0)). \quad (85)$$

Here $b(\tilde{H}_0)$ is an integral constant defined by

$$b(\alpha) \equiv \int dZ\frac{(\alpha+Z^2)^{1/2}}{(1+Z^2)^{5/6}}\left[\mathcal{F}^2\left(\arctan Z, \sqrt{1-\frac{1}{\alpha}}\right) - \mathcal{F}_0^2\right]^2. \quad (86)$$

Comparing this result with the Skyrme model [35] in which the action is

$$S = \int d^4x\mathrm{Tr}\left(\frac{f_\pi^2}{4}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e^2}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2\right), \quad (87)$$

we can read off f_π^2 and the dimensionless e^2,

$$f_\pi^2 = 4\hat{T}(2\pi\alpha')^2 A = \frac{\tilde{H}_0^2(0)}{108\pi^3\mathcal{F}_0}\lambda N_c M_{\mathrm{KK}}^2,$$
$$e^2 = \frac{1}{32\hat{T}(2\pi\alpha')^2 B} = \frac{216\pi^3\mathcal{F}_0^4}{\tilde{H}_0(0)b(\tilde{H}_0(0))}\frac{1}{\lambda N_c}. \quad (88)$$

Now the pion decay constant f_π and the parameter e are both affected by the glue condensate $\tilde{\kappa}$. As before, we use ξ defined in (28) instead of $\tilde{\kappa}$, and the ξ dependence of f_π and e is shown in Fig. 8 where we have defined

$$\tilde{f}_\pi \equiv \sqrt{\frac{108\pi^3}{\lambda N_c M_{\mathrm{KK}}^2}}f_\pi = \frac{\tilde{H}_0(0)}{\sqrt{\mathcal{F}_0}},$$
$$\tilde{e} \equiv \sqrt{\frac{\lambda N_c}{216\pi^3}}e = \frac{\mathcal{F}_0^2}{\sqrt{\tilde{H}_0^{1/2}(0)b(\tilde{H}_0(0))}} \quad (89)$$

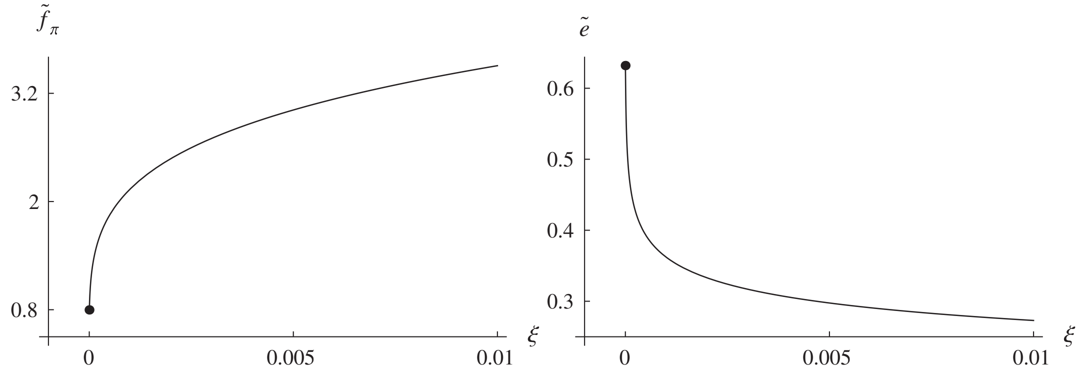

FIG. 8. The ξ dependence of $\tilde{f}_\pi$ and $\tilde{e}$.

for convenience. We can see that f_π goes up while e declines with ξ.

B. Vector mesons

Next we consider the first excited vector mode $B_\mu^{(1)}$ which is identified as the ρ meson. In the $\xi_+^{-1}(x^\mu) = \xi_-(x^\mu) = \exp(i\pi(x^\mu)/f_\pi)$ gauge, A_μ can be expanded as

$$A_\mu(x^\mu, z) = \frac{i}{f_\pi}\partial_\mu \pi(x^\mu)\hat{\psi}_0(z) + \frac{1}{2f_\pi^2}[\pi(x^\mu), \partial_\mu \pi(x^\mu)] + v_\mu(x^\mu)\psi_1(z) \tag{90}$$

where $v_\mu = B_\mu^{(1)}$. Thus the field strength is

$$\begin{aligned} F_{\mu\nu} &= \frac{i}{f_\pi}([\partial_\mu\pi, v_\nu] + [v_\mu, \partial_\nu\pi])\psi_1\hat{\psi}_0 \\ &\quad + \frac{1}{f_\pi^2}[\partial_\mu\pi, \partial_\nu\pi](1-\hat{\psi}_0^2) + (\partial_\mu v_\nu - \partial_\nu v_\mu)\psi_1 \\ &\quad + [v_\mu, v_\nu]\psi_1^2 + \mathcal{O}((\pi, v_\mu)^3), \\ F_{z\mu} &= \frac{i}{f_\pi}\partial_\mu\pi\partial_z\hat{\psi}_0 + v_\mu\partial_z\psi_1. \end{aligned} \tag{91}$$

The effective action involving π and v_μ up to $\mathcal{O}((\pi, v_\mu)^3)$ can be obtained

$$\begin{aligned} S_{\mathrm{D8}} = \int d^4x\{ &-a_{\pi^2}\mathrm{Tr}(\partial_\mu\pi\partial^\mu\pi) \\ &+ a_{v^2}(\mathrm{Tr}(\partial_\mu v_\nu - \partial_\nu v_\mu)^2 + m_v^2\mathrm{Tr}v_\mu^2) \\ &+ a_{v^3}\mathrm{Tr}([v^\mu, v^\nu](\partial_\mu v_\nu - \partial_\nu v_\mu)) \\ &+ a_{v\pi^2}\mathrm{Tr}([\partial^\mu\pi, \partial^\nu\pi](\partial_\mu v_\nu - \partial_\nu v_\mu)) \\ &+ \mathcal{O}((\pi, v_\mu)^4)\}. \end{aligned} \tag{92}$$

Then we determine all the coefficients one by one. The coefficient before the kinetic term of pion is

$$a_{\pi^2} = \frac{2\hat{T}(2\pi\alpha')^2}{f_\pi^2}\int dz \frac{9}{8}\frac{U_z^3}{U_{\mathrm{KK}}}H_0^{1/2}(U_z)(\partial_z\hat{\psi}_0)^2 = 1 \tag{93}$$

due to the definition of f_π in (88). Next, we redefine

$$\Psi_1(Z) \equiv \sqrt{\hat{T}(2\pi\alpha')^2R^3}\,\psi_1(U_{\mathrm{KK}}Z), \tag{94}$$

so that it is properly normalized and the coefficient before the vector kinetic term is

$$\begin{aligned} a_{v^2} &= \hat{T}(2\pi\alpha')^2\int dz\frac{R^3}{U_z}H_0^{1/2}(U_z)\psi_1^2(z) \\ &\equiv \int dZ K^{-1/3}(Z)\tilde{H}_0^{1/2}(Z)\Psi_1^2(Z) = 1 \end{aligned} \tag{95}$$

by the orthogonal condition (63). This leads to

$$\begin{aligned} m_v^2 &= a_{v^2}m_v^2 \\ &= \hat{T}(2\pi\alpha')^2\int dz\frac{9}{4}\frac{U_z^2}{U_{\mathrm{KK}}}H_0^{1/2}(U_z)\left(\frac{d\psi_1(z)}{dz}\right)^2 \\ &= \Lambda_1 M_{\mathrm{KK}}^2\tilde{H}_0(0) \equiv M_1^2, \end{aligned} \tag{96}$$

which is in agreement with Eq. (66) except for a redefinition of $\hat{T}$. So the $\tilde{\kappa}$ dependence of the vector mass is the same as in the one-flavor case. The three-point self-coupling for the vector field is

$$\begin{aligned} a_{v^3} &= \hat{T}(2\pi\alpha')^2\int dz\frac{R^3}{U_z}H_0^{1/2}(U_z)\psi_1^3(z) \\ &= \frac{(6\pi)^{3/2}}{\sqrt{\lambda N_c}}I_{v^3}(\tilde{H}_0(0)). \end{aligned} \tag{97}$$

Similarly, the vector-Goldstone-Goldstone(VGG) three-point coupling is

$$\begin{aligned} a_{v\pi^2} &= \frac{\hat{T}(2\pi\alpha')^2}{f_\pi^2}\int dz\frac{R^3}{U_z}H_0^{1/2}(U_z)\psi_1(1-\hat{\psi}_0^2) \\ &= \frac{\pi(3\pi)^{3/2}}{M_{\mathrm{KK}}^2\sqrt{2\lambda N_c}}I_{v\pi^2}(\tilde{H}_0(0)). \end{aligned} \tag{98}$$

Here we have defined

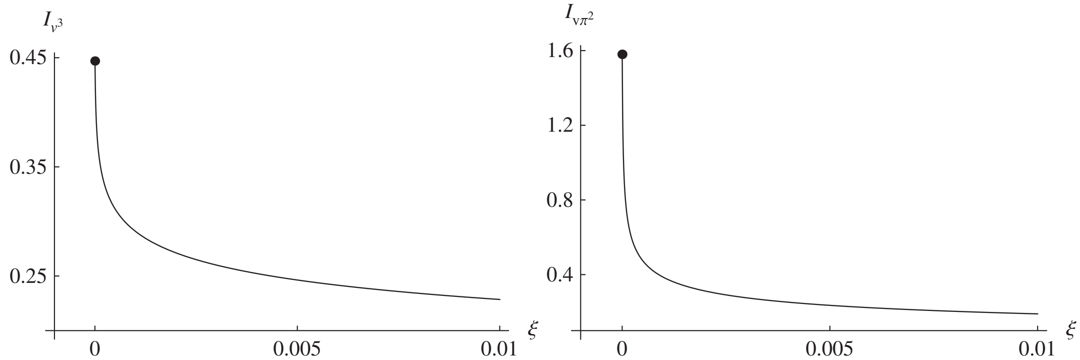

FIG. 9. The ξ dependence of I_{v^3} and $I_{v\pi^2}$.

$$I_{v^3}(\tilde{H}_0(0)) = \frac{1}{\tilde{H}_0^{1/4}(0)} \int dZ \frac{(\tilde{H}_0(0) + Z^2)^{1/2}}{(1 + Z^2)^{5/6}} \Psi_1^3(Z),$$
$$I_{v\pi^2}(\tilde{H}_0(0)) = \frac{2\mathcal{F}_0}{\pi \tilde{H}_0^{7/4}(0)} \int dZ \left[1 - \frac{1}{\mathcal{F}_0^2} \mathcal{F}^2\left(\arctan Z, \sqrt{1 - \tilde{H}_0^{-1}(0)}\right)\right] \frac{(\tilde{H}_0(0) + Z^2)^{1/2}}{(1 + Z^2)^{5/6}} \Psi_1(Z). \quad (99)$$

We can see that the couplings depend on $\tilde{\kappa}$ both explicitly in $\tilde{H}_0(0)$ in the integrands and the coefficients before the integrals, and implicitly in eigenfunction ψ_1 through the appearance of $\tilde{H}_0(Z)$ in the eigenvalue equation. The dependence is illustrated in Fig. 9. So both the three-point self-interaction of the vector meson and the VGG coupling are becoming weaker when $\tilde{\kappa}$ is turned on.

VII. CONCLUSION AND DISCUSSION

In this paper, we have studied the S-S model in the D0-D4 background which corresponds to the gauge theory in an excited state with a nonzero expectation value of $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$. The effects of this quantity on the meson spectra, pion decay constant, and the lowest-lying three-vector and vector-Goldstone-Goldstone couplings are studied. The dependence of these quantities on $\tilde{\kappa}$ comes in two parts: one is an explicit dependence in the $H_0(U_{\mathrm{KK}})$ factor in the formulas and the other is implicit in the eigenvalues and eigenfunctions [see (52), (65), and (99)]. The $\tilde{\kappa}$ dependence of the mass spectra are different from the D(-1)-D3 case in [14]. On the gravity side, this $\tilde{\kappa}$ dependence comes from the backreaction of the $D0$ charges to the metric and takes effect by coupling the metric to the flavor branes in the DBI action. On the field theory side, the glue condensate comes into play through its backreaction on the glue fluctuations, which couple to the flavors through glue-quark couplings. It must be a strong coupling nonperturbative phenomenon to have sizable effects on the mass spectra and the couplings. So these two pictures seem to be consistent. However, the $\tilde{\kappa}$ dependence in the metric always appears in $\tilde{\kappa}^2$, the squared form. The Chern-Simons terms with C_1 form field for D8 is zero since it involves fluctuations in the S_4 directions. So there is no explicit P or CP breaking terms in the effective Lagrangian from DBI action—no P or CP violating mixings and interactions, which seem to be strange since P or CP is broken due to nonzero condensate $\tilde{\kappa}$. One tends to give a hand-waving argument as follows: Since the condensate is in the pure glue sector, there could be P or CP violating mixings of glueballs when $\tilde{\kappa}$ is nonzero. The P or CP violating mixings of mesons in this model only happen through intermediate glueball mixing, and due to the OZI rule, this process may be suppressed in the large N_c.

In this paper we have not studied the Chern-Simons term containing f_4. This term can produce more interaction terms [27] and is also related to the baryons in this model [28,29]. It is easy to extend the discussions to this term in the D0-D4 background, which allows one to learn more interactions and the baryon properties with regard to the condensate $\tilde{\kappa}$. However, to introduce deconfinement temperature into this model is a little difficult since this needs a background with a horizon to give the deconfinement temperature in the four-spacetime also with the form field $C_\tau d\tau$ in the fifth direction, which may not be easy to find.

The string theory background used in this paper corresponds to a gauge theory with a *real* Euclidean condensate $\langle \mathrm{tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})\rangle$ as in [10] in which the gauge theory background was claimed to be self-dual. Since our background here is not supersymmetric, the self-dual property is not clear at present. Considering the quantum effect there could be some modification to $\tilde{\kappa}$. The situation that a nonextremal gravity background can lead to a non-self-dual field strength has been studied in [36] in the context of the localized instanton case. We leave this direction for future research. The absolute value of the quantities studied in this paper may not be of much significance. But the tendency of these quantities as $\tilde{\kappa}$ is turned on may capture the qualitative effect of the real Euclidean condensate in this model. However, a real Euclidean condensate may not be realistic in the real world, which is Minkowski. The string theory background for gauge theory with real Minkowski condensates can also be found. However, there could arise some other problems. We are still working on this possibility. The preliminary result is that the real Minkowski condensate may have opposite effects, compared to the Euclidean one, on the quantities studied in this paper. This needs to be confirmed in future work.

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