Bound on noncommutative standard model with hybrid gauge transformation via lepton flavor conserving *Z* decay

Weijian Wang,^{1,2} Jia-Hui Huang,³ and Zheng-Mao Sheng^{4,1,[*](#page-0-0)}

 1 Department of Physics, Zhejiang Institute of Modern Physics, Zhejiang University,

Hangzhou 310027, People's Republic of China
²Department of Physics, North China Electric Power University, Baoding, People's Republic of China

³ Center of Mathematical Science, Zhejiang University, Hangzhou 310027, People's Republic of China ⁴ Department of Physics, Institute for Eusian Theory and Simulation, Zhejiang University

 4 Department of Physics, Institute for Fusion Theory and Simulation, Zhejiang University,

Hangzhou 310027, People's Republic of China

(Received 10 June 2013; published 18 July 2013)

The $Z \rightarrow e^+e^-$ decay is studied based on the noncommutative standard model with the hybrid gauge transformation. It is shown that if the latter is not included, the noncommutative correction to the amplitude of the $Z \rightarrow e^+e^-$ appears only as a phase factor, so that there is no new physical effect on the decay width. However, when the hybrid gauge transformation is included, the noncommutative effect appears in the two-body decay process. The discrepancy between the experimental branch ratio and the standard model prediction allows us to set the bound on the noncommutative parameters.

The concept of noncommutative (NC) space-time was firstly introduced by Snyder in 1947 [[1](#page-3-0)]. Interest in NC space-time was revived since it appeared in the string theory and other quantum gravity models as effective theories in the lower energy limit $[2-5]$ $[2-5]$ $[2-5]$. In a popular NC model the NC space-time is characterized by a coordinate operator satisfying

$$
[\hat{x}_{\mu}, \hat{x}_{\nu}] = i\theta_{\mu\nu} = \frac{ic_{\mu\nu}}{\Lambda_{\rm NC}^2},\tag{1}
$$

where $\theta_{\mu\nu}$ is a constant antisymmetric matrix. Its elements have a dimension of $(mass)^{-2}$. Here $c_{\mu\nu}$ is a real antisymmetric matrix, whose dimensionless elements are assumed to be of order unity, and the NC scale Λ_{NC} characterizes the threshold where the NC effect becomes relevant and its role can be compared to that of \hbar in quantum mechanics. The existence of a finite Λ_{NC} implies the existence of a fundamental space-time distance below which the spacetime coordinates become fuzzy. By Weyl-Moyal correspondence, the quantum field theory in NC space-time is equivalent to that in ordinary space-time with the normal product of the field variables replaced by the star product, defined by [[6\]](#page-3-3)

$$
\phi_1(x) * \phi_2(x) = \exp\left(\frac{i}{2} \theta^{\mu\nu} \partial^x_\mu \partial^y_\nu\right) \phi_1(x) \phi_2(y)|_{y \to x}.\tag{2}
$$

Using this method, a noncommutative extension of the standard model (NCSM) has been proposed [\[7](#page-3-4)], where the SU(N) Lie algebra is generalized to the enveloping algebra via the Seiberg-Witten map (SWM) [[3\]](#page-3-5). The SWM is a map between the noncommutative field and its counterpart in ordinary space-time as a power series

DOI: [10.1103/PhysRevD.88.025031](http://dx.doi.org/10.1103/PhysRevD.88.025031) PACS numbers: 11.10.Nx, 12.15.-y, 12.60.-i, 13.38.Dg

of the NC parameter $\theta_{\mu\nu}$. The NCSM predicts the NC-corrected particle vertex and many new interactions beyond the standard model, for instance, the $Z - \gamma - \gamma$ and $Z - g - g$ vertices. The rich phenomenological implication has been intensively examined in high energy processes for possible experimental signals or given a bound on the noncommutative scale Λ_{NC} [\[8](#page-3-6),[9](#page-4-0)].

On the other hand, the neutrino oscillation experiments have shown convincing evidence of massive neutrinos and leptonic flavor mixing $[10,11]$ $[10,11]$, so that in constructing NCSM the neutrino mass should be included. It is found that the hybrid gauge transformation and hybrid SWMs are needed to accommodate the seesaw mechanism [[11](#page-4-2)], the most popular mechanism for generating neutrino mass and the gauge invariance of NC gauge theory. The hybrid gauge transformation and hybrid SWM have been adopted in the Higgs sector of NCSM to ensure covariant Yukawa terms [\[7\]](#page-3-4). In this scenario, the Higgs fields feel a ''left'' charge and a ''right'' charge in the NC gauge theory and transform from the left side and right side correspondingly. Although it is only applied to the Higgs sector in Ref. [\[7](#page-3-4)], in Ref. [\[11\]](#page-4-2) it was shown that this method can in principle be extended to consider fermion fields. A new physics predicted by the hybrid gauge transformation is the tree-level coupling between the neutrino and the photon

$$
i\kappa e(\hat{A}_{\mu} * \nu - \nu * \hat{A}_{\mu}), \tag{3}
$$

where \hat{A} and $\hat{\nu}$ are the photon and neutrino fields, respectively. In NC quantum electrodynamics, to maintain the gauge invariance the charge is quantized to -1 , 0, and 1, corresponding to the interaction terms $e\hat{A}_{\mu} * \psi$, $e(\hat{A}_{\mu} * \psi - \psi * \hat{A}_{\mu})$, and $e \psi * \hat{A}_{\mu}$. However, in the NCSM based on the enveloping algebra, the Seiberg-Witten map [*z](#page-0-1)msheng@zju.edu.cn can overcome the constraint of charge quantization and

guarantee the gauge invariance at the same time. Therefore one can loosen the constraint on charge quantization and arbitrarily set the electric charge in (3) (3) (3) as κe . The photon-neutrino interaction can lead to interesting phenomena, and has been discussed by many authors (see Ref. [\[12\]](#page-4-3) and the references therein).

It is interesting to see if the hybrid gauge transformation will lead to other phenomenological effects. In Ref. [\[13\]](#page-4-4), the anomalous $Z - \nu - \nu$ interaction is derived and the invisible Z decay process $Z \rightarrow \nu \bar{\nu}$ is studied. It is shown that for $\kappa = 1$, the current experimental result $\Gamma_{\text{invisible}} =$ (499.0 ± 1.5) MeV [[14\]](#page-4-5) allows us to set the bound $\Lambda_{NC} \geq 140$ GeV on the noncommutative scale.

Besides the invisible decay, it is also of interest to investigate the $Z \rightarrow l^+l^-$ channel. In the standard model, the Z boson decays into lepton pairs through the lepton flavor conserving interaction at the tree level. Up to now, the current experimental data produces $Br(Z \rightarrow e^+e^-) =$
3.363 \pm 0.004%, $Br(Z \rightarrow \mu^+\mu^-) =$ 3.366 \pm 0.007%. $Br(Z \to \mu^+ \mu^-) = 3.366 \pm 0.007\%,$ and $Br(Z \to \tau^+\tau^-) = 3.370 \pm 0.0023\%$ [\[14\]](#page-4-5). On the other hand, the theoretical prediction from SM, including the loop correction, is $Br(Z \to e^+e^-) = Br(Z \to \mu^+\mu^-) =$ 3.3346% and Br $(Z \to \tau^+ \tau^-) = 3.3338\%$ [\[15\]](#page-4-6). In this paper, we focus on the $Z \rightarrow e^+e^-$ and in the following calculation the zero lepton mass approximation is adopted. The gap between the experimental results and the theoretical prediction is of order 0.03% and exhibits possible existence of new physics beyond the standard model. Motivated by this, various models beyond the SM have been discussed $[15–17]$ $[15–17]$ $[15–17]$ $[15–17]$. In Ref. $[17]$, the same issue has been discussed in the NCSM framework without the hybrid gauge transformation. However, our detailed analysis [\[9\]](#page-4-0) showed that the NC effect only appears in the $Z - l - l$ vertex as a phase factor, so that no physical deviation appears. Here we study $Z \rightarrow e^+e^-$ in the framework of NCSM with hybrid gauge transformations. From the viewpoint of gauge invariance, the hybrid feature also affects the charged lepton interaction. To see this, we briefly review our earlier results [\[9\]](#page-4-0). The action of a lepton in NCSM can be written as

$$
\hat{S}_{\text{lepton}} = i \int d^4x [\bar{\hat{\Psi}}_L \gamma^\mu D_{\mu L} \hat{\Psi}_L + \bar{\hat{I}}_R \gamma^\mu D_{\mu R} \hat{I}_R] \quad (4)
$$

with $\hat{\Psi}_L$ and \hat{l}_R denoting the doublet lepton

$$
\hat{\Psi}_L = \begin{pmatrix} \hat{\nu}_L \\ \hat{l}_L \end{pmatrix},\tag{5}
$$

and the right-handed singlet lepton, respectively. Under the hybrid gauge transformation, the $\hat{\Psi}_L$ and $\hat{\ell}_R$ transform as

$$
\delta_{\hat{\Lambda}} \left(\frac{\hat{\nu}_L}{\hat{l}_L} \right) = ig_Y \Big[\Big(-\frac{1}{2} + \kappa \Big) \hat{\Lambda} * \Big(\frac{\hat{\nu}_L}{\hat{l}_L} \Big) - \kappa \Big(\frac{\hat{\nu}_L}{\hat{l}_L} \Big) * \hat{\Lambda} \Big],
$$

$$
\delta_{\hat{\Lambda}} \hat{l}_R = ig_Y \Big[(-1 + \kappa) \hat{\Lambda} * \hat{l}_R - \kappa \hat{l}_R * \hat{\Lambda} \Big],
$$
 (6)

where $\hat{\Lambda}$ is the gauge parameter. The parameter κ denotes the freedom between left and right hypercharge of matter fields. One can think of the $\hat{\Psi}_L$ as having a "left" hypercharge $-\frac{1}{2} + \kappa$ and a "right" hypercharge κ . In the limit of $\theta \rightarrow 0$, the NC gauge transformation ([6\)](#page-1-0) reduces to the ordinary ones with the right hypercharge in commutative space-time. The exact value of κ cannot be constrained by NC gauge invariance itself. However, as we have mentioned in [[9\]](#page-4-0), the freedom would play an important role in the renormalization of NC fields where the κ deformed terms can be fixed to cancel the extra UV divergence. In this paper, we treat it as a free parameter and explore the phenomenological implication on it. Under the gauge transformation above, the covariant derivatives in Eq. (4) (4) are

$$
D_{\mu L}\hat{\Psi}_L = \partial_{\mu}\hat{\Psi}_L - ig_L \hat{A}^a_{\mu} T^a * \hat{\Psi}_L - \left(-\frac{1}{2} + \kappa\right) g_Y \hat{B}_{\mu} * \hat{\Psi}_L + i\kappa g_Y \hat{\Psi}_l * \hat{B}_{\mu},
$$
\n(7)

$$
D_{\mu R}\hat{l}_R = \partial_{\mu}\hat{l}_R - i\kappa g_Y B_{\mu} * \hat{l}_R + i\kappa g_Y \hat{l}_R * B_{\mu}, \qquad (8)
$$

where \hat{A}^a_μ and g_L are the $SU(2)_L$ gauge fields and a coupling constant. To get the appropriate particle vertex, we should replace the fermion and gauge fields in Eqs. ([4\)](#page-1-1), [\(7\)](#page-1-2), and ([8\)](#page-1-3) by their classical counterparts via appropriate Seiberg-Witten maps. The detailed formation of the Seiberg-Witten map is given in Ref. [[11](#page-4-2)], where the so-called θ exact formation is adopted to include the contribution of all θ orders. From the deformed Lagrangian [\[9\]](#page-4-0) one can then obtain the Feynman rule of the $Z - l - l$ interaction

$$
\frac{ie}{\sin 2\theta_W} \gamma^{\mu} (C_V - C_A \gamma^5) e^{\frac{i}{2} p_1 \theta_{P_2}} \n+ \frac{2\kappa e \sin \theta_W}{\cos \theta_W} \gamma^{\mu} \sin \left(\frac{1}{2} p_1 \theta_{P_2}\right),
$$
\n(9)

where p_1 (p_2) is the ingoing (outgoing) lepton momentum, $p_1 \theta p_2 \equiv p_1^{\mu} \theta_{\mu \nu} p_2^{\nu}, C_V = -\frac{1}{2} + 2\sin^2 \theta_W, C_A = -\frac{1}{2}$, and θ_W denotes the Weinberg angle. We have applied the equation of motion to the electron external line and omitted the vanishing terms due to the on-shell condition.

Using the Feynman rule in Eq. (9) (9) , the derivative decay width of $Z \rightarrow e^+e^-$ can be easily obtained in the Z boson rest frame

$$
\frac{d\Gamma}{d\cos\theta d\phi} = \frac{M_Z}{48\pi^2} \left[\frac{e^2}{\sin^2 2\theta_W} (C_V^2 + C_A^2) + \left(4\kappa^2 e^2 \tan^2 \theta_W - 2\kappa C_V \frac{e^2}{\cos^2 \theta_W} \right) \right. \\
\left. \times \sin^2 \left(\frac{1}{2} p_1 \theta p_2\right) \right].
$$
\n(10)

In the calculation, we omit the lepton mass. As mentioned, the NC parameter $\theta_{\mu\nu}$ is a fundamental constant that breaks the Lorentz symmetry. Following the method adopted in Ref. [\[18\]](#page-4-8), one can decompose $\theta_{\mu\nu}$ into two types: the electric-like components $\theta_E = (\theta_{01}, \theta_{02}, \theta_{03})$

and the magnetic-like components $\theta_B = (\theta_{23}, \theta_{31}, \theta_{12})$. Both of them are assumed to be directionally fixed in a primary, unrotated reference. That is, when discussing phenomena in the laboratory frame, the Earth's rotation should be included. Defining $(\hat{X}, \hat{Y}, \hat{Z})$ to be the orthonormal basis of this primary frame, θ_E and θ_B are

$$
\theta_E = \frac{1}{\Lambda_E^2} (\sin \eta_E \cos \xi_E \hat{X} + \sin \eta_E \sin \xi_E \hat{Y} + \cos \eta_E \hat{Z}),
$$
\n(11)

$$
\theta_B = \frac{1}{\Lambda_B^2} (\sin \eta_B \cos \xi_B \hat{X} + \sin \eta_B \sin \xi_B \hat{Y} + \cos \eta_B \hat{Z}),
$$
\n(12)

where η and ξ denote the polar angular and azimuth angular of NC parameter with $0 \le \eta \le \pi$ and $0 \le \xi \le$ 2π , respectively. Since we are in the $(\hat{x},\hat{y},\hat{z})$ frame on Earth, it is necessary to find an appropriate transformation matrix correlating the primary and laboratory reference frames. Following Ref. [\[19\]](#page-4-9), we have

$$
\begin{pmatrix}\n\hat{X} \\
\hat{Y} \\
\hat{Z}\n\end{pmatrix} = \begin{pmatrix}\nc_a s_\zeta + s_\delta s_a c_\zeta & c_\delta c_\zeta & s_a s_\zeta - s_\delta c_a c_\zeta \\
-c_a s_\zeta + c_\delta s_a s_\zeta & \delta s_\zeta & -s_a c_\zeta - s_\delta c_a s_\zeta \\
-c_\delta s_a & s_\delta & c_\delta c_a\n\end{pmatrix}
$$
\n
$$
\times \begin{pmatrix}\n\hat{X} \\
\hat{Y} \\
\hat{Z}\n\end{pmatrix},
$$
\n(13)

where the abbreviation $c_{\alpha} = \cos \alpha$ and $s_{\alpha} = \sin \alpha$, with $\alpha = a, \delta, \text{ and } \zeta$, respectively, are used. Here, δ and a define the location and orientation of the experiment site, with $-\frac{\pi}{2} \le \delta \le \frac{\pi}{2}$ and $0 \le a \le 2\pi$, $\zeta = \omega t$ is the rotation angle, and $\omega = 2\pi/23h56m4.09s$ is the Earth's angular velocity. Ignoring the Earth's revolution, the collider machine returns to its original position after one day. Using Eqs. (11) (11) (11) – (13) , we get

$$
p_2 \theta p_1 = -\frac{s}{2\Lambda_{\rm NC}^2} (\sin \theta \cos \phi \Theta_E^x + \sin \theta \sin \phi \Theta_E^y + \cos \theta \Theta_E^z)
$$
(14)

with

$$
\Theta_E^x = s_\eta c_\xi (c_a s_\zeta + s_\delta s_a c_\zeta) + s_\eta s_\xi (-c_a s_\zeta + c_\delta s_a s_\zeta)
$$

\t\t\t
$$
- c_\eta c_\delta c_a,
$$

\n
$$
\Theta_E^y = s_\eta c_\xi c_\delta c_\zeta + s_\eta s_\xi c_\delta c_\zeta + c_\eta c_\delta,
$$

\n
$$
\Theta_E^z = s_\eta c_\xi (s_a s_\zeta - s_\delta c_a c_\zeta) + s_\eta s_\xi (-s_a c_\zeta - s_\delta c_a s_\zeta)
$$

\t\t\t
$$
+ c_\eta c_\delta c_a.
$$

\t\t\t(15)

Substituting Eq. (14) (14) (14) into Eq. (10) , we obtain the decay width of $Z \rightarrow e^+e^-$ in the laboratory frame.

FIG. 1 (color online). NC correction of the branch ratio $Z \rightarrow e^+e^-$ as a function of Λ_{NC} .

Due to the Earth's rotation, any observable calculated in the NC space-time frame should depend on time. On the other hand, it is difficult to follow the experiments in time. It is therefore reasonable to average the cross section or decay width over a full day. For our problem, the time-averaged decay width is

$$
\langle \Gamma \rangle_T = \frac{1}{T_{day}} \int_0^{T_{day}} dt \int_{-1}^1 d(\cos \theta) \int_0^{2\pi} d\phi \frac{d\Gamma}{d\cos \theta d\phi}.
$$
 (16)

In particular, we are interested in the NC correction of the branch ratio

$$
\Delta BR = \frac{\delta \Gamma}{\Gamma_0} \equiv \frac{\Gamma - \Gamma_{SM}}{\Gamma_{SM}}.
$$
 (17)

The behavior of Δ BR for different κ , as well as the current experimental uncertainty, is shown in Fig. [1](#page-2-3) as a function

FIG. 2 (color online). The η dependence of \triangle BR ($Z \rightarrow e^+e^-$) for $\kappa = 1$.

FIG. 3 (color online). Bound on Λ_{NC} as the function of κ where we set the range of κ at [0, 1].

of the NC scale Λ_{NC} . In the numerical analysis, we use the input parameters of Ref. [[15](#page-4-6)]. The location and orientation of laboratory frame are set to be $(\delta, a) = (\frac{\pi}{4}, \frac{\pi}{4})$, where the Large Electron-Positron collider experiment measures the decay width of $Z \rightarrow e^+e^-$. We can see from Fig. [1](#page-2-3) that ΔBR is sensitive to both κ and Λ_{NC} . Clearly, the NC correction is significantly enhanced as Λ_{NC} or κ decreases. Compared with the experimental branch ratio $Br(Z \rightarrow e^+e^-) = 3.363 \pm 0.004\%$, for the choice of $\kappa =$ 1, a bound on the noncommutative scale $\Lambda_{NC} \ge 150 \text{ GeV}$ is obtained by imposing the constraint $\Delta BR \leq 3 \times 10^{-4}$. As seen from Eq. ([8\)](#page-1-3), the NC correction of decay width also depends on the orientation of θ_E , i.e., the parameter η . In Fig. [1,](#page-2-3) we have set $\eta = \frac{\pi}{2}$. It is thus necessary to investigate the sensitivity of $\Delta \overline{BR}$ on η . The NC correction of the time-averaged decay width is presented as the function of parameter η in Fig. [2.](#page-2-4) One can see from Fig. [2](#page-2-4) that the NC effect produces a positive deviation from the SM branch ratio for the whole range of η . Despite the fact that a slightly peaked distribution appears, the curve is not sensitive to η . In this sense, the bound obtained from Fig. [1](#page-2-3) should be credible.

In Fig. [3,](#page-3-7) we show the allowed region of ΔBR in the (κ, Λ_{NC}) plane for $0 < \kappa < 1$. We see that as κ increases, a higher bound on the NC scale appears. Furthermore, a lower limit $\kappa \ge 0.04$ is found for the forbidden region when we set κ to zero. This means that in Eq. [\(9](#page-1-4)), for $\kappa = 0$ the NC correction to the magnitude of $Z \rightarrow e^+e^-$ only appears as a phase factor, indicating that there is no NC deviation to the decay width. Thus if we assume that the discrepancy between the experimental and the SM results is fully induced by the noncommutative effect, the value of κ cannot be arbitrarily small. In the hybrid feature, additional sin-type deformation shows up in the Feynman rule of NC $Z - e - e$ interaction and leads to NC correction which is potentially detectable or allows us to set bounds on the NC parameters in high-accuracy measurements of Z decay width.

In conclusion, the $Z \rightarrow e^+e^-$ channel provides an ideal process to understand not only the space-time noncommutativity, but also the mathematical structure of the corresponding gauge theory. We showed that the decay width is sensitive to both Λ_{NC} and the parameter κ for the freedom of the hybrid gauge transformation. In terms of the NC effect, the discrepancy between the experimental and SM results allows us to set a bound on the noncommutative parameters. Although the current experimental uncertainty is still a little large, the next generation Z factory with the Giga-Z option of the International Linear Collider can generate 2×10^9 Z events at resonance energy [\[20,](#page-4-10)[21\]](#page-4-11). We therefore expect that the high-luminosity Z factory can significantly enhance the sensitivity to probe the noncommutative model via Z boson decays.

The authors would like to thank H. S. Xie and M. Y. Yu for useful discussion during this work. This work is supported by the Fundamental Research Funds for the Central Universities.

- [1] H. Snyder, *Phys. Rev.* **71**[, 38 \(1947\)](http://dx.doi.org/10.1103/PhysRev.71.38); **72**[, 68 \(1947\)](http://dx.doi.org/10.1103/PhysRev.72.68).
- [2] A. Connes, M. R. Douglas, and A. Schwarz, [J. High](http://dx.doi.org/10.1088/1126-6708/1998/02/003) [Energy Phys. 02 \(1998\) 003.](http://dx.doi.org/10.1088/1126-6708/1998/02/003)
- [3] N. Seiberg and E. Witten, [J. High Energy Phys. 09 \(1999\) 032.](http://dx.doi.org/10.1088/1126-6708/1999/09/032)
- [4] M. R. Douglas and N. A. Nekrasov, [Rev. Mod. Phys.](http://dx.doi.org/10.1103/RevModPhys.73.977) **73**, [977 \(2001\)](http://dx.doi.org/10.1103/RevModPhys.73.977).
- [5] R.J. Szabo, *Phys. Rep.* **378**[, 207 \(2003\)](http://dx.doi.org/10.1016/S0370-1573(03)00059-0).
- [6] J. E. Moyal, [Math. Proc. Cambridge Philos. Soc.](http://dx.doi.org/10.1017/S0305004100000487) 45, 99 [\(1949\)](http://dx.doi.org/10.1017/S0305004100000487).
- [7] X. Calmet, B. Jurco, P. Schupp, J. Wess, and M. Wohlgenannt, [Eur. Phys. J. C](http://dx.doi.org/10.1007/s100520100873) 23, 363 (2002).
- [8] W. Behr, N. G. Deshpande, G. Duplancic, P. Schupp, J. Trampetic, and J. Wess, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s2003-01207-4) 29, 441 (2003);

M. Buric, D. Latas, V. Radovanovic, and J. Trampetic, Phys. Rev. D 75[, 097701 \(2007\);](http://dx.doi.org/10.1103/PhysRevD.75.097701) B. Melic, K. Passek-Kumericki, and J. Trampetic, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.72.054004) 72, 054004 [\(2005\)](http://dx.doi.org/10.1103/PhysRevD.72.054004); M. Haghighat, M. M. Ettefaghi, and M. Zeinali, Phys. Rev. D 73[, 013007 \(2006\)](http://dx.doi.org/10.1103/PhysRevD.73.013007); A. Alboteanu, T. Ohl, and R. Ruckl, Phys. Rev. D 76[, 105018 \(2007\);](http://dx.doi.org/10.1103/PhysRevD.76.105018) P. K. Das, N. G. Deshpande, and G. Rajasekaran, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.77.035010) 77, [035010 \(2008\);](http://dx.doi.org/10.1103/PhysRevD.77.035010) A. Prakash, A. Mitra, and P. K. Das, [Phys.](http://dx.doi.org/10.1103/PhysRevD.82.055020) Rev. D 82[, 055020 \(2010\);](http://dx.doi.org/10.1103/PhysRevD.82.055020) R. Horvat, D. Kekez, and J. Trampetic, Phys. Rev. D 83[, 065013 \(2011\)](http://dx.doi.org/10.1103/PhysRevD.83.065013); W. Wang, F. Tian, and Z. M. Sheng, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.84.045012) 84, 045012 [\(2011\)](http://dx.doi.org/10.1103/PhysRevD.84.045012); S. K. Garg, T. Shreecharan, P. K. Das, N. G. Deshpande, and G. Rajasekaran, [J. High Energy Phys.](http://dx.doi.org/10.1007/JHEP07(2011)024)

07 (2011[\) 024](http://dx.doi.org/10.1007/JHEP07(2011)024); N. G. Deshpande and S. K. Garg, [Phys.](http://dx.doi.org/10.1016/j.physletb.2012.01.049) Lett. 708B[, 150 \(2012\).](http://dx.doi.org/10.1016/j.physletb.2012.01.049)

- [9] W. Wang, J. H. Huang, and Z. M. Sheng, *[Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.86.025003)* 86, [025003 \(2012\)](http://dx.doi.org/10.1103/PhysRevD.86.025003).
- [10] Q. R. Ahmad et al. (SNO Collaboration), *[Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.89.011301)* 89[, 011301 \(2002\)](http://dx.doi.org/10.1103/PhysRevLett.89.011301); K. Eguchi et al. (KamLAND Collaboration), Phys. Rev. Lett. 90[, 021802 \(2003\)](http://dx.doi.org/10.1103/PhysRevLett.90.021802); M. H. Ahn et al. (K2K Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.90.041801) 90, 041801 [\(2003\)](http://dx.doi.org/10.1103/PhysRevLett.90.041801); F.P. An et al. (DAYA-BAY Collaboration), Phys. Rev. Lett. 108[, 171803 \(2012\);](http://dx.doi.org/10.1103/PhysRevLett.108.171803) J. K. Ahn (RENO Collaboration), Phys. Rev. Lett. 108[, 191802 \(2012\)](http://dx.doi.org/10.1103/PhysRevLett.108.191802).
- [11] R. Horvat, A. Ilakovac, P. Schupp, J. Trampetic, and J. You, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2012.07.046) 715, 340 (2012).
- [12] J. Trampetic, [Int. J. Geom. Methods Mod. Phys.](http://dx.doi.org/10.1142/S0219887812610166) 09, [1261016 \(2012\)](http://dx.doi.org/10.1142/S0219887812610166).
- [13] R. Horvat, A. Ilakovac, P. Schupp, J. Trampetic, and J. You, [arXiv:1204.6201](http://arXiv.org/abs/1204.6201).
- [14] J. Beringer et al. (Particle Data Group), *[Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.86.010001)* 86, [010001 \(2012\).](http://dx.doi.org/10.1103/PhysRevD.86.010001)
- [15] E. O. Iltan, [Mod. Phys. Lett. A](http://dx.doi.org/10.1142/S0217732309027455) 24, 377 (2009); C. X. Yue, J. Y. Liu, and S. H. Zhu, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.78.095006) 78, 095006 [\(2008\)](http://dx.doi.org/10.1103/PhysRevD.78.095006).
- [16] M. Maya and O. G. Miranda, Z. Phys. C 68[, 481 \(1995\)](http://dx.doi.org/10.1007/BF01620725); E. O. Iltan, Phys. Rev. D 65[, 036003 \(2002\)](http://dx.doi.org/10.1103/PhysRevD.65.036003); A. G. Dias, C. A. de, S. Pires, and P. S. Rodrigues da Silva, [Phys.](http://dx.doi.org/10.1103/PhysRevD.77.055001) Rev. D 77[, 055001 \(2008\)](http://dx.doi.org/10.1103/PhysRevD.77.055001);
- [17] E.O. Iltan, *Phys. Rev. D* **66**[, 034011 \(2002\).](http://dx.doi.org/10.1103/PhysRevD.66.034011)
- [18] Y. M. Fu and Z. M. Sheng, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.75.065025) 75, 065025 [\(2007\)](http://dx.doi.org/10.1103/PhysRevD.75.065025); Z. M. Sheng, Y. M. Fu, and H. B. Yu, [Chin.](http://dx.doi.org/10.1088/0256-307X/22/3/012) Phys. Lett. 22[, 561 \(2005\).](http://dx.doi.org/10.1088/0256-307X/22/3/012)
- [19] J. i. Kamoshita, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s10052-007-0371-y) 52, 451 (2007).
- [20] J. A. Aguilar et al., [arXiv:hep-ph/0106315.](http://arXiv.org/abs/hep-ph/0106315)
- [21] G. Wilson, in DESY-EFCA LC Workshops at Frascati, 1998 and Oxford, 1999 (unpublished).