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Relativistic path integral and relativistic Hamiltonians in QCD and QED

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The proper-time 4D path integral is used as a starting point to derive the new explicit parametric form of the quark-antiquark Green's function in gluonic and QED fields entering as a common Wilson loop. The subsequent vacuum averaging of the latter allows us to derive the instantaneous Hamiltonian. The explicit form and solutions are given in the case of the $q\bar{q}$ mesons in magnetic field.

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I. INTRODUCTION

The path-integral formalism in quantum mechanics created by Feynman [1,2] is an important benchmark in the development and in our understanding of quantum theory. Many varieties of this formalism and new methods to solve the problems, which seemed before unsolvable, have been suggested since then; see the books and review papers [3–9] summarizing the modern achievements in this field.

The extension of the path-integral formalism to the quantum field theory was done in several directions. One of the most known line of development was started already in Ref. [2], where field variables, i.e., electromagnetic potentials $A_i(\mathbf{x}, t)$, $\varphi(\mathbf{x}, t)$, play the role of quantum spatial coordinates $q(t)$, and the resulting path integral is becoming the functional integral. This line is now a part of standard lore present in many textbooks, see, e.g., Refs. [5–9].

However, it is important that in all works of this direction, the path integration concerns spatial coordinates and/or field variables, but not the time coordinate, and in this way one can say that this development is similar to the path integrals in quantum mechanics, where time plays the ordering role and stays outside of the realm of fluctuating variables.

Another and more general approach unifying space and time coordinates in the path integral is based on the proper time coordinate. The latter was introduced by Fock [10] and Schwinger [11], who used proper time formalism for the field theory in external electromagnetic fields; however, it did not exploit path integrals.

In QED, path integrals based on the proper time were suggested in Ref. [12] and developed in Ref. [13].

Path integrals for QCD both in time and space variables using proper time as an ordering variable were suggested in Refs. [14–16]. The first use of the QCD path integral for quarks and gluons was done in Ref. [17] and exploited to demonstrate the confinement due to field correlators (stochastic confinement) (for a review, see Ref. [18]).

The full form of the path integral in QCD for quarks and gluons, based on the proper time ordering, was given in Ref. [17] for $T = 0$ and in Ref. [19] for $T > 0$, and different approximations were reviewed in Ref. [20] for some relativistic models and in Ref. [21] for QCD. It was called the Fock-Feynman-Schwinger representation, and we retain this name in what follows.

Based on Fock-Feynman-Schwinger representation, a new relativistic Hamiltonian was derived in Refs. [22,23] for quarks and in Refs. [24,25] for gluons, where a new important variable ω was introduced, playing the role of the einbein variable [26]. Its average value ω_0 is the average quark (or gluon) energy and explains the appearance of the notion of constituent mass in earlier models. The relativistic Hamiltonian with einbeins ω_i allows us to calculate all low-lying states in QCD—mesons, glueballs, baryons, and hybrids—from the first-principle input: current quark masses, α_s, and string tension σ, see Refs. [27–29] for reviews.

However, the introduction of ω_i as einbein variables, being successful, is an approximate procedure, and its limitations and corrections were not clarified enough in the literature. An attempt in this direction was done in Ref. [30], where the fluctuation of the time coordinate in the path integration was substituted by the fluctuation's integration in $\Delta\omega_i$. The resulting expressions for quark decay constants of mesons in Ref. [30] are quite successful in comparison with experiment; however, the exact scheme of approximations was not clearly stated.

An additional impulse for a development in this area was given recently by the inclusion of high magnetic field B in the dynamics of QCD and QED, see Refs. [31–35] for recent papers. In this case, ω_i depend on B and might vanish or grow fast (depending on quark spin projection), which calls for a careful analysis of all corrections.

In the present paper, we undertake such an analysis and rederive different forms of path integrals and relativistic Hamiltonians for the QCD and QED systems, typically for quark-antiquark or atoms, taking into account both QCD and QED dynamics in the first case.

The first thing we meet confronting 4D path integrals is the problem of the time-coordinate fluctuation, which necessarily requires distinguishing average (ordering) time and fluctuating time, similar to the old notion of the *Zitterbewegung*. We analyze this phenomenon by comparing the Bethe-Salpeter and path-integral formalisms and show how the latter can be developed using the fact that all dynamics is contained in the Wilson loop formalism augmented by spin insertions.

II. PATH INTEGRAL: TREATING TIME FLUCTUATIONS

We start with the simplest example of a scalar particle in external field; this problem was considered for QED by Feynman in Ref. [12].

The scalar one particle Green's function is (in Euclidean space-time)

$$g(x,y) = \left(\frac{1}{m^2 - D_\mu^2}\right)_{xy} = \int_0^\infty ds (D^4 z)_{xy} \exp(-K)\Phi(x,y), \tag{1}$$

where $D_\mu = \partial_\mu - ieA_\mu$,

$$K = m^2 s + \frac{1}{4}\int_0^s d\tau \left(\frac{dz_\mu}{d\tau}\right)^2, \tag{2}$$

$$\Phi(x,y) = \exp ie \int_y^x A_\mu dz_\mu, \tag{3}$$

and

$$(D^4 z)_{xy} \simeq \lim_{N\to\infty} \prod_{n=1}^N \int \frac{d^4 z(n)}{(4\pi\varepsilon)^2} \int \frac{d^4 p}{(2\pi)^4} e^{ip\left(\sum_{n=1}^N z(n) - (x-y)\right)}, \quad N\varepsilon = s. \tag{4}$$

At this point, it is important to stress the difference between the nonrelativistic quantum-mechanical and relativistic path integration. In the first case, one has $(D^3 z) = (D^3 z(t))$ in (1), and the time variable t has the ordering character: the consecutive pieces of trajectory $\mathbf{z}(t)$ are ordered by time. In the relativistic path integral, this role is given to the proper time τ, s while the "time" $z_4(\tau)$ is fluctuating together with spatial coordinates $\mathbf{z}(\tau)$. In terms of any local field theory and Bethe-Salpeter type of equation, this is allowable and necessary, since any moment of time z_4 appears in the amplitude with a new interaction point, which may happen before or after the previous interaction point; thus, the points of interaction lie chaotically on the time axis. However, from the point of view of a stationary process, which creates the system with a given quantized energy state in the limit of a long time interval, one may think of an averaged progressive time and averaged trajectories of constituents, where stochastic time fluctuations are dealt with in a well-defined averaging process. In this way, the time-fluctuating relativistic trajectories are averaged into stationary time-ordered trajectories, similar to the quantum mechanical ones, where fluctuations are allowed for spatial coordinates. Correspondingly, one can write

$$z_4(\tau) = \bar{z}_4(\tau) + \tilde{z}_4(\tau), \tag{5}$$

where $\bar{z}_4(\tau) \equiv t_E = 2\omega\tau$ is the averaged time proportional to the proper time, while the fluctuating time $\tilde{z}_4(\tau)$ can be written as a sum of one-step fluctuations:

$$\tilde{z}_4(\tau) = \sum_{k=1}^n \Delta z_4(k), \qquad \tau = n\varepsilon, \qquad N\varepsilon = s, \qquad \sum_{k=1}^N \Delta z_4(k) = 0. \tag{6}$$

The proper time s is expressed via the total Euclidean time $T = x_4 - y_4$ and the new variable ω,

$$s = T/2\omega, \tag{7}$$

and hence the scalar Green's function (1) can be rewritten in the form

$$g(x,y) = T\int_0^\infty \frac{d\omega}{2\omega^2} D^3 z e^{-K(\omega)} \langle \Phi(x,y)\rangle_{\Delta z_4}. \tag{8}$$

Here,

$$K(\omega) = \int_0^T dt_E \left(\frac{\omega}{2} + \frac{m^2}{2\omega} + \frac{\omega}{2}\left(\frac{d\mathbf{z}}{dt_E}\right)^2\right), \tag{9}$$

while

$$\langle \Phi(x,y)\rangle_{\Delta z_4} = D\Delta z_4 \exp\left[ie\int A_i(\mathbf{z}(t_E), t_E + \tilde{z}_4)dz_i + ie\int A_4 dt_E + ie\int A_4 d\Delta z_4\right], \tag{10}$$

$$D\Delta z_4 \equiv \int \frac{dp_4}{2\pi} \prod_{k=1}^n \frac{d\Delta z_4(k)}{\sqrt{4\pi\varepsilon}} \exp\left\{\sum_{k=1}^N \left[ip_4 \Delta z_4(k) - \frac{1}{4}\frac{(\Delta z_4(k))^2}{\varepsilon} + ie\Delta z_4(k) A_4\right]\right\}. \tag{11}$$

The result of integration in (11) can be written as

$$\langle \Phi(x,y)\rangle_{\Delta z_4} = \sqrt{\frac{\omega}{2\pi T}}\bar{\Phi}(x,y), \tag{12}$$

where $\bar{\Phi}(x,y)$ is the averaged Wilson line augmented by the fluctuation,

$$\bar{\Phi}(x,y) = \exp\left[ie\int_{\mathbf{y}}^{\mathbf{x}} A_i(\mathbf{z}(t_E), t_E) d\mathbf{z}_i + ie\int_{y_4}^{x_4} A_4(\mathbf{z}(t_E), t_E) dt_E\right] \exp(\Delta S). \tag{13}$$

In the simplest case of the free scalar Green's function, $A_\mu \equiv 0$ and $\bar{\Phi}(x,y) = 1$, hence,

$$g_0(x,y) = \sqrt{\frac{T}{8\pi}}\int_0^\infty \frac{d\omega}{\omega\sqrt{\omega}}(D^3 z)_{\mathbf{xy}} e^{-K(\omega)}$$
$$= \frac{1}{8\pi^2 T}\int_0^\infty d\omega \exp\left[-\frac{m^2 T}{2\omega} - \frac{(\mathbf{x}-\mathbf{y})^2}{2T}\omega - \frac{\omega T}{2}\right]$$
$$= \frac{1}{4\pi^2}\frac{m}{u}K_1(mu),$$
$$u^2 = T^2 + (\mathbf{x}-\mathbf{y})^2, \tag{14}$$

where $K_1(x)$ is the Bessel function of the second kind.

At this point the role of ω becomes clear, since for large T the integral in (14) can be taken by the stationary point method with the action

$$S(\omega, T) = \frac{m^2 T}{2\omega} + \frac{(\mathbf{x}-\mathbf{y})^2}{2T}\omega + \frac{\omega T}{2}; \quad \frac{\partial S(\omega, T)}{\partial \omega}\bigg|_{\omega=\omega_0} = 0 \tag{15}$$

and

$$\omega_0 = \frac{mT}{\sqrt{(\mathbf{x}^2 - \mathbf{y})^2 + T^2}} \to m, \quad \text{for } T \gg |\mathbf{x}-\mathbf{y}|,$$

and one finally obtains the standard answer for large T yielding the asymptotics of the rhs of (14) at large T,

$$g_0(x,y) = \frac{\sqrt{m}}{4\pi^{3/2}T^{3/2}}\exp(-mT). \tag{16}$$

Another form exploits the Hamiltonian in (14); namely, one can use the relation

$$\int (D^3 z)_{\mathbf{xy}} e^{-K(\omega)} = \langle \mathbf{x}|e^{-H(\omega)T}|\mathbf{y}\rangle, \tag{17}$$

where

$$H(\omega) = \frac{\mathbf{p}^2 + m^2}{2\omega} + \frac{\omega}{2}. \tag{18}$$

Applying the stationary point method to the integrals (14) and (17), one obtains at large T the energy eigenvalue

$$\frac{\partial H(\omega)}{\partial \omega}\bigg| \omega = \omega_0 = 0; \quad \omega_0 = \sqrt{\mathbf{p}^2 + m^2}. \tag{19}$$

From (19), one can understand that ω plays the role of a virtual particle energy, and the condition (19) has the meaning of the energy shell condition. This interpretation holds also for the case of N particles with interaction, when the integrals aver $\prod_{i=1}^N d\omega_i$ are involved. Note that in this way, ω_i are no longer approximate einbein variables, as in our previous works (see, e.g., Refs. [18,23]).

We now turn to the case of $A_\mu \neq 0$ and remark that $A_\mu(\mathbf{z}, z_4)$ are functions of coordinates, which will be used later in the process of vacuum averaging, yielding points of interaction, correlators, etc., but at this moment in (13), $\bar{\Phi}(x,y)$ is a set of all possible Wilson lines obtained by time

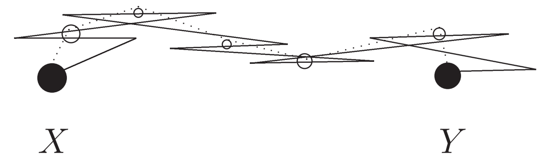

FIG. 1. The time-fluctuating trajectory in the z_1, z_4 plane. The points $z_1(t_E)$, t_E are marked by circles and are connected by the average trajectory depicted by the dotted line.

fluctuations with the weight given in (11), see Fig. 1 as an illustration.

In one particular case, when $\mathbf{z}(t_E)$ is fixed, i.e., the trajectory is parallel to the z_4 axis, all Δz_4 fluctuations are washed out, since all fluctuations cancel each other,

$$\exp\left(ie\int A_4(\mathbf{z}, z_4)dz_4\right) = \exp\left(ie\int A_4(\mathbf{z}, t_E)dt_E\right). \tag{20}$$

The same would happen in the case of QCD, where again overlapping pieces of the Wilson line cancel each other. We shall come back to the problem of fluctuating Wilson lines when we consider gauge invariant two-body Green's functions.

In the case of the white system of the quark and antiquark of opposite charges, one must start with the one-body Green's function

$$S_i(x,y) = (m_i + \hat{\partial} - ig\hat{A} - ie_i\hat{A}^{(e)})^{-1}_{xy}$$
$$\equiv (m_i + \hat{D}^{(i)})^{-1}_{xy}$$
$$= (m_1 - \hat{D}^{(i)})(m_i^2 - (\hat{D}^{(i)})^2)^{-1}_{xy}. \tag{21}$$

The path-integral representation for S_i is [8]

$$S_i(x,y) = (m_i - \hat{D}^{(i)})\int_0^\infty ds_i (Dz)_{xy} e^{-K_i}\Phi_\sigma^{(i)}(x,y)$$
$$\equiv (m_i - \hat{D}^{(i)})G_i(x,y), \tag{22}$$

where

$$K_i = m_i^2 s_i + \frac{1}{4}\int_0^{s_i} d\tau_i\left(\frac{dz_\mu^{(i)}}{d\tau_i}\right)^2, \tag{23}$$

$$\Phi_\sigma^{(i)}(x,y) = P_A P_F \exp\left(ig\int_y^x A_\mu dz_\mu^{(i)} + ie_i\int_y^x A_\mu^{(e)} dz_\mu^{(i)}\right)$$
$$\times \exp\left(\int_0^{s_i} d\tau_i \sigma_{\mu\nu}(gF_{\mu\nu} + e_i B_{\mu\nu})\right). \tag{24}$$

Here, $F_{\mu\nu}$ and $B_{\mu\nu}$ are correspondingly gluon and c.m. field tensors, and P_A, P_F are ordering operators, $\sigma_{\mu\nu} = \frac{1}{4i}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. Equations (21)–(24) hold for the quark, $i = 1$, while for the antiquark one should reverse the signs of e_i and g. In explicit form, one writes

$$\sigma_{\mu\nu}F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{H} & \boldsymbol{\sigma}\mathbf{E} \\ \boldsymbol{\sigma}\mathbf{E} & \boldsymbol{\sigma}\mathbf{H} \end{pmatrix}, \qquad \sigma_{\mu\nu}B_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma}\mathbf{B} & 0 \\ 0 & \boldsymbol{\sigma}\mathbf{B} \end{pmatrix}. \tag{25}$$

The two-body $q_1 q_2$ Green's function can be written as [17,21]

$$G_{q_1\bar{q}_2}(x,y) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz^{(1)})_{xy}(Dz^{(2)})_{xy}\langle \hat{T} W_\sigma(A)\rangle_A \times \exp\Big(ie_1\int_y^x A_\mu^{(e)} dz_\mu^{(1)} - ie_2\int_y^x A_\mu^{(e)} dz_\mu^{(2)} + e_1\int_0^{s_1} d\tau_1(\boldsymbol{\sigma}\mathbf{B}) - e_2\int_0^{s_2} d\tau_2(\boldsymbol{\sigma}\mathbf{B})\Big), \tag{26}$$

where

$$\hat{T} = \mathrm{tr}(\Gamma_1(m_1 - \hat{D}_1)\Gamma_2(m_2 - \hat{D}_2)), \tag{27}$$

"tr" is the trace over Dirac and color indices acting on all terms. Here $\langle W_\sigma(A)\rangle$ is the closed Wilson loop with the spin insertions, and one should have in mind that color and electromagnetic (e.m.) spin insertions in general do not commute, which should be taken into account when computing the spin-dependent part of the interaction, see Ref. [36]; in (26), this fact was disregarded,

$$W_\sigma(A) = P_a P_F \exp\Big[ig\oint A_\mu dz_\mu + g\int_0^{s_1}\sigma_{\mu\nu}^{(1)}F_{\mu\nu}d\tau_1 - g\int_0^{s_2}\sigma_{\mu\nu}^{(2)}F_{\mu\nu}d\tau_2\Big]. \tag{28}$$

It is important that the physically meaningful result for the Green's function is obtained by two different averaging procedures applied to the total Wilson loop $W = \Phi_\sigma^{(1)}(x,y)\Phi_\sigma^{(2)}(y,x)$:

(1) one should average W over all time fluctuations;
(2) one should average W over all nonperturbative (np) and perturbative (pert) field configurations with the weight given by the standard QED + QCD field actions so that the final result is

$$\langle\langle W\rangle\rangle \equiv \langle\langle W\rangle_{\Delta z_4}\rangle_{A,A^{(e)}}. \tag{29}$$

However, the class of processes of interest in QCD is very wide, since any process starting and finishing with definite hadron states, such as form factors, decays, and hadron reactions, needs an explicit definition of initial and final states as eigenstates of the Hamiltonian $H(\omega_1,\omega_2)$, and therefore can use the formalism discussed in this paper.

It is clear that in the fluctuation averaging $\langle W\rangle_{\Delta_z 4}$ the result is an average Wilson loop passing through the points $\{\mathbf{z}(t_E) + \Delta\mathbf{z}(t_E), t_E + \Delta t_E\}$, $t_E \epsilon (0,T)$, where $\Delta\mathbf{z}(t_E)$, Δt_E depend on T, m_1, m_2 and also on the concrete field configuration. The latter dependence goes away after the next averaging process, over vacuum fields.

One can estimate the average time fluctuation Δt_E in the case of the free relativistic particle propagation.

E.g., assuming the correlation function has the form

$$f(z_4^{(1)} - z_4^{(2)}) = \exp\Big(-\frac{(z_4^{(1)} - z_4^{(2)})^2}{(\Delta\bar{z})^2}\Big), \tag{30}$$

and taking into account time fluctuations $z_4^{(1)} = t_E^{(1)} + \tilde{z}_4(t_E)$, and integrating over Δz_4, one obtains the increase of the correlation time

$$(\Delta\bar{z})^2 \to (\Delta z)^2_{\Delta z_4} = (\Delta\bar{z})^2 + \frac{t_E^{(1)}}{2\omega} \sim (\Delta\bar{z})^2 + \frac{T}{2m}. \tag{31}$$

However, this result is an artifact of the inaccurate definition of the path-integration measure, when at the ends of the time interval Δt_E the path can change the direction, implying infinite time derivative. Imposing a proper condition on the magnitude of the derivative, i.e., with smooth trajectories, the result would be different. From the point of view of the relation $\Delta M \Delta t \gtrsim 1$, one can in principle calculate however accurate values of masses M for large T, and only the coupling to decay channels, i.e., the width Γ, should put a lower limit on the accuracy ΔM.

It is interesting how this problem occurs in our path-integral formalism. Indeed, the basic dynamics which is contained in $\langle\langle W\rangle\rangle$ when the time fluctuation is supported by the interaction, Eq. (30), can be described by the diagram in Fig. 2. Now, from the point of view of Hamiltonian dynamics with the trace of the hypersurface shown in Fig. 2 by a dotted line, the Hamiltonian becomes a matrix with Fock states, numerating columns, and rows,

$$H_{q\bar{q}} \to \begin{pmatrix} H_{q\bar{q}} & \hat{V}_{12} & \cdots \\ \hat{V}_{21} & H_{(q\bar{q}),(q\bar{q})_2} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}, \tag{32}$$

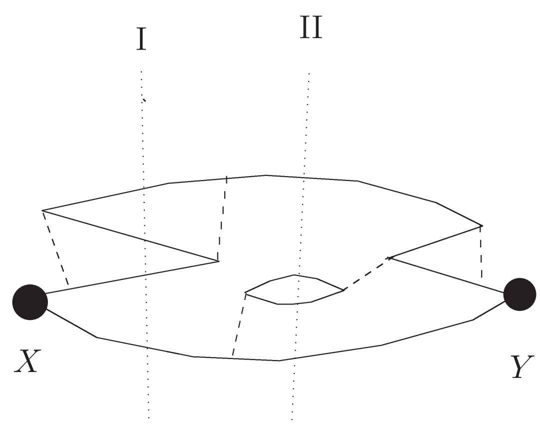

FIG. 2. The vacuum averaged Wilson lines displaying pair creation in the time fluctuation process. The hypersurface traces I and II mark the double quark pair state of the Hamiltonian.

where nondiagonal elements are transition operators and the diagonal ones define dynamics (and masses) of more and more complex systems. Therefore, e.g., $\hat{V}_{13}$ is responsible for the decay $q\bar{q} \to (q\bar{q})_1 + (q\bar{q})_2$, and hence defines the accuracy of the possible mass determination of the incident state $(q\bar{q})$.

The wave functions of (32) are actually Fock columns of different states, e.g., $\{\Psi_{q\bar{q}}, \Psi_{(q\bar{q})(q\bar{q})},\ldots\}$, and therefore the $(q\bar{q})$ eigenstates $\{\Phi^{(n)}_{q\bar{q}}\}$ are no longer an orthonormal set of states. As we shall see, the eigenstates $\Psi_n(\omega_1, \omega_2)$ will not be orthonormal on the energy shells $(\omega_1^{(0)}, \omega_2^{(0)})$, different for each n. In this way, going from the 4D path integral to the relativistic Hamiltonian 3D formalism, one naturally meets the many-channel Hamiltonian, where diagonal elements correspond to the fluctuation-averaged trajectories.

III. FROM PATH INTEGRAL TO INSTANTANEOUS DYNAMICS

As a result of two averaging processes, time fluctuation and vacuum averaging, the basic dynamical input of the resulting 3D path integral—the doubly renormalized Wilson loop—can be written as

$$\langle\langle W\rangle\rangle = Z_W \exp\left\{-\frac{1}{2}\iint d\pi_{\mu\nu}(1)d\pi_{\lambda\sigma}(2) \times [g^2\langle F_{\mu\nu}(1)F_{\lambda\sigma}(2)\rangle + e^2\langle F^{(e)}_{\mu\nu}(1)F^{(e)}_{\lambda\sigma}(2)\rangle] + O(FFF)\right\}, \tag{33}$$

where

$$d\pi_{\mu\nu} \equiv ds_{\mu\nu} + \sigma^{(1)}_{\mu\nu}\frac{dt_E^{(1)}}{2\omega_1} - \sigma^{(2)}_{\mu\nu}\frac{dt_E^{(2)}}{2\omega_2},$$

and the integration $ds_{\mu\nu}$ is done over the minimal area $S_{\min}$ inside the time-averaged trajectories of quark and antiquark $\bar{L}_1$ and $\bar{L}_2$. Note that in addition to the time-fluctuation smearing discussed above, there is also nonperturbative smearing provided by the np field correlators.

Indeed, the quadratic (Gaussian) color field correlators can be written as [19]

$$\begin{aligned}
\frac{g^2}{N_c}\langle\langle \mathrm{Tr}E_i(x)\Phi E_j(y)\Phi^\dagger\rangle\rangle &= \delta_{ij}\left(D^E(u) + D_1^E(u) + u_4^2\frac{\partial D_1^E}{\partial u^2}\right) + u_iu_j\frac{\partial D_1^E}{\partial u^2},\\
\frac{g^2}{N_c}\langle\langle \mathrm{Tr}H_i(x)\Phi H_j(y)\Phi^\dagger\rangle\rangle &= \delta_{ij}\left(D^H(u) + D_1^H(u) + \mathbf{u}^2\frac{\partial D_1^H}{\partial \mathbf{u}^2}\right) - u_iu_j\frac{\partial D_1^H}{\partial u^2},\\
\frac{g^2}{N_c}\langle\langle \mathrm{Tr}H_i(x)\Phi E_j(y)\Phi^\dagger\rangle\rangle &= \varepsilon_{ijk}u_4u_k\frac{\partial D_1^{EH}}{\partial u^2},
\end{aligned} \tag{34}$$

where D^E, D^H are purely np correlators, and $D_1^{E,H}$ contain a perturbative part. The same type of equations but with replacement $\frac{g^2}{N_c} \to e^2$ and keeping only $D_1^{E,H}$ holds also for e.m. correlators. Note that at zero temperature, color-electric and color-magnetic correlators coincide; note also that np correlators D^E, D^H are due to Euclidean vacuum fields.

The explicit form of perturbative correlators $D_1^{E,H}$ to lowest order in α_s is

$$D_1^E(x) = D_1^H(x) = \frac{16\alpha_s}{3\pi x^4} + O(\alpha_s^2), \tag{35}$$

while for e.m. fields one should replace $\frac{4}{3}\alpha_s \to \alpha$.

At this point it is important to realize that the correlators depend on space and time intervals, e.g., $D(\mathbf{z}^{(1)} - \mathbf{z}^{(2)}, t_E^{(1)} - t_E^{(2)})$ and $\langle\langle W\rangle\rangle$ in Eq. (34) even after fluctuation averaging implies nonlocal in time dynamics; e.g., the term $\iint e^z\langle F^{(e)}_{\mu\nu}(1)F^{(e)}_{\lambda\sigma}(2)\rangle ds_{\mu\nu}(1)ds_{\lambda\sigma}(2)$ stands actually for a photon exchange diagram. We are now going to replace this time nonlocal interaction by the instantaneous one, which is easily done in the correlator language, simply by integrating in (34) all correlators over time differences, $t_E^{(1)} - t_E^{(2)}$,

$$dt_E^{(1)}dt_E^{(2)} = dt_Ed(t_E^{(1)} - t_E^{(2)}), \qquad dt_E = d\frac{t_E^{(1)} + t_E^{(2)}}{2}.$$

It is important that the main part of our interaction, the confining interaction, is ensured by the correlator $D^E(\sqrt{t2 + r2})$, which has a very small correlation length λ, as was shown on the lattice [36] and analytically [37]. $D^E(t) \sim e^{-t/\lambda}$, $t \gtrsim \lambda$, $\lambda \sim 0.1$ fm, and therefore the transition to the instantaneous dynamics is done on small averaging interval $\Delta t \sim \lambda$. Therefore, for all processes with momentum (energy) transfer ΔQ satisfying $\Delta Q\lambda \lesssim 1$, this transition of a np confining mechanism to the instantaneous dynamics is allowable. The case of gluon exchange is similar to the Coulomb interaction, where the instantaneous approximation in the Bethe-Salpeter equation is known as the Salpeter equation and is widely used in the literature. We shall mostly use the one-gluon exchange (OGE) interaction as a perturbation, and therefore our transition to the instantaneous dynamics is justified.

For the case of the zero angular momentum (see in Ref. [23] the general derivation), one can write for the instantaneous straight line $w_\mu(t,\beta) = z_\mu^{(1)}(t)\beta = z_\mu^{(2)}(t)(1-\beta)$, $0 \le \beta \le 1$, and, e.g., $ds_{\mu 4} = (z_\mu^{(1)}(t) - z_\mu^{(2)}(t))d\beta dt$.

For zero angular momentum, one can simplify the integration over the area of the minimal surface in (33) and obtain the result neglecting spin-containing terms in (33) for the moment,

$$\langle\langle W\rangle\rangle = Z_W \exp\left(-\int_0^T [V_0(r(t_E))]dt_E\right), \tag{36}$$

where $r(t_E) = |\mathbf{z}_1(t_E) - \mathbf{z}_2(t_E)|$, and

$$V_0(r) = V_{\rm conf}(r) + V_{\rm OGE}(r), \tag{37}$$

$$V_{\rm conf}(r) = 2r\int_0^r d\lambda \int_0^\infty d\nu D(\lambda,\nu) \to \sigma r, \quad (r\to\infty), \tag{38}$$

$$\sigma = 2\int_0^\infty d\nu \int_0^\infty d\lambda D(\nu,\lambda), \tag{39}$$

$$V_{\rm OGE} = \int_0^r \lambda d\lambda \int_0^\infty d\nu D_1^{\rm pert}(\lambda,\nu) = -\frac{4}{3}\frac{\alpha_s}{r}. \tag{40}$$

As a result, one can write for the product of $q\bar{q}$ Green's functions (we omit renormalization Z factors, Fock amplitude coefficients, and ordering operators for simplicity),

$$\left(\frac{1}{(m_1^2-\hat{D}_1^2)(m_2^2-\hat{D}_2^2)}\right)_{xy} = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}}(D^3z_1)_{\mathbf{xy}}(D^3z_2)_{\mathbf{xy}}e^{-A(\omega_1,\omega_2,\mathbf{z}_1,\mathbf{z}_2)}, \tag{41}$$

where $A \equiv K_1(\omega_1) + K_2(\omega_2) + \int V_0(r(t_E))dt_E$, and

$$K_i(\omega_i) = \frac{m_i^2+\omega_i^2}{2\omega_i}T + \int_0^T dt_E \frac{\omega_i}{2}\left(\frac{d\mathbf{z}^{(i)}}{dt_E}\right)^2.$$

We can also introduce here the two-body 3D Hamiltonian $H(\omega_1,\omega_2,\mathbf{p}_1,\mathbf{p}_2)$ and rewrite (41) as

$$\left(\frac{1}{(m_1^2-\hat{D}_1^2)(m_2^2-\hat{D}_2^2)}\right)_{xy} = \frac{T}{8\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}}\langle\mathbf{x}|e^{-H(\omega_1,\omega_2,\mathbf{p}_1,\mathbf{p}_2)T}|_{\mathbf{y}}\rangle, \tag{42}$$

where H is obtained in a standard way from the action $A(\omega_1,\omega_2,\mathbf{z}_1,\mathbf{z}_2)$ [we omit all e.m. fields except for external magnetic fields (m.f.) $\mathbf{B}$],

$$H = \sum_{i=1}^2 \frac{(\mathbf{p}^{(i)} - \frac{e_i}{2}(\mathbf{B}\times\mathbf{z}^{(i)}))^2 + m_i^2 + \omega_i^2 - e_i\boldsymbol{\sigma}^i\mathbf{B}}{2\omega_i} + V_0(r) + V_{ss} + \Delta M_{SE}, \tag{43}$$

and V_0 is given in (37). The spin-dependent part of H, V_{ss} is obtained perturbatively from $\sigma_{\mu\nu}F_{\mu\nu}$ terms in (28) and is calculated in the presence of m.f. in Ref. [36]. It is considered as a perturbative correction and is a relativistic generalization of the standard hyperfine interaction,

$$V_{ss}(r) = \frac{1}{4\omega_1\omega_2}\int \langle\sigma_{\mu\nu}^{(i)}F_{\mu\nu}(x)\sigma_{\rho\lambda}^{(2)}F_{\rho\lambda}(y)\rangle d(x_4-y_4).$$

Its explicit form is given in Ref. [38]. Finally, the correction $\frac{\langle\sigma(i)F(x)\sigma(i)F(y)\rangle}{4_\omega 1_\omega 2}$, where i refers to the same quark (antiquark) yields the spin-independent self-energy correction ΔM_{SE} which was calculated earlier [39] and for zero mass quarks and no m.f. is

$$\Delta M_{SE} = -\frac{3\sigma}{2\pi\omega_1} - \frac{3\sigma}{2\pi\omega_2}. \tag{44}$$

For the case of nonzero m.f., the resulting ΔM_{SE} is given in Ref. [38]. We can now write the total Green's function of $q_1\bar{q}_2$ system denoting by Y the product of projection operators $Y = \Gamma(m_1 - \hat{D}_1)\Gamma(m_2 - \hat{D}_2)$,

$$\begin{aligned} m_1 - \hat{D}_1 &= m_1 - i\hat{p}_1 = m_1 + \omega_1\gamma_4 - i\mathbf{p}\boldsymbol{\gamma}, \\ m_2 - \hat{D}_2 &= m_2 - \omega_2\gamma_4 - i\mathbf{p}\boldsymbol{\gamma}, \end{aligned} \tag{45}$$

where $\mathbf{p}$ is the quark 3-momentum in the c.m. system.

As a result, one has

$$\int d^3(\mathbf{x}-\mathbf{y})G(x,y) = \int d^3(\mathbf{x}-\mathbf{y})\mathrm{tr}\left(\frac{4Y_\Gamma}{(m_1^2-\hat{D}_1^2)(m_2^2-\hat{D}_2^2)}\right)_{xy} = \frac{T}{2\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}}\langle Y_\Gamma\rangle\langle\mathbf{x}|e^{-H(\omega_1,\omega_2,\mathbf{p}_1,\mathbf{p}_2)T}|_{\mathbf{y}}\rangle. \tag{46}$$

We have used in (46) the relations $4\langle Y\rangle = \mathrm{tr}\langle\Gamma(m_1 - i\hat{p}_1)\Gamma(m_2 - i\hat{p}_2)\rangle$, and neglected spin-dependent terms in H; we have taken into account that D_μ acting on the Wilson line, i.e., $D_\mu \exp(ig\int^x A_\mu dz_\mu)\Lambda$, yields $\exp(ig\int^x A_\mu dz_\mu)\partial_\mu\Lambda$. The c.m. projection of the Green's function yields

$$\int d^3(\mathbf{x}-\mathbf{y})\langle\mathbf{x}|e^{-H(\omega_1,\omega_2,\mathbf{p}_1,\mathbf{p}_2)T}|_{\mathbf{y}}\rangle = \sum_n \varphi_n^2(0)e^{-M_n(\omega_1,\omega_2)T}, \tag{47}$$

see the Appendix for explicit separation of the relative coordinates, Eqs. (A9)–(A12). Here, $M_n(\omega_1,\omega_2)$ is the eigenvalue of $H(\omega_1,\omega_2,\mathbf{p}_1,\mathbf{p}_2)$ in the c.m. system, where $\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 = 0$; $\mathbf{p}_1 = \mathbf{p} = -\mathbf{p}_2$.

The integrals over $d\omega_1$, $d\omega_2$ for $T\to\infty$ can be performed by the stationary point method. Namely, one has

$$\int G(x,y)d^3(\mathbf{x}-\mathbf{y}) = \frac{T}{2\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty \frac{d\omega_2}{\omega_2^{3/2}}\sum_n e^{-M_n(\omega_1,\omega_2)T}\varphi_n^2(0)\langle Y\rangle = \sum_n \frac{e^{-M_n(\omega_1^{(0)},\omega_2^{(0)})T}\varphi_n^2(0)\langle Y\rangle}{\omega_1^{(0)}\omega_2^{(0)}\sqrt{(\omega_1^{(0)}M_n^{"}(1))(\omega_2^{(0)}M_n^{"}(2))}}, \quad (48)$$

where

$$\frac{\partial M_n(\omega_1,\omega_2)}{\partial\omega_i}\bigg|_{\omega_i=\omega_i^{(0)}}=0,\quad M_n^{"}(i)=\frac{\partial M_n(\omega_1,\omega_2)}{\partial\omega_i^2}\bigg|_{\omega_i=\omega_i^{(0)}}, \quad (49)$$

and we have neglected the mixed terms $\frac{\partial 2_M n}{\partial_\omega 1\partial_\omega 2}$ for simplicity; however, we should keep them in concrete calculations (see the exact result in the Appendix). Comparing the results of (47) and (48) with the definitions of quark decay constants f_Γ^n,

$$\int G_\Gamma(x)d^3\mathbf{x} = \sum_n\int d^3\mathbf{x}\langle 0|j_\Gamma|n\rangle\langle n|j_\Gamma|0\rangle e^{i\mathbf{P}\mathbf{x}-M_nT}\frac{d^3\mathbf{P}}{2M_n(2\pi)^3} = \sum_n \varepsilon_\Gamma\otimes\varepsilon_\Gamma\frac{(M_nf_\Gamma^n)^2}{2M_n}e^{-M_nT}, \quad (50)$$

where for $\Gamma=\gamma_\mu,\gamma_\mu\gamma_5$,

$$\sum_{k=1,2,3}\varepsilon_\mu^{(k)}(q)\varepsilon_\nu^{(k)}(q)=\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}, \quad (51)$$

and $\varepsilon_\Gamma=1$ for $\Gamma=1,\gamma_5$, one obtains the expression for f_Γ^n (to lowest order in V_{ss}),

$$(f_\Gamma^n)^2=\frac{N_c\langle Y_\Gamma\rangle|\varphi_n(0)|^2}{\omega_1^{(0)}\omega_2^{(0)}M_n\xi_n}, \quad \xi_n\equiv\sqrt{(\omega_1^{(0)}M_n^{"}(1))(\omega_2^{(0)}M_n^{"}(2))}. \quad (52)$$

It is interesting that numerical estimates using (52) and (A20) are close to those obtained in Ref. [30].

IV. RELATIVISTIC HAMILTONIANS OF A MESON IN MAGNETIC FIELD

The resulting relativistic Hamiltonian in the instantaneous limit is given in (43) and can be written as

$$H=\sum_{i=1}^2\frac{(\mathbf{p}^{(i)}-\frac{e_i}{2}(\mathbf{B}\times\mathbf{z}^{(i)}))^2+m_i^2+\omega_i^2-e_i\boldsymbol{\sigma}^{(i)}B}{2\omega_i} + U(\mathbf{z}^{(1)}-\mathbf{z}^{(2)},\boldsymbol{\sigma}^{(1)},\boldsymbol{\sigma}^{(2)},\omega_1,\omega_2), \quad (53)$$

where

$$U=V_0(r)+V_{ss}+\Delta M_{SE}. \quad (54)$$

We shall be interested in the spectrum of the $q_1\bar{q}_2$ system in the magnetic field $\mathbf{B}$, but before that we shall test the general form of the Hamiltonian $H(\omega_1,\omega_2,\mathbf{p}_1\mathbf{p}_2)$ and its eigenvalues obtained at the stationary point values $\omega_1^{(0)}$, $\omega_2^{(0)}$.

We start with the case of $\mathbf{B}=0$ and $U=-\frac{Z\alpha}{|\mathbf{z}(1)-\mathbf{z}(2)|}$. Separating the total and relative momenta and coordinates,

$$\mathbf{R}=\frac{\omega_1\mathbf{z}^{(1)}+\omega_2\mathbf{z}^{(2)}}{\omega_1+\omega_2},\quad \boldsymbol{\eta}=\mathbf{z}^{(1)}-\mathbf{z}^{(2)};\quad \boldsymbol{\pi}=\frac{1}{i}\frac{\partial}{\partial\boldsymbol{\eta}}, \quad (55)$$

and $\mathbf{P}=\mathbf{p}^{(1)}+\mathbf{p}^{(2)}$, one obtains in (53) with $\mathbf{B}=0$,

$$H=\frac{\mathbf{P}^2}{2(\omega_1+\omega_2)}+\frac{\boldsymbol{\pi}^2}{2\tilde{\omega}}+U(\boldsymbol{\eta})+\sum_{i=1,2}\frac{m_i^2+\omega_i^2}{2\omega_i}. \quad (56)$$

(1) The first example is the relativistic electron with mass m_1 in the Coulomb field of a heavy atom of mass m_2 with charge Ze, $U(\eta)=-\frac{Z\alpha}{\eta}$. For $\mathbf{P}=0$, one has for the ground state

$$M(\omega_1,\omega_2)=\sum_{i=1,2}\frac{m_i^2+\omega_i^2}{2\omega_i}-\frac{\tilde{\omega}(Z\alpha)^2}{2}. \quad (57)$$

Minimizing in ω_1 for $m_2\gg m_1$, one obtains

$$M\approx m_2+m_1\sqrt{1-(Z\alpha)^2}, \quad (58)$$

which coincides with the exact answer from the Dirac equation.

(2) As a second example, we consider an electron-positron system; then from the same Hamiltonian (56) for $\mathbf{P}=0$ and $m_1=m_2=m$, one obtains after minimization

$$M=2m\sqrt{1-\frac{\alpha^2}{4}}\approx 2m-\frac{m\alpha^2}{4}, \quad (59)$$

which looks correct, at least in the expansion in α.

(3) In the next example, we consider the noninteracting $q_1\bar{q}_2$ system in constant magnetic field $\mathbf{B}$ along the z axis. For $U=0$, one can solve the one-body problem for each quark in m.f. with the result for the lowest Landau levels

$$M(\omega_1,\omega_2)=\sum_i\frac{m_i^2+\omega_i^2+eB(2n_i+1)-e_i\boldsymbol{\sigma}^{(i)}\mathbf{B}+(\mathbf{p}_z^{(i)})^2}{2\omega_i}, \quad (60)$$

and after minimization one has

$$M(\omega_1^{(0)},\omega_2^{(0)}) = \sum_i \sqrt{(\mathbf{p}_z^{(i)})^2 + m_i^2 + eB(2n_i+1) - e_i\boldsymbol{\sigma}^{(i)}\mathbf{B}}. \tag{61}$$

We turn now to the general case of the $q_1\bar{q}_2$ system and consider first the case of a neutral system, $e_1 = -e_2 = e$. In terms of total and relative momenta, the Hamiltonian has the form

$$H_{q_1q_2} = H_B + H_\sigma + U, \tag{62}$$

$$H_B = \frac{1}{2\omega_1}\left[\frac{\tilde{\omega}}{\omega_2}\mathbf{P} + \boldsymbol{\pi} - \frac{e_1}{2}\mathbf{B}\times\left(\mathbf{R} + \frac{\tilde{\omega}}{\omega_1}\boldsymbol{\eta}\right)\right]^2 + \frac{1}{2\omega_2}\left[\frac{\tilde{\omega}}{\omega_1}\mathbf{P} - \boldsymbol{\pi} - \frac{e_2}{2}\mathbf{B}\times\left(\mathbf{R} - \frac{\tilde{\omega}}{\omega_2}\boldsymbol{\eta}\right)\right]^2, \tag{63}$$

$$H_\sigma = \sum_{i=1,2}\frac{m_i^2 + \omega_i^2 - e_i\boldsymbol{\sigma}^{(i)}\mathbf{B}}{2\omega_i}. \tag{64}$$

The $\mathbf{R}$ dependence for (62) in the case when $e_1 = -e_2$ can be factorized out in the way discovered long ago [40],

$$\Psi(\mathbf{R},\boldsymbol{\eta}) = \varphi(\boldsymbol{\eta})\exp\left(i\mathbf{P}\mathbf{R} - \frac{ie}{2}(\mathbf{B}\times\boldsymbol{\eta})\mathbf{R}\right), \tag{65}$$

and for $\varphi(\boldsymbol{\eta})$, one obtains the equation

$$(H_0 + H_\sigma + U)\varphi(\eta) = M\varphi(\eta), \tag{66}$$

where H_0 is

$$H_0 = \frac{1}{2\tilde{\omega}}\left(-\frac{\partial^2}{\partial\boldsymbol{\eta}^2} + \frac{e^2}{4}(\mathbf{B}\times\boldsymbol{\eta})^2\right). \tag{67}$$

One can replace for simplicity the linear confining term by the oscillator potential, $V_{\rm conf} = \sigma\eta \to \tilde{V}_{\rm conf} \equiv \frac{\sigma}{2}(\frac{\eta^2}{\gamma} + \gamma)$, where γ satisfies stationary point condition $\frac{\partial M}{\partial\gamma}|_{\gamma=\gamma_0} = 0$, which ensures some 5% accuracy of this replacement. Then the lowest eigenvalue $\bar{M}$ of the basic part of Hamiltonian $\bar{H} = H_0 + H_\sigma + \tilde{V}_{\rm conf}$ is

$$\bar{M}(\omega_1,\omega_2,\gamma) = \varepsilon_{n_\perp,n_z} + \sum_{i=12,}\frac{m_i^2 + \omega_i^2 - e_i\boldsymbol{\sigma}^{(i)}\mathbf{B}}{2\omega_i}, \tag{68}$$

where $e_1 = e = -e_2$, and

$$\varepsilon_{n_\perp,n_z} = \frac{1}{2\tilde{\omega}}\left[\sqrt{e^2B^2 + \frac{4\sigma\tilde{\omega}}{\gamma}}(2n_\perp + 1) + \sqrt{\frac{4\sigma\tilde{\omega}}{\gamma}}\left(n_z + \frac{1}{2}\right)\right] + \frac{\gamma\sigma}{2}. \tag{69}$$

We turn now to the case of the charged two-body system in m.f., and here one can consider two different situations. In the first case, when $e_1 = e_2 = e$ and also $m_1 = m_2$ [and hence $\omega_1^{(0)} = \omega_2^{(0)}$], one can do an exact factorization of $\mathbf{R}$ and $\boldsymbol{\eta}$,

$$H_B = \frac{P^2}{2(\omega_1+\omega_2)} - \frac{e\mathbf{P}(\mathbf{B}\times\mathbf{R})}{\omega_1+\omega_2} + \frac{e^2}{8\tilde{\omega}}(\mathbf{B}\times\mathbf{R})^2 + \frac{\pi^2}{2\tilde{\omega}} + \frac{e^2(\mathbf{B}\times\boldsymbol{\eta})^2(\omega_1^3+\omega_2^3)}{8(\omega_1+\omega_2)^2\omega_1\omega_2} + \Delta H_B(\omega_1,\omega_2). \tag{70}$$

H_σ is given in (64) and ΔH_B is

$$\Delta H_B(\omega_1,\omega_2) = -\frac{\omega_2^2-\omega_1^2}{\omega_1\omega_2(\omega_1+\omega_2)}\frac{e}{2}\boldsymbol{\pi}(\mathbf{B}\times\boldsymbol{\eta}) - \frac{\omega_2-\omega_1}{\omega_1\omega_2}\frac{e}{2}\boldsymbol{\pi}(\mathbf{B}\times\mathbf{R}) - \frac{\omega_2-\omega_1}{(\omega_1\omega_2)}\frac{e}{2}\mathbf{P}(\mathbf{B}\times\boldsymbol{\eta}) + \frac{(\omega_2^2-\omega_1^2)}{(\omega_1+\omega_2)^2\omega_1\omega_2}\frac{e^2}{4}(\mathbf{B}\times\mathbf{R})(\mathbf{B}\times\boldsymbol{\eta}). \tag{71}$$

For $\omega_1 = \omega_2$, ΔH_B vanishes and the Hamiltonian has the form

$$H = \frac{P^2}{4\omega} - \frac{e(\mathbf{P}(\mathbf{B}\times\mathbf{R}))}{2\omega} + \frac{e^2}{4\omega}(\mathbf{B}\times\mathbf{R})^2 + \frac{\pi^2}{\omega} + \frac{e^2}{16\omega}(\mathbf{B}\times\boldsymbol{\eta})^2 + \frac{2m^2 + 2\omega^2 - e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\mathbf{B}}{2\omega} + \frac{\sigma}{2}\left(\frac{\eta^2}{\gamma} + \gamma\right) + V_{\rm OGE} + V_{ss} + \Delta M_{SE}. \tag{72}$$

The lowest eigenvalues of the Hamiltonian (72) are

$$M = \frac{m^2+\omega^2}{\omega} + \langle V_{\rm OGE}\rangle + \langle V_{ss}\rangle + \langle \Delta M_{SE}\rangle + \frac{eB}{2\omega}(2N_\perp+1) + \sqrt{\left(\frac{eB}{2\omega}\right)^2 + \frac{2\sigma}{\gamma_0\omega}}(2n_\perp+1) + \left(n_\parallel + \frac{1}{2}\right)\sqrt{\frac{2\sigma}{\gamma_0\omega}} - \frac{e(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\mathbf{B}}{2\omega} + \frac{\gamma_0\sigma}{2}. \tag{73}$$

We now turn to the general case of a charged $q_1\bar{q}_2$ system, when $e_1 \neq e_2$, and write the full instantaneous Hamiltonian as in Eqs. (62)–(64) but with arbitrary e_1 and e_2, $e_1 + e_2 = e$, and e is the total charge of the meson.

In this case, the simple factorization form (65) does not work, and one must instead make a first step towards factorization; namely, one must associate the c.m. motion in m.f. with the total charge e of the system. This is done in the following form discussed previously in Ref. [33]:

$$\Psi(\boldsymbol{\eta}, \mathbf{R}) = \exp(i\Gamma)\varphi(\boldsymbol{\eta}, \mathbf{R}), \tag{74}$$

$$\Gamma = \mathbf{PR} - \frac{\bar{e}}{2}(\mathbf{B}\times\boldsymbol{\eta})\mathbf{R}, \qquad \bar{e} = \frac{e_1-e_2}{2}, \tag{75}$$

and the resulting Hamiltonian from the relation $H_0\Psi = \exp(i\Gamma)H_0'\varphi$ is

$$\begin{aligned} H_0' = {}& \frac{\mathbf{P}^2}{2(\omega_1+\omega_2)} + \frac{(\omega_1+\omega_2)\Omega_R^2\mathbf{R}_\perp^2}{2} + \frac{\boldsymbol{\pi}^2}{2\tilde{\omega}} + \frac{\tilde{\omega}\Omega_\eta^2\boldsymbol{\eta}_\perp^2}{2} \\ & + X_{LP}\mathbf{BL}_P + X_{L_\eta}\mathbf{BL}_\eta + X_1\mathbf{P}(\mathbf{B}\times\boldsymbol{\eta}) \\ & + X_2(\mathbf{B}\times\mathbf{R})\cdot(\mathbf{B}\times\boldsymbol{\eta}) + X_3\boldsymbol{\pi}(\mathbf{B}\times\mathbf{R}) \\ & + \frac{m_1^2+\omega_1^2}{2\omega_1} + \frac{m_2^2+\omega_2^2}{2\omega_2}, \end{aligned} \tag{76}$$

$$\Omega_R^2 = B^2\frac{(e_1+e_2)^2}{16\omega_1\omega_2}, \tag{77}$$

$$\Omega_\eta^2 = \frac{B^2}{2\tilde{\omega}(\omega_1+\omega_2)^2}\left[\frac{(e_1\omega_2+\bar{e}\omega_1)^2}{2\omega_1} + \frac{(e_2\omega_1-\bar{e}\omega_2)^2}{2\omega_2}\right]. \tag{78}$$

Here, all coefficients $X_i (i = 1, 2, 3)$ are given explicitly in Appendix 2 of Ref. [33].

Treating the terms X_1, X_2, X_3 as a perturbation ΔM_X,

$$\Delta M_X = \langle X_1\mathbf{P}(\mathbf{B}\times\boldsymbol{\eta}) + X_2(\mathbf{B}\times\mathbf{R})(\mathbf{B}\times\boldsymbol{\eta}) + X_3\boldsymbol{\pi}(\mathbf{B}\times\mathbf{R})\rangle, \tag{79}$$

one can write the total energy eigenvalues $M_n^{(0)}$ of the Hamiltonian $\bar{H}$ in (68) as

$$M_n^{(0)} = M^{(0)}(\mathbf{P}) + M^{(0)}(\boldsymbol{\pi}) + \Delta M_X + H_\sigma, \tag{80}$$

where

$$M^{(0)}(\mathbf{P}) = \frac{P_z^2}{2(\omega_1+\omega_2)} + \Omega_R(2n_{R_\perp}+1) + X_{LP}\mathbf{L}_P\mathbf{B}, \tag{81}$$

and $M^{(0)}(\boldsymbol{\pi})$ is the eigenvalue of the operator H_π,

$$H_\pi = \frac{\boldsymbol{\pi}^2}{2\tilde{\omega}} + \frac{\tilde{\omega}\Omega_\eta^2\boldsymbol{\eta}_\perp^2}{2} + X_{L_\eta}\mathbf{BL}_\eta + V_{\rm conf} + V_{\rm OGE}. \tag{82}$$

We have written above the most general forms of instantaneous Hamiltonians in the external m.f. It is seen that to a good accuracy the dynamical contributions of e.m. and color fields can be separated, except in the OGE and spin-dependent terms, and as shown in Ref. [38], the m.f. contribution to the both terms is decisive at large eB.

V. DISCUSSION OF RESULTS

We have started with the general 4D proper-time path integral for the Green's function of a quark and an antiquark in gluonic $(A_\mu, F_{\mu\nu})$ and e.m. $(A_\mu^e, B_{\mu\nu})$ fields. These fields are contained in the generalized Wilson loop W with the inclusion of spin-field operators $(\sigma_{\mu\nu}(F_{\mu\nu} + B_{\mu\nu}))$.

After a vacuum averaging procedure in the partition function, the averaged Wilson loop $\langle W\rangle_{A,A^{(e)}}$ contains all possible interactions including internal quark loops from the terms $\mathrm{tr}\ln(m_i^2 - \hat{D}_i^2)$ in the partition function.

As a first step, we have traded the particle proper times for the Euclidean (ordering) times $t_E^{(1)}$, $t_E^{(2)}$ and performed path integration over fourth particle coordinates z_4, $\bar{z}_4$, which is physically the time fluctuations around $t_E^{(1)}$, $t_E^{(2)}$. We have shown that this time-fluctuation integration leads to the 3D path integrals with the action (or Hamiltonian in the Hamiltonian form of path integral) which is a matrix in the Fock states. The resulting 3D path integrals are integrals over new parameters ω_1, ω_2, and the spectrum of the $q_1\bar{q}_2$ system can be found for large times by a stationary point procedure in ω_1, ω_2.

In this way, one is going from the 4D formalism to the multichannel 3D formalism with an additional ω integration for each particle.

As a next step, we have observed that the interaction appearing in the averaged Wilson loop $\langle W\rangle_{A,A^{(e)}}$ has the form of field correlators $\langle F_{\mu\nu}(x)F_{\lambda\sigma}(y)\rangle$, $\langle B_{\mu\nu}(x)B_{\lambda\sigma}(y)\rangle$, and the first correlator has a very small correlation length $\lambda \sim 0.1$ fm (found on the lattice [36] and in analytic calculations [37]). This allows us to go over to the instantaneous dynamics when the bilocal or multilocal interaction $\langle F(x)F(y)\rangle$ is replaced by the time-averaged potentials $V(\mathbf{x}-\mathbf{y}) = \int d(x_4-y_4)\langle F(x)F(y)\rangle$, and this is valid when the basic parameter defining the quark trajectory, string tension σ satisfies $\sigma\lambda^2 \ll 1$ so that the typical time length on trajectory $t_0 \sim \frac{1}{\sqrt{\sigma}}$ is much larger than λ. Note that this condition is opposite to the one used for validity of the OPE and QCD sum rules.

As a result, one obtains the instantaneous relativistic Hamiltonian $H(\omega_1, \omega_2)$ depending on two parameters ω_1, ω_2 [for the $(q_1\bar{q}_2)$ Hamiltonian matrix element], and the actual spectrum is obtained from the eigenvalues $M_n(\omega_1, \omega_2)$ at the stationary points $\omega_1^{(0)}$, $\omega_2^{(0)}$. Note that these points are different for different $n = 0, 1, 2\ldots$

We have checked the results in Sec. IV for several simple systems and found good agreement with the known results. Moreover, this formalism for eigenvalues has been used for more than 20 years in many papers, a small part of which were cited here, and the results in all systems—mesons, baryons, hybrids and glueballs—are well compared with the experimental and lattice ones.

The important new element in this paper is the rigorous derivation of the integral representation for the $(q_1\bar{q}_2)$ Green's function Eqs. (41), (42), and (48), which gives a new meaning to the parameters ω_1, ω_2, and allows us not only to calculate the spectrum but also the Green's function itself.

As an important application of the developed formalism, we have derived in Sec. IV the explicit form of Hamiltonians of the $(q_1\bar{q}_2)$ system in the constant m.f. $\mathbf{B}$, and defined the main part of the spectrum for neutral and charged mesons.

These results have been used for the explicit numerical evaluation of the ρ-meson spectra in Ref. [41], which are in reasonable agreement with existing lattice data. Moreover, the same formalism was extensively exploited in Ref. [35] for the calculation of chiral condensate and in Ref. [33] for magnetic moments.

Actually, the field of possible applications of our method in QCD and QED is enormous, and the method is especially simple in the cases when only spectral properties are of interest. This is clearly seen when one compares this method with the Bethe-Salpeter equation. In the last case, one is facing the problems of the relative time and insufficiency of the ladder kernel already in the QED case.

In the QCD case, the use of the Bethe-Salpeter equation is in addition associated with the vector propagator form of confinement, which is physically not consistent, or with some phenomenological form, and in this way the method loses its fundamental character. On the contrary, the very short-correlation property of confinement suits perfectly to establish the validity of instantaneous Hamiltonian formalism and allows for an accurate and simple procedure.

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APPENDIX: DERIVATION OF THE GENERAL EXPRESSION FOR THE $q_1\bar{q}_2$ GREEN'S FUNCTION

We start with the general definition for the $q_1\bar{q}_2$ Green's function in the vacuum gluonic and external e.m. fields,

$$G(x, y) = \langle \mathrm{tr}\Gamma S_1(x, y)\bar{\Gamma}\bar{S}_2(y, x)\rangle_A = \left\langle \mathrm{tr}\Gamma \frac{(m_1 - \hat{D}_1)}{m_1^2 - \hat{D}_1^2}\bar{\Gamma}\frac{(m_2 - \hat{\bar{D}}_2)}{(m_2^2 - \hat{\bar{D}}_2^2)}\right\rangle_A \tag{A1}$$

$$= 4\int_0^\infty ds_1 \int_0^\infty ds_2 (D^4 z^{(1)} D^4 z^{(2)})_{xy} e^{-K_1 - K_2}\langle YW_F\rangle, \tag{A2}$$

where $\langle YW_F\rangle = \frac{1}{4}\,\mathrm{tr}[\Gamma(m_1 - i\hat{p}_1)\bar{\Gamma}(m_2 - i\hat{p}_2)\langle W_F\rangle_A]$ and $W_F \equiv \langle\langle W\rangle\rangle$ given in (33); the spin operator ordering in (A2) is not written explicitly. Neglecting spin dependence, one has a purely scalar function W_F, which is proportional to a unit 4×4 matrix.

Introducing the effective energies $\omega_i = \frac{T}{2_s i}$, $T \equiv |x_4 - y_4|$, one can rewrite (A1) as

$$G(x, y) = \frac{T}{2\pi}\int_0^\infty \frac{d\omega_1}{\omega_1^{3/2}}\frac{d\omega_2}{\omega_2^{3/2}} \times (D^3 z^{(1)} D^3 z^{(2)})_{\mathbf{xy}} e^{-K_1(\omega_1) - K_2(\omega_2)}\langle\langle YW_F\rangle\rangle_{\Delta z_4}, \tag{A3}$$

and we have taken into account that

$$\int (Dz_4^{(1)} Dz_4^{(2)})_{x_4y_4}\langle YW_F\rangle e^{-\frac{1}{4}\int_0^{s_1}\left(\frac{dz_4^{(1)}}{d\tau_1}\right)^2 d\tau_1 - \frac{1}{4}\int_0^{s_2}\left(\frac{dz_4^{(2)}}{d\tau_2}\right)^2 d\tau_2} = \frac{\sqrt{\omega_1\omega_2}}{2\pi T}\langle\langle YW_F\rangle\rangle_{\Delta z_4}. \tag{A4}$$

Here, $\langle\langle YW_F\rangle\rangle_{\Delta_z 4}$ corresponds to the time-fluctuating Wilson loop average, as in Fig. 1, renormalized and normalized by the condition

$$\langle\langle YW_F\rangle\rangle_{\Delta z_4}(g = e = 0) = 1. \tag{A5}$$

We omit in what follows the Fock column structure of the corresponding particle contents in our averaged Wilson loop $\langle\langle YW_F\rangle\rangle_{fl}$ with the corresponding Z_i factors for each Fock line and concentrate on the simplest case of one renormalized closed $(q_1\bar{q}_2)$ loop depending on t_E, as shown in Fig. 2. In the neutral case, $e_1 = -e_2$, $\langle\langle YW_F\rangle\rangle_{\Delta_z 4}$ depends only on coordinate differences $\boldsymbol{\eta}(t_E) = \mathbf{z}^{(1)}(t_E) - \mathbf{z}^{(2)}(t_E)$ defined at the same moment t_E, and one can proceed integrating out the c.m. motion. K_1, K_2 in (A3) are

$$K_1(\omega_1)+K_2(\omega_2) = \left(\frac{m_1^2+\omega_1^2}{2\omega_1}+\frac{m_2^2+\omega_2^2}{2\omega_2}\right)T + \int_0^T dt_E\left[\frac{\omega_1}{2}\left(\frac{d\mathbf{z}^{(1)}}{dt_E}\right)^2+\frac{\omega_2}{2}\left(\frac{d\mathbf{z}^{(2)}}{dt_E}\right)^2\right]. \quad (A6)$$

Introducing now the coordinates

$$\boldsymbol{\eta}(t_E)=\mathbf{z}^{(1)}-\mathbf{z}^{(2)}, \quad \boldsymbol{\rho}(t_E)=\frac{\omega_1}{\omega_1+\omega_2}\mathbf{z}^{(1)}(t_E)+\frac{\omega_2}{\omega_1+\omega_2}\mathbf{z}^{(2)}(t_E), \quad (A7)$$

one can rewrite the last term in (A6) as

$$\int_0^T dt_E\left(\frac{\omega_1+\omega_2}{2}\left(\frac{d\boldsymbol{\rho}}{dt_E}\right)^2+\frac{\tilde{\omega}}{2}\left(\frac{d\boldsymbol{\eta}}{dt_E}\right)^2\right), \quad (A8)$$

and the path integral $(D^3z^{(1)}D^3z^{(2)})_{\mathbf{xy}}$ as

$$(D^3z^{(1)}D^3z^{(2)})_{\mathbf{xy}} = \int\frac{d^3\mathbf{p}_1}{(2\pi)^3}\int\frac{d^3\mathbf{p}_2}{(2\pi)^3}e^{i\mathbf{p}_1\left(\sum\Delta\mathbf{z}^{(1)}-(\mathbf{x}-\mathbf{y})\right)+i\mathbf{p}_2\left(\sum\Delta\mathbf{z}^{(2)}-(\mathbf{x}-\mathbf{y})\right)} \times\frac{d^3\Delta z^{(1)}}{(4\pi\varepsilon_1)^{3/2}}\frac{d^3\Delta z^{(2)}}{(4\pi\varepsilon_2)^{3/2}} = (D^3\rho)_{\mathbf{xy}}(D^3\eta)_{00}, \quad (A9)$$

where

$$(D^3\boldsymbol{\rho})_{\mathbf{xy}}=\int\frac{d^3\mathbf{P}}{(2\pi)^3}\prod_k e^{i\mathbf{P}\left(\sum\Delta\boldsymbol{\rho}_k-(\mathbf{x}-\mathbf{y})\right)}\frac{d^3\Delta\boldsymbol{\rho}k}{\left(\frac{2\pi\Delta t_E}{\omega_1+\omega_2}\right)^{3/2}}, \quad (A10)$$

$$(D^3\boldsymbol{\eta})_{00}=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\prod_k e^{i\mathbf{q}\sum_k\Delta\boldsymbol{\rho}k}\frac{d^3\Delta\boldsymbol{\eta}_k}{\left(\frac{2\pi\Delta t_E}{\tilde{\omega}}\right)^{3/2}}. \quad (A11)$$

In absence of an external magnetic field, which acts on c.m. coordinate $\boldsymbol{\rho}$, it is convenient to consider the $\mathbf{P}=0$ projection of the Green's function

$$\int G(x,y)d^3(\mathbf{x}-\mathbf{y}) = \frac{T}{2\pi}\int_0^\infty\frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty\frac{d\omega_2}{\omega_2^{3/2}}(D^3\boldsymbol{\eta})_{00}e^{-K(\eta)}\langle\langle YW_F\rangle\rangle_{\Delta z_4} = \frac{T}{2\pi}\int_0^\infty\frac{d\omega_1}{\omega_1^{3/2}}\int_0^\infty\frac{d\omega_2}{\omega_2^{3/2}}\langle 0|\langle Y\rangle e^{-HT}|0\rangle, \quad (A12)$$

where

$$K(\eta)=\left(\frac{m_1^2+\omega_1^2}{2\omega_1}+\frac{m_2^2+\omega_2^2}{2\omega_2}\right)T+\int_0^T dt_E\frac{\tilde{\omega}}{2}\left(\frac{d\boldsymbol{\eta}}{dt_E}\right)^2, \quad (A13)$$

$$\langle 0|e^{-HT}|0\rangle=\sum_{n=0}^\infty|\varphi_n(0)|^2e^{-M_n(\omega_1,\omega_2)T}. \quad (A14)$$

Here, $\varphi_n(0)=\varphi_n(\omega_1,\omega_2,\boldsymbol{\eta})|_{\boldsymbol{\eta}=0}$, and $M_n(\omega_1,\omega_2)$ is the eigenvalue of the Hamiltonian

$$H\equiv H(\omega_1,\omega_2), \qquad H\varphi_n=M_n(\omega_1,\omega_2)\varphi_n. \quad (A15)$$

Assuming that $\langle\langle W_F\rangle\rangle_{\Delta_z 4}$ can be represented as

$$\langle\langle W_F\rangle\rangle_{\Delta z_4}=\exp\left(-\int\hat{V}(\eta,\omega)dt_E\right), \quad (A16)$$

the Hamiltonian can be written in the form

$$H(\omega_1,\omega_2)=\sum_{i=1,2}\frac{m_i^2+\omega_i^2}{2\omega_i}+\frac{\mathbf{p}^2}{2\tilde{\omega}}+\hat{V}(\eta,\omega_1,\omega_2). \quad (A17)$$

At this point one can define the so-called quark decay constants $f_\Gamma^{(n)}$ [30],

$$\int G(x,y)d^3(\mathbf{x}-\mathbf{y})=\sum_n\varepsilon_\Gamma\bigotimes\varepsilon_\Gamma\frac{\bar{M}_n(f_\Gamma^{(n)})^2}{2}e^{-\bar{M}_nT}, \quad (A18)$$

where $\varepsilon_\Gamma=1$ for the S and P channels, and $\varepsilon_\Gamma=\varepsilon_\mu^{(k)}$ for the V, A channels,

$$\sum_{k=1,2,3}\varepsilon_\mu^{(k)}(q)\varepsilon_\nu^{(k)}(q)=\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}, \quad (A19)$$

and hence $f_\Gamma^{(n)}$ can be found from (A12) as

$$(f_\Gamma^{(n)})^2e^{-\bar{M}_nT}=\frac{T}{2\pi}\frac{2\langle Y\rangle}{\bar{M}_n}\int_0^\infty\frac{d\omega_1}{\omega_1^{3/2}} \times\int_0^\infty\frac{d\omega_2}{\omega_2^{3/2}}\varphi_n^2(0)e^{-M_n(\omega_1,\omega_2)T}. \quad (A20)$$

Here, T on both sides is assumed to tend to ∞, and one can calculate the integral on the rhs of (A20) by the stationary point method,

$$M_n(\omega_1,\omega_2) = M_n(\omega_1^{(0)},\omega_2^{(0)})+M_n^{(11)}(\omega_1^{(0)},\omega_2^{(0)})\frac{(\omega_1-\omega_1^{(0)})2}{2} + M_n^{(22)}(\omega_1^{(0)},\omega_2^{(0)})\frac{(\omega_2-\omega_2^{(0)})2}{2} + M_n^{(12)}(\omega_1^{(0)},\omega_2^{(0)})(\omega_1-\omega_1^{(0)})(\omega_2-\omega_2^{(0)}), \quad (A21)$$

where

$$M_n^{(ik)}=\frac{\partial^2M_n}{\partial\omega_i\partial\omega_k}\Bigg|_{\omega_i=\omega_i^{(0)},\omega_k=\omega_k^{(0)}} \quad (A22)$$

and

$$\frac{\partial M_n}{\partial\omega_i}\Bigg|_{\omega_i=\omega_i^{(0)}}=0, \quad i=1,2. \quad (A23)$$

Doing the integration in (A20) with the help of (A21), one obtains

$$(f_\Gamma^{(n)})^2 = \frac{N_c \langle Y \rangle \varphi_n^2(0)}{(\omega_1^{(0)} \omega_2^{(0)}) \bar{M}_n \xi_n},$$

where

$$\xi_n = \sqrt{\omega_1^{(0)} \omega_2^{(0)} \Omega_n},$$

with

$$\Omega_n = \frac{\alpha\beta(\alpha-\beta)^2}{(\alpha-\beta)^2+\gamma^2} + \frac{\gamma^2[(\alpha+\beta)^2 - 2(\alpha-\beta)^2 - \gamma^2]}{4[(\alpha-\beta)^2+\gamma^2]}, \quad \text{(A24)}$$

where we have denoted

$$\alpha = \frac{1}{2} M_n^{(11)}, \qquad \beta = \frac{1}{2} M_n^{(22)}, \qquad \gamma = M_n^{(12)}, \quad \text{(A25)}$$

and finally

$$\bar{M}_n = M_n(\omega_1^{(0)}, \omega_2^{(0)}), \quad \bar{Y} = \frac{1}{4}\operatorname{tr}_D(\Gamma(m_1 - i\hat{p}_1)\bar{\Gamma}(m_2 - i\hat{p}_2), \quad \text{(A26)}$$

and tr_D denotes the trace over Dirac 4×4 indices. It is instructive to compare (A24) with the old result obtained in Ref. [30] using approximate path integrals over $(D\Delta\omega)$,

$$(f_\Gamma^{(n)})^2_{\Delta\omega} = \frac{2N_c \langle Y \rangle \varphi_n^2(0)}{\bar{M}_n \omega_1^{(0)} \omega_2^{(0)}}. \quad \text{(A27)}$$

As one can see, comparing (A24) and (A27), in the first case (the time-fluctuation approach of the present paper) the factor $\frac{1}{\sqrt{\Omega\omega_1^{(0)}\omega_2^{(0)}}}$ should be equal to 2 for both expressions to coincide. In practice, for the $(q_1\bar{q}_2)$ state made of zero mass quarks, $m_1 = m_2 = 0$, and with the total mass made of confining interaction, see Ref. [28] for details, one has

$$M_n(\omega_1, \omega_2) = \sum_{i=1}^{2} \frac{m_i^2 + \omega_i^2}{2\omega_i} + (2\tilde{\omega})^{-1/3}\sigma^{2/3} a_n, \quad a_0 = 2,338, \quad \text{(A28)}$$

and for $m_1 = m_2 = 0$ one obtains

$$(\Omega_0 \omega_1^{(0)} \omega_2^{(0)})^{-1/2} = 3, \quad \text{(A29)}$$

while for $m_1 = 0$, $m_2 \ll \sqrt{\sigma}$, the result is

$$(\Omega_0 \omega_1^{(0)} \omega_2^{(0)})^{-1/2} \cong 2.34. \quad \text{(A30)}$$

This implies that the quark decay constants $f_\Gamma^{(n)}$ obtained in the new method will be larger by (10–20)% as compared with previous calculations in Ref. [30].

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