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D-term triggered dynamical supersymmetry breaking

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We present the mechanism of the dynamical supersymmetry breaking at the metastable vacuum recently uncovered in the $\mathcal{N} = 1$ $U(N)$ supersymmetric gauge theory that contains adjoint superfields and that is specified by Kähler and noncanonical gauge kinetic functions and a superpotential whose tree vacua preserve $\mathcal{N} = 1$ supersymmetry. The overall $U(1)$ serves as the hidden sector, and no messenger superfield is required. The dynamical supersymmetry breaking is triggered by the nonvanishing D term coupled to the observable sector and is realized by the self-consistent Hartree-Fock approximation of the Nambu-Jona-Lasinio type while it eventually brings us the nonvanishing F term as well. It is shown that theoretical analysis is resolved as a variational problem of the effective potential for three kinds of background fields, namely, the complex scalar, and the two order parameters D and F of supersymmetry, the last one being treated perturbatively. We determine the stationary point and numerically check the consistency of such treatment as well as the local stability of the scalar potential. The coupling to the $\mathcal{N} = 1$ supergravity is given, and the gravitino mass formula is derived.

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I. INTRODUCTION

Spontaneous breaking of rigid supersymmetry occurs much less frequently compared with that of internal symmetry in quantum field theory and has attracted much interest [1,2] of theorists for over the three decades. Mass hierarchy in elementary particle physics indicates that it is most desirable to break $\mathcal{N} = 1$ supersymmetry dynamically. In fact, under the nonrenormalization theorem [3], no holomorphic operator is generated in perturbation theory, and instanton generated nonperturbative superpotentials have been the major source of dynamical supersymmetry breaking (DSB).

In this paper, we focus our attention on general rigid $\mathcal{N} = 1$ theory in four spacetime dimensions consisting of vector superfields and chiral superfields in the adjoint representation which permits a noncanonical gauge kinetic function τ_{ab} (that may follow from the second derivative of the prepotential) and hence the D term-gaugino-matter fermion (or D term-Dirac gauginos) nonrenormalizable coupling. It has recently been shown in Refs. [4,5] that, in this general situation, supersymmetry is dynamically broken in the metastable vacuum. The mechanism that triggers the DSB is the condensate of the Dirac bilinear above, forcing one of the order parameters D of supersymmetry to be nonvanishing. This is very much reminiscent of the Nambu-Jona-Lasinio (NJL) theory [6,7] of broken chiral symmetry and hence the BCS superconductivity [8,9], being formulated in terms of the effective action of the auxiliary field whose stationary value is the order parameter. The method of approximation employed is the self-consistent Hartree-Fock approximation where the tree and the one-loop contributions are regarded as comparable. Once this mechanism operates, a nonvanishing F term is shown to be induced and contributes, for instance, to the mass of the fermions. The mechanism requires massive adjoint scalars, in particular, the scalar gluons, and, together with the feature that the D term triggers the breaking, is quite distinct from the previous proposals [10–14] of DSB both from theoretical and experimental perspectives. The overall $U(1)$ where the nonvanishing D and the Nambu-Goldstone (NG) fermion reside serves as the hidden sector, and no messenger field is necessary [4] as nonvanishing third prepotential derivatives connect the $U(1)$ sector with the observable $SU(N)$ sector [15,16].

While our treatment of the theory bears much resemblance with that of the NJL theory, there is one important complexity which has no counterpart in the NJL and which we did not emphasize in Ref. [4]. In NJL, aside from the pseudoscalar auxiliary massless singlet field commonly denoted by π, there is only one singlet auxiliary scalar field denoted by σ in the effective action, which is the order parameter of chiral symmetry. (See Appendix A.) In other words, the stationary condition of energy with respect to the scalar is at the same time the stationarity with respect to the order parameter (the gap equation). This is not the case here. After treating the $U(N)$ singlet real auxiliary field F as perturbation, we have two kinds of background fields in the effective potential: these are the singlet complex scalar φ and the singlet auxiliary field D. The stationarity of energy with respect to the scalar and

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that with respect to the order parameter are one and the other, and both must be imposed simultaneously. In this paper, we will mainly deal with such a multivariable variational problem in depth and present the solution which is the local minimum of the scalar potential. We will also include a few other materials which have phenomenological implications. We work in the unbroken phase of $U(N)$ and invoke $U(N)$ invariance of the expectation values to suppress indices often.

In the next section, we start out from exhibiting the component action from that of the superspace, state the set of assumptions we have made in Refs. [4,5] and in this paper, and give the Noether current associated with $\mathcal{N} = 1$ rigid supersymmetry. We review the original reasoning that has led us to the D-term triggered dynamical supersymmetry breaking. We set up the background field formalism to be used in the subsequent sections, separating the three kinds of background from the fluctuations. The action can be coupled to $\mathcal{N} = 1$ supergravity, and we derive the gravitino mass formula via the super-Higgs mechanism associated with the nonvanishing D-term. The action contains a sequence of special cases in which the gauge coupling function and the superpotential are related in a specific form, including the one where the rigid $\mathcal{N} = 2$ supersymmetry is partially broken to $\mathcal{N} = 1$ at the tree level [15,17]. In section three, we elaborate upon our treatment of the effective potential with the three kinds of background fields as well as the point of the Hartree-Fock approximation in Refs. [4,5]. Section IV is the main thrust of this paper. We present our variational analyses of the effective potential in full detail. Treating one of the order parameters F as an induced perturbation, we demonstrate that the stationary values $(D_*, \varphi_*, \bar{\varphi}_*)$ are determined by the intersection of the two real curves, namely, the simultaneous solution to the gap equation and the equation of φ stationarity (the energy condition). Numerical analysis is provided that demonstrates the existence of such a solution as well as the self-consistency of our analysis. The second variation of the scalar potential is computed, and the local stability of the vacuum is shown from the numerical data. We finish our paper with a summary and brief comments on the issue of regularization and subtraction schemes.

In two of the appendices on rudimentary materials to be referred to in the text, we take a brief look at the NJL effective action and recall a formula of the second variation of a multivariable function. Phenomenological applications of our finding and the estimate of the longevity of our metastable vacuum have been given in Refs. [4,5], which we do not repeat in this paper.

II. ACTION, ASSUMPTIONS, AND SOME PROPERTIES

The action we work with in this paper is the general $\mathcal{N} = 1$ supersymmetric action consisting of chiral superfield Φ^a in the adjoint representation and the vector superfield V^a with three input functions, the Kähler potential $K(\Phi^a, \bar{\Phi}^a)$ with its gauging, the gauge kinetic superfield $\tau_{ab}(\Phi^a)$ that follows from the second derivatives of a generic holomorphic function $\mathcal{F}(\Phi^a)$, and the superpotential $W(\Phi^a)$,

$$\mathcal{L} = \int d^4\theta K(\Phi^a, \bar{\Phi}^a) + (\text{gauging}) + \int d^2\theta\, \mathrm{Im}\frac{1}{2}\tau_{ab}(\Phi^a)\mathcal{W}^{\alpha a}\mathcal{W}^b_\alpha + \left(\int d^2\theta W(\Phi^a) + \text{c.c.}\right). \tag{2.1}$$

The gauge group is taken to be $U(N)$, and, for simplicity, we assume that the theory is in the unbroken phase of the entire gauge group, which can be accomplished by tuning the superpotential. We also assume that third derivatives of $\mathcal{F}(\Phi^a)$ at the scalar vacuum expectation values (vev's) are nonvanishing.

A. Action and component expansion

The component Lagrangian of Eq. (2.1) reads

$$\mathcal{L}_{U(N)} = \mathcal{L}_{\text{Kähler}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{sup}}, \tag{2.2}$$

where

$$\mathcal{L}_{\text{Kähler}} = g_{ab}\mathcal{D}_\mu\phi^a\mathcal{D}^\mu\bar{\phi}^b - \frac{i}{2}g_{ab}\psi^a\sigma^\mu\mathcal{D}'_\mu\bar{\psi}^b + \frac{i}{2}g_{ab}\mathcal{D}'_\mu\psi^a\sigma^\mu\bar{\psi}^b + g_{ab}F^a\bar{F}^b - \frac{1}{2}g_{ab,\bar{c}}F^a\bar{\psi}^b\bar{\psi}^c - \frac{1}{2}g_{bc,a}\bar{F}^c\psi^a\psi^b + \frac{1}{\sqrt{2}}g_{ab}(\lambda^c\psi^a k_c^{*b} + \bar{\lambda}^c\bar{\psi}^b k_c{}^a) + \frac{1}{2}D^a\mathfrak{D}_a, \tag{2.3}$$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2}\mathcal{F}_{ab}\lambda^a\sigma^\mu\mathcal{D}_\mu\bar{\lambda}^b - \frac{1}{2}\bar{\mathcal{F}}_{ab}\mathcal{D}_\mu\lambda^a\sigma^\mu\bar{\lambda}^b - \frac{1}{4}(\Im\mathcal{F})_{ab}F^a_{\mu\nu}F^{b\mu\nu} - \frac{1}{8}(\Re\mathcal{F})_{ab}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma} - \frac{\sqrt{2}i}{8}(\mathcal{F}_{abc}\psi^c\sigma^\nu\bar{\sigma}^\mu\lambda^a - \bar{\mathcal{F}}_{abc}\bar{\lambda}^a\bar{\sigma}^\mu\sigma^\nu\bar{\psi}^c)F^b_{\mu\nu} + \frac{1}{2}(\Im\mathcal{F})_{ab}D^aD^b + \frac{\sqrt{2}}{4}(\mathcal{F}_{abc}\psi^c\lambda^a + \bar{\mathcal{F}}_{abc}\bar{\psi}^c\bar{\lambda}^a)D^b + \frac{i}{4}\mathcal{F}_{abc}F^c\lambda^a\lambda^b - \frac{i}{4}\bar{\mathcal{F}}_{abc}\bar{F}^c\bar{\lambda}^a\bar{\lambda}^b - \frac{i}{8}\mathcal{F}_{abcd}\psi^c\psi^d\lambda^a\lambda^b + \frac{i}{8}\bar{\mathcal{F}}_{abcd}\bar{\psi}^c\bar{\psi}^d\bar{\lambda}^a\bar{\lambda}^b, \tag{2.4}$$

$$\mathcal{L}_{\text{sup}} = F^a \partial_a W - \frac{1}{2}\partial_a \partial_b W \psi^a \psi^b + \text{c.c.}, \tag{2.5}$$

where

$$\mathfrak{D}_a = -\frac{1}{2}(\mathcal{F}_b f^b_{ac} \bar{\phi}^c + \bar{\mathcal{F}}_b f^b_{ac} \phi^c) \tag{2.6}$$

and f^b_{ac} is the structure constant of $SU(N)$. Note that an equation of motion for F^a is $F^a = -g^{ab}\overline{\partial_b W}$ + fermions. We also assume $\langle F^a \rangle_{\text{tree}} = -\langle g^{ab}\overline{\partial_b W}\rangle_{\text{tree}} = 0$ at the tree level. At the lowest order in perturbation theory, there is no source which gives vev's to the auxiliary field D^0: $\langle D^0 \rangle_{\text{tree}} = 0$. The $U(N)$ gaugino is massless at the tree level while the fermionic partner of the scalar gluon receives the tree-level mass $m_a = m_0 = \langle g^{00}\partial_0 \partial_0 W\rangle_{\text{tree}}$.

B. Assumptions

While we have already stated, it is useful to recapitulate here a set of assumptions made in order to address better the question of dynamical supersymmetry breaking within our framework:

(1) A general $\mathcal{N} = 1$ supersymmetric action of chiral superfield Φ^a in the adjoint representation and the vector superfield V^a with the three input functions, namely, the Kähler potential $K(\Phi^a, \bar{\Phi}^a)$ with its gauging, the gauge kinetic superfield $\tau_{ab}(\Phi^a)$ that follows from the second derivatives of a generic holomorphic function $\mathcal{F}(\Phi^a)$, and the superpotential $W(\Phi^a)$.

(2) Third derivatives of $\mathcal{F}(\Phi^a)$ at the scalar vev's are nonvanishing.

(3) The superpotential at tree level preserves $\mathcal{N} = 1$ supersymmetry.

(4) The gauge group is $U(N)$ and the vacuum is taken to be in the unbroken phase of $U(N)$. It is in principle straightforward to extend this to the (partially) broken cases where $U(N)$ is broken into the product groups. The variational analyses we carry out in Sec. IV, however, become more complex, and we will not address this in this paper, See the comment at Eq. (3.22).

C. Supercurrent

We give here an off-shell form of the $\mathcal{N} = 1$ supercurrent:

$$\begin{aligned}\eta_1 \mathcal{S}^{(1)\mu} &= \sqrt{2} g_{ab} \eta_1 \sigma^\nu \bar{\sigma}^\mu \psi^a \mathcal{D}_\nu \bar{\phi}^b + \sqrt{2} i g_{ab} \eta_1 \sigma^\mu \bar{\psi}^a F^b \\ &\quad - i \mathcal{F}_{ab} \eta_1 \sigma_\nu \bar{\lambda}^a F^{\mu\nu b} + \frac{1}{2}\mathcal{F}_{ab}\epsilon^{\mu\nu\rho\delta}\eta_1 \sigma_\nu \bar{\lambda}^a F^{\rho\delta b} \\ &\quad - \frac{i}{2}\bar{\mathcal{F}}_{ab}\eta_1 \sigma^\mu \bar{\lambda}^a D^b + \frac{\sqrt{2}}{4}(\mathcal{F}_{abc}\psi^c \sigma^\nu \bar{\sigma}^\mu \lambda^b \\ &\quad - \bar{\mathcal{F}}_{abc}\bar{\lambda}^c \bar{\sigma}^\mu \sigma^\nu \bar{\psi}^b)\eta_1 \sigma_\nu \bar{\lambda}^a. \end{aligned} \tag{2.7}$$

Equations of motion for auxiliary fields are

$$D^a = -\frac{1}{2} g^{ab}\mathfrak{D}_b - \frac{1}{2\sqrt{2}} g^{ab}(\mathcal{F}_{bcd}\psi^d \lambda^c + \bar{\mathcal{F}}_{bcd}\bar{\psi}^d \bar{\lambda}^c),$$

$$F^a = -g^{ab}\overline{\partial_b W} - \frac{i}{4} g^{ab}(\mathcal{F}_{bcd}\psi^c \psi^d - \bar{\mathcal{F}}_{bcd}\bar{\lambda}^c \bar{\lambda}^d). \tag{2.8}$$

Once the $U(N)$ invariant components of the auxiliary fields, D^0 and F^0, receive nonvanishing vev's together with $U(N)$ invariant scalar vev's, the second and the fifth terms of the rhs of Eq. (2.7) at these vev's clearly develop a one-body fermionic operator nonvanishing at zero momentum [18–20]: this particular combination of $\bar{\psi}^0$ and $\bar{\lambda}^0$ creates the one-particle state which is identified with the Nambu-Goldstone fermion [19].

D. Original reasoning of D-term triggered dynamical supersymmetry breaking (DDSB)

In Ref. [4], it was shown that the vacuum state of the theory, albeit being metastable, develops a nonvanishing vev of an auxiliary field D^0 in the Hartree-Fock approximation. The theory, therefore, realizes the D-term dynamical supersymmetry breaking. The relatively simple estimate has shown that the vacuum can be made long lived. Let us recall a few more key aspects.

The part of the Lagrangian which produces the fermion mass matrix of size $2N$ is

$$-\frac{1}{2}(\lambda^a, \psi^a)\begin{pmatrix} 0 & -\frac{\sqrt{2}}{4}\mathcal{F}_{abc}D^b \\ -\frac{\sqrt{2}}{4}\mathcal{F}_{abc}D^b & \partial_a \partial_c W \end{pmatrix}\begin{pmatrix}\lambda^c \\ \psi^c\end{pmatrix} + (\text{c.c.}). \tag{2.9}$$

It was observed that the auxiliary D^a field, which is an order parameter of $\mathcal{N} = 1$ supersymmetry, couples to the fermionic (but not bosonic) bilinears through the third prepotential derivatives: the nonvanishing vev of D^0 immediately gives a Dirac mass to the fermions. Equation (2.8) implies

$$\langle D^0 \rangle = -\frac{1}{2\sqrt{2}}\langle g^{00}(\mathcal{F}_{0cd}\psi^d \lambda^c + \bar{\mathcal{F}}_{0cd}\bar{\psi}^d \bar{\lambda}^c)\rangle, \tag{2.10}$$

telling us that the condensation of the Dirac bilinear is responsible for $\langle D^0 \rangle \neq 0$.

We diagonalize the holomorphic part of the mass matrix:

$$M_{Fa} \equiv \begin{pmatrix} 0 & -\frac{\sqrt{2}}{4}\langle \mathcal{F}_{0aa}D^0 \rangle \\ -\frac{\sqrt{2}}{4}\langle \mathcal{F}_{0aa}D^0 \rangle & \langle \partial_a \partial_a W \rangle \end{pmatrix}. \tag{2.11}$$

Note that the nonvanishing third prepotential derivatives are $\mathcal{F}_{0aa}$ where a refers to the generators of the unbroken gauge group. By an orthogonal transformation, we obtain the two eigenvalues of Eq. (2.11) for each generator, which are mixed Majorana-Dirac type :

$$\Lambda^{(\pm)}_{a\mathbf{11}} = \frac{1}{2}\langle \partial_a \partial_a W \rangle\left(1 \pm \sqrt{1 + \frac{\langle \mathcal{F}_{0aa}D^0\rangle^2}{2\langle \partial_a \partial_a W\rangle^2}}\right). \tag{2.12}$$

Introducing

$$\lambda_{a\mathbf{11}}^{(\pm)} \equiv \frac{1}{2}\Big(1 \pm \sqrt{1+\Delta_{\mathbf{11}}^2}\Big), \qquad \Delta_{a\mathbf{11}}^2 \equiv \frac{\langle \mathcal{F}_{0aa} D^0\rangle^2}{2\langle \partial_a \partial_a W\rangle^2}, \tag{2.13}$$

we obtain

$$|\Lambda_{a\mathbf{11}}^{(\pm)}|^2 = |\langle \partial_a \partial_a W\rangle| |\lambda_{a\mathbf{11}}^{(\pm)}|^2. \tag{2.14}$$

It was also shown in Ref. [4] that the nonvanishing F^0 term is induced by the consistency of our procedure of computation. (See also Refs. [21,22].) This is because the stationary value of the scalar fields gets shifted upon the variation (the vacuum condition). The final mass formula for the $SU(N)$ fermions is to be read off from

$$\begin{aligned}\mathcal{L}_{\rm mass}^{(\rm holo)} &= -\frac{1}{2}\langle g_{0a,a}\rangle\langle \bar F^0\rangle \psi^a\psi^a + \frac{i}{4}\langle \mathcal{F}_{0aa}\rangle\langle F^0\rangle \lambda^a\lambda^a \\ &\quad -\frac{1}{2}\langle \partial_a\partial_a W\rangle \psi^a\psi^a + \frac{\sqrt{2}}{4}\langle \mathcal{F}_{0aa}\rangle \psi^a\lambda^a\langle D^0\rangle \\ &\equiv -\frac{1}{2}\sum_{a=1}^{N^2-1}\Psi(x)^{at}M_{a,a}\Psi^a(x), \\ \Psi^a(x) &= \begin{pmatrix}\lambda^a(x)\\ \psi^a(x)\end{pmatrix}.\end{aligned} \tag{2.15}$$

We will write down the explicit form in the next subsection. See Eqs. (2.17), (2.18), (2.19), and (2.20). A main remaining point is how to establish the procedure in which the stationary values of the scalar fields, D^0 and F^0 perturbatively induced are determined, which we will resolve in this paper.

E. Quadratic part of the quantum action

In this subsection, we write down parts of the action with the background fields for the computation of the one-loop determinant in the next section. The linear terms that arise upon separation into quantum fields and background fields are dropped as they always cancel with source terms in $\Gamma_{\rm 1PI}$.

1. Fermionic part

Let us extract the fermion bilinears from Eqs. (2.3), (2.4), and (2.5) which are needed for our analysis in what follows. Rescaling the fermion fields so that their kinetic terms become canonical, we obtain

$$\begin{aligned}\mathcal{L}_F &= -\frac{i}{2}\psi^a\sigma^\mu\partial_\mu\bar\psi^a + \frac{i}{2}(\partial_\mu\psi^a)\sigma^\mu\bar\psi^a - \frac{i}{2}\lambda^a\sigma^\mu\partial_\mu\bar\lambda^a + \frac{i}{2}(\partial_\mu\lambda^a)\sigma^\mu\bar\lambda^a - \frac{1}{2}(g^{bb}g_{0b,\bar b}F^0)\bar\psi^b\bar\psi^b \\ &\quad -\frac{1}{2}(g^{bb}g_{0b,b}\bar F^0)\psi^b\psi^b + \frac{\sqrt{2}}{4}\Big(\mathcal{F}_{0aa}\sqrt{g^{aa}\,{\rm Im}\mathcal{F}^{aa}}D^0\Big)\psi^a\lambda^a + \frac{\sqrt{2}}{4}\Big(\bar{\mathcal{F}}_{0aa}\sqrt{g^{aa}\,{\rm Im}\mathcal{F}^{aa}}D^0\Big)\bar\psi^a\bar\lambda^a \\ &\quad + \frac{i}{4}(\mathcal{F}_{0aa}g^{aa}F^0)\lambda^a\lambda^a - \frac{i}{4}(\bar{\mathcal{F}}_{0aa}g^{aa}\bar F^0)\bar\lambda^a\bar\lambda^a - \frac{1}{2}(g^{aa}\partial_a\partial_a W)\psi^a\psi^a - \frac{1}{2}(g^{aa}\overline{\partial_a\partial_a W})\bar\psi^a\bar\psi^a.\end{aligned} \tag{2.16}$$

Here the fermion fields ψ^a, $\bar\psi^a$, λ^a, $\bar\lambda^a$ are to be integrated to make a part of the effective potential, while the gauge kinetic function $\mathcal{F}_{aa}$, the Kähler metric g_{aa}, and their derivatives are functions of the $U(N)$ singlet c-number background scalar field φ^0. The order parameters of supersymmetry F^0, $\bar F^0$, and D^0 are taken as background fields as well.

From the Lagrangian $\mathcal{L}_F$, the holomorphic part of the mass matrix is read off as

$$\mathcal{M}_a = \begin{pmatrix} -\frac{i}{2}g^{aa}\mathcal{F}_{0aa}F^0, & -\frac{\sqrt{2}}{4}\sqrt{g^{aa}({\rm Im}\mathcal{F})^{aa}}\mathcal{F}_{0aa}D^0 \\ -\frac{\sqrt{2}}{4}\sqrt{g^{aa}({\rm Im}\mathcal{F})^{aa}}\mathcal{F}_{0aa}D^0, & g^{aa}\partial_a\partial_a W + g^{aa}g_{0a,a}\bar F^0\end{pmatrix} = \begin{pmatrix} m^a_{\lambda\lambda} & m^a_{\lambda\psi} \\ m^a_{\psi\lambda} & m^a_{\psi\psi}\end{pmatrix}. \tag{2.17}$$

We parametrize this matrix such that, in the case of $F^0 = \bar F^0 = 0$, its form reduces to that of Refs. [4,5]. The quantities having multiple indices such as $\mathcal{F}_{0aa}$ receive $U(N)$ invariant expectation values: $\langle\mathcal{F}_{0aa}\rangle = \langle\mathcal{F}_{000}\rangle$, etc. See, for instance, Ref. [16]. We suppress the indices as we work with the unbroken $U(N)$ phase in this paper,

$$\Delta \equiv -\frac{2m_{\lambda\psi}}{m_{\psi\psi}}, \qquad f \equiv \frac{2im_{\lambda\lambda}}{{\rm tr}\mathcal{M}}. \tag{2.18}$$

The two eigenvalues of the holomorphic mass matrix are written as

$$\Lambda^{(\pm)} \equiv ({\rm tr}\mathcal{M})\lambda^{(\pm)}, \tag{2.19}$$

where

$$\lambda^{(\pm)} = \frac{1}{2}\left(1 \pm \sqrt{(1+if)^2 + \Big(1+\frac{i}{2}f\Big)^2\Delta^2}\right). \tag{2.20}$$

These provide the masses for the two species of $SU(N)$ fermions once the stationary values are determined.

2. Bosonic part

Likewise, we extract the bosonic quantum bilinears from Eqs. (2.3), (2.4), and (2.5). Let

$$\phi^a = \delta^a_0\varphi^0 + \sqrt{g^{aa}(\varphi)}\tilde\varphi^a, \tag{2.21}$$

$$A^a_\mu = \sqrt{({\rm Im}\mathcal{F})^{aa}}\tilde A^a_\mu, \tag{2.22}$$

$$F^a = \sqrt{g^{aa}(\varphi)}\tilde{F}^a, \tag{2.23}$$

$$D^a = \sqrt{(\mathrm{Im}\mathcal{F})^{aa}}\tilde{D}^a, \tag{2.24}$$

where φ^0 are the background c-number field while $\tilde{\varphi}^a$, $\tilde{A}^a_\mu$, $\tilde{F}^a$, and $\tilde{D}^a$ are the quantum scalar, vector, and auxiliary fields, respectively.

We obtain

$$\mathcal{L}_B^{(1)} = \partial_\mu \tilde{\varphi}^a \partial^\mu \tilde{\varphi}^{*a} - \frac{1}{4}\tilde{F}^a_{\mu\nu}\tilde{F}^{a\mu\nu} + \tilde{F}^a \bar{\tilde{F}}^a + \frac{1}{2}\tilde{D}^a\tilde{D}^a + \tilde{F}^a((\sqrt{g^{aa}}\partial_a W) + (g^{aa}\partial_a\partial_a W)\tilde{\varphi}^a) + \bar{\tilde{F}}^a((\overline{\sqrt{g^{aa}}\partial_a W}) + (g^{aa}\overline{\partial_a\partial_a W})\tilde{\varphi}^{a*}). \tag{2.25}$$

We have also ignored $-\frac{1}{8}(\mathrm{Re}\mathcal{F})_{ab}\epsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^b_{\rho\sigma}$ as we eventually set φ^a to be constant in our analysis, and this term becomes a total derivative.

F. Coupling to $\mathcal{N} = 1$ supergravity and the super-Higgs mechanism

If Eq. (2.1) couples to $\mathcal{N} = 1$ supergravity, the Lagrangian is augmented to become the following one [23]:

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\Big[\frac{3}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8\mathcal{R})\exp\Big\{-\frac{1}{3}[K(\Phi,\Phi^\dagger) + \Gamma(\Phi,\Phi^\dagger,V)]\Big\} + \frac{1}{16g^2}\tau_{ab}(\Phi)W^{\alpha a}W^b_\alpha + W(\Phi)\Big] + \mathrm{H.c.} \tag{2.26}$$

The fermionic part of the Lagrangian relevant to the super-Higgs mechanism is given by

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{fermionic}} = &-i\bar{\psi}_a\bar{\sigma}^\mu\tilde{\mathcal{D}}_\mu\psi^a + \epsilon^{\mu\nu\alpha\beta}\bar{\psi}_\mu\bar{\sigma}_\nu\tilde{\mathcal{D}}_\alpha\psi_\beta - \frac{i}{2}[\lambda_a\sigma^\mu\tilde{\mathcal{D}}_\mu\bar{\lambda}^a + \bar{\lambda}_a\bar{\sigma}^\mu\tilde{\mathcal{D}}_\mu\lambda^a] - \frac{i}{2\sqrt{2}}g\partial_c\tau_{ab}D^a\psi^c\lambda^b \\ &+ \frac{i}{2\sqrt{2}}g\partial_{c^*}\tau^*_{ab}D^a\bar{\psi}^c\bar{\lambda}^a - \frac{1}{2}gD_a\psi_\mu\sigma^\mu\bar{\lambda}^a + \frac{1}{2}gD_a\bar{\psi}_\mu\bar{\sigma}^\mu\lambda^a - e^{K/2}\Big[W^*\psi_\mu\sigma^{\mu\nu}\psi_\nu + W\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu \\ &+ \frac{i}{\sqrt{2}}D_aW\psi^a\sigma^\mu\bar{\psi}_\mu + \frac{i}{\sqrt{2}}D_{a^*}W^*\bar{\psi}^a\bar{\sigma}^\mu\psi_\mu + \frac{1}{2}\mathcal{D}_aD_bW\psi^a\psi^b + \frac{1}{2}\mathcal{D}_{a^*}D_{b^*}W^*\bar{\psi}^a\bar{\psi}^b \\ &- \frac{1}{4}g^{ab^*}D_{b^*}W^*\partial_a\tau_{cd}\lambda^c\lambda^d - \frac{1}{4}g^{ab^*}D_aW\partial_{b^*}\tau^*_{cd}\bar{\lambda}^c\bar{\lambda}^d\Big], \end{aligned} \tag{2.27}$$

where e is the determinant of the vierbein and the covariant derivatives of several kinds are defined as follows:

$$\tilde{\mathcal{D}}_\mu\psi_\nu = \partial_\mu\psi_\nu + \omega_\mu\psi_\nu + \frac{1}{4}(\partial_aK\tilde{\mathcal{D}}_\mu\phi^a - \partial_{a^*}K\tilde{\mathcal{D}}_\mu\phi^{a*})\psi_\nu + \frac{i}{2}gA^a_\mu\,\mathrm{Im}F_a\psi_\nu, \tag{2.28}$$

$$\tilde{\mathcal{D}}_\mu\lambda^a = \partial_\mu\lambda^a + \omega_\mu\lambda^a - gf^{abc}A^b_\mu\lambda^c + \frac{1}{4}(\partial_bK\tilde{\mathcal{D}}_\mu\phi^b - \partial_{b^*}K\tilde{\mathcal{D}}_\mu\phi^{b*})\lambda^a + \frac{i}{2}gA^b_\mu\,\mathrm{Im}F_b\lambda^a, \tag{2.29}$$

$$D_aW = \partial_aW + (\partial_aK)W, \tag{2.30}$$

$$\mathcal{D}_aD_bW = \partial_a\partial_bW + (\partial_a\partial_bK)W + 2(\partial_aK)D_bW - (\partial_aK)(\partial_bK)W. \tag{2.31}$$

Now, we focus on the gravitino mass terms to discuss the super-Higgs mechanism associated with Eq. (2.26),

$$e^{-1}\mathcal{L}_{\text{gravitino mass}} = -e^{K/2}W^*\psi_\mu\sigma^{\mu\nu}\psi_\nu + \frac{i}{\sqrt{2}}\psi_\mu\sigma^\mu\Big[i\frac{g}{\sqrt{2}}D_a\bar{\lambda}^a + e^{K/2}D_aW^*\bar{\psi}^a\Big] + \mathrm{H.c.} \tag{2.32}$$

The field redefinition of the gravitino

$$\psi'_\mu = \psi_\mu + i\frac{\sqrt{2}}{6W^*e^{K/2}}\sigma^\mu\bar{\psi}_{\mathrm{NG}} + \frac{\sqrt{2}}{3W^{*2}e^K}\partial_\mu\bar{\psi}_{\mathrm{NG}} \tag{2.33}$$

eliminates the mixing terms of the gravitino with the gaugino λ and the adjoint fermion ψ:

$$e^{-1}\mathcal{L}_{\text{gravitino mass}} = -e^{K/2}W^*\psi'_\mu\sigma^{\mu\nu}\psi'_\nu + \frac{1}{2W^*e^{K/2}}\bar{\psi}^2_{\mathrm{NG}} + \mathrm{H.c.}, \tag{2.34}$$

where the NG fermion ψ_{NG} absorbed in the massive gravitino is read

$$\bar{\psi}_{\mathrm{NG}} \equiv i\frac{g}{\sqrt{2}}D_a\bar{\lambda}^a + e^{K/2}D_{a^*}W^*\bar{\psi}^a. \tag{2.35}$$

Equation (2.34) tells us that the gravitino mass is given by

$$m_{3/2} = e^{\langle K\rangle/2}\frac{\langle W\rangle}{M_P^2}. \tag{2.36}$$

Requiring the cosmological constant to be almost vanishing

$$0 \simeq \langle V\rangle = \frac{g^2}{2}(D^a)^2 + e^K\left[|D_a W|^2 - \frac{3}{M_P^2}|W|^2\right], \tag{2.37}$$

the gravitino mass can be expressed in terms of the vev's of the auxiliary fields

$$m_{3/2} \simeq e^{\langle K\rangle/2}\frac{\sqrt{|\langle D_a W\rangle|^2 + \frac{g^2}{2}\langle D^a\rangle^2}}{\sqrt{3}M_P}. \tag{2.38}$$

G. Special cases

As is mentioned in the introduction, the theory permits a sequence of interesting limiting cases. If we demand the Kähler function K to be special Kähler, K are expressible in terms of $\mathcal{F}$ as

$$K = \mathrm{Im}\,\mathrm{Tr}\bar{\Phi}\frac{\partial\mathcal{F}(\Phi)}{\partial\Phi}, \tag{2.39}$$

$g_{ab} = \mathrm{Im}\mathcal{F}_{ab}$, etc.. If we further demand such that the action possesses the rigid $\mathcal{N} = 2$ supersymmetry with one input function by choosing the superpotential to be a particular form, the tree vacua are shown to break $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$ spontaneously [15–17,24].[1] We list the transformation laws for the doublet of fermions in this special case,

$$\delta\begin{pmatrix}\lambda^a\\ \psi^a\end{pmatrix} = F^a_{\mu\nu}\sigma^{\mu\nu}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} - i\sqrt{2}\sigma^\mu\begin{pmatrix}\bar{\eta}_2\\ -\bar{\eta}_1\end{pmatrix}\mathcal{D}_\mu\phi^a + \begin{pmatrix}iD^a & -\sqrt{2}\tilde{F}^a\\ \sqrt{2}F^a & i\tilde{D}^a\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}, \tag{2.40}$$

where

$$\tilde{D}^a = -\frac{1}{2}g^{ab}\mathfrak{D}_b + \frac{1}{2\sqrt{2}}g^{ab}(\mathcal{F}_{bcd}\psi^d\lambda^c + \bar{\mathcal{F}}_{bcd}\bar{\psi}^d\bar{\lambda}^c),$$
$$\tilde{F}^a = -\sqrt{2N}g^{ab}(e\delta^0_b + m\bar{\mathcal{F}}_{0b}) - \frac{i}{4}g^{ab}(\mathcal{F}_{bcd}\lambda^c\lambda^d - \bar{\mathcal{F}}_{bcd}\bar{\psi}^c\bar{\psi}^d). \tag{2.41}$$

H. Connection with the previous work

We here stop shortly to address the connection of Ref. [4] with the previous work. Models of dynamical supersymmetry breaking with nonvanishing F and D terms have been previously proposed: they are, for instance, the 3-2 model [12] and the 4-1 model in Ref. [21].[2] In these models, supersymmetry is unbroken at the tree level and is broken by the nonvanishing vev of the F-term through instanton generated superpotentials. A nonvanishing vev of the D term is also induced but is much smaller than that of the F term.

In our mechanism, supersymmetry is unbroken at the tree level and is broken in a self-consistent Hartree-Fock approximation of the NJL type that produces a nonvanishing vev for the D term. A nonvanishing vev for the F term is induced in our Hartree-Fock vacuum that shifts the tree vacuum, and we explore the region of the parameter space in which the F-term vev is treated perturbatively.

We should mention that the way in which the two kinds of gauginos (or the gaugino and the adjoint matter fermion) receive masses is an extension of that proposed in Ref. [30]: the pure Dirac-type gaugino mass is generated in Ref. [30],[3] while the mixed Majorana-Dirac–type gaugino masse is generated in our case, the Majorana part being given by the second derivative of the superpotential. In Ref. [30], the dynamical origin of the nonvanishing D-term vev was not addressed.

As for the application to dynamical chiral symmetry breaking, a supersymmetric NJL-type model has been considered [45–48]. Chiral symmetry is not spontaneously broken in a supersymmetric case. Even in softly broken supersymmetric theories, the chiral symmetry broken phases are degenerate with the chirally symmetric ones. Thus, in supersymmetric theories, the phase with broken chiral symmetry is no longer the energetically preferred ground state.

III. EFFECTIVE POTENTIAL IN THE HARTREE-FOCK APPROXIMATION

The goal of this section is to determine the effective potential to the leading order in the Hartree-Fock approximation. We will regulate the one-loop integral by the dimensional reduction [49]. We prepare a supersymmetric counterterm, setting the normalization condition. We make brief comments on regularization and subtraction schemes in the end of Sec. IV. We also change the notation for expectation values in general from $\langle\ldots\rangle$ to $\ldots_*$ as our main thrust of this paper is the determination of the stationary values from the variational analysis.

A. Point of the approximation

In the Hartree-Fock approximation, one begins with considering the situation where one-loop corrections in

[1]This superpotential consists of the terms referred to as the electric and magnetic Fayet-Iliopoulos terms. This $\mathcal{N} = 2$ Fayet-Iliopoulos term is very special in the sense that, by the $SU(2)_R$ rigid rotation, it can be represented as a part of the superpotential. In this way, it avoids the difficulty (see, for instance, Ref. [25] for a recent discussion) of coupling the system to $\mathcal{N} = 2$ supergravity [26,27].

[2]For the application of these models to the mediation mechanism; see, for example, Refs. [22,28,29].

[3]Attention has been paid to Dirac gaugino in many papers [28,29,31–44].

the original expansion in $\hbar$ become large and are comparable to the tree contribution. The optimal configuration of the effective potential to this order is found by matching the tree against one loop, varying with respect to the auxiliary fields. In this section, we start the analysis of this kind for our effective potential. There are three constant background fields as arguments of the effective potential: $\varphi \equiv \varphi^0$ (complex), $U(N)$ invariant background scalar, $D \equiv D^0$ (real) and $F \equiv F^0$ (complex). The latter two are the order parameters of $\mathcal{N} = 1$ supersymmetry.

We vary our effective potential with respect to all these constant fields and examine the stationary conditions. We also examine a second derivative at the stationary point along the constraints of the auxiliary fields to understand better the Hartree-Fock corrected mass of the scalar gluons. Let us denote our effective potential by V. It consists of three parts:

$$V = V^{\rm tree} + V_{\rm c.t.} + V_{\rm 1\text{-}loop}. \tag{3.1}$$

The first term is the tree contributions, the second one is the counterterm, and the last one is the one-loop contributions. After the elimination of the auxiliary fields, the effective potential is referred to as the scalar potential so as to be distinguished from the original V.

B. Tree part

To begin with, let us write down the tree part and find a parametrization by two complex and one real parameters. We also introduce simplifying notation $g_{00}(\varphi, \bar{\varphi}) \equiv g(\varphi, \bar{\varphi})$, $({\rm Im}\mathcal{F}(\varphi))_{00} \equiv {\rm Im}\mathcal{F}''(\varphi)$, $\partial_0 W(\varphi) = W'(\varphi)$, $g_{00,0} \equiv \partial g$, etc.,

$$V^{\rm tree}(D, F, \bar{F}, \varphi, \bar{\varphi}) = -gF\bar{F} - \frac{1}{2}({\rm Im}\mathcal{F}'')D^2 - FW' - \bar{F}\bar{W}'. \tag{3.2}$$

As a warm up, let us determine the vacuum configuration by a set of stationary conditions at the tree level:

$$\frac{\partial V^{\rm tree}}{\partial D} = 0, \tag{3.3}$$

$$\frac{\partial V^{\rm tree}}{\partial F} = 0, \quad \text{as well as its complex conjugate}, \tag{3.4}$$

$$\frac{\partial V^{\rm tree}}{\partial \varphi} = 0, \quad \text{as well as its complex conjugate}. \tag{3.5}$$

Equation (3.3) determines the stationary value of D:

$$D = 0 \equiv D_*, \tag{3.6}$$

while from Eq. (3.4), we obtain

$$F = -g^{-1}(\varphi, \bar{\varphi})\bar{W}'(\bar{\varphi}) \equiv F_*(\varphi, \bar{\varphi}). \tag{3.7}$$

Equation (3.5) together with these two gives

$$W'(\varphi_*) = 0 \quad \text{and therefore } F_*(\varphi, \bar{\varphi}) = 0 \tag{3.8}$$

as well as

$$\begin{aligned} &V^{\rm tree}_{\rm scalar}(\varphi, \bar{\varphi}) \\ &\quad \equiv V^{\rm tree}(\varphi, \bar{\varphi}, D_* = 0, F = F_*(\varphi, \bar{\varphi}), \bar{F} = \overline{F_*(\varphi, \bar{\varphi})}) \\ &\quad = g^{-1}(\varphi, \bar{\varphi})|W'(\varphi)|^2. \end{aligned} \tag{3.9}$$

The negative coefficients of the rhs of Eq. (3.2) imply that both D and F profiles of the potential have a maximum for a given φ. These signs are, of course, the right signs for the stability of the scalar potential as is clear by completing the square. This is a trivial comment to make here but will become less trivial later. The mass of the scalar gluons at tree level $|m_{s*}|^2$ is read off from the second derivative at the stationary point:

$$\frac{\partial^2 V^{\rm tree}(\varphi, \bar{\varphi})}{\partial\varphi\partial\bar{\varphi}}\bigg|_{\varphi_*, \bar{\varphi}_*} = g^{-1}(\varphi_*, \bar{\varphi}_*)|W''(\varphi_*)|^2, \tag{3.10}$$

$$m_s(\varphi, \bar{\varphi}) \equiv g^{-1}(\varphi, \bar{\varphi})W''(\varphi), \tag{3.11}$$

$$m_{s*} = m_s(\varphi_*, \bar{\varphi}_*). \tag{3.12}$$

As we have already introduced in Eq. (2.18), Δ and r are defined by

$$\Delta \equiv -2\frac{m_{\lambda\psi}}{m_{\psi\psi}} = \frac{\sqrt{2}}{2}\frac{\sqrt{g^{-1}({\rm Im}\mathcal{F}'')^{-1}}\mathcal{F}'''}{g^{-1}W'' + g^{-1}\partial g\bar{F}}D \equiv r(\varphi, \bar{\varphi}, F, \bar{F})D. \tag{3.13}$$

Recall that we have suppressed the indices, invoking the $U(N)$ invariance of the expectation values. Also

$$f_3 \equiv \frac{g^{-1}\mathcal{F}'''F}{g^{-1}W'' + g^{-1}\partial g\bar{F}}, \tag{3.14}$$

where f_3 differs from f in Eq. (2.18) by

$$\begin{aligned} &(g^{-1}W'' + g^{-1}\partial g\bar{F})f_3 \\ &\quad = \left(g^{-1}W'' + g^{-1}\partial g\bar{F} - \frac{i}{2}g^{-1}\mathcal{F}'''F\right)f. \end{aligned} \tag{3.15}$$

We obtain

$$F = \frac{m_s}{g^{-1}\mathcal{F}'''}\varepsilon, \quad \bar{F} = \frac{\bar{m}_s}{g^{-1}\bar{\mathcal{F}}'''}\varepsilon, \quad \varepsilon = \frac{f_3 + \frac{\bar{m}_s}{m_s}\frac{g^{-1}\partial g}{g^{-1}\bar{\mathcal{F}}'''}|f_3|^2}{1 - |\frac{g^{-1}\partial g f_3}{g^{-1}\bar{\mathcal{F}}'''}|^2}. \tag{3.16}$$

While we will not make exploit in this paper, $V^{\rm tree}$ can be written as a function of φ complex, $|\Delta|$ real, f_3 complex:

$$\begin{aligned} V^{\rm tree} = &-\left|\bar{m}_s + \frac{g^{-1}\bar{\partial}g}{g^{-1}\mathcal{F}'''}m_s\varepsilon\right|^2|\mathcal{F}'''|^{-2}g^2(g^{-1}({\rm Im}\mathcal{F}'')|\Delta|^2 \\ &+ g|f_3|^2) - \frac{m_s}{g^{-1}\mathcal{F}'''}\varepsilon W' - \frac{\bar{m}_s}{g^{-1}\bar{\mathcal{F}}'''}\bar{\varepsilon}\bar{W}', \end{aligned} \tag{3.17}$$

where m_s, ε, g, $\mathcal{F}$ (and their derivatives) are the functions listed above and undergo the variations to be carried out in the subsequent subsections. We also see that the mass scales of the problem are set by m_{s*}, the scalar gluon mass and $g^{-1}\bar{\mathcal{F}}_*'''$, the third prepotential derivative (and $g^{-1}\partial g$), once the stationary value of the scalar is determined.

C. Treatment of UV infinity

In the NJL theory [6,7], there is only one coupling constant carrying dimension -1, and the dimensionless quantity is naturally formed by combining it with the relativistic cutoff, which is interpreted as the onset of UV physics. In the theory under our concern, UV physics is specified by the three input functions, K, $\mathcal{F}$, W, and the UV scales and infinities reside in some of the coefficients. Our supersymmetric counterterm [4,5] is

$$V_{\rm c.t.} = -\frac{1}{2}{\rm Im}\int d^2\theta \Lambda \mathcal{W}^{0\alpha}\mathcal{W}_{0\alpha} = -\frac{1}{2}({\rm Im}\Lambda)D^2. \tag{3.18}$$

It is a counterterm associated with ${\rm Im}\mathcal{F}''$. We set up a renormalization condition,

$$\frac{1}{N^2}\frac{\partial^2 V}{(\partial D)^2}\bigg|_{D=0,\varphi=\varphi_*,\bar{\varphi}=\bar{\varphi}_*} = 2c, \tag{3.19}$$

and relate (or transmute) the original infinity of the dimensional reduction scheme with that of ${\rm Im}\mathcal{F}''$. We have indicated that this condition is set up at $D = 0$ and the stationary point of the scalar which we will determine. We stress again that the entire scheme is supersymmetric.

D. One-loop part

The entire contribution of all particles in the theory to $i\cdot$[the one-particle irreducible part (1PI) to one-loop] $\equiv i\Gamma_{\text{1-loop}}$ is easy to compute, knowing Eqs. (2.19) and (2.20), and (2). It is given by

$$i\Gamma_{\text{1-loop}} = \left(\int d^4x\right)\sum_a \int \frac{d^4k}{(2\pi)^4} \times \ln\left(\frac{(|\Lambda_a^{(+)}|^2 - k^2 - i\varepsilon)(|\Lambda_a^{(-)}|^2 - k^2 - i\varepsilon)}{(|m_{s,a}|^2 - k^2 - i\varepsilon)(-k^2 - i\varepsilon)}\right). \tag{3.20}$$

In the unbroken $U(N)$ phase, it is legitimate to replace $\sum_a$ by N^2 and drop the index a as we have said before.[4] We obtain

$$V_{\text{1-loop}} \equiv (-i)\frac{1}{(\int d^4x)}\Gamma_{\text{1-loop}} \tag{3.21}$$

$$= -N^2|{\rm tr}\mathcal{M}|^4\int\frac{d^4l^\mu}{(2\pi)^4 i} \times \ln\left(\frac{(|\lambda^{(+)}|^2 - l^2 - i\varepsilon)(|\lambda^{(-)}|^2 - l^2 - i\varepsilon)}{(|\frac{m_s}{{\rm tr}\mathcal{M}}|^2 - l^2 - i\varepsilon)(-l^2 - i\varepsilon)}\right) \equiv N^2|{\rm tr}\mathcal{M}|^4 J. \tag{3.22}$$

Note that $|m_s|^2$, whose stationary value gives the tree mass squared of the scalar gluon, differ from $|{\rm tr}\mathcal{M}|^2$:

$$|{\rm tr}\mathcal{M}|^2 = \left|m_s - \frac{i}{2}(g^{-1}\mathcal{F}''')F + (g^{-1}\partial g)\bar{F}\right|^2. \tag{3.23}$$

To evaluate the integral in d dimensions, we just quote

$$I(x^2) \equiv -\int\frac{d^4l^\mu}{(2\pi)^4 i}\log(x^2 - l^2 - i\varepsilon), \tag{3.24}$$

$$I(x^2) - I(0) = \frac{1}{32\pi^2}[A(\varepsilon,\gamma)(x^2)^2 - (x^2)^2\log(x^2)], \tag{3.25}$$

where

$$A(\varepsilon,\gamma) = \frac{1}{2} - \gamma + \frac{1}{\varepsilon}, \qquad \varepsilon = 2 - \frac{d}{2}. \tag{3.26}$$

We obtain

$$V_{\text{1-loop}} = \frac{N^2|{\rm tr}\mathcal{M}|^4}{32\pi^2}\bigg[A(\varepsilon,\gamma)\bigg(|\lambda^{(+)}|^4 + |\lambda^{(-)}|^4 - \left|\frac{m_s}{{\rm tr}\mathcal{M}}\right|^4\bigg) - |\lambda^{(+)}|^4\log|\lambda^{(+)}|^2 - |\lambda^{(-)}|^4\log|\lambda^{(-)}|^2 + \left|\frac{m_s}{{\rm tr}\mathcal{M}}\right|^4\log\left|\frac{m_s}{{\rm tr}\mathcal{M}}\right|^4\bigg]. \tag{3.27}$$

This again depends upon Δ, f, and φ.

IV. STATIONARY CONDITIONS AND GAP EQUATION

A. Variational analyses

Now we turn to our variational problem. It is stated as in the tree case as

$$\frac{\partial V}{\partial D} = 0, \tag{4.1}$$

$$\frac{\partial V}{\partial F} = 0 \text{ and its complex conjugate}, \tag{4.2}$$

$$\frac{\partial V}{\partial \varphi} = 0 \text{ and its complex conjugate}. \tag{4.3}$$

We will regard the solution to be obtained by considering Eqs. (4.1) and (4.3) first and solving D and φ for F and $\bar{F}$:

$$D = D_*(F,\bar{F}), \qquad \varphi = \varphi_*(F,\bar{F}), \qquad \bar{\varphi} = \bar{\varphi}_*(F,\bar{F}). \tag{4.4}$$

Equation (4.2) is then

[4]In those cases where the $U(N)$ is broken to the product group $\prod_{\alpha=1}^n U(N_\alpha)$, we need not only replace $\sum_a\cdots$ by $\sum_\alpha\cdots\alpha$ but also must treat the $\mathcal{N}=1$ multiplet of the broken generators that receives the mass by the Higgs mechanism [16].

$$\left.\frac{\partial V(D=D_*(F,\bar{F}),\varphi=\varphi_*(F,\bar{F}),\bar{\varphi}=\bar{\varphi}_*(F,\bar{F}),F,\bar{F})}{\partial F}\right|_{D,\varphi,\bar{\varphi},\bar{F}\ \text{fixed}}=0 \tag{4.5}$$

and its complex conjugate. These will determine $F = F_*$, $\bar{F} = \bar{F}_*$.

In this paper, we are going to work in the region where the strength $\|F_*\|$ is small and can be treated perturbatively. This means that, in the leading order, the problem posed by Eq. (4.1) and (4.3) becomes

$$\frac{\partial V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0)}{\partial D}=0, \tag{4.6}$$

$$\frac{\partial V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0)}{\partial \varphi}=\frac{\partial V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0)}{\partial \bar{\varphi}}=0, \tag{4.7}$$

and this problem does not involve the tree potential Eq. (3.2) except the last D^2 term, as F and $\bar{F}$ are set zero. Equation (4.6) is nothing but the gap equation given in Refs. [4,5], while Eq. (4.7) is the stationary conditions for the scalar. This is the variational problem which is analyzed in this paper. A set of stationary values $(D_*, \varphi_*, \bar{\varphi}_*)$ is determined as the solution.

B. Analysis in the region $F_* \approx 0$

Let us first determine $V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0)$ explicitly. We need to solve the normalization condition,

$$2cN^2=\left.\frac{\partial^2 V}{(\partial D)^2}\right|_{D=0,*}=-(\mathrm{Im}\mathcal{F}''_*)-(\mathrm{Im}\Lambda)+N^2|\mathrm{tr}\mathcal{M}|^4\left.\frac{\partial^2 J}{(\partial D)^2}\right|_{D=0}, \tag{4.8}$$

where J has been introduced in Eq. (3.22). At $F, \bar{F} \to 0$,

$$\Delta\to\Delta_0\equiv r_0(\varphi,\bar{\varphi})D,\quad \text{where } r_0=\frac{\sqrt{2}}{2}\frac{\sqrt{g^{-1}(\mathrm{Im}\mathcal{F}'')^{-1}}\mathcal{F}'''}{g^{-1}W''}, \tag{4.9}$$

$$\lambda^{(\pm)}\to\lambda_0^{(\pm)}=\frac{1}{2}\left(1\pm\sqrt{1+\Delta_0^2}\right), \tag{4.10}$$

where

$$\lambda_0^{(+)}+\lambda_0^{(-)}=1,\qquad \lambda_0^{(+)}\lambda_0^{(-)}=-\frac{1}{4}\Delta_0^2,\qquad \lambda_0^{(+)}-\lambda_0^{(-)}=\sqrt{1+\Delta_0^2}, \tag{4.11}$$

$$\frac{m_s}{\mathrm{tr}\mathcal{M}}\to 1, \tag{4.12}$$

$$J\to J_0\equiv\frac{1}{32\pi^2}\Big[A(\varepsilon,\gamma)\Big\{\frac{1}{2}\Big(1+\frac{1}{2}\Delta_0^2\Big)\Big(1+\frac{1}{2}\bar{\Delta}_0^2\Big)+\frac{1}{2}\sqrt{1+\Delta_0^2}\sqrt{1+\bar{\Delta}_0^2}-1\Big\}-|\lambda_0^{(+)}|^4\log|\lambda_0^{(+)}|^2-|\lambda_0^{(-)}|^4\log|\lambda_0^{(-)}|^2\Big], \tag{4.13}$$

essentially reducing the situation to that of Refs. [4,5].

Note, however, that r and Δ (or r_0, Δ_0) are complex in general except those special cases which include the case of the rigid $\mathcal{N}=2$ supersymmetry partially broken to $\mathcal{N}=1$ at the tree vacua. For $|\Delta_0| \ll 1$,

$$J_0\approx\frac{1}{32\pi^2}\Big[A(\varepsilon,\gamma)\frac{1}{2}(\Delta_0^2+\bar{\Delta}_0^2)-\frac{1}{4}(\Delta_0^2+\bar{\Delta}_0^2)+\mathcal{O}(|\Delta_0|^{4-\varepsilon})\Big]. \tag{4.14}$$

We solve the normalization condition for the number A to obtain

$$A=\frac{1}{2}+\frac{32\pi^2}{|m_{s*}|^4(r_{0*}^2+\bar{r}_{0*}^2)}\Big(2c+\frac{\mathrm{Im}\mathcal{F}''_*}{N^2}+\frac{\mathrm{Im}\Lambda}{N^2}\Big)\equiv\tilde{A}(c,\Lambda,\varphi_*,\bar{\varphi}_*). \tag{4.15}$$

We obtain

$$\begin{aligned}V_0&=V(D,\varphi,\bar{\varphi},F=0,\bar{F}=0)\\&=-\frac{1}{2}\mathrm{Im}\mathcal{F}''D^2-\frac{1}{2}(\mathrm{Im}\Lambda)D^2\\&\quad+\frac{N^2|m_s|^4}{32\pi^2}\Big[\tilde{A}(c,\Lambda,\varphi_*,\bar{\varphi}_*)\Big\{\frac{1}{2}\Big(1+\frac{1}{2}\Delta_0^2\Big)\Big(1+\frac{1}{2}\bar{\Delta}_0^2\Big)+\frac{1}{2}\sqrt{1+\Delta_0^2}\sqrt{1+\bar{\Delta}_0^2}-1\Big\}-|\lambda_0^{(+)}|^4\log|\lambda_0^{(+)}|^2\\&\quad-|\lambda_0^{(-)}|^4\log|\lambda_0^{(-)}|^2\Big].\end{aligned} \tag{4.16}$$

After some calculation, this is found to be expressible as

$$\begin{aligned}\frac{V_0}{N^2|m_s|^4}&=\Big(\frac{1}{64\pi^2}+\tilde{c}-\tilde{\delta}(\varphi,\bar{\varphi})\Big)\Big(\frac{\Delta_0+\bar{\Delta}_0}{2}\Big)^2+\frac{1}{32\pi^2}\tilde{A}\Big(\frac{1}{8}|\Delta_0|^4+f(\Delta_0,\bar{\Delta}_0)\Big)\\&\quad-\frac{1}{32\pi^2}\big(|\lambda_0^{(+)}|^4\log|\lambda_0^{(+)}|^2+|\lambda_0^{(-)}|^4\log|\lambda_0^{(-)}|^2\big),\end{aligned} \tag{4.17}$$

where

$$\tilde{c} = \frac{c}{|m_{s*}|^4\left(\frac{r_{0*}^2+\bar{r}_{0*}^2}{2}\right)}, \tag{4.18}$$

$$\tilde{\delta}(\varphi,\bar{\varphi}) = \frac{1}{2}\left(\frac{\frac{\mathrm{Im}\mathcal{F}_*''}{N^2}+\frac{\mathrm{Im}\Lambda}{N^2}}{\frac{(r_{0*}^2+\bar{r}_{0*}^2)}{2}|m_{s*}|^4}\right)\left[\frac{\frac{\mathrm{Im}\mathcal{F}''/N^2+\mathrm{Im}\Lambda/N^2}{\mathrm{Im}\mathcal{F}_*''/N^2+\mathrm{Im}\Lambda/N^2}}{\frac{|m_s|^4}{|m_{s*}|^4}\frac{(\frac{r_0+\bar{r}_0}{2})^2}{\left(\frac{r_{0*}^2+\bar{r}_{0*}^2}{2}\right)}}-1\right], \tag{4.19}$$

$$f(\Delta_0,\bar{\Delta}_0) = \frac{1}{2}\left(\sqrt{1+\Delta_0^2}\sqrt{1+\bar{\Delta}_0^2}-|\Delta_0|^2-1\right). \tag{4.20}$$

Note that

$$\tilde{\delta}_* \equiv \tilde{\delta}(\varphi_*,\bar{\varphi}_*) \neq 0, \tag{4.21}$$

and

$$|f(\Delta_0,\bar{\Delta}_0)| \leq \text{const} \quad \text{for } |\Delta_0| \gg 1. \tag{4.22}$$

If r_0 (and Δ_0) is real, this is rewritten as

$$\frac{V_0}{N^2|m_s|^4} = \left(\left(c'+\frac{1}{64\pi^2}\right)-\delta\right)\Delta_0^2 + \frac{1}{32\pi^2}\left[\frac{\tilde{A}}{8}\Delta_0^4 - \lambda_0^{(+)4}\log\lambda_0^{(+)2} - \lambda_0^{(-)4}\log\lambda_0^{(-)2}\right], \tag{4.23}$$

where $c' \equiv \frac{c}{r_{0*}^2|m_{s*}|^4}$ is the rescaled number, and

$$\delta(\varphi,\bar{\varphi}) \equiv \frac{1}{2}\left(\frac{\frac{\mathrm{Im}\mathcal{F}_*''}{N^2}+\frac{\mathrm{Im}\Lambda}{N^2}}{r_{0*}^2|m_{s*}|^4}\right)\left[\frac{\frac{\mathrm{Im}\mathcal{F}''/N^2+\mathrm{Im}\Lambda/N^2}{\mathrm{Im}\mathcal{F}_*''/N^2+\mathrm{Im}\Lambda/N^2}}{\frac{r_0^2|m_s|^4}{r_{0*}^2|m_{s*}|^4}}-1\right] \tag{4.24}$$

and $m_s(\varphi,\bar{\varphi}) = g^{-1}W''$ are the functions of φ, $\bar{\varphi}$. Clearly, there are two scales in our current problem $|r_{0*}|^{-1/2}$ and $|m_{s*}|$, which are controlled by the second superpotential derivative and the third prepotential derivative at the stationary value φ_*.

Let us turn to the gap equation

$$\left.\frac{\partial V_0}{\partial D}\right|_{\varphi,\bar{\varphi}} = 0. \tag{4.25}$$

For Eq. (4.17), scaling out $|r_0|^2$, we obtain

$$\begin{aligned}0 = D\Bigg[&\left(\frac{1}{64\pi^2}+\tilde{c}-\tilde{\delta}\right)(1+\cos 2\theta)+\frac{\tilde{A}}{32\pi^2}\left\{\frac{1}{2}|\Delta_0|^2-(1-\cos 2(\theta-\theta'))\right\}\\ &-\frac{1}{32\pi^2}\Bigg\{(2\log|\lambda_0^{(+)}|^2+1)\frac{1}{2}\left(\frac{e^{2i\theta}\bar{\lambda}_0^{(+)}}{\sqrt{1+\Delta_0^2}}+\frac{e^{-2i\theta}\lambda_0^{(+)}}{\sqrt{1+\bar{\Delta}_0^2}}\right)|\lambda_0^{(+)}|^2\\ &-(2\log|\lambda_0^{(-)}|^2+1)\frac{1}{2}\left(\frac{e^{2i\theta}\bar{\lambda}_0^{(-)}}{\sqrt{1+\Delta_0^2}}+\frac{e^{-2i\theta}\lambda_0^{(-)}}{\sqrt{1+\bar{\Delta}_0^2}}\right)|\lambda_0^{(-)}|^2\Bigg\}\Bigg],\end{aligned} \tag{4.26}$$

where

$$\Delta_0 = |\Delta_0|e^{i\theta}, \qquad r_0 = |r_0|e^{\imath\theta}, \qquad \tan 2\theta' = \frac{|\Delta_0|^2\sin 2\theta}{1+|\Delta_0|^2\cos 2\theta}. \tag{4.27}$$

Note that $|1-\cos 2(\theta-\theta')| \to 0$ in the region $\theta \sim 0$ or $|\Delta_0| \gg 1$.

On the other hand, for Eq. (4.23) with Δ_0 being real, $N^2|m_s|^4$ is scaled out, and it is simply given by the Δ_0 derivative:

$$\begin{aligned}0 &= \Delta_0\left[2\left(\left(c'+\frac{1}{64\pi^2}\right)-\delta\right)+\frac{1}{32\pi^2}\left\{\frac{\tilde{A}}{2}\Delta_0^2+\frac{1}{\Delta_0}\frac{d}{d\Delta_0}(-\lambda_0^{(+)4}\log\lambda_0^{(+)2}-\lambda_0^{(-)4}\log\lambda_0^{(-)2})\right\}\right]\\ &= \Delta_0\left[2\left(\left(c'+\frac{1}{64\pi^2}\right)-\delta\right)+\frac{1}{32\pi^2}\left\{\frac{\tilde{A}}{2}\Delta_0^2-\frac{1}{\sqrt{1+\Delta_0^2}}(\lambda_0^{(+)3}(2\log\lambda_0^{(+)2}+1)-\lambda_0^{(-)3}(2\log\lambda_0^{(-)2}+1))\right\}\right],\end{aligned} \tag{4.28}$$

which is our original gap equation.[5] In both cases, the solutions are given by the extremum of the potential $V_0(D,\varphi,\bar{\varphi})$ in its D profile. We stress again that the D profile is not a direct stability criterion of the vacua, which is to be discussed with regard to the scalar potential $V_0(D_*(\varphi,\bar{\varphi}),\varphi,\bar{\varphi})$.

[5]We have introduced $\delta(\varphi,\bar{\varphi})$ such that its stationary value $\delta(\varphi_*,\bar{\varphi}_*) = 0$, which can therefore be ignored in analyzing Eq. (4.28).

We next examine $\frac{\partial V_0}{\partial \varphi}|_{D,\bar{\varphi}} = 0$ and its complex conjugate. For Eq. (4.17), we obtain

$$\begin{aligned} 2\frac{\partial}{\partial\varphi}(\ln|m_s|^2)\frac{V_0}{N^2|m_s|^4} &= \left(\frac{\partial\tilde{\delta}}{\partial\varphi}\right)\left(\frac{\Delta_0+\bar{\Delta}_0}{2}\right)^2 \\ &\quad - D\left[\frac{\partial\ln r_0}{\partial\varphi}\bigg|_{\bar{\varphi}} + \frac{\partial\ln\bar{r}_0}{\partial\bar{\varphi}}\bigg|_{\varphi}\right]\frac{\partial}{\partial D}\left(\frac{V_0}{N^2|m_s|^4}\right) \\ &\quad - D\hat{\mathcal{P}}\left(\frac{V_0}{N^2|m_s|^4}\right) \text{ and its complex conjugate,} \end{aligned} \tag{4.29}$$

where

$$\hat{\mathcal{P}} = i\left(\frac{\partial\theta}{\partial\varphi}\right)\left(r_0\frac{\partial}{\partial\Delta_0} - \bar{r}_0\frac{\partial}{\partial\bar{\Delta}_0}\right). \tag{4.30}$$

The second term of the rhs of Eq. (4.29) is proportional to the gap equation, Eq. (4.26). As for the third term, after some calculation, we obtain

$$\begin{aligned} \frac{-\hat{\mathcal{P}}(\frac{V_0}{N^2|m_s|^4})}{(\frac{\partial\theta}{\partial\varphi})|\Delta||r_0|} &= \left(\frac{1}{64\pi^2} + \tilde{c} - \tilde{\delta}\right)\sin 2\theta + \frac{1}{32\pi^2}\tilde{A}\sin 2(\theta-\theta') \\ &\quad - \frac{1}{32\pi^2}\frac{1}{2}\left(\frac{\sin(2\theta-\theta')}{|1+\Delta_0^2|^{1/2}} + \sin 2(\theta-\theta')\right) \\ &\quad \times|\lambda_0^{(+)}|^2(2\log|\lambda_0^{(+)}|^2+1) \\ &\quad + \frac{1}{32\pi^2}\frac{1}{2}\left(\frac{\sin(2\theta-\theta')}{|1+\Delta_0^2|^{1/2}} - \sin 2(\theta-\theta')\right) \\ &\quad \times|\lambda_0^{(-)}|^2(2\log|\lambda_0^{(-)}|^2+1) \\ &\equiv C(\theta,|\Delta_0|). \end{aligned} \tag{4.31}$$

In the rhs of Eqs. (4.29) and (4.31), we have regarded Δ_0, $\bar{\Delta}_0$, φ, and $\bar{\varphi}$ as independent variables.

For Eq. (4.23), with Δ_0 real, we obtain

$$2\partial(\ln|m_s|^2)\frac{V_0}{N^2|m_s|^4} = \left(\frac{\partial\delta}{\partial\varphi}\right)\Delta_0^2 - \frac{\partial\Delta_0}{\partial\varphi}\frac{\partial}{\partial\Delta_0}\left(\frac{V_0}{N^2|m_s|^4}\right) \tag{4.32}$$

and its complex conjugate. Here in the last term of the rhs, we have regarded Δ_0, φ, $\bar{\varphi}$ as independent variables.

Finally the stationary values $(D_*, \varphi_*, \bar{\varphi}_*)$ are determined by Eqs. (4.26) and (4.29) or by Eqs. (4.28) and (4.32). Let us discuss the latter case first. As the second term of the rhs in Eq. (4.32) is nothing but the gap equation, Eqs. (4.28) and (4.32) can be safely replaced by

$$\frac{V_0}{N^2|m_s|^4} = \frac{\frac{\partial\delta}{\partial\varphi}}{2\partial(\ln|m_s|^2)}\Delta_0^2, \tag{4.33}$$

φ being real. The solution to Eq. (4.33) in the Δ_0 profile is determined as the point of intersection of the potential with the quadratic term having $\varphi = \bar{\varphi}$ dependent coefficients. Actually, it is a real curve in the full $(\Delta_0, \varphi = \bar{\varphi})$ plane. Likewise, the solution to the gap equation, Eq. (4.28), the condition of the Δ_0 extremum of the potential, provides us with another real curve in the $(\Delta_0, \varphi = \bar{\varphi})$ plane. The values $(\Delta_{0*}, \varphi_* = \bar{\varphi}_*)$ are the intersection of these two. The schematic figure of the intersection is displayed in Fig. 1. By tuning our original input functions, it is possible to arrange such intersection. Conversely, as an inverse problem, for given Δ_{0*} and the height of the Δ_0 profile, one can always find the values of the coefficients in Eq. (4.28) and the coefficient function in Eq. (4.33) that accomplish this. Dynamical supersymmetry breaking has been realized.

As for the former case, as in the latter case, we can safely replace Eq. (4.29) by

$$\frac{V_0}{N^2|m_s|^4} = \frac{(\frac{\partial\tilde{\delta}}{\partial\varphi})}{2\partial(\ln|m_s|^2)}\left(\frac{\Delta_0+\bar{\Delta}_0}{2}\right)^2 + \frac{(\frac{\partial\theta}{\partial\varphi})|\Delta_0|^2}{2\partial(\ln|m_s|^2)}C. \tag{4.34}$$

The values $(\Delta_{0*}, \bar{\Delta}_{0*}, \varphi_*, \bar{\varphi}_*)$ can be determined by the intersection of Eqs. (4.26) and (4.34). We will not carry out the (numerical) analysis for this case further in this paper.

C. Determination of F_*

Let us now turn to the analysis of the remaining equation of our variational problem, Eq. (4.2). In our current treatment,

$$F = -\frac{1}{g}\bar{W}' + \frac{1}{g}\frac{\partial}{\partial\bar{F}}V_{1\text{-loop}} \approx -\frac{1}{g}\bar{W}' + \frac{1}{g}\frac{\partial}{\partial\bar{F}}V_{1\text{-loop}}\bigg|_{F=0}. \tag{4.35}$$

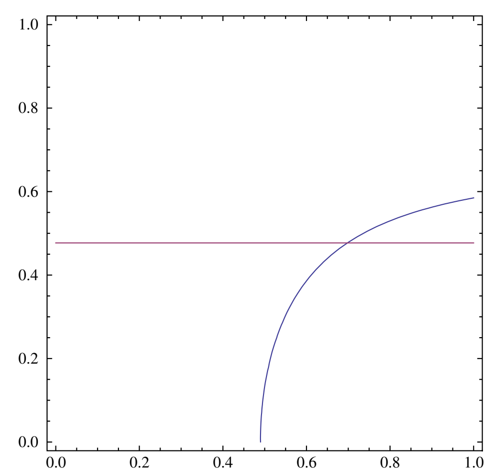

FIG. 1 (color online). The schematic picture of the intersection of the two curves which represent the solution to the gap equation (the straight one) and the φ flat condition (the curved one). The horizontal axis is denoted by φ/M and the vertical one by Δ_0. The values at the stationary point $(\Delta_{0*}, \varphi_* = \bar{\varphi}_*)$ are read off from the intersection point.

As the stationary values $(D_*, \varphi_*, \bar{\varphi}_*)$ are already determined, this equation and its complex conjugate determine F_* and $\bar{F}_*$:

$$F_* = \frac{1}{g(\varphi_*, \bar{\varphi}_*)}\bigg(-\bar{W}'(\varphi_*, \bar{\varphi}_*) + \frac{\partial}{\partial \bar{F}} V_{1\text{-loop}}(D_*, \varphi_*, \bar{\varphi}_*, F, \bar{F})\bigg|_{F=\bar{F}=0}\bigg). \quad (4.36)$$

Note that, knowing $V_{1\text{-loop}}$ explicitly in Eq. (3.27), the rhs can be evaluated. We can check the consistency of our treatment through f_3 in Eq. (3.14) by $|f_3| \ll 1$.

D. Numerical study of the gap equation

In this subsection, we study some numerical solutions to the gap equation, Eq. (4.28), and the stationary condition for φ, Eq. (4.33), in the real Δ_0 case. The equations we should study are

$$0 = 2\left(c' + \frac{1}{64\pi^2}\right) + \frac{1}{32\pi^2}\left\{\frac{\tilde{A}}{2}\Delta_{0*}^2 - \frac{1}{\sqrt{1+\Delta_{0*}^2}}(\lambda_0^{(+)3}(2\log\lambda_0^{(+)2}+1) - \lambda_0^{(-)3}(2\log\lambda_0^{(-)2}+1))\bigg|_{\Delta_0=\Delta_{0*}}\right\}, \quad (4.37)$$

$$\frac{V_0}{N^2|m_{s*}|^4} = \frac{\frac{\partial\delta(\varphi,\bar{\varphi})}{\partial\varphi}|_{\varphi_*,\bar{\varphi}_*}}{2\partial(\ln|m_{s*}|^2)}\Delta_{0*}^2, \quad (4.38)$$

where we note that $\delta(\varphi, \bar{\varphi})$ in the gap equation (4.28) vanishes at the stationary point in the real Δ_0 case. By using Eqs. (4.23) and (4.24), the second condition can be rewritten after dividing by Δ_{0*}^2 as

$$\left(c' + \frac{1}{64\pi^2}\right) + \frac{1}{32\pi^2}\left[\frac{\tilde{A}}{8}\Delta_{0*}^2 - \frac{1}{\Delta_{0*}^2}(|\lambda_0^{(+)}|^4\log|\lambda_0^{(+)}|^2 - |\lambda_0^{(-)}|^4\log|\lambda_0^{(-)}|^2)\bigg|_{\Delta_0=\Delta_{0*}}\right] = \frac{1}{4N^2\partial\ln|m_{s*}|^2}\frac{\mathrm{Im}(\mathcal{F}''_* + \Lambda)}{r_{0*}^2|m_{s*}|^4}\left[\partial\ln\mathrm{Im}(\mathcal{F}''+\Lambda)|_{\varphi_*,\bar{\varphi}_*} - \frac{\partial(r_0|m_s|^2)^2|_*}{(r_{0*}^2|m_{s*}|^4)^2}\right]. \quad (4.39)$$

The nontrivial solution $\Delta_{0*} \neq 0$ to the gap equation (4.37) is found by some region of the parameters c' and $\tilde{A}$, which was already done in Ref. [4]. This solution fixes the lhs of Eq. (4.39) and φ_* is determined by solving Eq. (4.39) in principle. In order to find φ_* explicitly, the form of the prepotential $\mathcal{F}$ and that of the superpotential W must be specified. Here, we take a simple prepotential and a superpotential of the following type (with some abuse of the notation):

$$\mathcal{F} = \frac{c}{2N}\mathrm{tr}\varphi^2 + \frac{1}{3!MN}\mathrm{tr}\varphi^3 \equiv \frac{1}{2}c\varphi^2 + \frac{1}{3!M}\varphi^3, \quad (4.40)$$

$$W = \frac{m^2}{N}\mathrm{tr}\varphi + \frac{d}{3!N}\mathrm{tr}\varphi^3 \equiv m^2\varphi + \frac{d}{3!}\varphi^3, \quad (4.41)$$

where c, d are dimensionless constants while m, M carry dimensions. In particular, M is a cutoff scale of the theory. This prepotential is minimal for DDSB. As for the superpotential, at least two terms are required to be supersymmetric at tree level. We can take a quadratic term φ^2 instead of the cubic one, but in that case, the rhs of Eq. (4.39) becomes singular because of $\partial\ln|m_s|^2 = 0$.

Substituting these $\mathcal{F}$ and W into Eq. (4.39), we obtain

$$\left(c' + \frac{1}{64\pi^2}\right) + \frac{1}{32\pi^2}\left[\frac{\tilde{A}}{8}\Delta_{0*}^2 - \frac{1}{\Delta_{0*}^2}(|\lambda_0^{(+)}|^4\log|\lambda_0^{(+)}|^2 - |\lambda_0^{(-)}|^4\log|\lambda_0^{(-)}|^2)\bigg|_{\Delta_0=\Delta_{0*}}\right] = -\frac{\mathrm{Im}(c+\Lambda)(\mathrm{Im}c)^4}{N^2}\frac{1}{(d\varphi_*/M)^2}, \quad (4.42)$$

where we used the fact that $1/M$, d, φ_* are real and c is pure imaginary, which are necessary for $\Delta_0 = \bar{\Delta}_0$. If we take the coefficients $c = i$, $d = 1$ for further simplification, we can easily obtain a solution by tuning N and $\mathrm{Im}\Lambda$. We note $0 \le \varphi_*/M \le 1$ for our effective theory to be valid. In our analysis carried out in this paper, we consider the region where the magnitude of the F term is smaller compared to that of the D term. Therefore, we need to check whether our solutions satisfy this property consistently. Let us consider the ratio of the auxiliary fields:

$$\left|\frac{F_*}{D_*}\right| = \left|\frac{-g^{-1}\bar{W}'(\bar{\varphi}_*) + g^{-1}\frac{\partial}{\partial\bar{F}}V_{1\text{-loop}}(D_*, \varphi_*, \bar{\varphi}_*, F, \bar{F})|_{F=\bar{F}=0}}{\Delta_{0*}/r_{0*}}\right|$$
$$= \left|\frac{1}{\sqrt{2}\Delta_{0*}\frac{\varphi_*}{M}}\left[\left(\frac{m}{M}\right)^2 + \frac{1}{2}\left(\frac{\varphi_*}{M}\right)^2\right] + i\frac{N^2}{\sqrt{2}\Delta_{0*}}\left(\frac{\varphi_*}{M}\right)^2\left[\frac{\tilde{A}}{128\pi^2}\Delta_{0*}^2 - \frac{1}{32\pi^2}(|\lambda_{0*}^+|^4\log|\lambda_{0*}^+|^2 + |\lambda_{0*}^-|^4\log|\lambda_{0*}^-|^2 + 1) + \frac{1+\frac{\Delta_{0*}^2}{2}}{32\pi^2\sqrt{1+\Delta_{0*}^2}}\left\{(\lambda_{0*}^+)^3\left(\log|\lambda_{0*}^+|^2 + \frac{1}{2}\right) - (\lambda_{0*}^-)^3\left(\log|\lambda_{0*}^-|^2 + \frac{1}{2}\right)\right\}\right]\right|, \quad (4.43)$$

TABLE I. Samples of numerical solutions for the gap equation and the stationary condition for φ. The ratio $|F_*/D_*|$ and $|f_{3*}|$ are also evaluated for a consistency check.

$c' + \frac{1}{64\pi^2}$	$\tilde{A}/(4\cdot 32\pi^2)$	Δ_{0*}	$\varphi_*/M(-\frac{N^2}{\mathrm{Im}(i+\Lambda)})$	$\lvert F_*/D_*\rvert$	$\lvert f_{3*}\rvert$
0.002	0.0001	0.477	0.707 (10,000)	2.621 ($m = M$)	1.77
0.002	0.0001	0.477	0.707 (10,000)	0.524 ($m \ll M$)	0.35
0.002	0.0001	0.477	0.707 (10,000)	0.860 ($m = 0.4M$)	0.58
0.003	0.001	1.3623	0.8639 (2000)	0.825 ($m = M$)	>1
0.003	0.001	1.3623	0.8639 (2000)	0.224 ($m \ll M$)	0.43
0.003	0.001	1.3623	0.5464 (5000)	1.092 ($m = M$)	>1
0.003	0.001	1.3623	0.5464 (5000)	0.142 ($m \ll M$)	0.27
0.003	0.001	1.3623	0.5464 (5000)	0.911 ($m = 0.9M$)	1.76
0.003	0.001	1.3623	0.3863 (10,000)	1.444 ($m = M$)	>1
0.003	0.001	1.3623	0.3863 (10,000)	0.100 ($m \ll M$)	0.19
0.003	0.001	1.3623	0.3863 (10,000)	0.960 ($m = 0.8M$)	1.85

where the form of the prepotential and that of the superpotential in Eqs. (4.40) and (4.66) are assumed and we have put $c = i$, $d = 1$ in the second equality.

Now, the numerical solutions to the gap equation and the stationary condition for φ are listed in Table I. In these examples, we have taken some values of $-\frac{N^2}{\mathrm{Im}(i+\Lambda)}$ and m just for an illustration, and the ratio $|F_*/D_*|$ and $|f_{3*}|$ are evaluated. We can find that the F term is smaller than the D term in some of these examples.

E. Second variation of the potential and the mass of the scalar gluons

We now turn to the question of the second variations of the scalar potential

$$V_{\mathrm{scalar}} = V(D = D_*(\varphi, \bar{\varphi}), F = F_*(\varphi, \bar{\varphi}) \approx 0, \bar{F} = \bar{F}_*(\varphi, \bar{\varphi}) \approx 0, \varphi, \bar{\varphi}) \tag{4.44}$$

at the stationary point $(D_*(\varphi_*, \bar{\varphi}_*), 0, 0, \varphi_*, \bar{\varphi}_*)$. It is convenient to separate $V(D, F, \bar{F}, \varphi, \bar{\varphi})$ into two parts:

$$V = \mathcal{V} + V_0. \tag{4.45}$$

Here

$$\mathcal{V}(F, \bar{F}, \varphi, \bar{\varphi}) \approx -gF\bar{F} - FW' - \bar{F}\bar{W}' + (\partial_F V_{1\text{-loop}})_* F + (\partial_{\bar{F}} V_{1\text{-loop}})_* \bar{F} + \frac{1}{2}(\partial_F^2 V_{1\text{-loop}})_* F^2 + \frac{1}{2}(\partial_{\bar{F}}^2 V_{1\text{-loop}})_* \bar{F}^2 + (\partial_F \partial_{\bar{F}} V_{1\text{-loop}})_* F\bar{F}, \tag{4.46}$$

and

$$V_0(D, \varphi, \bar{\varphi}) = V(D, \varphi, \bar{\varphi}, F = 0, \bar{F} = 0). \tag{4.47}$$

In Eq. (4.46), we have extracted the F, $\bar{F}$ dependence of $V_{1\text{-loop}}$ [Eq. (3.27)] as series, and $*$ indicates that they are evaluated at $(D_*, \varphi_*, \bar{\varphi}_*, 0, 0)$ after the derivatives are taken. Equation (4.47) has been computed in Eqs. (4.16) and (4.23). We will compute the second partial derivatives and the second variations of V_{scalar}, using the formula in the appendix.

For $\mathcal{V}$, $\vec{y}_L = (F, \bar{F})$, $\vec{y}_R = (\varphi, \bar{\varphi})$,

$$M_{RR_*} \equiv \begin{pmatrix} \partial^2 \mathcal{V}, & \partial\bar{\partial}\mathcal{V} \\ \bar{\partial}\partial\mathcal{V}, & \bar{\partial}^2\mathcal{V} \end{pmatrix}_* \approx 0, \tag{4.48}$$

$$M_{RL_*} \equiv \begin{pmatrix} \partial\partial_F \mathcal{V}, & \partial\partial_{\bar{F}}\mathcal{V} \\ \bar{\partial}\partial_F\mathcal{V}, & \bar{\partial}\partial_{\bar{F}}\mathcal{V} \end{pmatrix}_* \approx \begin{pmatrix} -W'' + (\partial\partial_F V_{1\text{-loop}}), & (\partial\partial_{\bar{F}} V_{1\text{-loop}}) \\ (\bar{\partial}\partial_F V_{1\text{-loop}}), & -\bar{W}'' + (\bar{\partial}\partial_{\bar{F}} V_{1\text{-loop}}) \end{pmatrix}_*, \tag{4.49}$$

$$M_{LR_*} = M_{RL_*}^t, \tag{4.50}$$

$$M_{LL_*} \equiv \begin{pmatrix} \partial_F^2 \mathcal{V}, & \partial_F\partial_{\bar{F}}\mathcal{V} \\ \partial_{\bar{F}}\partial_F\mathcal{V}, & \partial_{\bar{F}}^2\mathcal{V} \end{pmatrix}_* \approx \begin{pmatrix} (\partial_F^2 V_{1\text{-loop}}), & -g + (\partial_F\partial_{\bar{F}} V_{1\text{-loop}}) \\ -g + (\partial_{\bar{F}}\partial_F V_{1\text{-loop}}), & (\partial_{\bar{F}}^2 V_{1\text{-loop}}) \end{pmatrix}_*. \tag{4.51}$$

Here we have denoted by $*$ that the derivatives are evaluated at the stationary point.

We obtain, after some computation,

$$\delta^2\mathcal{V}_* \approx \frac{1}{2}\delta\vec{y}_R^t M_{RL_*}(-M_{LL_*}^{-1}) M_{LR_*}\delta\vec{y}_R \equiv \frac{1}{2}\delta\vec{y}_R^\dagger \begin{pmatrix} \mathcal{M}_{\varphi\bar{\varphi}} & \mathcal{M}_{\varphi\varphi} \\ \mathcal{M}_{\bar{\varphi}\bar{\varphi}} & \mathcal{M}_{\bar{\varphi}\varphi} \end{pmatrix}_* \delta\vec{y}_R. \tag{4.52}$$

Here

$$\mathcal{M}_{\varphi\bar{\varphi}} = \frac{1}{g(G^2 - |C|^2)}(G(|A|^2 + |B|^2) + CA\bar{B} + \bar{C}\bar{A}B), \tag{4.53}$$

$$\mathcal{M}_{\varphi\varphi} = \frac{1}{g(G^2 - |C|^2)}(2GAB + CA^2 + \bar{C}B^2), \tag{4.54}$$

$$G \equiv 1 - \frac{\partial_F \partial_{\bar{F}} V_{1\text{-loop}}}{g}, \qquad C \equiv \frac{\partial_{\bar{F}}^2 V_{1\text{-loop}}}{g}, \\ A \equiv W'' - \partial\partial_F V_{1\text{-loop}}, \qquad B \equiv -\partial\partial_{\bar{F}} V_{1\text{-loop}}. \tag{4.55}$$

Here in the last line of Eq. (4.52), we have changed the real quadratic form into the complex one. We see that in the region $|(\partial_F\partial_{\bar{F}}V)_0|_*, |(\partial_{\bar{F}}^2 V)_0|_*, \ll g_*$, the matrix $\mathcal{M}_*$ is well approximated by

$$\mathcal{M}_* \approx \frac{1}{g}\begin{pmatrix} |A|^2 + |B|^2, & 2AB \\ 2\bar{A}\bar{B}, & |A|^2 + |B|^2 \end{pmatrix}_*. \tag{4.56}$$

The two eigenvalues are

$$\frac{1}{g}(|A| \pm |B|)_*^2 = \frac{1}{g}(|W'' - (\partial\partial_F V_{1\text{-loop}})| \pm |(\partial\partial_{\bar{F}} V_{1\text{-loop}})|)_*^2, \tag{4.57}$$

respectively, ensuring the positivity of Eq. (4.52).

For V_0, $y_L = D$, $\vec{y}_R = (\varphi, \bar{\varphi})$,

$$M_{RR_*} = \begin{pmatrix} \partial^2 V_0, & \partial\bar{\partial}V_0 \\ \bar{\partial}\partial V_0, & \bar{\partial}^2 V_0 \end{pmatrix}_*, \qquad M_{RL_*} = \begin{pmatrix} \partial\partial_D V_0 \\ \bar{\partial}\partial_D V_0 \end{pmatrix}_*, \tag{4.58}$$

$$M_{LR_*} = M_{RL}^*, \qquad M_{LL_*} = \partial_D^2 V_{0*}. \tag{4.59}$$

We know that the D profile of $V_0(D, \varphi, \bar{\varphi})$ near the stationary point is convex to the top, and we fit this by

$$V_0 = V_h(\varphi, \bar{\varphi}) - \frac{\alpha(\varphi, \bar{\varphi})}{2}(D - D_*(\varphi, \bar{\varphi}))^2 \\ + \mathcal{O}((D - D_*(\varphi, \bar{\varphi}))^4). \tag{4.60}$$

Here α is a positive real function of φ, $\bar{\varphi}$ and $V_h(\varphi, \bar{\varphi}) = V_0(D_*(\varphi, \bar{\varphi}), \varphi, \bar{\varphi})$. One can check

$$-M_{RL_*} M_{LL_*}^{-1} M_{LR*} = \alpha_* \begin{pmatrix} \partial D_* \\ \bar{\partial} D_* \end{pmatrix}_* ((\partial D_*), (\bar{\partial} D_*))_*, \tag{4.61}$$

while

$$M_{RR*} = \begin{pmatrix} \partial^2 V_h & \partial\bar{\partial}V_h \\ \bar{\partial}\partial V_h & \bar{\partial}^2 V_h \end{pmatrix}_* - \alpha \begin{pmatrix} (\partial D_*)^2 & |\partial D_*|^2 \\ |\partial D_*|^2 & (\bar{\partial} D_*)^2 \end{pmatrix}_* \tag{4.62}$$

and

$$\delta^2 V_{0*} = \frac{1}{2}\delta\vec{y}_R(M_{RR*} - M_{RL*}M_{LL*}^{-1}M_{LR*})_*\delta\vec{y}_R \\ = \delta\vec{y}_R^\dagger \begin{pmatrix} \partial\bar{\partial}V_h & \partial^2 V_h \\ \bar{\partial}^2 V_h & \partial\bar{\partial}V_h \end{pmatrix}_* \delta\vec{y}_R \equiv \delta\vec{y}_R^\dagger \mathcal{M}_{h*}\vec{y}_R. \tag{4.63}$$

The entire contribution of the second variation $\delta^2 V_* = \delta^2 \mathcal{V}_* + \delta^2 V_{0*}$ to the leading order in the Hartree-Fock approximation is given by Eqs. (4.52) and (4.63). The mass of the scalar gluons squared is obtained by multiplying the combined mass matrix by g_*^{-1}:

$$g_*^{-1}(\mathcal{M}_* + \mathcal{M}_{h*}), \tag{4.64}$$

generalizing the tree formula. In practice, we just need a well-approximated formula valid in the region we work with, and one can invoke the $U(1)$ invariance to ensure that the two eigenvalues of the complex scalar gluons are degenerate. Let us, therefore, use the expression

$$\begin{aligned} \frac{1}{g}|W'' - (\partial\partial_F V_{1\text{-loop}})|_*^2 + \partial\bar{\partial}V_{h*} = {} & \Bigg|\frac{\varphi_*}{M} - iN^2\left(\frac{\varphi_*}{M}\right)^2\Bigg[\frac{A(\varepsilon,\gamma)}{32\pi^2}\Big\{(\lambda_{0*}^+)^4 + (\lambda_{0*}^-)^4 + \frac{2}{\varphi_*/M} - 2 - \frac{3}{4}\Delta_{0*}^2 + \frac{1}{8}\Delta_{0*}^4\Big\} \\ & - \frac{1}{32\pi^2}\Big\{(\lambda_{0*}^+)^4\log(\lambda_{0*}^+)^2 + (\lambda_{0*}^-)^4\log(\lambda_{0*}^-)^2 + \frac{3(1+\frac{\Delta_{0*}^2}{2})}{\sqrt{1+\Delta_{0*}^2}}\left((\lambda_{0*}^-)^3\left(\log(\lambda_{0*}^-)^2 + \frac{1}{2}\right)\right. \\ & \left.\left. - (\lambda_{0*}^+)^3\left(\log(\lambda_{0*}^+)^2 + \frac{1}{2}\right)\right)\Big\} - \frac{2}{32\pi^2}\frac{1}{\varphi_*/M}\Bigg]\Bigg|^2 M^2 + 2N^2\left(\frac{\varphi_*}{M}\right)^2\Bigg[2\left(c' + \frac{1}{64\pi^2}\right)\Delta_{0*}^2 \\ & + \frac{2}{32\pi^2}\left(\frac{\tilde{A}}{8} - (\lambda_{0*}^+)^4\log(\lambda_{0*}^+) - (\lambda_{0*}^-)^4\log(\lambda_{0*}^-)^2\right) + \frac{\mathrm{Im}(i+\Lambda)}{N^2}\frac{1}{\varphi_*/M}\Bigg]M^2 \end{aligned} \tag{4.65}$$

to check the local stability of the potential and the mass. The above expression is obtained for our simple example of $\mathcal{F}$ and W,

$$\mathcal{F} = \frac{i}{2}\varphi^2 + \frac{1}{3!M}\varphi^3, \qquad W = m^2\varphi + \frac{1}{3!}\varphi^3, \tag{4.66}$$

and the real case $\Delta_0 = \bar{\Delta}_0$ is applied. Using the numerical analyses carried out in the last subsection, we have made a list of data on Eq. (4.65). Except for the last case in Table II, the scalar gluon masses squared are found to be positive for any N, which implies that our stationary points are locally stable. Even in the last case, the stability is ensured for small N. In these data, we have checked that the inequalities $|(\partial_F\partial_{\bar{F}}V_{1\text{-loop}})|_*, |(\partial_{\bar{F}}^2 V_{1\text{-loop}})|_* \ll g_*$ are in fact satisfied. As a summary of our understanding, a schematic figure is drawn in Fig. 2, which illustrates the local

TABLE II. Samples of numerical values for the scalar gluon masses.

$c' + \frac{1}{64\pi^2}$	$\tilde{A}/(4 \cdot 32\pi^2)$	Δ_{0*}	$\varphi_*/M(-\frac{N^2}{\mathrm{Im}(i+\Lambda)})$	Scalar gluon mass
0.002	0.0001	0.477	0.707 (10,000)	$0.4998 + 0.0056N^2 + 8.607 \times 10^{-7}N^4$
0.003	0.001	1.3623	0.8639 (2000)	$0.7463 + 0.0106N^2 + 2.653 \times 10^{-4}N^4$
0.003	0.001	1.3623	0.5464 (5000)	$0.2986 + 0.0008N^2 + 4.694 \times 10^{-5}N^4$
0.003	0.001	1.3623	0.3863 (10,000)	$0.1492 - 0.0024N^2 + 7.235 \times 10^{-5}N^4$

stability of the scalar potential at the vacuum of dynamically broken supersymmetry in comparison with the well-known NJL potential.

F. Summary and choice of regularization and subtraction scheme

In this paper, we have considered the theory specified by the general $\mathcal{N} = 1$ supersymmetric Lagrangian, Eq. (2.1); have regularized the theory by the supersymmetric dimensional regularization (dimensional reduction); and have subtracted the part of the $1/(\epsilon)$ poles of the regularized one-loop effective action in Eq. (3.27) by the supersymmetric subtraction scheme defined by the condition (3.19). The upshot is an effective potential, Eqs. (4.17) and (4.23), as a function of the background constant scalar and the order parameter D of supersymmetry, with another order parameter F of supersymmetry being induced and treated perturbatively. Supersymmetry is dynamically broken as is represented by the nonvanishing value of the order parameters at the stationary point. The original infinity is transmuted into the infinite constant Λ which is the coefficient of the counterterm, and the effective potential has been recast to describe the behavior of the theory well below the UV cutoff residing in the prepotential function. As the theory is perturbatively nonrenormalizable, Λ is still present in our final expressions of the effective potential, and we regard it to take a large value.

We now make brief comments on other regularizations and subtraction schemes which we did not employ in this paper. The relativistic momentum cutoff is a natural choice of the NJL theory as we mentioned earlier, but regularizing the integral Eq. (3.20) by the momentum cuoff leads us to a rather unwieldy expression. See Ref. [5]. Unlike supersymmetric dimensional reduction [49], the momentum cutoff *per se*, while preserving the equality between the Bose and Fermi degrees of freedom, does not have a firm basis on the regularized action which the supersymmetry algebra acts on. Moreover, as is clear from Eq. (A.1) of Ref. [5], the result violates the positivity of the effective potential in the vicinity of the origin in the Δ profile. This violation is a necessity in the broken chiral symmetry of the NJL theory, but here it contradicts with the positive semidefiniteness of energy that the rigid supersymmetric theory possesses. Turning to the choice of the subtraction scheme, one might also like to apply the "(modified) minimal subtraction scheme" in our one-loop integral, Eq. (3.27). While we do not know how to justify this prescription here, the subsequent analyses proceed almost in the same way, and the main features of the equations obtained from our variational analyses and the conclusions are unchanged.

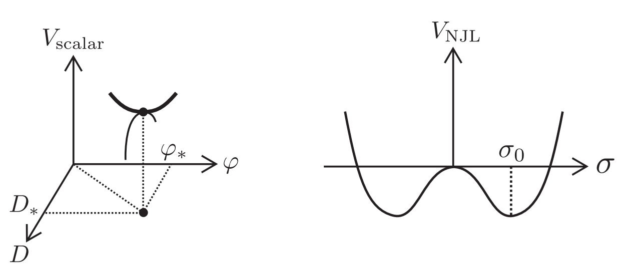

FIG. 2. Comparison of V_{scalar} around the stationary value (D_*, φ_*) with V_{NJL}.

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APPENDIX A: NJL EFFECTIVE ACTION

In this appendix, we briefly recall a few aspects of the $SU(N)$ Nambu-Jona-Lasinio model,

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\Psi} i \partial\!\!\!/ \Psi + \frac{\lambda/N}{2}[(\bar{\Psi}\Psi)^2 + (\bar{\Psi} i\gamma_5 \Psi)^2]. \tag{A1}$$

The equivalent Lagrangian is

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{\mathrm{NJL}} - \frac{1}{2}\frac{1}{\lambda/N}\left[\left(\sigma + \frac{\lambda}{N}\bar{\Psi}\Psi\right)^2 + \left(\pi + \frac{\lambda}{N}\bar{\Psi} i\gamma_5\Psi\right)^2\right] \\ &= -\frac{1}{2}\frac{1}{\lambda/N}\sigma^2 - \frac{1}{2}\frac{1}{\lambda/N}\pi^2 + \bar{\Psi}(i\partial\!\!\!/ - \sigma - i\gamma_5\pi)\Psi.\end{aligned} \tag{A2}$$

The 1PI vertex function (or the effective action) $\Gamma_{\mathrm{1PI}}[\sigma, \pi]$ to one-loop (or leading order in $1/N$) reads

$$i\Gamma_{1\mathrm{PI}}[\sigma,\pi] = -\frac{i}{2}\frac{1}{\lambda/N}\int d^4x(\sigma^2+\pi^2) + N\ln\,\det(i\partial\!\!\!/ - \sigma(x) - i\gamma_5\pi(x)). \quad \text{(A3)}$$

The gap equation is

$$0 = \frac{i\delta\Gamma_{1\mathrm{PI}}}{\delta\sigma(x)}\bigg|_{\sigma(x)=\langle\sigma\rangle=\sigma_0,\pi(x)=0} = -i\frac{1}{\lambda/N}\sigma_0 - N\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\frac{1}{k\!\!\!/ - \sigma_0} = \sigma_0\left(-i\frac{1}{\lambda/N} - 4N\int\frac{d^4k}{(2\pi)^4}\times\frac{(-1)}{k^2-\sigma_0^2}\right). \quad \text{(A4)}$$

APPENDIX B: FORMULA FOR THE SECOND VARIATION

In this appendix, we recall the formula for the second variation of a multivariable function subject to a set of stationary constraints. Let V be the function of two sets of variables: $\{\{y^1,\ldots,u^{n(L)}\}\} = \mathcal{D}_L$, $\{\{y^{n(L)+1},\ldots,y^{n(L)+n(R)}\}\} = \mathcal{D}_R$. Namely,

$$V = V(y^1,\ldots,y^{n(L)},y^{n(L)+1},\ldots,y^{n(L)+n(R)}) \quad \text{(B1)}$$

under

$$\frac{\partial V}{\partial y^i} = 0, \quad i = 1,\ldots,n(L). \quad \text{(B2)}$$

Let the second variation of V be

$$\delta^2V \equiv \frac{1}{2}\sum_{y^i,y^j\in\mathcal{D}_L\cup\mathcal{D}_R}\frac{\partial^2V}{\partial y^i\partial y^j}\delta y^i\delta y^j, \quad \text{(B3)}$$

but $\delta y^i \in \mathcal{D}_L$ are not independent variations.

It is convenient to introduce a new vector notation:

$$\vec{y}_L = (y^1,\ldots,y^{n(L)})^t, \quad \vec{y}_R = (y^{n(L)+1},\ldots,y^{n(L)+n(R)})^t, \text{ etc.} \quad \text{(B4)}$$

Define

$$M_{X,X'} = \left(\frac{\partial^2V}{\partial y^i\partial y^j}\right), \quad y^i\in\mathcal{D}_X, \quad y^j\in\mathcal{D}_{X'} X, X' \text{ are either } L \text{ or } R. \quad \text{(B5)}$$

Equation (B3) reads

$$\delta^2V = \frac{1}{2}(\delta\vec{y}_R, M_{RR}\delta\vec{y}_R) + (\delta\vec{y}_R, M_{RL}\delta\vec{y}_L) + \frac{1}{2}(\delta\vec{y}_L, M_{LL}\delta\vec{y}_L). \quad \text{(B6)}$$

Varying Eq. (B2) with respect to $\vec{y}_L$ and $\vec{y}_R$, we obtain

$$M_{LL}\delta\vec{y}_L + M_{LR}\delta\vec{y}_R = 0. \quad \text{(B7)}$$

Hence,

$$\delta\vec{y}_L = -M_{LL}^{-1}M_{LR}\delta\vec{y}_R. \quad \text{(B8)}$$

Substituting this into Eq. (B6), we obtain

$$\delta^2V = \frac{1}{2}(\delta\vec{y}_R, (M_{RR} - M_{RL}M_{LL}^{-1}M_{LR})\delta\vec{y}_R). \quad \text{(B9)}$$

The generic scalar mass matrix in the text can be read off $M_{RR} - M_{RL}M_{LL}^{-1}M_{LR}$ at the stationary value.

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