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Strange quark matter in strong magnetic fields within a confining model

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We construct an equation of state of strange quark matter in a strong magnetic field within a confining model. The confinement is modeled by means of the Richardson potential for quark-quark interaction modified suitably to account for a strong magnetic field. We compare our results for the equation of state and magnetization of matter to those derived within the MIT bag model. The differences between these models arise mainly due to the momentum dependence of the strong interaction between quarks in the Richardson model. Specifically, we find that the magnetization of strange quark matter in this model has much more pronounced de Haas-van Alfvén oscillations than in the MIT bag model, which is the consequence of the (static) gluon-exchange structure of the confining potential.

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I. INTRODUCTION

Compact stellar objects can be tentatively divided into two broad classes: one includes stars made of the ordinary baryonic matter either in the confined (hadronic) or deconfined (quark-gluon) state, the second includes stars made of strange matter. The latter possibility goes back to Witten's idea [1] that the deconfined quark matter composed of an equal number of up, down, and strange quarks may be the true ground state of matter at high density. Since then, the possibility of strange quark matter (SQM) and strange stars made of SQM, as an alternative to hadronic/quark compact objects, has been continuously explored.

Soft γ-ray repeaters and anomalous x-ray pulsars are commonly identified with compact stars with surface magnetic fields $B_s \sim 10^{14}$–10^{15} G. These objects, which feature the largest stationary B fields observed in Nature to date, are collectively termed as "magnetars." The interpretation of astrophysical manifestations of magnetars requires good knowledge of the properties of dense matter in the presence of a large magnetic field. There have been some recent advances in this context in our understanding of the properties of strange quark matter in strong magnetic fields. The stationary properties, hydrodynamics, transport, and macroscopic dynamics have been studied in Refs. [2–7]. More general but related aspects of the physics of fermionic (quark) matter in strong fields have been discussed recently in, e.g., Refs. [8–13].

In the present work we study the effect of a large magnetic field on SQM. The properties of cold quark matter at large baryon density is poorly known due the nonperturbative nature of quantum chromodynamics (QCD) at densities and temperatures relevant for compact stars. Because the *ab initio* lattice calculations at low temperatures and finite chemical potentials presently encounter serious problems, effective phenomenological models are commonly used. Among the the most popular ones are the MIT bag model [14] and the Nambu-Jona-Lasinio model [15]. Both models have some merits and some disadvantages. For example, the Nambu-Jona-Lasinio model exhibits chiral symmetry breaking but does not account for the confinement property of QCD. On the other hand, the bag models are built to confine through the introduction of an *ad hoc* bag pressure but are unable to account for the chiral symmetry breaking. An alternate to the bag model way to introduce the confinement is to take density-dependent quark masses. Many phenomenological models have been proposed in the past that are based on density-dependent quark masses [16–19]. We will base our discussion of quark matter in a strong magnetic field on one such model, that was originally introduced by Dey *et al.* [18]. In this model, the quarks interact among themselves through the Richardson potential [20], in which the asymptotic freedom and confinement is built in. Initially, it was used in the meson phenomenology and later tested in the baryon sector [21]. This latter model will serve as a basis for studying confining strange matter at nonzero temperatures.

Substantial changes in the strange matter properties appear when the electromagnetic scales become of the order of the nuclear scales, which is the case for fields $B \geq 10^{18}$ G. Such fields have not been observed directly in astrophysics, but theoretical extrapolations of surface fields observed in magnetars suggest that the fields of this magnitude can be reached in the deep interiors of compact objects. An upper value of the B field is set by the equilibrium that can be sustained by the gravitational forces and pressure components of matter in a strong magnetic field. The anticipated value of the maximal field is in the range $10^{18} \leq B_{\max} \leq 10^{20}$ G, but the precise value of $B_{\max}$ remains uncertain (Ref. [9] and references therein).

This work is organized as follows. In Sec. II we introduce the Richardson-potential model (hereafter RP model) and demonstrate its modifications due to the strong magnetic fields. The results of our numerical computations are shown in Sec. III. Finally, our findings are summarized in Sec. IV.

II. MODEL

We consider SQM in a strong magnetic field at high densities and nonzero temperature. The u, d, and s quarks interact via the Richardson potential [20]

$$V(q^2) = -\frac{4}{9}\frac{\pi}{\ln[1 + (q^2 + m_g^2)/\Lambda^2]}\frac{1}{(q^2 + m_g^2)}, \quad (1)$$

where m_g is gluon mass and Λ is a scale parameter. The finite gluon mass is responsible for screening in medium and is related to the screening length D via

$$m_g^2 = D^{-2} = \frac{2\alpha_0}{\pi}\sum_{i=u,d,s} k_F^i \mu_i^*, \quad (2)$$

where α_0 is the perturbative quark gluon coupling, $\mu_i^* \equiv \sqrt{(k_F^i)^2 + m_i^2}$, k_F^i is the Fermi momentum, and m_i the quark mass. The index i labels quark flavors. An important feature of our model is that quark masses depend on the density. We parametrize this dependence as

$$m_i = M_i + M_q \mathrm{sech}\left(\nu\frac{n_b}{n_0}\right), \qquad i = u, d, s, \quad (3)$$

where $n_b = (n_u + n_d + n_s)/3$ is the baryon number density, n_0 is the normal nuclear matter density, and ν is a parameter. At large n_b the second term in (3) decays exponentially and the quark mass m_i falls off from its constituent value M_q to its current value M_i.

The number and energy densities of each quark flavor in the absence of quantizing magnetic field are given by

$$n = \frac{6}{(2\pi)^3}\int_0^\infty f(\epsilon) d^3k, \quad (4)$$

$$\varepsilon = \frac{6}{(2\pi)^3}\int_0^\infty f(\epsilon)\epsilon d^3k, \quad (5)$$

where ϵ is the single particle energy, $f(\epsilon) = \{1 + \exp[(\epsilon - \mu)/T]\}^{-1}$ is the Fermi distribution function, with μ being the chemical potential and T the temperature; factor 6 is the sum over the spin and color degrees of freedom. Note that the full single-particle energy ϵ consists of the kinetic energy of relativistic particle with mass m_i and the potential energy arising from the interaction with other quarks via the Richardson potential (1). As is well known, in a magnetic field the motion of charged particles is Landau quantized in the direction perpendicular to the field. For sufficiently large magnetic fields one needs to take into account the modification of the single particle energies and the phase space due to the Landau quantization of quark orbitals.

We assume that the field is along the z direction of the Cartesian coordinate system, $\mathbf{B} = B\hat{z}$. Then, the motion is quantized in the x-y plane and the momentum of quarks of mass m_i and charge eQ_i can be decomposed into components parallel and perpendicular to the z direction, $\mathbf{k} \equiv (k_z, k_\perp)$, with $k_\perp^2 = 2ne|Q|B$, where e is the (positive) unit of charge. Consequently, the single particle kinetic energy in the nth Landau level is given by

$$\epsilon = \sqrt{k_z^2 + m^2 + 2ne|Q|B}. \quad (6)$$

The number density of any quark flavor is then given by

$$n = \frac{3}{(2\pi)^3} e|Q|B \sum_{n=0}^{\infty}(2 - \delta_{n,0})\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} f(\epsilon) dk_z. \quad (7)$$

The kinetic part of the energy density for a particular quark flavor is given by

$$\varepsilon_{\mathrm{kin}} = \frac{3}{(2\pi)^3} e|Q|B \sum_{n=0}^{\infty}(2 - \delta_{n,0})\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} f(\epsilon)\epsilon dk_z. \quad (8)$$

The potential part of the energy density due to interaction between the flavors i and j is given by

$$\varepsilon_{\mathrm{pot}}^{ij} = \frac{e^2|Q_i||Q_j|}{(2\pi)^5} B^2 \sum_{n_i}\sum_{n_j}(2 - \delta_{n_i,0})(2 - \delta_{n_j,0})\int_0^{2\pi} d\phi_i \times \int_0^{2\pi} d\phi_j \int_{-\infty}^{\infty} dk_z^i \int_{-\infty}^{\infty} dk_z^j f(\epsilon_i) f(\epsilon_j) N V(q^2) S, \quad (9)$$

where

$$N = \frac{(\epsilon_i + m_i)(\epsilon_j + m_j)}{4\epsilon_i\epsilon_j},$$

$$S = 1 + \frac{k_i^2 k_j^2}{(\epsilon_i + m_i)^2(\epsilon_j + m_j)^2} + \frac{2\mathbf{k}_i \cdot \mathbf{k}_j}{(\epsilon_i + m_i)(\epsilon_j + m_j)}.$$

The total energy density is obtained, after summation over the quark flavors, as

$$\varepsilon = \sum_i \varepsilon_{\mathrm{kin}} + \frac{1}{2}\sum_{i,j}\varepsilon_{\mathrm{pot}}^{ij}, \qquad i, j = u, d, s. \quad (10)$$

The net entropy density is given by the combinatorial expression for quark quasiparticles

$$s = -\frac{3}{(2\pi)^3} e \sum_i |Q_i| B \sum_{n=0}^{\infty}\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dk_z \{f(\epsilon_i)\ln f(\epsilon_i) + [1 - f(\epsilon_i)]\ln[1 - f(\epsilon_i)]\}, \quad (11)$$

where i summation is over the quark flavors. Then, the thermodynamic pressure is given by

$$p = \sum_i \mu_i n_i + Ts - \varepsilon, \qquad i = u, d, s. \quad (12)$$

The magnetization of the matter at a given temperature and constant baryon number density is given by

$$M = \frac{dp}{dB}. \quad (13)$$

A number of authors [2,5,9–11] have noticed that in the presence of a strong magnetic field the pressure is anisotropic and it is useful to decompose the pressure in components along ($p_{\|}$) and perpendicular ($p_{\perp}$) to the field as

$$p_{\|} = p, \qquad p_{\perp} = p - MB. \quad (14)$$

In strange quark matter the β equilibrium can be sustained among the quark flavors; therefore, the abundances of leptons (electrons and muons) are negligible. The charge neutrality condition can be written as

$$n_u = \frac{1}{2}(n_d + n_s) = n_b. \quad (15)$$

The weak interactions establish an equilibrium among the quark flavors via the nonleptonic weak process $u + d \rightleftharpoons u + s$. Thus, the equilibrium with respect to these weak reactions requires that the chemical potentials of quark flavors obey the condition

$$\mu_d = \mu_s. \quad (16)$$

To summarize, the key equations of our model are Eqs. (6)–(12) that are subject to the constraints (15) and (16). These equations are solved self-consistently.

III. RESULTS

In this section we discuss the results of a numerical solution of the self-consistent equations presented above. Our main focus will be the effect of the Richardson potential on the properties of strange matter in strong magnetic fields at finite temperature. The numerical values of the parameters of our model are $\Lambda = 100$ MeV, $\nu = 0.333$, $\alpha_0 = 0.2$, $M_q = 310$, $M_u = 4$, $M_d = 7$, and $M_s = 150$ with all masses given in MeV. A discussion of the feasible parameter space can be found in Ref. [18].

In Fig. 1 we show the function $p(T)$ for the RP model together with the results obtained with the MIT bag model with two values of the bag constant along with the result for noninteracting matter. The case of nonmagnetized and strongly magnetized matter ($B = 3 \times 10^{19}$ G) are displayed.

The bag model and noninteracting gas results are self-similar, because they differ only by a temperature-independent constant. In the absence of a magnetic field the pressure shows T^2 power-law behavior with temperature. In the magnetic field the temperature dependence is nonmonotonic in the bag model, but in the RP model the temperature dependence shows the same features as in the absence of a magnetic field.

In Fig. 2 we show the equation of state of SQM in the RP model and the bag model for fixed $T = 20$ MeV. The bag model equations of state show $p \propto n^{4/3}$ scaling inherent to the ultrarelativistic noninteracting gas. In the case of the RP model the scaling is different because the Richardson potential introduces additional momentum dependence in the single particle energies, which results in nearly linear dependence of pressure of density. Furthermore, in the absence of a magnetic field the equation of state in the RP model is softer than in the bag model at low densities and reaches asymptotically the equation of state with $B_{\mathrm{MIT}} = 110$ MeV fm^{-3} at high densities. While the high values of bag constant can mimic the RP model, for such large values of B_{MIT} the strange matter is not the absolute ground state of matter.

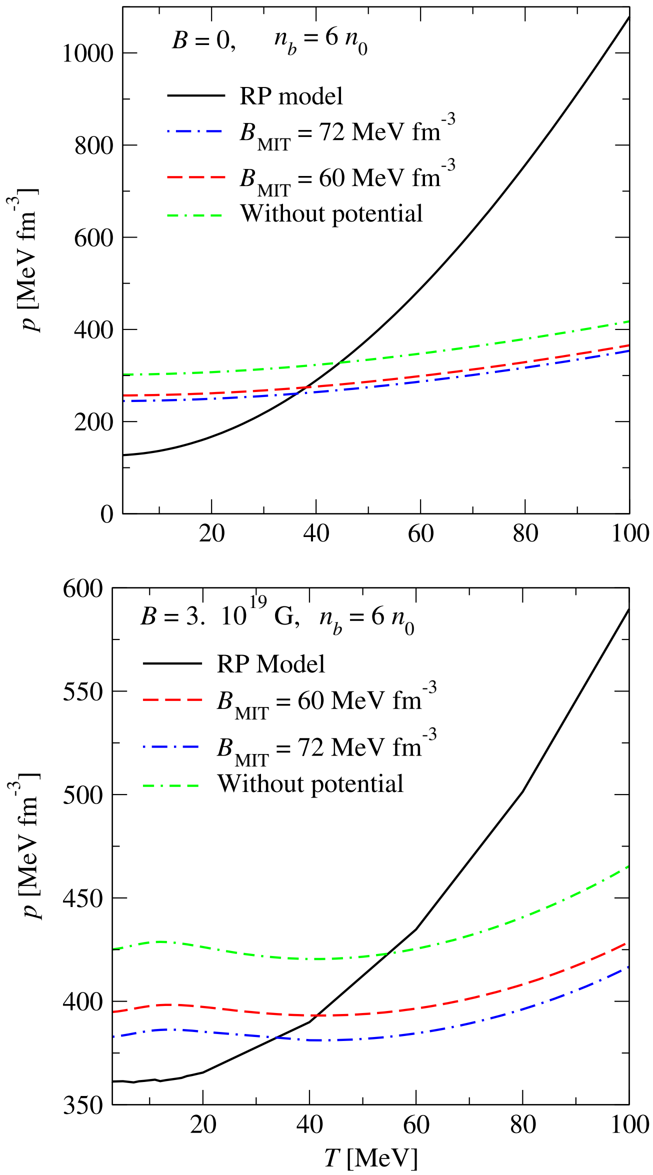

FIG. 1 (color online). Dependence of the thermodynamic pressure p on the temperature at fixed baryon number density $n_b = 6n_0$ for the RP model (solid, black line), for the MIT bag model with $B_{\mathrm{MIT}} = 60$ MeV fm^{-3} (dashed, red line), and for $B_{\mathrm{MIT}} = 72$ MeV fm^{-3} (dash dotted, blue line) and without potential (double-dash-dotted, green line). The upper and lower panels correspond to the field values $B = 0$ and $B = 3 \times 10^{19}$ G.

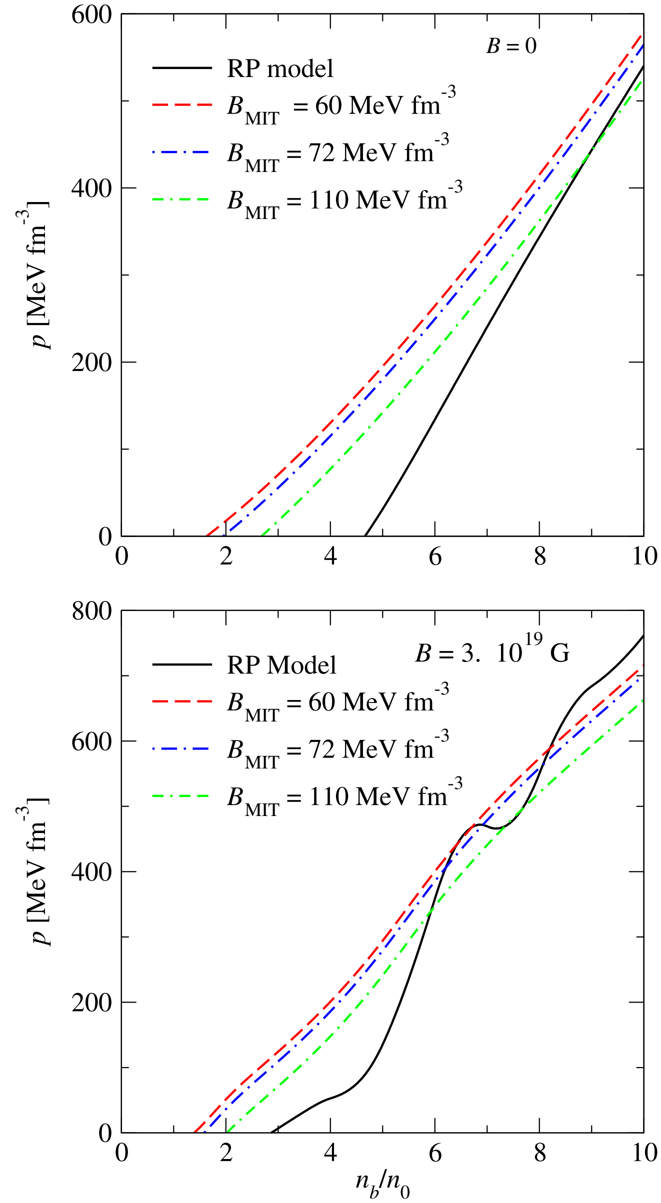

FIG. 2 (color online). Dependence of the thermodynamic pressure p on the normalized baryon number density at $T = 20$ MeV for $B = 0$ (upper panel) and $B = 3 \times 10^{19}$ G (lower panel). The pressure is shown for the RP model (solid, black line), for the MIT bag model with $B_{\mathrm{MIT}} = 60$ MeV fm^{-3} (dashed, red line), $B_{\mathrm{MIT}} = 72$ MeV fm^{-3} (dash-dotted, blue line), and $B_{\mathrm{MIT}} = 110$ MeV fm^{-3} (double dash-dotted, green line).

The upper and lower panels display the differences arising due to the strong magnetic field ($B=3\times10^{19}$ G). The magnetic field introduces some oscillations in the pressure with density; in each case the increase of the pressure after a plateau is caused by the opening of a new Landau level. The oscillations are much stronger in the RP model and this can be traced back to the momentum dependence of the potential. The major contribution comes from the static gluon propagator part of the potential [the term $(q^2 + m_g^2)^{-1}$], while the logarithmic factor in the potential weakly depends on momentum. Note that at some density the pressure has a plateau and slight negative downturn, which can be interpreted as an instability of homogeneous magnetized matter towards phase separation.

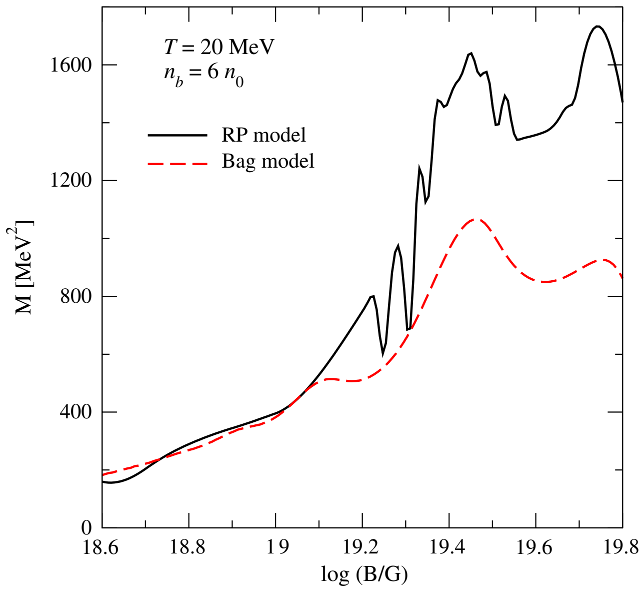

FIG. 3 (color online). Dependence of the magnetization on the magnetic field at baryon number density $n_b = 6n_0$ and $T = 20$ MeV for the RP model (solid line, black line) and MIT bag model (dashed, red line). The bag model result does not depend on the value of the bag constant.

Figure 3 displays the magnetization of matter as a function of the magnetic field for the bag model and RP model at fixed $n = 6n_0$ and $T = 20$ MeV. Note that the magnetization does not depend on the bag constant. For fields $B > 10^{19}$ G the magnetization shows de Haas–van Alfven oscillations in both models. However, the oscillations are much more pronounced in the RP model than in the bag model. This is the consequence of the momentum dependence of the RP interaction, which has the structure of the static gluon exchange. A similar effect was observed in Ref. [2] in a noninteracting strange quark matter model. Note also, the absolute value of the magnetization is by a factor 2 lager in the RP model for sufficiently large fields.

At large magnetic fields the anisotropy due to the magnetic field is important. The pressure components in parallel and perpendicular direction to the magnetic field are not the same. We show the variations of $p_{\parallel}$ and $p_{\perp}$ with B in the RP and bag models at $n = 6n_0$ in Fig. 4. We note that below $B = 3 \times 10^{18}$ G, both $p_{\parallel}$ and $p_{\perp}$ are practically equal to the pressure of matter in absence of a magnetic field. Hence, for the SQM with the model under consideration the effect of the magnetic field is not significant below $B \sim 10^{18}$ G. With the increase of B, $p_{\parallel}$ increases

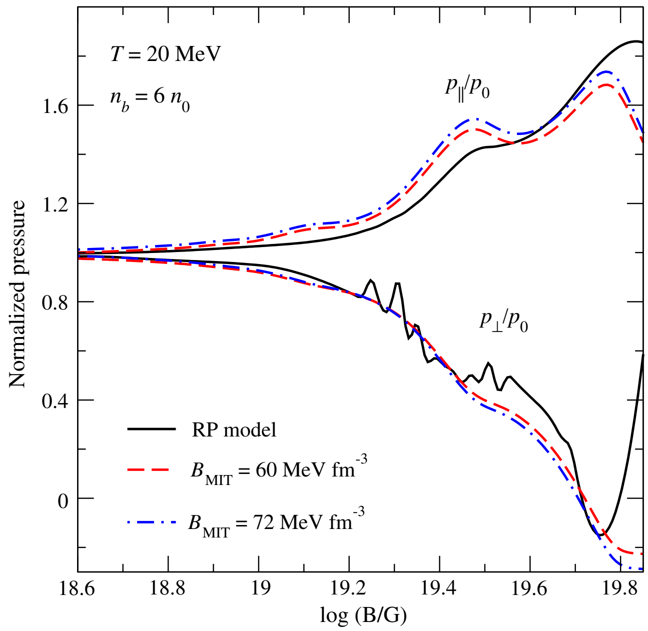

FIG. 4 (color online). Dependence of the normalized pressure in parallel and perpendicular directions to the magnetic field on the strength of the magnetic field at $n_b = 6n_0$ and $T = 20$ MeV for various models (the curves are labeled as in Fig. 2). The upper three curves correspond to the parallel pressure and the lower three curves to the perpendicular pressure.

whereas $p_\perp$ decreases for both models. For large fields, at a certain value of B, $p_\perp$ becomes negative and this critical value is almost the same in both models. Recalling that without the confining potential, at a very large magnetic field $p_\perp \to 0$ [2,10], we see that the confining potential provides additional "attraction" inside the SQM, and its effect becomes more transparent at larger B. The oscillations of the function $p_\perp$ reflect the oscillations in the magnetization.

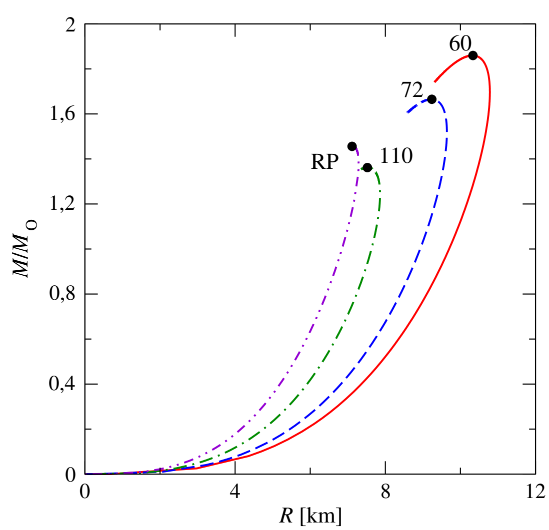

FIG. 5 (color online). Mass-radius relation for strange stars in the absence of magnetic fields. The MIT bag model based results are labeled by the value of the $B_{\rm MIT}$; those based on the RP model by "RP".

The mass-radius relation for the underlying models in the absence of a magnetic field are shown in Fig. 5, which demonstrates the key difference between the RP and bag models in the astrophysics context. Because of the softer equation of state of the RP model the strange stars are more compact (the radii are smaller) and their maximum mass is by about 20% smaller than for the models with $B_{\rm MIT} \sim$ 60–70 MeV fm^{-3}. The computation of the mass-radius relation in the case of strongly magnetized matter can be carried out on the basis of the equations of state obtained in this work. Such calculation requires the solution of Einstein's equations in axial symmetry, because of the anisotropy in the pressure induced by the magnetic field and is beyond the scope of this work (see e.g., [22]).

IV. SUMMARY

In this work we studied the effects of strong magnetic fields, quark-quark confining interaction, and chiral symmetry restoration on the equation of state of the charge neutral strange quark matter. The confining interaction is modeled by the Richardson potential (RP) which features both the asymptotic freedom and the confinement. The chiral symmetry restoration is parametrized as a smooth crossover of the quark masses from their constituent values at low baryon densities to their current ones at large baryon densities. We compared the RP model to the MIT bag model. We find significant differences between the equation of state and the magnetization of the strange quark matter predicted by these models. This is the result of the intrinsic momentum dependence in the interaction of the RP model, which mimics the one-gluon-exchange interaction of the QCD. Specifically, we find that (a) the thermodynamic pressure in the RP model is more sensitive to temperature and baryon density when the magnetic field is strong; (b) the magnetization is larger in the RP model than in the bag model in the limit of large fields, $B > 10^{19}$ G; (c) the de Haas-van Alfvén oscillations in the magnetization and in the transverse pressure $p_\perp$ is more pronounced in the RP model.

Furthermore, we find that the presence of a confining potential, modeled either in terms of the RP potential or the MIT bag, suppresses the pressure components $p_\parallel$ and $p_\perp$ and, at large B, the anisotropy in the equation of state. The splitting between the longitudinal pressure $p_\parallel$ and the transverse pressure $p_\perp$ was found to be weaker than that in free (noninteracting) SQM. This underlines the importance of taking into account the confining potential in studies of strongly magnetic SQM matter in cores of neutron stars and in strange stars. It remains an interesting task to explore the effects of the confining potential in a strong magnetic field on the structure and geometry of such stars.

The strong magnetic fields in the interiors of strange stars will affect the transport process and weak interaction rates. The strong de-Haas–van Alfvén oscillations in the magnetic field will induces oscillations in, for example, the transport coefficients, as demonstrated for the bulk viscosity in Ref. [2]. They will affect the kinematics of Urca processes, as in the case of nucleonic matter [23] and may open an additional channel of neutrino bremsstrahlung due to the Pauli paramagnetic shift in the Fermi levels of quarks [24].

In this work we assumed that the strange matter is in the normal (unpaired) state. It is likely that the flavor symmetric quark matter at low temperatures will be a superfluid. The interplay between the superfluidity and magnetism in quark matter has been studied in a number of contexts [25–31]; however, much remains still unexplored, one possible subject being the extension of the present setup to the case of superfluidity of strange matter.

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