

Supersymmetrizing massive gravity

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When four scalar fields with global Lorentz symmetry are coupled to gravity and take a vacuum expectation value, breaking diffeomorphism invariance spontaneously, the graviton becomes massive. This model is supersymmetrized by considering four $N = 1$ chiral superfields with global Lorentz symmetry. The global supersymmetry is promoted to a local one using the rules of tensor calculus of coupling the $N = 1$ supergravity Lagrangian to the four chiral multiplets. When the scalar components of the chiral multiplets z^A acquire a vacuum expectation value, both diffeomorphism invariance and local supersymmetry are broken spontaneously. The global Lorentz index A becomes identified with the space-time Lorentz index, making the scalar fields z^A vectors and the chiral spinors ψ^A spin-3/2 Rarita-Schwinger fields. We show that the spectrum of the model in the broken phase consists of a massive spin-2 field, two massive spin-3/2 fields with different mass and a massive vector.

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I. INTRODUCTION

Massive gravity was the attraction of many authors. In 1970, van Dam, Veltman and Zakharov [1,2] showed that there is a discrete difference between theories with a zero-mass graviton and theories with a small but nonzero mass. It was concluded that the graviton mass is not some extreme small value and that it should be rigorously zero. They used the action of Fierz and Pauli [3], where general coordinate invariance is broken by mass terms. Their results do not go over into those of general relativity, in particular for the bending of light by the Sun. Massive gravity was first thought to be not physically possible because of this van Dam-Veltman-Zakharov discontinuity [2]. However, Vainshtein showed that the perturbation theory is not suitable when the mass goes to zero because of the singularity in the graviton mass in the higher-order contributions. Then, he resolved this problem [4] by finding that the massive graviton behaves like a massless particle below a certain distance scale. So, the graviton could have a small mass without contradicting experiments. Further developments of this scale were considered in Ref. [5] (see also Ref. [6]).

However, Boulware and Deser [7] investigated the behavior of the massive Einstein theory, and they concluded that it is ill behaved since the ghost scalar does not decouple at the nonlinear level. They deduced that general relativity is an isolated theory. Isham, Salam and Strathdee [8] formulated a Lagrangian theory describing the mixing of the graviton with a massive 2^+f meson. Chamseddine, Salam and Strathdee [9] generalized this by introducing the mixing terms through a spontaneous symmetry-breaking mechanism. Dvali, Gabadadze, and Porrati [10] considered theories with extra dimensions where they considered a five-dimensional model. Their

theory seems to be free of ghosts when considered around a true background, but only in the decoupling limit.

It was believed that since there is no Higgs mechanism that is free of ghosts and returns a mass for the graviton, it fails to obtain consistent massive general relativity free of ghosts in four dimensions. Siegel [11] considered open-string field theories. He used four scalars to restore diffeomorphism invariance. However, studying his theory around a trivial background shows that it is not free of ghosts. Then, Arkani-Hamed, Georgi and Schwartz [12] applied this to massive gravity to introduce general coordinate invariance, but they did not obtain a ghost-free model. Later, 't Hooft [13] (see also Ref. [14]) considered the use of four scalar fields breaking general coordinate invariance by their vacuum expectation value. These scalars give mass to the graviton where at the end a massive spin-2 boson and a massive scalar survive. However, in the unbroken phase, the scalar fields kinetic energies include a ghost. In the broken symmetry phase, the massive graviton does not have a Fierz-Pauli term, and the ghost state remains coupled.

In Ref. [15], Chamseddine and Mukhanov used four scalars with global Lorentz symmetry. They showed how to form massive gravity by using the Higgs mechanism. The graviton will get mass after the four scalar fields acquire nonzero expectation values as a consequence of spontaneous symmetry breaking. Three scalar degrees of freedom are absorbed by the graviton, while one remains coupled. The graviton then becomes massive, with a Fierz-Pauli mass term, and thus has five degrees of freedom. The action is simply given by Einstein action plus the action of the four extra scalar particles. The resulting theory is ghost free below scales related to Vainshtein scales. In Ref. [16], the limit of massive gravity, as the mass of the graviton goes to zero, was studied below the Vainshtein scale. It was shown that it goes smoothly to Einstein gravity. In Ref. [17], massive gravity is presented by a simplified reformulation where a simpler quadratic action is found.

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Ghost-free theories of massive gravity were proposed in four dimensions in Refs. [18,19]. In Ref. [19], nonlinear theories were constructed and it was shown that the Hamiltonian constraint which projects out the Boulware-Deser ghost is maintained up to the quartic order. In Ref. [20], nonlinear massive gravity models, for a flat fiducial metric, were proposed, and the absence of the Boulware-Deser ghost was proven (see also Ref. [21] for a curved fiducial metric). Recently, the characteristics of the ghost-free Wess-Zumino massive gravity model with five degrees of freedom were analyzed in Ref. [22] (see also Ref. [23]). It was shown that it admits superluminal shock-wave solutions and is accordingly acausal, where ironically this originates from the constraint that eliminates the Boulware-Deser ghost.

In this paper, we generalize the Higgs mechanism used in the formulation of massive gravity to obtain a theory of massive supergravity. When massive gravity is supersymmetrized, the graviton and the gravitino both become massive due to the breakdown of diffeomorphism invariance. This theory is interesting because it will end up with a massive spin-3/2 particle in addition to the gravitino. Therefore, two spin-3/2 particles exist, which is similar to what we have in $N = 2$ supergravity. To write our globally supersymmetric action, we use superfields and write an action in superspace using D terms and F terms. Then, we use the rules of tensor calculus to promote global invariance to a local one. We will couple the supergravity Lagrangian [24] to the chiral and vector multiplets by the rules given in Ref. [25] (see also Ref. [24]) to get our final action that is restricted by certain conditions that are discussed below. In Sec. II, we generalize the bosonic case and write down the possible D terms and F terms. In Sec. III, we use tensor calculus to couple to supergravity and find the action in the linearized approximation that is ghost free and returns a Fierz-Pauli term for the vierbein. Section IV is the conclusion. In Appendix A, we list all the possible D -type terms that can be used to form our action. Appendix B presents the notation and convention used.

II. GENERALIZING THE BOSONIC CASE

To generalize the bosonic case, we use instead of four scalar fields a set of four chiral superfields $\Phi^A(x, \theta, \bar{\theta})$ subject to the conditions

$$\bar{D}_{\dot{\alpha}} \Phi^A(x, \theta, \bar{\theta}) = 0,$$

where $A = 0, 1, 2, 3$ is a global Lorentz index. These chiral superfields are given by

$$\begin{aligned} \Phi_A = & \varphi_A + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\varphi_A - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial_\mu\partial^\mu\varphi_A + \sqrt{2}\theta\psi_A \\ & - \frac{i}{\sqrt{2}}\theta\theta(\partial_\mu\psi_A\sigma^\mu\bar{\theta}) + \theta\theta F_A, \end{aligned} \quad (1)$$

where φ_A are complex scalar fields.

In the bosonic case [15], the action is written in terms of an induced metric $H^{AB} = g^{\mu\nu}\partial_\mu\varphi^A\partial_\nu\varphi^B$, and it is found to be

$$\begin{aligned} S = & -\frac{1}{2\kappa^2} \int d^4x\sqrt{-g}R + \frac{m^2}{8} \int d^4x\sqrt{-g}(H^2 - H_B^A H_A^B) \\ & + 3\left(\frac{1}{16}H^2 - 1\right)^2, \end{aligned}$$

where $H = H_A^A$ and $\kappa^2 = 8\pi G$.

There are many other actions, all of which agree at the second-order level but differ at cubic or higher orders. Expanding

$$\varphi^A = (x^A + \chi^A), \quad g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$$

after defining

$$H^{AB} = \eta^{AB} + \bar{h}^{AB},$$

we find that

$$\bar{h}^{AB} = h^{AB} + (\partial^A\chi^B + \partial^B\chi^A) + \dots$$

The action then takes the form

$$\begin{aligned} S = & -\frac{1}{2} \int d^4x\sqrt{-g}R \\ & + \frac{m^2}{8} \int d^4x\sqrt{-g}[(\bar{h}^2 - \bar{h}_B^A\bar{h}_A^B) + \dots], \end{aligned}$$

where $\bar{h} = \bar{h}_A^A$.

The full action can be written in terms of \bar{h}_B^A , since the metric perturbations around Minkowski transform similarly to the infinitesimal transformations which keep Einstein action invariant, $\tilde{x}^A = x^A + \xi^A$, with χ^A instead of ξ^A . Therefore, the action up to second-order terms is given by

$$\begin{aligned} S = & \frac{1}{2} \int d^4x[\bar{h}_B^{A,C}\bar{h}_{A,C}^B - 2\bar{h}_C^{A,C}\bar{h}_{A,D}^D + 2\bar{h}_C^{A,C}\bar{h}_{A,A} - \bar{h}_{A,A}\bar{h}^A \\ & - m^2(\bar{h}_B^A\bar{h}_A^B - \bar{h}^2)]. \end{aligned}$$

The field H is quadratic in the field φ^A , and thus the action is at least quartic in the fields φ^A . We note that the zero and linear terms are canceled through the higher-order term (in the above case it is quartic).

It would be interesting to generalize to the supersymmetric case what was done in Ref. [19] using the quadratic formulation in Ref. [17]; however, this is much more complicated, and such a formulation needs a superspace formulation of supergravity [26].

To form our generalized induced metric, we start by writing a quartic interaction

$$D_\alpha\Phi_A D_\beta\Phi_B \bar{D}^{\dot{\alpha}}\Phi^* \bar{D}^{\dot{\beta}}\Phi^* M_{\dot{\alpha}\dot{\beta}CD}^{\alpha\beta AB},$$

where $M_{\dot{\alpha}\dot{\beta}CD}^{\alpha\beta AB}$ is a multispinor constructed in such a way as to make the action invariant under Lorentz transformations. There are two possible strategies to adopt: to

symmetrize and antisymmetrize with respect to the fermionic indices $\alpha\beta$ and $\dot{\alpha}\dot{\beta}$, or to use the equivalence of $\alpha\dot{\alpha}$ to a vector index

$$V_{\alpha\dot{\alpha}} = \sigma_{\alpha\dot{\alpha}}^{\mu} V_{\mu}.$$

We thus define H_{ABC} as the basic field

$$H_{ABC} = D^{\alpha}\Phi_A(\sigma_B)_{\alpha\dot{\alpha}}\bar{D}^{\dot{\alpha}}\Phi_C^* = D\Phi_A\sigma_B\bar{D}\Phi_C^*. \quad (2)$$

Its Hermitian conjugate is

$$H_{ABC}^* = D\Phi_C\sigma_B\bar{D}\Phi_A^* = H_{CBA}.$$

We also denote $H_{ABC}\eta^{AB}$ by H_{AAC} , and we define the contracted field

$$H_C = H_{AAC}, \quad H_A^* = H_{ACC} \quad (3)$$

to simplify our expressions. The products that could be formed from this H field are given in Appendix A.

We can therefore start from the action

$$\begin{aligned} & c_1 H_{ABC} H_{ABC} + c_2 H_{ABC} H_{ACB} + c_2^* H_{ABC} H_{BAC} \\ & + c_3 H_{ABC} H_{BCA} + c_4 H_{ABC} H_{CAB} + c_5 H_{ABC} H_{CBA} \\ & + c_6 H_A H_A^* + c_7 H_A H_A + c_7^* H_A^* H_A^* \\ & + \epsilon^{ABCD} H_{ABC} (c_8 H_D - c_8^* H_D^*) + \epsilon^{ABCD} (c_9 H_{ABE} H_{CDE} \\ & + c_9^* H_{EAB} H_{ECD} + c_{10} H_{AEB} H_{CED} + c_{11} H_{AEB} H_{ECD} \\ & + c_{11}^* H_{AEB} H_{CDE} + c_{12} H_{EAB} H_{CDE}). \end{aligned}$$

In addition to the D -type terms, we can add to our action F -type terms such as

$$\begin{aligned} & c_{13} \bar{D}^2 (D\Phi_A \sigma^{AB} D\Phi_B) + c_{13}^* D^2 (\bar{D}\Phi_A^* \bar{\sigma}^{AB} \bar{D}\Phi_B^*) \\ & + c_{14} \bar{D}^2 (D\Phi_A D\Phi^A \bar{D}\Phi_B^* \bar{D}\Phi^{B*}) \\ & + c_{14}^* D^2 (\bar{D}\Phi_A^* \bar{D}\Phi^{A*} D\Phi_B D\Phi^B), \end{aligned} \quad (4)$$

where all the constants c_i are real except for those whose conjugates appear (i.e., c_2 , c_7 , c_8 , c_9 , c_{11} , c_{13} , c_{14} are complex). Many more F -type terms can be written, but we wrote only those we are going to use.

In the bosonic case [15] we have seen that the action with the correct behavior is expressed in terms of the field \bar{h}_{AB} , where

$$\bar{h}_{AB} = H_{AB} - \eta_{AB} \equiv H_{AB} - \partial_{\mu} x_A \partial^{\mu} x_B,$$

so that in this case there is no need to consider higher-order terms in H_{AB} and it is enough to consider the terms

$$(\bar{h}_B^A \bar{h}_A^B - \bar{h}^2).$$

For this, we instead work with

$$\bar{H}_{ABC} = H_{ABC} - D x_A \sigma_B \bar{D} x_C^*,$$

where x_A are the coordinates, since this will avoid including higher-order terms in H_{ABC} . As we will prove below, at the end the action will be formed of three D -type terms and two F -type terms. It will be given by

$$\begin{aligned} & m^4 \int (c_1 \bar{H}_{ABC} \bar{H}_{BCA} + c_2 \bar{H}_{ABB} \bar{H}_{CCA} \\ & + c_3 \bar{H}_{AB} \bar{H}_{AB}^*) d\theta^2 d\bar{\theta}^2 d^4 x \\ & + \frac{m^2}{\kappa} \int (c_4 \bar{D}^2 (D\Phi_A \sigma^{AB} D\Phi_B) d\theta^2 d^4 x + \text{H.c.}) \\ & + m^4 \int (c_5 \bar{D}^2 (D\Phi_A D\Phi^A \bar{D}\Phi_B^* \bar{D}\Phi^{B*}) d\theta^2 d^4 x + \text{H.c.}), \end{aligned} \quad (5)$$

where $H_{AB} = D\Phi_A D\Phi_B$, and m and κ are used to fix the dimensions.

III. COUPLING TO SUPERGRAVITY

To couple our supersymmetric action to supergravity, we first start by writing down the supergravity Lagrangian. We need first to define the vierbein

$$e_a^{\mu} = g^{\mu\nu} e_{\nu a} = g^{\mu\nu} \eta_{ab} e_b^{\nu}, \quad (6)$$

and its relation to the spin connection $w_{\nu a}{}^b$ is given by the equation

$$\partial_{\nu} e_a^{\mu} = -w_{\nu a}{}^b e_b^{\mu} - \Gamma_{\gamma\nu}^{\mu} e_a^{\gamma}. \quad (7)$$

The supergravity Lagrangian field content consists of the spin-2 field $e_{a\mu}$, the spin-3/2 field ψ_{μ} , and the auxiliary fields S , P , A_{μ} . This Lagrangian is given by [24]

$$L_{S,G} = -\frac{e}{2\kappa^2} R(e, w) - \frac{e}{3} |u|^2 + \frac{e}{3} A_{\mu} A^{\mu} - \frac{1}{2} \bar{\phi}_{\mu} R^{\mu}, \quad (8)$$

where

$$u = S - iP, \quad (9)$$

$$R_{\mu\nu}{}^{rs} = \partial_{\mu} w_{\nu}{}^{rs} + w_{\mu}{}^{rp} w_{\nu p}{}^s - \mu \leftrightarrow \nu, \quad (10)$$

$$R^{\mu} = \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_{\nu} D_{\rho}(w) \phi_{\sigma}, \quad (11)$$

$$R = e_r{}^{\mu} e_s{}^{\nu} R_{\mu\nu}{}^{rs}, \quad (12)$$

$$D_{\mu} = \partial_{\mu} + (1/2) w_{\mu rs} \sigma^{rs}, \quad (13)$$

$$w_{\mu rs} = w_{\mu rs}(e) + K_{\mu rs}(e, \phi_{\mu}), \quad (14)$$

$$K_{\mu rs}(e, \phi_{\mu}) = (\kappa^2/4) (\bar{\phi}_{\mu} \gamma_r \phi_s - \bar{\phi}_{\mu} \gamma_s \phi_r + \bar{\phi}_r \gamma_{\mu} \phi_s), \quad (15)$$

and e is the determinant of the vierbein. This Lagrangian is invariant under local supersymmetry transformations up to a total divergence.

Next, we couple this to the supersymmetric action using the rules of tensor calculus. These rules provide us with equations showing how to couple the components of chiral and vector multiplets to supergravity [24] (see also Ref. [25]). The global supersymmetry will then be

promoted to a local one. Below is a review of how this is done. The component fields of a vector multiplet are Majorana spinors (ξ and λ), two scalars (C and M), and one auxiliary scalar field (D). It is given by

$$V = (C, \xi, M, V_\mu, \lambda, D). \quad (16)$$

A left-handed chiral multiplet (F -type) contains a complex scalar field z , left-handed Weyl spinors X_L , and a complex auxiliary field h . Then it is

$$F = (z, X_L, h). \quad (17)$$

For the F -type multiplet the action formula is

$$e^{-1}L_F = h + \kappa u z + \kappa \bar{\phi}_\mu \gamma^\mu \chi + i \kappa^2 \bar{\phi}_\mu \gamma^{\mu\nu} \phi_{\nu R} z + \text{H.c.}, \quad (18)$$

while for the D -type multiplet one has

$$\begin{aligned} e^{-1}L_D = & D + \frac{i\kappa}{2} \bar{\phi}_\mu \gamma^5 \gamma^\mu \lambda - \frac{\kappa}{3} (u M^* + u^* M) \\ & + \frac{i\kappa^2}{8} \epsilon^{\mu\nu\rho\sigma} \bar{\phi}_\mu \gamma_\nu \phi_\rho \bar{\xi} \phi_\sigma \\ & + \frac{2}{3} \kappa V_\mu \left(A^\mu + \frac{3}{8} i e^{-1} \epsilon^{\mu\rho\sigma\tau} \bar{\phi}_\rho \gamma_\tau \phi_\sigma \right) \\ & - i \frac{\kappa}{3} e^{-1} \bar{\xi} \gamma_5 \gamma_\mu R^\mu - \frac{2}{3} \kappa^2 C e^{-1} L_{S.G.} + e^{-1} L_{S.G.} \end{aligned} \quad (19)$$

To eliminate the auxiliary fields u and A_μ , we first find their equations of motion. These are given, respectively, by

$$\kappa z - \frac{\kappa}{3} M^* - \frac{1}{3} u^* = 0, \quad \frac{2}{3} \kappa V_\mu + \frac{1}{3} A_\mu = 0. \quad (20)$$

Plugging back the auxiliary fields u and A_μ (using their equations of motion) into the Lagrangian, we get

$$\begin{aligned} e^{-1}L_F + e^{-1}L_D = & D - \frac{1}{2\kappa^2} R(e, w) - \frac{1}{2} e^{-1} \bar{\phi}_\mu R^\mu + h \\ & + h^* - \kappa^2 (M z + M^* z^*) + 3\kappa^2 z z^* \\ & + (\kappa \bar{\phi}_\mu \gamma^\mu \chi + i \kappa^2 \bar{\phi}_\mu \gamma^{\mu\nu} \phi_{\nu R} z + \text{H.c.}) \end{aligned} \quad (21)$$

Therefore, to write down the full Lagrangian, we have first to express the supermultiplets in terms of their component fields. Substituting for the metric $g^{\mu\nu} = e_A^\mu e^{\nu A}$ and expanding the fields around the vacuum solution

$$\varphi^A = x^A + \chi^A, \quad e_A^\mu = \delta_A^\mu + \bar{e}_A^\mu, \quad (22)$$

the components of our superfields, ignoring terms higher than quadratic order, are given by

(1) For the superfield $\bar{H}_{ABC} \bar{H}^{BCA}$:

$$\begin{aligned} C = 0, \quad \xi = 0, \quad M = & -8(\psi_A \sigma_B \bar{\sigma}^A \psi^B), \quad V_\mu = \text{quadratic}, \quad \lambda = \text{quadratic}, \\ D = & -16(\partial_\mu \chi_A \partial^\mu \chi^A + \partial_\mu \chi_A^* \partial^\mu \chi^{A*}) + 32(\partial_\mu \chi_A \partial^\mu \chi^{A*} + \partial^A \chi_A \partial^B \chi_B^*) + 80F_A F^{A*} - 8\epsilon^{ABCD} \psi_A \sigma_B \partial_C \bar{\psi}_D \\ & - 8i(\psi_A \sigma^A \partial_B \bar{\psi}^B + \psi_A \sigma^B \partial^A \bar{\psi}_B) - 56i \psi_A \sigma^\mu \partial_\mu \bar{\psi}^A + 32\bar{e} \partial_A \chi^A + 32\bar{e} \partial_A \chi^{A*} + 32\bar{e}^2, \end{aligned} \quad (23)$$

where V_μ and λ do not affect our results, since they will give terms with higher orders. In these calculations, indices are lowered and raised by $\eta^{\mu A}$. Similarly, the components of the other vector multiplets are calculated. Below we list only the two supermultiplets that at the end will enter into the action.

(2) For $\bar{H}_{AB}{}^B \bar{H}_C{}^{CA}$:

$$\begin{aligned} C = 0, \quad \xi = 0, \quad M = & 16(\psi_A \sigma^A \bar{\sigma}^B \psi_B), \quad V_\mu = \text{quadratic}, \quad \lambda = \text{quadratic}, \\ D = & 32(\partial_\mu \chi_A \partial^\mu \chi^A + \partial_\mu \chi_A^* \partial^\mu \chi^{A*}) + 32\partial^A \chi_A \partial^B \chi_B^* + 128\partial_\mu \chi_A \partial^\mu \chi^{A*} + 272F_A F^{A*} \\ & + 8\epsilon^{ABCD} \psi_A \sigma_B \partial_C \bar{\psi}_D - 8i(\psi_A \sigma^A \partial_B \bar{\psi}^B + \psi_A \sigma^B \partial^A \bar{\psi}_B) - 200i \psi_A \sigma^\mu \partial_\mu \bar{\psi}^A + 96\bar{e}^2 + 128\bar{e}_a^a \bar{e}_a^\mu \\ & + 96\bar{e} \partial_A \chi^A + 96\bar{e} \partial_A \chi^{A*} + 128\bar{e}_A^\mu \partial_\mu \chi^A + 128\bar{e}_A^\mu \partial_\mu \chi^{A*}. \end{aligned} \quad (24)$$

(3) For the third, $(D\Phi_A D\Phi_B)(\bar{D}\Phi^{B*} \bar{D}\Phi^{A*})$:

$$\begin{aligned} C = 0, \quad \xi = 0, \quad M = 0, \quad V_\mu = 0, \quad \lambda = & \text{quadratic}, \\ D = & 32(\partial^A \chi_A \partial^B \chi_B^* + \partial_\mu \chi_A \partial^\mu \chi^{A*}) + 80F_A F^{A*} - 8\epsilon^{ABCD} \psi_A \sigma_B \partial_C \bar{\psi}_D - 8i(\psi_A \sigma^A \partial_B \bar{\psi}^B + \psi_A \sigma^B \partial^A \bar{\psi}_B) \\ & - 56i \psi_A \sigma^\mu \partial_\mu \bar{\psi}^A + 64\bar{e}_\mu^a \bar{e}_a^\mu + 64\bar{e}_A^\mu \partial_\mu \chi^A + 64\bar{e}_A^\mu \partial_\mu \chi^{A*}. \end{aligned} \quad (25)$$

From the above vector multiplets, we can form an action with the following required conditions:

- (1) It has a Fierz-Pauli term for the vierbeins ($\bar{e}_A^\mu \bar{e}_\mu^A - \bar{e}^2$). This choice decouples the sixth degree of freedom of the massive graviton.
- (2) It contains no linear vierbein term.
- (3) It gives a Maxwell form for the χ_A fields,

$$l(\partial_\mu \chi_A \partial^\mu \chi^{A*} - \partial_A \chi^A \partial_B \chi^{B*}), \quad (26)$$

where l is a constant.

- (4) It is ghost free where there should be no terms like

$$\partial_\mu \chi_A \partial^\mu \chi^A \quad \text{or} \quad \partial_A \chi^A \partial_B \chi^{B*}. \quad (27)$$

- (5) The gravitinos should be massive.

The first four conditions are well satisfied if we only consider D -type terms. However, to make the gravitino massive, F -type terms should be included. It is found out that only such terms will return a mass term for the gravitino.

Several F -type terms can be written. Calculations show that the two terms listed above will give us the required results. Their components are given by

- (1) $\bar{D}^2(D\Phi_A \sigma^{AB} D\Phi_B)$:

$$\begin{aligned} z &= -96i - 48i\partial^A \chi_A - 48i\bar{e} - 16i\bar{e}\partial^A \chi_A - 32i\bar{e}_A^\mu \partial_\mu \chi^A + 8i\bar{e}_\mu^A \bar{e}_A^\mu - 8i\bar{e}^2, \\ X_\alpha &= -4\sqrt{2}(\sigma^{AB} \psi_A)_\alpha \partial_\mu \partial^\mu \chi_B - 16\sqrt{2}i(\partial^A \psi_A)_\alpha \partial^B \chi_B - 48\sqrt{2}i(\partial^A \psi_A)_\alpha + 16\sqrt{2}i(\partial^A \psi_B)_\alpha \partial^B \chi_A, \\ h &= \text{total derivative.} \end{aligned} \quad (28)$$

h is a total derivative; therefore it will not affect our calculations.

- (2) $\bar{D}^2(D\Phi_A D\Phi^A \bar{D}\Phi_B^* \bar{D}\Phi^{B*})$:

$$\begin{aligned} z &= -128\bar{\psi}_A \bar{\psi}^A, \\ X_\alpha &= 128\sqrt{2}i(\sigma^B \bar{\psi}_B)_\alpha \partial^A \chi_A + 64 \times 4\sqrt{2}i(\sigma^\nu \bar{\psi}_B)_\alpha \partial_\nu \chi_B^*, \\ h &= -64 \times 4(\partial_\mu \chi_A \partial^\mu \chi^A + \partial_\mu \chi_A^* \partial^\mu \chi^{A*} + \partial^A \chi_A \partial^B \chi_B^*) - 64 \times 2F_A F^{A*} - 64\epsilon^{ABCD} \psi_A \sigma_B \partial_C \bar{\psi}_D \\ &\quad + 64i(\psi_A \sigma^A \partial_B \bar{\psi}^B + \psi_A \sigma^B \partial^A \bar{\psi}_B + \psi_A \sigma^\mu \partial_\mu \bar{\psi}^A) - 64 \times 16 - 64 \times 16\bar{e} - 64 \times 8\bar{e}_\mu^A \bar{e}_A^\mu \\ &\quad - 64 \times 4\bar{e}^2 + 64(\bar{e}\partial_A \chi^A + \bar{e}\partial_A \chi^{A*}) + 64 \times 4(\bar{e}_A^\mu \partial_\mu \chi^A + \bar{e}_A^\mu \partial_\mu \chi^{A*}). \end{aligned} \quad (29)$$

Forcing the constraints mentioned above to obtain a well-behaved action, we can write a system of equations to solve for the constants c_1, c_2, c_3, c_4 and c_5 . The equations are found to be

- (1) No ghost: $-16c_1 + 32c_2 - 64 \times 4(c_5 + c_5^*) = 0$.
- (2) Maxwell: $32c_1 + 64 \times 2c_2 + 32c_3 = l$ and $32c_1 + 32c_2 + 32c_3 + 48^2 \times 3c_4 c_4^* - 64 \times 4(c_5 + c_5^*) = -l$.
- (3) Constant: $-64 \times 16(c_5 + c_5^*) + 96^2 \times 3c_4 c_4^* = 0$.

The solution is

$$c_1 = \frac{l}{24} - 432c_4 c_4^*, \quad c_2 = \frac{l}{48}, \quad c_3 = -\frac{3l}{32} + 432c_4 c_4^*, \quad c_5 = \frac{27}{2} c_4 c_4^*. \quad (30)$$

By normalizability of the kinetic term of the χ^A field, we have $l = -1/2$. Moreover, c_4 is arbitrary, but we can choose it such that the term $\bar{\psi}_A \gamma^A \gamma^B \psi_B$ cancels out. This sets c_4 to be $\frac{i}{48\sqrt{6}}$ and reduces the full Lagrangian ($e^{-1}L_F + e^{-1}L_D$) to

$$\begin{aligned}
& -\frac{1}{2}m^4(\partial_\mu\chi_A\partial^\mu\chi^{A*} - \partial^A\chi_A\partial^B\chi_B^*) + \frac{7}{3}m^4(\bar{e}_A^\mu\bar{e}_A^\mu - \bar{e}^2) - m^4F_AF^{A*} - \frac{7}{3}m^4(\bar{e}\partial_A\chi^A + \bar{e}\partial_A\chi^{A*}) \\
& + \frac{7}{3}m^4(\bar{e}_A^\mu\partial_\mu\chi^A + \bar{e}_A^\mu\partial_\mu\chi^{A*}) - \frac{5}{24}m^4\epsilon^{ABCD}\bar{\psi}_A\gamma_B\gamma_5\partial_C\psi_D - \frac{29\sqrt{6}i}{36}m^6\kappa\bar{\psi}_A\gamma^{AB}\psi_B + \frac{3i}{8}m^4\bar{\psi}_A\gamma_\mu\partial^\mu\psi^A \\
& - \frac{5\sqrt{6}}{24}m^6\kappa\bar{\psi}_A\gamma^A\gamma^B\psi_B + \frac{1}{2}e^{-1}\epsilon^{\mu\nu\rho\sigma}\bar{\phi}_\mu\gamma_\nu\gamma_5\partial_\rho\phi_\sigma + \frac{\sqrt{6}i}{3}m^2\kappa\bar{\phi}_\mu\gamma^{\mu\nu}\phi_\nu + \frac{\sqrt{2}i}{4}m^4\kappa\bar{\phi}_\mu\gamma^\mu\gamma^A\psi_A \\
& - \frac{\sqrt{2}i}{4}m^4\kappa\bar{\psi}_A\gamma^A\gamma^\mu\phi_\mu + \frac{\sqrt{3}}{6}m^2\bar{\phi}_\mu\gamma^\mu\partial^A\psi_A + \frac{\sqrt{3}}{6}m^2\partial^A\bar{\psi}_A\gamma^\mu\phi_\mu + \frac{\sqrt{3}}{12}m^2\kappa\bar{\phi}_\mu\gamma^\mu\gamma^A\gamma^B\partial_B\psi_A \\
& + \frac{\sqrt{3}}{12}m^2\kappa\partial_B\bar{\psi}_A\gamma^B\gamma^A\gamma^\mu\phi_\mu - \frac{1}{2\kappa^2}R(e, w). \tag{31}
\end{aligned}$$

Now it is clear how m and κ fix the dimensions, where we have $[\chi_A] = -1$, $[\bar{e}] = 0$, $[F_A] = 0$, $[\psi_A] = -1/2$ and the gravitino $[\phi_\mu] = 3/2$.

The equations of motion for $\bar{\psi}_A$ and $\bar{\phi}_\mu$ are, respectively,

$$\begin{aligned}
& -\frac{5}{24}m^4\epsilon^{ABCD}\gamma_B\gamma_5\partial_C\psi_D + \frac{3i}{8}m^4\gamma_\mu\partial^\mu\psi^A - \frac{\sqrt{6}}{8}m^6\kappa\psi^A - \frac{5\sqrt{6}}{36}m^6\kappa\gamma^B\gamma^A\psi_B + \frac{\sqrt{6}}{18}m^6\kappa\gamma^A\gamma^B\psi_B \\
& - \frac{\sqrt{3}}{6}m^2\gamma^\mu\partial^A\phi_\mu - \frac{\sqrt{2}i}{4}m^4\kappa\gamma^A\gamma^\mu\phi_\mu - \frac{\sqrt{3}}{12}m^2\kappa\gamma^B\gamma^A\gamma^\mu\partial_B\phi_\mu = 0 \tag{32}
\end{aligned}$$

and

$$\frac{\sqrt{3}}{6}m^2\gamma^\mu\partial_A\psi^A + \frac{\sqrt{2}i}{4}m^4\kappa\gamma^\mu\gamma^\nu\psi_\nu + \frac{\sqrt{3}}{12}m^2\gamma^\mu\gamma^A\gamma^B\partial_B\psi_A + \frac{\sqrt{6}i}{3}m^2\kappa\gamma^{\mu\nu}\phi_\nu + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\phi_\sigma = 0. \tag{33}$$

Next, we can decompose ψ_A into a spin-3/2 helicity, $\hat{\psi}_A$, and a spin-1/2 helicity, λ :

$$\psi_A = \hat{\psi}_A + \frac{1}{4}\gamma_A\gamma_5\lambda \Rightarrow \bar{\psi}_A = \bar{\hat{\psi}}_A + \frac{1}{4}\bar{\lambda}\gamma_A\gamma_5, \tag{34}$$

where $\gamma_A\hat{\psi}^A = 0$. Similarly, we decompose ϕ_μ

$$\phi_\mu = \hat{\phi}_\mu + \frac{1}{4}\gamma_\mu\gamma_5\eta \Rightarrow \bar{\phi}_\mu = \bar{\hat{\phi}}_\mu + \frac{1}{4}\bar{\eta}\gamma_\mu\gamma_5, \tag{35}$$

where $\gamma_\mu\hat{\phi}^\mu = 0$ and again $\hat{\phi}_\mu$ is a spin-3/2 helicity, while η is spin-1/2 helicity. Using this decomposition, the equations of motion become

$$\begin{aligned}
& -\frac{5}{24}m^4\epsilon^{ABCD}\gamma_B\gamma_5\partial_C\hat{\psi}_D + \frac{5}{96}m^4\epsilon^{ABCD}\gamma_B\gamma_D\partial_C\lambda + \frac{3i}{8}m^4\gamma_\mu\partial^\mu\hat{\psi}^A + \frac{3i}{32}m^4\gamma_\mu\gamma^A\gamma^5\partial^\mu\lambda + \frac{3\sqrt{6}}{32}m^6\kappa\gamma^A\gamma^5\lambda \\
& - \frac{29\sqrt{6}}{72}m^6\kappa\hat{\psi}^A - \frac{\sqrt{3}}{6}m^2\gamma^5\partial^A\eta - \frac{\sqrt{2}i}{4}m^4\kappa\gamma^A\gamma^5\eta - \frac{\sqrt{3}}{12}m^2\kappa\gamma^B\gamma^A\gamma^5\partial_B\eta = 0 \tag{36}
\end{aligned}$$

and

$$\frac{\sqrt{3}}{3}m^2\gamma^\mu\partial_A\hat{\psi}^A + \frac{\sqrt{2}i}{4}m^4\kappa\gamma^\mu\gamma^5\lambda + \frac{\sqrt{6}}{6}m^2\kappa\hat{\phi}^\mu - \frac{\sqrt{6}}{8}m^2\kappa\gamma^\mu\gamma^5\eta + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\partial_\rho\hat{\phi}_\sigma + \frac{1}{2}\gamma^5\gamma^{\mu\rho}\partial_\rho\eta = 0. \tag{37}$$

To simplify these field equations, we multiply Eqs. (36) and (37) by γ_A and γ_μ , respectively. Also, we trace these equations by ∂_A and ∂_μ , respectively. We can then write $\hat{\psi}^A$ in terms of λ and η :

$$\partial_A\hat{\psi}^A = \frac{3}{8}\gamma^5\gamma^A\partial_A\lambda + \frac{9\sqrt{6}i}{8}m^2\kappa\gamma^5\lambda + 3\sqrt{2}\kappa\gamma^5\eta, \tag{38}$$

and a similar equation for $\partial_A\hat{\phi}^A$:

$$\partial_A\hat{\phi}^A = \frac{-\sqrt{3}i}{2}m^2\gamma^5\gamma^A\partial_A\lambda + \frac{11\sqrt{2}}{2}m^4\kappa\gamma^5\lambda - \frac{3}{4}\gamma^5\gamma_A\partial^A\eta - \frac{7\sqrt{6}i}{2}m^2\kappa\gamma^5\eta. \tag{39}$$

Also, an equation relating λ and η is found. This is given by

$$\begin{aligned}
 & -\frac{5\sqrt{6}}{24}m^4\kappa\gamma^A\partial_A\lambda - \frac{109i}{4}m^6\kappa^2\lambda - 4\sqrt{2}im^2\kappa\gamma^A\partial_A\eta \\
 & - \frac{137\sqrt{3}}{6}m^4\kappa^2\eta - 2\sqrt{3}\partial_A\partial^A\eta = 0.
 \end{aligned} \tag{40}$$

Since the action is invariant under local supersymmetry transformations, we can choose the gauge to be $\eta = 0$. Then we get

$$\gamma_A\partial^A\lambda + \frac{109\sqrt{6}}{5}im^2\kappa\lambda = 0. \tag{41}$$

This gives a Dirac-type equation for the spin-1/2 helicities.

It should be noted that the divergence of $\hat{\phi}$ is found in terms of λ . However, we can find a combination of $\hat{\phi}_A$ and $\hat{\psi}_A$,

$$\hat{\phi}'_A = \hat{\phi}_A + \alpha\hat{\psi}_A, \tag{42}$$

such that the divergence of $\hat{\phi}'$ equals zero ($\partial^A\hat{\phi}'_A = 0$) [27]. Then $\hat{\phi}'$ has two helicities, 3/2 and $-3/2$.

To count degrees of freedom, we are coupling supergravity to a $N = 1$ supersymmetry model similar to the Wess-Zumino model. Before the coupling, supergravity contains two bosonic degrees of freedom (massless spin-2 graviton) and two fermionic degrees of freedom (one massless spin-3/2 gravitino). The $N = 1$ supersymmetry model has four spin-0 particles, φ^A , with only six degrees of freedom (3 times 2), since φ^0 decouples due to Fierz-Pauli choice. For this, we have six fermionic degrees of freedom forming a multiplet. Therefore, we started with an overall eight fermionic degrees of freedom and eight bosonic degrees of freedom.

After coupling to supergravity, we obtain $N = 1$ massive representation, having the same number of degrees of freedom as before coupling. Starting with the bosonic degrees of freedom, we have a massive spin-2 particle, with five degrees of freedom. Also, when coordinate invariance is broken by the four scalars, the Lorentz symmetry of the tangent manifold gets identified with that of space-time, and the four scalars become four-vectors. Therefore, we add to the bosonic degrees of freedom a massive vector field (spin-1 particle), having three degrees of freedom. This will constitute the overall eight bosonic degrees of freedom. The fermionic degrees of freedom arise from having two massive spin-3/2 particles, ϕ_μ and ψ_A , with four degrees of freedom each.

At the end, we are left with two massive spin-3/2 particles, ϕ_μ and ψ_A . From the full Lagrangian [Eq. (31)], we can get the mass of each by comparing with the Rarita-Schwinger Lagrangian. The gravitino, ψ_A , can be rescaled to get a normalized kinetic term, and then its mass value is found to be $\frac{58\sqrt{6}}{15}m^2\kappa$, while the mass of the ϕ_μ is $\frac{2\sqrt{6}}{3}m^2\kappa$. This is similar to the $N = 2$ supersymmetry, in which we have two gravitinos, but there they have the same

mass. However, in our case, supersymmetry is completely broken. Since it is a space-time symmetry, it is broken exactly at the same scale as the diffeomorphism breaking. Before diffeomorphism breaking, we had spin-1/2 and not spin-3/2; therefore the two gravitinos would not have the same mass. We coupled a pure supergravity multiplet to a matter multiplet, and we are left with two massive spin-3/2 particles having completely different masses. ϕ_μ is a genuine gravitino, while the other, ψ_A , becomes identified with a gravitino after the breaking.

IV. CONCLUSION

In this paper, we gave a detailed derivation of supersymmetrizing massive gravity. Generalizing the Higgs mechanism used before to make the graviton massive, we were able to form a massive supergravity action. We started with four $N = 1$ chiral superfields that break diffeomorphism invariance and local supersymmetry by the scalar component taking a vacuum expectation value. To write the full Lagrangian, we wrote the supermultiplets in terms of their component fields. First, we started by writing down all the possible D -type terms. For this we added F -type terms to satisfy all required conditions. This was coupled to supergravity using the rules of tensor calculus for chiral and vector multiplets. At the end, the degrees of freedom were analyzed and the equations of motion were obtained. In what done, we were not able to see the ghost, because we were not going to the nonlinear level, and any emergence of such ghosts occurs at higher orders.

Much work remains to be done, as this paper shows that it is possible to construct a sensible theory of supersymmetric massive gravity with a Higgs mechanism. It remains to be seen whether it is possible to construct the action from the basic field H_{ABC} by adding higher-order terms. Another possibility is to generalize the simpler quadratic action [17] to the supersymmetric case. Also, one could analyze higher orders where ghosts may be present.

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APPENDIX A: D -TYPE AND F -TYPE TERMS

The products that could be formed as D -type terms are given by

- (1) $H_{ABC}H_{ABC}$, where $(H_{ABC}H_{ABC})^* = H_{CBA}H_{CBA} = H_{ABC}H_{ABC}$ and is self-adjoint.
- (2) $H_{ABC}H_{ACB}$, where $(H_{ABC}H_{ACB})^* = H_{CBA}H_{BCA} = H_{ABC}H_{BAC}$.
- (3) $H_{ABC}H_{BCA}$, where $(H_{ABC}H_{BCA})^* = H_{CBA}H_{ACB} = H_{ABC}H_{BCA}$ and is self-adjoint.

- (4) $H_{ABC}H_{CAB}$, where $(H_{ABC}H_{CAB})^* = H_{CBA}H_{BAC} = H_{ABC}H_{CAB}$ is self-adjoint.
- (5) $H_{ABC}H_{CBA}$, where $(H_{ABC}H_{CBA})^* = H_{CBA}H_{ABC}$ and is self-adjoint.
- (6) $H_A H_A^*$ is self-adjoint.
- (7) $H_A H_A$, where $(H_A H_A)^* = H_A^* H_A^*$.
- (8) $\epsilon^{ABCD} H_{ABC} H_D$, where $(\epsilon^{ABCD} H_{ABC} H_D)^* = -\epsilon^{ABCD} H_{ABC} H_D^*$.
- (9) $\epsilon^{ABCD} H_{ABE} H_{CDE}$, where $(\epsilon^{ABCD} H_{ABE} H_{CDE})^* = \epsilon^{ABCD} H_{EBA} H_{EDC} = \epsilon^{ABCD} H_{EAB} H_{ECD}$.
- (10) $\epsilon^{ABCD} H_{AEB} H_{CED}$, where $(\epsilon^{ABCD} H_{AEB} H_{CED})^* = \epsilon^{ABCD} H_{AEB} H_{CED}$ is self-adjoint.
- (11) $\epsilon^{ABCD} H_{AEB} H_{ECD}$, where $(\epsilon^{ABCD} H_{AEB} H_{ECD})^* = \epsilon^{ABCD} H_{AEB} H_{CDE}$.
- (12) $\epsilon^{ABCD} H_{EAB} H_{CDE}$, where $(\epsilon^{ABCD} H_{EAB} H_{CDE})^* = \epsilon^{ABCD} H_{BAE} H_{EDC} = \epsilon^{ABCD} H_{EAB} H_{CDE}$ is self-adjoint.

APPENDIX B: NOTATION AND CONVENTION

The conventions used are those by Wess and Bagger [26] with some differences, which are as follows:

- (1) $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$.
- (2) $\epsilon_{\alpha\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix};$
 $\epsilon^{\alpha\beta} = \epsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$
- (3) $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}.$
- (4) $(\sigma^{\mu\nu})_{\alpha}^{\beta} \equiv \frac{i}{4}(\sigma^{\mu}\bar{\sigma}^{\nu} - \sigma^{\nu}\bar{\sigma}^{\mu})_{\alpha}^{\beta};$
 $(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \equiv \frac{i}{4}(\bar{\sigma}^{\mu}\sigma^{\nu} - \bar{\sigma}^{\nu}\sigma^{\mu})^{\dot{\alpha}}_{\dot{\beta}}.$
- (5) $\epsilon_{0123} = +1.$

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