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Presence or absence of light moduli: The controlling feature for supersymmetry phenomenology

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Supersymmetry and string theory suggest the existence of light moduli. Their presence, or absence, controls the realization of supersymmetry at low energies. If there are no such fields, or if all such fields are fixed in a supersymmetric fashion, the conventional thermal production of lightest supersymmetric particle dark matter is possible, as is an anomaly-mediated (''mini-split'') spectrum. On the other hand, the axion solution to the strong *CP* problem is not operative, and slow roll inflation appears difficult to implement. If there are light moduli, a mini-split spectrum is less generic, weakly interacting dark matter appears atypical, and the supersymmetry scale is likely tens of TeV or higher.

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I. INTRODUCTION

Both the relatively large mass of the Standard Model–like Higgs boson discovered at the LHC [1,2] and current bounds on superpartners place tension on models of weak-scale supersymmetry. It is possible to achieve a Higgs mass of 125 GeV in nonminimal models while placing superpartners just beyond the reach of current searches and minimizing the usual measures of fine-tuning [3]. However, within the minimal model, a Standard Model–like Higgs boson at 125 GeV is suggestive of SUSY-breaking in the range of 10s of TeV or higher [4–9], corresponding in the most naive estimate to parameter tuning at a level of part-in-10^4 if the cutoff scale is high. Thus it is worth considering the possibility that if supersymmetry plays a role in nature, the scale of its breaking is higher than expected from conventional ideas of naturalness.

Prior to the LHC exclusions and the Higgs discovery, at least two arguments pointed to a SUSY-breaking scale above (likely well above) 10 TeV. The first is the experimental constraints on flavor-changing neutral currents and *CP* violation. If phases and mixings are $\mathcal{O}(1)$, these processes already probe SUSY scales in the range of hundreds of TeV (see, for example, the recent study of [10]). However, if phases and mixings are small, either accidentally or due to some structure, the bounds are diluted.

The second argument arises from the cosmological moduli problem [11,12]. Moduli (strictly speaking pseudomoduli) are ubiquitous in string compactifications. They are characterized by the property that sufficiently far away in the field space, the potential for these fields vanishes. In theories of low-scale supersymmetry breaking, one expects the masses of these fields to be of order of the gravitino mass or larger. While on the one hand such fields might seem problematic, on the other they might play a role in understanding two pressing problems in particle physics and cosmology: the strong *CP* problem and inflation. Many string moduli respect discrete shift symmetries, which have the potential to give rise to accidental, continuous Peccei-Quinn symmetries. Moreover, fields with very flat potentials would seem desirable to account for slow roll inflation. But quite generally, if present, moduli have (other) profound consequences for cosmology. Except under special circumstances, before these fields settle into their ground state, for a long period they dominate the energy density of the universe. If the moduli have Planck-suppressed couplings, then unless they are quite heavy, they decay long after nucleosynthesis, destroying light elements and spoiling the success of this pillar of the Big Bang theory. Moduli masses must be at least 10s of TeV and probably larger if their lifetimes are to be sufficiently short. In this case, however, lightest supersymmetric particle (LSP) dark matter is not produced thermally. The possibility that a stable LSP might be nonthermally produced via moduli decays has been widely discussed (see, for example, [13,14]).

Therefore several considerations point to a surprisingly high scale of supersymmetry breaking. Adopting this ''heavy SUSY'' viewpoint raises the following questions:

(1) Even if supersymmetry is broken at a high scale, might some states remain parametrically lighter and be directly visible at the LHC? Are the heavy states indirectly visible in experiments searching for electric dipole moments or rare decays?

(2) Once one has admitted some degree of fine-tuning, how much fine-tuning might there be? Are there any upper bounds on the supersymmetry breaking scale, following (for example) from cosmological considerations or coupling unification?

One interesting suggestion as to how LHC-observable phenomena might emerge from supersymmetry breaking at a very high scale has been dubbed ''mini-split supersymmetry'' [5,6,15]. In these models, gauginos are significantly lighter than other superpartners, typically with a winolike LSP. It is possible in such cases that the gauginos may be seen at the LHC, even though the scale of the other superpartners is much higher. More generally, the following arguments for a split structure have been advanced:

(1) Because of symmetries, the gaugino/sfermion mass hierarchy is generic (it is probably not meaningful to

say this is "natural," as the structure requires significant fine-tuning).

(2) The split spectrum is compatible with unification, perhaps even more compatible than more "natural" models.

(3) The lightest gaugino yields a dark matter candidate.

These features are present in models of anomaly-mediated SUSY-breaking [16–20], as well as in attempts to build string models based on G_2 manifolds [7] and on variations of the KKLT scenario [21].

In this paper, we point out that the existence of moduli, and their nature, is a controlling issue in the realization of supersymmetry: the breaking scale of supersymmetry, possible hierarchies of supersymmetric particles, and the nature of and production of dark matter are governed by the presence or absence of moduli. In this framework, our main focus will be gravity mediation, though we will make some comments on gauge mediation (see also [22]).

Our principle observation is that one can enumerate three possibilities for moduli in supersymmetric theories, each with distinct consequences for low-energy physics:

(1) No moduli: In this case, dark matter can be produced thermally. Split supersymmetry is a likely outcome, but there is no compelling setting either for inflation or the resolution of the strong CP problem. A variant is the possibility that all moduli are charged under an unbroken (or nearly unbroken) symmetry [23].

(2) Supersymmetric moduli (only): Here (all of) the moduli ρ have $|F_\rho| \ll m_{3/2}M_p$. Thermal production of dark matter requires extremely heavy moduli; nonthermal production in moduli decays requires a slightly lower scale. A split spectrum is likely, but the anomaly-mediated contributions are not necessarily dominant. Again, there is no attractive setting for the Peccei-Quinn solution of the strong CP problem, but supersymmetric moduli are candidate inflatons.

(3) Nonsupersymmetric moduli: an anomaly-mediated spectrum appears nongeneric. If a stable LSP is kinematically accessible, it is overproduced. The dark matter, then, needs to be something other than weakly interacting massive particles (WIMPs), such as axions. For fixed axion decay constant f_a, there is an upper limit on the modulus mass.

The first two possibilities do not violate any obvious principle of theories of gravity, but string theory examples of these phenomena are hard to come by. The third scenario requires moduli masses in the 100 TeV range, and was initially viewed with skepticism because of the resulting fine-tuning. But given the lack of compelling examples of the first two solutions, the possibility of heavy nonsupersymmetric moduli and correspondingly high scale of supersymmetry breaking has always demanded serious attention. This paper will elaborate these points.

In Secs. II, III, and IV we discuss the cases of no moduli, supersymmetric moduli, and nonsupersymmetric moduli. In Sec. V we discuss constraints on moduli if dark matter is composed of axions and in particular upper bounds on the moduli masses, and in Sec. VI we discuss briefly the case of scalar fields with stronger-than-Planck-strength interactions [21,23,24]. In Sec. VII we conclude, summarizing how the moduli scenarios we have outlined control the nature and phenomenology of low-energy supersymmetry.

II. ABSENCE OF MODULI

The moduli observed in string models might be artifacts of theorists' efforts to construct weak coupling quantum gravity theories. It is possible that nature may exhibit low-energy supersymmetry without moduli. Supersymmetry may be broken in a sector of the theory without gauge singlet fields with large F components. This is typical of many models of dynamical supersymmetry breaking such as the ISS models with metastable supersymmetry breaking and models with stable dynamical supersymmetry breaking. In these models one would expect the leading contribution to gaugino masses to arise through anomaly mediation, leading to a split spectrum. References [5,25,26] have stressed that certain threshold effects can lead to a spectrum which is not strictly anomaly mediated. In the next section, we will see another way in which a more "compressed" spectrum might readily arise.

In a theory without moduli, if the universe was already in thermal equilibrium at very high energies, a viable thermal relic abundance of a winolike LSP may be produced. As noted in [14,27], the wino mass in this case tends to be large (2.7–3 TeV), and, with a strictly anomaly-mediated spectrum, all of the gauginos are far beyond the reach of the LHC.

While in some respects very simple, the no-modulus scenario has unappealing features. First, as we have noted, moduli would seem likely candidates for axions, and this possibility is unavailable. As we will describe in Sec. VI, a more complicated (and perhaps less plausible) structure is necessary to implement the Peccei-Quinn solution of the strong CP problem in such theories. Second, from the point of view of slow roll inflation, the absence of moduli is troubling. One could certainly imagine that the role of the inflation is played by a field with a potential which is flat in some suitable region of field space, but moduli appear ready-made to satisfy the conditions for slow roll inflation.

III. SUPERSYMMETRIC MODULI

By a supersymmetric modulus, we mean a modulus with a mass parametrically larger than $m_{3/2}$. Because the mass is supersymmetry preserving, it should arise from a mass term in the superpotential, while in order that the field be considered a modulus, its higher order couplings must be small. We can parametrize the superpotential as

$$W_\phi = m_\phi M_p^2 w(\phi/M_p). \tag{1}$$

Perhaps the most well-known model containing supersymmetric moduli is the KKLT scenario, for which the superpotential has this form, as we will review shortly. First let us consider a simple toy model. We can define the origin for ϕ so that W contains no linear term in ϕ. To determine the typical size of $\langle\phi\rangle$ and $\langle F_\phi\rangle$, we need to include supersymmetry-breaking dynamics. Suppose W has the form of Eq. (1), and an additional piece responsible for supersymmetry breaking,

$$W = W_\phi + W_0 + fX. \tag{2}$$

We suppose that the Kähler potential is such that X is stabilized at the origin. Then including a general Kähler potential for ϕ,

$$K = (k_1^\phi \phi + \text{c.c.}) + \phi^\dagger\phi + (k_3^\phi \phi^\dagger\phi\phi + \text{c.c.}). \tag{3}$$

At the minimum,

$$\phi \simeq k_1^\phi \frac{m_{3/2}}{m_\phi} M_p, \tag{4}$$

and the F component of ϕ is of order

$$F_\phi \simeq k_1^\phi \frac{m_{3/2}}{m_\phi} m_{3/2} M_p. \tag{5}$$

In such models, ϕ can couple to W_α^2 with $\mathcal{O}(1)$ coefficient and maintain the minimal anomaly-mediated spectrum as long as m_ϕ is at least two or three orders of magnitude larger than $m_{3/2}$.

A. KKLT

The scenario popularized by KKLT provides a model for supersymmetric moduli of the type we have described. It also illustrates possible additional problems with such cosmologies. The model is described by an effective Lagrangian for a field, ρ, with superpotential

$$W = e^{-b\rho} + W_0, \tag{6}$$

with small W_0. The Kähler potential is

$$K = -\ln(\rho + \rho^\dagger). \tag{7}$$

The model has a supersymmetric minimum with

$$\rho \approx \frac{1}{b}\log(W_0/b). \tag{8}$$

At the minimum, ρ is large. Supersymmetry must be broken by some other dynamics. It is often argued that there can be explicit breaking by D branes, but it is not clear that this is consistent. A simple possibility is that there are some other light degrees of freedom which spontaneously break supersymmetry [21,28,29]. For example, introduce a field X with superpotential

$$W_X = fX \tag{9}$$

and a Kähler potential

$$K_X = aX + \text{c.c.} + X^\dagger X + \cdots \tag{10}$$

where the higher-order terms are chosen so that $X = 0$ at the minimum of the potential (this is a definition of the zero of X). Then we can relate m_ρ to $m_{3/2}$:

$$m_\rho^2 = \rho^2 m_{3/2}^2. \tag{11}$$

Supersymmetry breaking induces a shift in ρ of order

$$\delta\rho \sim \frac{1}{\rho} \tag{12}$$

and a corresponding shift in F_ρ. F_ρ is suppressed relative to $m_{3/2}M_p$. In particular,

$$e^K |F_\rho|^2 g^{\rho\rho^\dagger} \sim m_{3/2}^2 M_p^2 \left(\frac{m_{3/2}}{m_\rho}\right)^2. \tag{13}$$

If we suppose that $m_{3/2} \approx 10$ TeV, and that $\rho \sim \frac{4\pi}{\alpha_{\text{gut}}}$, then the reheating temperature (assuming ρ is the only modulus) is greater than 5 GeV, in a range such that one can produce a suitable dark matter density. Of course, it is critical that ρ is the *only* light modulus; other moduli [30,31] breaking supersymmetry lead to cosmological difficulties. The X field above is such a modulus and would need to be replaced by sector which dynamically breaks SUSY without moduli. As in the no-modulus case, this could be a theory with stable, dynamical supersymmetry breaking, or a theory with metastable breaking, such as ISS.[1]

We might expect a ρW_α^2 coupling. The nonzero F_ρ will then contribute to gaugino masses. This contribution is of order

$$m_\lambda \approx \frac{m_{3/2}}{\rho}. \tag{14}$$

The anomaly-mediated contributions then only dominate if ρ is sufficiently large (or equivalently the modulus is quite heavy compared to $m_{3/2}$).

There are other cosmological issues associated with such moduli, particularly the problem of overshoot [32] and related destabilization issues. Various solutions to this problem have been proposed. Specifically in the framework of KKLT models, "racetrack" type superpotentials [33–35] may naturally lead to heavy moduli which avoid these difficulties. They are also argued to lead to anomaly-mediated gaugino masses [21,36]. Other solutions have been discussed, for example, in [37]; as our focus is on somewhat different issues, we will not assess these scenarios further here.

[1]One of the scenarios discussed in [21] is a realization of this latter possibility.

One can contemplate variants of the scenario where the would-be modulus acquires mass comparable to the Planck mass [28].[2] This would be a realization of the no-moduli scenario (in the absence of a pseudomodulus, i.e. replacing X by a model of dynamical supersymmetry breaking without moduli).

B. Consequences of supersymmetry for moduli decays

It is straightforward to show that the decay rates into the scalar and fermionic components of a lighter multiplet are related in specific ways by supersymmetry, up to corrections proportional to the soft masses. In Appendix A we discuss how this works at tree level for the dimension-five operators mentioned previously. In Appendix B we outline a more general argument from the unitary representations of the SUSY algebra. Here, for brevity, we sketch an argument from field theory for the case of decays to a massless multiplet. Consider first a simple Wess-Zumino model with a heavy field, Φ, and a massless field, ϕ. For the superpotential, take

$$W = \frac{1}{2} M \Phi^2 + \lambda \Phi \phi \phi. \tag{15}$$

Supersymmetry relates the Green's functions:

$$\langle F^*_\Phi(x_1) \psi_\alpha(x_2) \psi_\beta(x_3)\rangle \epsilon^{\alpha\beta} = 2\langle \Phi(x_1)^* \partial_\mu \phi(x_2) \partial^\mu \phi(x_3)\rangle. \tag{16}$$

This relation can be proven easily, for example, by considering the superspace Green's function:

$$\langle \Phi^*(x_1, \theta_1) \phi(x_2, \theta_2) \phi(x_3, \theta_3)\rangle. \tag{17}$$

The left hand side of Eq. (16) is the coefficient of $\bar\theta_1^2 \theta_2 \theta_3$ in this Green's function; translating by θ_1 in superspace, the coefficient of this term is the right-hand side of the equation.

To extract the decay amplitudes, we can apply the LSZ formalism. First we note the relations for the Green's functions, in momentum space,

$$\langle F^\dagger F\rangle = p^2 \langle \phi^\dagger \phi\rangle. \tag{18}$$

So we can relate the single particle matrix elements needed for LSZ; those of ϕ and F differ by a factor of m^2, the physical on-shell mass. There are two possible initial states (which can be thought of as the scalar and its antiparticle) and two possible final states in either the two boson or two fermion channel. Combining the Ward identity for the Green's functions and the result for the single particle matrix elements demonstrates the equality of the two boson and two fermion matrix elements. The result is readily verified at tree level.

Similarly, for a scalar coupled to W_α^2, one can prove an equality for the matrix elements (and hence the rates) for the decays: $\phi \to A_\mu + A_\mu$ and $\phi \to \lambda\lambda$. When supersymmetry is broken these equalities will fail, but, except for tuned values of the parameters, we expect the rates to be comparable.

C. Moduli decays and the reheat temperature

We can consider, then, the lifetime of the moduli (first in the supersymetric case). The lifetimes depend on the kinetic terms for the moduli and their couplings to other fields and one can obtain quite different results with different choices. Given that much of the motivation to consider moduli comes from string theory, it seems appropriate to consider kinetic terms familiar from various string models. We take as a model the heterotic string compactified on a Calabi-Yau manifold, and take the modulus to be the so-called model-independent dilaton. Then the Kähler potential and gauge coupling functions are [38]:

$$K = -M_p^2 \ln(S + S^\dagger); \qquad f = S. \tag{19}$$

Here we have taken S to be dimensionless and indicated explicit factors of M_p. In this case where the decay is principally through the coupling SW_α^2, the decay rates to pairs of gauge bosons and gauginos are the same. At leading order, summing over the gauge multiplets of the MSSM, one obtains [39]

$$\Gamma(S \to gg) + \Gamma(S \to \tilde g \tilde g) = \frac{3}{4\pi} \frac{m_S^3}{M_p^2}. \tag{20}$$

This translates to a reheating temperature

$$T_R = 9.8 \times \left(\frac{g_*(T_R)}{10}\right)^{-1/4} \left(\frac{m_S}{10^5\ \mathrm{GeV}}\right)^{3/2} \mathrm{MeV}. \tag{21}$$

For reference, we note that the minimum temperature required to achieve successful nucleosynthesis is approximately 4 MeV [12].

An alternative model is provided by the "T modulus" of simple Calabi-Yau compactifications of the heterotic string [38]. Here:

$$K = -3M_p^2 \log\left(T + T^* - \frac{1}{3} \frac{\phi_i^* \phi_i}{M_p^2}\right); \qquad f = 0, \tag{22}$$

where the ϕ_i denote the matter fields. Writing $T = T_0 + \delta T$, after rescaling the δT and ϕ_i kinetic terms to make them canonical generates the couplings:

$$\mathcal{L}_{T\phi} = \frac{1}{\sqrt{3}} \delta T \phi_i^* \phi_i. \tag{23}$$

These are among the dimension-five operators listed in [13]. The decay rates to fermion and boson pairs are the same in the SUSY limit and are suppressed by m_ϕ^2/m_T^2.

[2] KKLT presumes that the superpotential for the modulus contains a small constant, W_0. It is conceivable that this constant is large, and that the effective low-energy theory, *after* integrating out this modulus, has a small $\langle W\rangle$, required for a small cosmological constant. Under these circumstances, the modulus could be quite heavy.

When a soft mass of order $m_{3/2}$ is present for the bosonic components, there is a contribution that independent of m_ϕ, but is still suppressed [13],

$$\Gamma(T \to \phi\phi) \sim \frac{1}{4\pi}\left(\frac{m_{3/2}}{m_T}\right)^4 \frac{m_T^3}{M_p^2}. \tag{24}$$

Shortly, we will be interested in the nonsupersymmetric case, and in particular the possibility that the decay channels to R-odd particles are not accessible. In that case, in Eq. (21), 9.8 is replaced by 6.9.

D. Decays and the relic density

The most urgent question in moduli decays is the resulting relic density. There is the possibility of overproduction of LSPs, if stable, and gravitinos. These lead to too-early matter domination, inconsistent with the observed light element abundances. We will focus principally in this subsection on models with a conserved R-parity and a stable LSP. We will remark at the end about the effects of R-parity violation, postponing more detailed analysis to a subsequent work.

We will first assume a conserved R-parity. In this case LSPs are produced (possibly overproduced) in decays of the modulus. It is also necessary to consider modulus decays to gravitinos. While a 10–100 TeV gravitino is relatively short-lived, its decay products include LSPs, which may be problematic.

As demonstrated in the previous section, in the SUSY limit, amplitudes for two-body decays to particles are identical to those for two-body decays to their supersymmetric partners. In particular, there are no helicity suppressions of decays to fermions compared to decays to bosons, as has been suggested in certain contexts.[3] With heavy supersymmetric moduli, all R-odd decays to partners of Standard Model fields are kinematically allowed and occur with rates approximately equal to the rates into their R-even partners, since the light MSSM fields appear supersymmetric to the moduli.

As a result, the number of LSPs produced per modulus decay is $\mathcal{O}(1)$. To keep the reheating temperature above the temperature of nucleosynthesis requires moduli masses above 30–100 TeV. In this range, the LSP density is an $\mathcal{O}(1)$ fraction of the total energy density at temperatures of order a few MeV, so matter domination occurs far too early. The weak interactions freeze out at

$$T_F \sim (M_p^{-2} G_F^{-4} T_R)^{\frac{1}{7}}, \tag{25}$$

which is about 1.8 MeV for $T_R \sim 5$ MeV, compared with freeze-out at 0.8 MeV in the ordinary radiation-dominated universe. The neutron-to-proton ratio thus increases from $n/p \approx 1/6$ at weak freeze-out to $n/p \approx 1/2$, increasing the abundance of helium.

A simple solution to this problem is that the moduli are heavier than 10^6 GeV, producing a reheating temperature of order a few hundred MeV or higher. For supersymmetric moduli, this large mass scale is not disturbing. If the dark matter annihilates effectively, the reheating temperature may be lower. In that sense an anomaly-mediated-type spectrum may in fact seem favored, since $\sigma_{\text{wino}} \sim m_{\tilde{W}}^{-2}$.

However, a related problem may still arise for supersymmetric moduli, dubbed the "moduli-induced gravitino problem" [40]. If moduli decays to gravitino pairs occur with $\mathcal{O}(1\%)$ branching fraction, the decays of gravitinos still typically overproduce dark matter, even if they avoid BBN constraints. As pointed out in [41], exploiting the Goldstino equivalence theorem allows analysis of this problem by considering couplings of the modulus S to Goldstinos. The branching fraction to gravitinos is controlled by Kähler potential couplings of S to the Goldstino superfield,[4] $S^\dagger ZZ + \text{c.c.}$ This coupling might be suppressed (see, for example, [43]); if not, the branching ratio of the modulus to gravitinos is of order one. So whether this is a problem depends on microscopic details of the theory.

We note in passing that with supersymmetric moduli, baryons might be produced coherently or in decays of the modulus. If there are ϵ baryons produced per modulus, the baryon to photon ratio is

$$\frac{n_B}{n_\gamma} \approx \epsilon \frac{T}{m_\phi} \approx \epsilon\left(\frac{T}{M_p}\right)^{1/3}. \tag{26}$$

So, for example, for a 1 GeV reheating temperature, we require $\epsilon \approx 10^{-4}$. Alternatively, if ϵ is fixed by the microscopic theory, the mass of the modulus is determined.

So far, we have assumed a conserved R-parity. If R-parity is violated, the role of the dark matter must be played by some other field. Provided the R-parity violating couplings are not too small, the lifetime of the would-be LSP is much shorter than that of the moduli, so their production is not a cosmological issue. For example, if the principle source of R breaking is the coupling

$$W_R = \lambda \bar{t}\,\bar{b}\,\bar{s} \tag{27}$$

then, unless $\lambda < 10^{-10}$ or so, gaugino decays are sufficiently rapid.

E. Summary

Supersymmetric moduli are a plausible outcome of moduli-fixing. They are suggestive of a split spectrum for superparticles, though anomaly-mediated contributions do not necessarily dominate the gaugino spectrum. In such

[3] Both fermionic and bosonic decays from the $\phi Q^* Q$ operator are proportional to small supersymmetric masses in the SUSY limit; therefore the leading effect of this operator may be the $m_{3/2}^4$ contribution to the bosonic final states.

[4] Here we mean in the sense of nonlinear realizations of supersymmetry, as in [42]; we are not assuming the gravitino has a light supersymmetric particle.

cases, avoiding overproduction of LSPs sets a lower bound on the modulus mass. Avoiding overproduction of dark matter through gravitinos places restrictions on the microscopic details of SUSY-breaking.

IV. NONSUPERSYMMETRIC MODULI

The KKLT scenario, with supersymmetry broken in a sector of the theory without flat directions, provides a model for supersymmetric moduli fixing. But there are a number of reasons to suspect that there should be moduli which gain mass only through supersymmetry breaking effects. The need for an axion to solve the strong *CP* problem provides one motivation; in a supersymmetric context, a nonsupersymmetric modulus seems to provide, as we have said, an ideal axion candidate. A second motivation is provided by metastable dynamical supersymmetry breaking, and especially the retrofitted models [44,45], where such moduli are an integral part of supersymmetry breaking. Inflation is also suggestive of relatively light moduli. Successful inflation requires a mass small compared to the Hubble constant during inflation. A nonsupersymmetric modulus automatically has mass of order the Hubble constant, so only a modest coincidence is required. Such a modulus also has only small self-interactions, so the mass can readily remain small throughout and the potential can be adequately flat.

A. *F*-terms

It is often assumed that there is only one modulus with an F-term of order $m_{3/2}M_p$. However, in a gravity-mediated theory, all moduli with masses of order $m_{3/2}$ will tend to have F components of this order, whether or not they appear explicitly in the superpotential. In supergravity, the F component of a modulus ϕ is

$$F_\phi = g^{-1}e^{K/2}\left(\frac{\partial W}{\partial\phi} + \frac{\partial K}{\partial\phi}W\right). \tag{28}$$

Here K is the Kähler potential and g is the inverse metric on the field space.[5] If ϕ does not appear in the superpotential, in the absence of symmetries, the second term is of order $m_{3/2}M_p$ from the linear term in K.

Of course, it is possible by a redefinition of the fields to simply *define* one field to have a nonvanishing auxiliary component, while all others vanish. But the redefinition will affect the couplings of these fields. For example, if originally only one field couples to W_α^2, then in general, all will, including the linear combination with nonvanishing F component. In this subsection, we will discuss in more detail the scaling of the moduli Kähler potentials and their implications for the spectrum.

[5]This is schematic; in the presence of multiple fields, one needs to consider diagonalization of g.

In the retrofitted models, there is a modulus X coupled to W_α^2 of some new gauge group. In the simplest case, this hidden sector is a pure gauge theory, and X does not couple directly to other light fields transforming under this group. Gaugino condensation in this group gives rise to a superpotential for X, and the X VEV is fixed by the Kähler potential. X can be defined so that the first derivative of the Kähler potential, K_1, vanishes.

Given that X couples to the kinetic term of one gauge group, it is likely to couple to the Standard Model gauge groups as well. As is typical of moduli of string theory, if we take the modulus to be dimensionless, its imaginary part is periodic with a period we can take to be a multiple of 2π. The gaugino masses then depend on the gauge coupling, g^2, and K_2, the second derivative of the Kähler potential at the minimum, as

$$m_\lambda \sim \frac{g^2}{\sqrt{K_2}}m_{3/2}. \tag{29}$$

The gaugino mass can be small if K_2 is large, or if the XW_α^2 coupling is for some reason suppressed.

It would seem that we are free to hypothesize whatever form for the Kähler potential we wish, but string theory provides some guidance. Typical Kähler potentials, as exemplified by the dilaton of the heterotic string or Type II theories, or the radial dilaton of each, behave like

$$K \sim -\ln(X + X^\dagger), \tag{30}$$

where the corresponding field obeys the periodicity property (with a suitable normalization)

$$X \to X + 2\pi i. \tag{31}$$

Because of the periodicity, X couples linearly to W_α^2. If there is a single field with such a coupling,

$$\langle X\rangle = g^{-2}. \tag{32}$$

Then K_2 is small and the gaugino mass is of order the gravitino mass.

With multiple fields, there are additional possibilities allowing for hierarchies between gaugino and scalar masses. For example, with two fields, X_1 and X_2, with $X_1 \gg X_2 \gg 1$ and $F_{X_1} \ll F_{X_2}$, then

$$m_\lambda = cm_{3/2}\frac{1}{X_1} \approx g^2 m_{3/2}. \tag{33}$$

An argument for moduli VEVs of this sort appears in [46]. What appears typical is that in the presence of nonsupersymmetric moduli, most soft SUSY-breaking masses will be of order $m_{3/2}$, without a large hierarchy.

B. Decays and the relic density

In order that nonsupersymmetric moduli decay before nucleosynthesis (implying a reheat temperature greater than about 10 MeV), they should decay through dimension-five operators; if they decay through dimension-six, their masses need to be of order 10^7 TeV or more. Possible dimension-five operators are listed in [13] and include the aforementioned coupling to W_α^2 as well as Kähler couplings to Q^*Q and $H_u H_d$.

If there is a conserved R-parity, and a modulus can decay to the LSP, then the number of LSPs produced in a single modulus decay ($N_{\rm LSP}$) is an important parameter controlling the cosmology. We have already shown in Sec. III. A that in the case of unbroken supersymmetry, the decays to pairs of particles and their supersymmetric partners occur at equal rates. For broken supersymmetry, these relations are corrected, but we do not expect qualitatively significant changes, except for kinematic reasons in particular regions of parameter space. Consequently, we expect $N_{\rm LSP}$ is typically ~ 1. In this situation, an acceptable cosmology only emerges for extremely heavy moduli.

When the temperature is around 10 MeV, a large fraction of the energy density is in LSPs. The relic density of a wino LSP for can be estimated by integrating simple Boltzmann equations [13]. In Fig. 1 we plot contours of the wino relic density as a function of the wino mass and the reheating temperature T_R, obtained by numerically integrating the Boltzmann equations. We fix $N_{\rm LSP} = 1$ and take

$$\Gamma_\phi \sim \frac{1}{2\pi}\frac{m_\phi^3}{M_p^2}, \tag{34}$$

as an estimate of the total width (compatible with our earlier discussion).

It is clear that in this scenario the reheating temperature should be over a few hundred MeV, corresponding to a SUSY-breaking scale above 10^6 GeV, and the wino should be extremely light compared to $m_{3/2}$, so that pair annihilation is more effective at reducing the density (a thermal abundance of $\Omega h^2 \sim 0.1$ is not achieved until the reheating temperature is of order several hundred GeV). We have

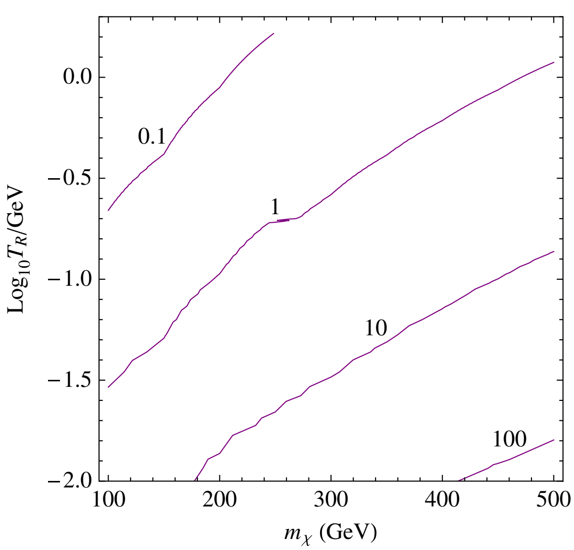

FIG. 1 (color online). Ωh^2 for the case $N_{\rm LSP} = 1$.

already argued that such a spectrum is atypical in the presence of nonsupersymmetric moduli.

Similar results have been obtained recently in [14], where it was also found that a Sommerfeld enhancement can greatly increase the wino pair annihilation rate and suppress the relic density to acceptable values for any reheating temperature. However, this effect is only present in a narrow window around $m_{\tilde W} \sim 2.4$ TeV and is thus highly nongeneric. The authors of this paper also stressed the clear degeneracy in Eq. (34): if the width is increased by decreasing the effective cutoff scale, the reheating temperature can be set high enough for any value of the modulus mass. We will discuss the possibility of modulus couplings stronger than expected with Planck suppression in Sec. VI.

To suppress the relic abundance without going to extreme regions of parameter space, the most obvious possibility is to make the moduli lighter than the LSP. An alternative possibility is that there is no conserved R-parity and no stable LSP. In either case, the universe might simply "reheat" to temperatures above nucleosynthesis, without leaving stable (or cosmologically long-lived) relics.

Another possibility is that the gauginos are light and modulus decays to these lighter states are somehow suppressed. For example, it might be that all couplings of the non-SUSY moduli to W_α^2 vanish or are highly suppressed [by a large value of K_2 in Eq. (29)]. Then the gauginos are indeed parametrically lighter than other superparticles. It is necessary that the moduli decay through other dimension-five operators, such as $\phi Q^* Q$ and $\phi^* H_u H_d$. If the Higgsinos and sfermions are heavier than m_ϕ, the width to gaugino final states *will* be suppressed; to keep the total width of order Eq. (34), decays to the Higgs bosons must be unsuppressed. The latter condition may be satisfied if Higgsino masses of order $m_{3/2}$ are obtained from an order-one $\phi^* H_u H_d$ coupling. Such a scenario can perhaps lead to an acceptable dark matter density, though the Higgsinos and sfermions may have to be rather heavier than the moduli. As we have remarked, this sort of spectrum seems surprising from the perspective of known string models, but it is a logical possibility.

We note that it has recently been pointed out in [47] that there is another cosmological problem with moduli that are stabilized by SUSY-breaking dynamics. The axion component of the modulus multiplet may remain very light and generically the branching ratio of moduli into axion pairs will be sizable. This scenario is constrained by the Planck measurement of $\Delta N_{\rm eff}$ and the authors of [47] emphasize that simply raising $m_{3/2}$ is insufficient to evade the bounds. The constraints imposed on particular types of microscopic models by this phenomenon will be described elsewhere.

C. Summary

The F-terms of nonsupersymmetric moduli are typically of order $m_{3/2}M_p$, which implies that the generic spectrum is not split. If R-parity is conserved and gaugino masses are comparable to those of other superparticles, viable cosmology demands kinematic suppression of the decays of moduli to the LSP. The simplest possibility is that any modulus in the theory must be lighter than the LSP. Alternatively, gauginos might be light, which requires that moduli couplings to W_α^2 must vanish or be quite small. If there is no branching ratio suppression, the reheat temperature should be quite high in order to avoid overclosure, corresponding to a high SUSY-breaking scale. Sufficient branching ratio suppression can be achieved for lower $m_{3/2}$ if decays to sfermions and higgsinos are kinematically forbidden and the coupling to W_α^2 is small (as it should be to keep the gauginos light). As pointed out in [47], decays of nonsupersymmetric moduli to axionlike objects place significant constraints on the microscopic theory, but suggest that the effective number of neutrinos at nucleosynthesis may be larger than three.

V. AXIONS AS DARK MATTER, BARYOGENESIS, AND UPPER BOUNDS ON $m_{3/2}$

Except for models with extremely heavy moduli, or no moduli at all, we have seen that it is challenging for the LSP to be the dark matter. An alternative dark matter candidate is the axion. In this section, we explore this possibility, discovering that for a fixed axion decay constant (or more precisely, $\theta_0 f_a$, where θ_0 is the initial axion "misalignment angle"), there is an upper bound on the mass of the modulus. We will also consider in this section the question of baryogenesis. Again, given the low reheat temperature, there appear to be two possibilities: baryon number violation in the moduli decays and Affleck-Dine baryogenesis [48].

We have stressed that the "no modulus" or "all moduli heavy" scenarios are unlikely settings for the axion solution to the strong *CP* problem, since if supersymmetry survives to the multi-TeV scale, the would-be modulus partner of the axion is missing. In such theories, one would need to introduce a Peccei-Quinn symmetry along the lines we will discuss in Sec. VI.

Therefore, we assume the existence of moduli with masses of order $m_{3/2}$, and an axion to solve the strong *CP* problem. We first recall some features of axion cosmology in supersymmetric theories [49]. Necessarily in such theories there is a modulus which can be thought of as the partner of the axion. For simplicity, we will assume its mass is of order $m_{3/2}$. This field starts to oscillate when $H \sim m_{3/2}$. The axion starts to oscillate when $m_a \approx H$. Assuming that the moduli dominate the energy at this time, we have that the axion energy density is of order $H^2 f_a^2$. On the other hand, the modulus energy density is of order $H^2 M_p^2$. So axions constitute a fraction $\theta_0^2 f_a^2 / M_p^2$ of the energy density. This is the fraction when the moduli decay (at, say, 10 MeV). In order that axions not dominate the energy density before temperatures of order 1 eV, we need

$$\frac{\theta_0^2 f_a^2}{M_p^2} < 10^{-7}\left(\frac{10\ \text{MeV}}{T_r}\right) \tag{35}$$

or

$$\theta_0 f_a < 10^{14.5}\ \text{GeV}\left(\frac{10\ \text{MeV}}{T_r}\right)^{1/2}. \tag{36}$$

If we suppose f_a is given, we have an upper bound on the reheat temperature, and correspondingly an upper bound on the mass m_ϕ. In particular, for $\theta_0 f_a = 10^{14.5}$,

$$m_\phi \lesssim 100\ \text{TeV}. \tag{37}$$

Another upper limit arises from baryogenesis. Consider first the possibility that baryons are produced in the decays of the ϕ particle; assume that there are ϵ baryons per decay (independent of the mass of ϕ). In that case, the baryon-to-photon ratio is of order

$$\frac{n_B}{n_\gamma} = \epsilon \frac{T_r}{m_\phi} \simeq \epsilon \left(\frac{90}{\pi^2 g_*}\right)^{1/4}\left(\frac{m_\phi}{2\pi M_p}\right)^{1/2}. \tag{38}$$

Alternatively, suppose that the baryons are produced by AD baryogenesis. We can again parameterize this process in terms of the number of baryons per modulus, ϵ, and we can again write the baryon-to-photon ratio as in Eq. (38). We can understand the parameter ϵ more microscopically in such a framework by assuming that the baryon number is generated along a flat direction described by a pseudomodulus Φ. We suppose that this field has a mass of order $m_{3/2}$, and that the flat direction is raised by the appearance in the superpotential of an operator,

$$W_B = \frac{1}{M_p^n}\Phi^{n+3}. \tag{39}$$

We also suppose that in the early universe, there is a term in the Φ potential, $-H^2|\phi|^2$. As a result, when $H \approx m_{3/2}$, the Φ field begins to oscillate. Its amplitude is of order

$$\Phi^{n+1} \approx M_p^n m_{3/2}. \tag{40}$$

Correspondingly, assuming that W_B is baryon-number violating and possesses phase δ, the baryon number density is

$$n_B \approx m_{3/2}(m_{3/2} M_p^n)^{\frac{2}{n+1}} \tan(\delta). \tag{41}$$

The number density of moduli at this time is of order $m_{3/2} M_p^2$ (including all moduli, so that H is of order $m_{3/2}$), so

$$\frac{n_B}{n_\gamma} \approx \left(\frac{m_{3/2}}{M_p}\right)^{\frac{2}{n+1}} \tan(\delta). \tag{42}$$

VI. MORE STRONGLY INTERACTING MODULI: AXIONS WITHOUT MODULI

In string theories, it is often true that at points (or on subspaces more generally) of the moduli space, there are light particles. At these points, the moduli interactions with themselves may be stronger than expected if they were simply described by Planck scale local operators. The modulus lifetime can be much shorter and reheating temperatures higher. Indeed, as these are typically points of enhanced symmetry, it is possible that the universe simply finds itself at such a point as inflation ends, and there is no moduli problem at all [23]. From the point of view of cosmology, this is similar to the "no modulus" case we have discussed.

More generally, one can wonder about our use of the Planck scale, as opposed to, say, a scale suppressed by powers of g^2. As we have just seen, if there is just one modulus, with a logarithmic Kähler potential, everything scales with M_p. With more moduli, different scalings are possible, as we saw in our discussion of gaugino masses in the previous section. Even with a single field, if one permits more general Kähler potential, there are other possibilities. Still, we view our estimates of lifetimes and masses as representing a "typical" behavior away from possible enhanced symmetry points.

To illustrate possible behaviors, suppose that at the enhanced symmetry point, the theory exhibits a linearly realized symmetry, under which the modulus (and other fields of the theory) transform by a phase. The modulus, X, might couple to messenger fields, as in gauge mediation, and other fields, so as to lead to a small breaking of the symmetry. The low energy theory would contain operators suppressed by powers of $1/\langle X\rangle$, rather than $1/M_p$ [21,23,24].

We can also contemplate axions which are not parts of moduli fields, according to our definition, but rather light fields with comparatively flat potentials, perhaps due to a discrete symmetry. These might resolve the strong CP problem in theories without moduli or with only supersymmetric moduli, but they must satisfy certain stringent requirements. These fields could also play a role in the transmission of supersymmetry breaking. Consider a model [49] with a field Φ coupling to a pair of vectorlike messenger fields and another gauge singlet S',

$$W \supset W_0 + \Phi\bar{Q}Q + \frac{1}{M_p^n}\Phi^{n+2}S', \qquad (43)$$

for some integer n. We can assume the model respects a discrete symmetry that accounts for this structure and forbids a linear term in the Kähler potential. If Φ obtains a negative mass from SUSY-breaking $\sim -m_{3/2}^2$, the global symmetry is spontaneously broken, generating a VEV $\langle\Phi\rangle \sim (m_{3/2}M_p^n)^{\frac{1}{n+1}}$. The F-term is then of order $F_\Phi \sim m_{3/2}\langle\Phi\rangle$ and produces gauge-mediated contributions to the soft masses in the visible sector when the Q, $\bar{Q}$ multiplets are integrated out. At 1-loop, there is a ΦW_α^2 coupling of order $(16\pi^2\langle\Phi\rangle)^{-1}$. This leads to gauge-mediated gaugino masses loop-suppressed relative to $m_{3/2}$, as in the anomaly-mediated contribution. The two-loop gauge-mediated scalar masses are of order the squared gaugino masses, and suppressed relative to supergravity contributions. Consequently it is possible for the visible spectrum to remain hierarchical, again bearing similarity to the "no modulus" case.

This model has an approximate $U(1)$ global symmetry which can play the role of the Peccei-Quinn symmetry. It is crucial that this be a very good symmetry; as is well known, this requires that the underlying discrete symmetry be quite large (e.g., Z_{12}). In models without generic Planck-suppressed moduli, such a structure is necessary to implement the Peccei-Quinn mechanism. The cosmological moduli problem is avoided, and the structure of the visible soft masses is model dependent.

VII. CONCLUSIONS

While the arguments for TeV scale supersymmetry have long seemed compelling, for some time there have been other reasons to contemplate the possibility that if supersymmetry plays a role in low-energy physics, the scale of supersymmetry breaking might be 10s of TeV or higher. The question of cosmological moduli has been among the most troubling of these. In this paper, we have seen that the presence or absence of moduli is a controlling consideration for supersymmetry phenomenology. If there are no moduli, a spectrum with gaugino masses smaller by a loop factor than scalar masses seems likely, and WIMP dark matter is produced by conventional thermal processes. On the other hand, the Peccei-Quinn solution to the strong CP problem is not easily embedded into a UV framework. In this case, needless to say, moduli cannot provide an explanation for an unexpectedly high scale of supersymmetry breaking. In such a picture, the LHC might find evidence for supersymmetry along the lines discussed in Ref. [5].

If there are only supersymmetric moduli, a split spectrum is again likely, but anomaly mediated contributions to gaugino masses only dominate for extremely heavy moduli. If WIMPs are the dark matter, they must be produced in moduli decays or afterwards. Either requires a high-mass scale. Again, the Peccei-Quinn solution to the strong CP problem cannot be provided by moduli, and the moduli do not provide an explanation of any particular scale of supersymmetry breaking. Avoiding overproduction of gravitinos places significant (but plausible) constraints on the microscopic theory.

Finally, in the case of nonsupersymmetric moduli, a hierarchical or split spectrum is not generic. If the theory contains a stable LSP, it is typically overproduced unless the LSP is heavier than the moduli. This, in turn, implies that the dark matter is likely to be in some form other than WIMPs. Assuming axion dark matter, for a fixed axion decay constant, there is an upper bound on the modulus mass and correspondingly on the scale of supersymmetry breaking. Such scenarios point to a supersymmetry breaking scale that is high compared to the TeV scale, but not arbitrarily high.

This latter picture suggests that the LHC, at least at 14 TeV, will not discover evidence for supersymmetry, and that direct and indirect detection experiments will not find evidence for dark matter. On the other hand, the next generation of charged lepton flavor violation experiments will permit a new probe of SUSY scales as high as $\mathcal{O}(150)$ TeV [10,50]. Such experiments might point to a particular energy scale. Eventually, a very high-energy hadron collider may be able to probe mass thresholds above 10 TeV directly and perhaps permit the study of supersymmetry breaking at the high scales contemplated here.

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APPENDIX A: SUPERSYMMETRIC AND NONSUPERSYMMETRIC DECAY AMPLITUDES I

In Sec. III A (also Appendix B), we showed that in a supersymmetric theory, the decay rates for moduli decays to pairs of particles are identical to those to their supersymmetric partners. In this section, we illustrate in detail how this works, in a way which indicates that the branching ratios are comparable for nonsupersymmetric moduli. We reanalyze each of the specific dimension-five couplings listed in [13].

Consider first the decay rate into the gauge multiplet. The coupling

$$-\frac{A}{4M_p}\phi W_\alpha^2 \tag{A1}$$

generates couplings to gauge bosons and gauginos. These include:

$$\frac{A}{M_p}\left(-\frac{1}{4}\phi F_{\mu\nu}^2+\frac{1}{4}F\tilde{F}+i\phi\lambda\sigma^\mu D_\mu\lambda^*+\frac{1}{4}F_\phi\lambda\lambda+\text{c.c.}\right). \tag{A2}$$

In [13], it was noted that the derivative coupling in Eq. (A2) to gauginos is suppressed if the gaugino mass is small. This can be understood by a helicity argument, or by using the gaugino equation of motion. But the term involving the auxiliary field was not considered, and it leads to a non-negligible coupling of the modulus to the gauginos, even if the expectation value of the auxiliary field vanishes, as discussed in [39]. This is in fact what happens in the supersymmetric case. Considering, first, global supersymmetry; for a massive field, $F_\phi = m_\phi\phi$, and one has a Yukawa coupling of the modulus to gauginos, with strength m_ϕ/M_p. In the supergravity case, with approximate supersymmetry, the same is readily shown to hold. If supersymmetry is broken in supergravity, even if ϕ does not appear in the superpotential, writing $\phi = \phi_0 + \delta\phi$, where ϕ_0 is the ϕ expectation value, one has

$$F_\phi = \left(\frac{\partial^2 W}{\partial^2\phi}+\frac{\partial^2 K}{\partial^2\phi}W+\cdots\right)\delta\phi K_2. \tag{A3}$$

For light moduli, the second term is typically of order $m_{3/2}$. This yields the coupling

$$\frac{A}{M_p}e^{K/2}WK_2K_2^{-1}\delta\phi\lambda\lambda+\text{c.c.} \tag{A4}$$

Scaling $\delta\phi$ so it has canonical kinetic term, and using the relation between $m_{3/2}$ and W, this coupling is then

$$\frac{A}{M_p}m_{3/2}\frac{g^2}{\sqrt{K_2}}\delta\phi\lambda\lambda+\text{c.c.} \tag{A5}$$

There is no parametric suppression of the branching ratio for light gauginos; of course, once the mass is close to the modulus mass, there will be phase space suppression.

A similar phenomenon occurs with the other dimension-five operators (again as expected from the supersymmetric case). Consider next the operator

$$\frac{B}{M_p}\phi^*H_UH_D+\text{c.c.} \tag{A6}$$

In [13], it is stated that this operator does not lead to decay to Higgsinos. But again this neglects the coupling to the auxiliary component of ϕ,

$$\frac{B}{M_p}F_\phi^*\psi_{H_U}\psi_{H_D}. \tag{A7}$$

Again, in the supersymmetric case, $F_\phi = m_\phi\phi$. In the nonsupersymmetric case, F_ϕ includes a term $K_2W\delta\phi$. This leads to a coupling, after rescalings (assuming canonical kinetic terms for the Higgs fields)

$$K_2^{-1/2}\delta\phi\psi_{H_U}\psi_{H_D}. \tag{A8}$$

If all of the Higgs scalars are lighter than the modulus, then the decay to these fields has the same parametric form. But even if only the lighter Higgs channel is available, one obtains a similar result, from the coupling:

$$\frac{B}{M_p}F_\phi^*(F_{H_U}H_D+F_{H_D}H_U)+\text{c.c.} \tag{A9}$$

So again, there is no parametric suppression of the decays to Higgsinos relative to Higgs scalars.

Finally, there are operators of the type:

$$\frac{C}{M_p}\phi QQ^*. \tag{A10}$$

The authors of [13] note that the decays to light sfermions are suppressed. After an integration by parts there is a component operator of the form

$$\frac{C}{M_p}\phi Q(\partial^2 Q^*) + \cdots, \tag{A11}$$

which gives an amplitude proportional to m_Q^2. The contribution to the rate is suppressed in the case of supersymmetric moduli. Including also the various auxiliary fields,

$$\frac{C}{M_p}(F_\phi F_Q^* Q + F_Q F_Q^* \phi + \cdots). \tag{A12}$$

If Q is massless, the decay amplitudes to either fermion or bose pairs again vanish to leading order. If Q is massive and supersymmetric, then the couplings $F_\phi Q F_Q^*$ and $\phi F_Q F_Q^*$ contribute to the decay amplitudes, as do the second derivative terms, leading to the expected equality of decay rates. In fact in this case the leading term in the amplitudes is proportional to m_Q, so the rate is suppressed only by two powers instead of four; however, generally the supersymmetric masses are extremely small compared to m_ϕ. If supersymmetry is broken and the moduli are nonsupersymmetric, both the F-terms ($F_\phi \approx K_2 W \delta\phi$, $F_Q \approx QW$) and the derivative terms give unsuppressed contributions to the decay rates into bosons governed by $(m_Q/m_\phi)^4$.

To summarize, this class of operators generally leads to suppressed decays to ordinary fermions. However, in general, the decays to sfermions are unsuppressed if $m_\phi \approx m_{3/2}$, so the operator is certainly problematic with regard to overclosure.

APPENDIX B: SUPERSYMMETRIC DECAY AMPLITUDES II

In this appendix we sketch for illustration a more primitive method to find relations unbroken supersymmetry implies between various decay rates and cross sections. We consider a simple example, the two-body decay of a heavy singlet scalar Φ_1 into two lighter charged scalars $\tilde{q}\,\bar{\tilde{q}}$ or fermions $q\bar{q}$. The argument is fundamentally equivalent to the Ward identity-LSZ approach given previously,[6] but does not use field theory.

Consider the sum of the squared matrix elements,

$$\int d\Omega \sum_i |\mathcal{M}|_i^2 = \int d\Omega \sum_i \left| {}_i\langle q\bar{q}|\frac{1}{2}\epsilon^{\alpha\beta} Q_\alpha^\dagger Q_\beta^\dagger e^{-iHt}|\Phi_1\rangle \right|^2. \tag{B1}$$

Here Q is the Weyl spinor of SUSY generators, Φ_1 is the lowest state (annihilated by both Q_α) in a chiral multiplet of mass M, and q, $\bar{q}$ are the spin-$1/2$ states of two additional CPT-conjugate chiral multiplets of mass $m < M/2$. The states $|q\bar{q}\rangle_i$ are the two-fermion states

$$|q\bar{q}\rangle_1 \equiv |q^\uparrow(p)\rangle|\bar{q}^\downarrow(-p)\rangle, \quad |q\bar{q}\rangle_2 \equiv |q^\downarrow(p)\rangle|\bar{q}^\uparrow(-p)\rangle, \tag{B2}$$

and the integration is over solid angle for the outgoing momenta p. Acting on the right, the SUSY generators raise $|\Phi_1\rangle$ to $|\Phi_2\rangle$, the highest state (annihilated by both $Q_\alpha^\dagger$) in the M multiplet,

$$|\mathcal{M}|_i^2 = 4M^2 |{}_i\langle q\bar{q}|e^{-iHt}|\Phi_2\rangle|^2. \tag{B3}$$

Inserting a factor of $(CPT)^2$,

$$\int d\Omega \sum_i |{}_i\langle q\bar{q}|e^{-iHt}(CPT)(CPT)|\Phi_2\rangle|^2 = \int d\Omega \sum_i |{}_i\langle q\bar{q}|e^{-iHt}|\Phi_1\rangle|^2, \tag{B4}$$

because CPT flips the sign of p in the two-fermion states and maps Φ_2 to Φ_1.

To act on the left with the Q, we first decompose the generators into

$$Q_\alpha = Q_\alpha^{(1)} + Q_\alpha^{(2)}(-1)^{F^{(1)}}, \tag{B5}$$

where the superscripts denote the one-particle subspaces on which the generators act, and $F^{(1)}$ is the fermion number operator on the first-particle space. The SUSY generators must be moved past the Lorentz generators that boost the fermion momenta to p and $-p$,

$$|q^\uparrow(p)\rangle|\bar{q}^\downarrow(-p)\rangle = U_p^{(1)} U_p^{(2)\dagger}|q^\uparrow(0)\rangle|\bar{q}^\downarrow(0)\rangle, \tag{B6}$$

where the axis of spin quantization is assumed for simplicity to lie parallel to p for each p. Us and Qs acting on different particle subspaces commute. In terms of spinor components, Us and Qs acting on the same subspaces obey

$$\begin{aligned} U_p Q^\dagger &= (\Lambda_{\frac{1}{2}} Q^\dagger) U_p = \begin{pmatrix} A Q_1^\dagger \\ A^{-1} Q_2^\dagger \end{pmatrix} U_p, \\ U_p^\dagger Q^\dagger &= (\Lambda_{\frac{1}{2}}^{-1} Q^\dagger) U_p^\dagger = \begin{pmatrix} A^{-1} Q_1^\dagger \\ A Q_2^\dagger \end{pmatrix} U_p^\dagger, \\ A &= \sqrt{\gamma(1-\beta)}, \qquad \gamma = E/m = M/2m, \\ \beta &= p/E. \end{aligned} \tag{B7}$$

After some algebra, we obtain

$$\int d\Omega \sum_i |\mathcal{M}|_i^2 = 4m^2(A^4 + A^{-4}) \int d\Omega |\langle \tilde{q}_1 \bar{\tilde{q}}_1|e^{-iHt}|\Phi_1\rangle|^2, \tag{B8}$$

where $\tilde{q}_1$, $\bar{\tilde{q}}_1$ are the lowest scalar states in the m multiplets and carry momentum p and $-p$. Equating (B4) and (B8) and reducing the prefactor, we find

[6] We note that both arguments are subject to the usual limitation that an unstable state cannot be made asymptotic, so manipulations that treat them as such are only valid in the spirit of the optical theorem and up to corrections of order Γ/M.

$$\frac{1}{2}(1+\beta^2)\int d\Omega|\langle\tilde{q}_1\bar{\tilde{q}}_1|e^{-iHt}|\Phi_1\rangle|^2 = \int d\Omega\sum_i|_i\langle q\bar{q}|e^{-iHt}|\Phi_1\rangle|^2, \quad \text{(B9)}$$

which relates the partial widths of the Φ_1 particle into scalar and fermionic final states. Note that the kinematic factor goes to 1 in the massless limit. Other relations can be derived similarly.

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