

Bouncing anisotropic universes with varying constantsJohn D. Barrow^{*} and David Sloan[†]*DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge University, Cambridge CB3 0WA, United Kingdom*

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We examine the evolution of a closed, homogeneous and anisotropic cosmology subject to a variation of the fine structure “constant” α within the context of the theory introduced by Bekenstein and Sandvik *et al.* which generalizes Maxwell’s equations and general relativity. The variation of α permits an effective ghost scalar field, whose negative energy density becomes dominant at small length scales, leading to a bouncing cosmology. A thermodynamically motivated coupling that describes energy exchange between the effective ghost field and the radiation field leads to an expanding, isotropizing sequence of bounces. In the absence of entropy production, we also find solutions with stable anisotropic oscillations around a static universe.

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I. INTRODUCTION

Spatially homogeneous cosmological models are a key area of study within relativity. The introduction of anisotropies gives rise to models in which a richer dynamical structure emerges, yet the cosmology remains simple enough to provide analytic and simple numerical results. These models serve as a test bed for physical theories and allow us to analyze questions about why the universe appears to be highly isotropic, whether inflation occurs for generic or stable sets of initial data, the effects of anisotropy on astronomical observables, and the behavior of cosmological models on the approach to spacetime singularities [1–3].

The idea that the fine structure constant α is a spacetime varying scalar field was first investigated by Bekenstein [4], who created a natural generalization of Maxwell’s equations to accommodate a varying electron charge. This idea was extended to include gravity and provide a theory to explore cosmological consequences of varying α by Sandvik *et al.* [5]. The resulting Bekenstein-Sandvik-Barrow-Magueijo (BSBM) isotropic cosmological models were found and used in conjunction with the astronomical data on varying α obtained from observations of high redshift quasar spectra [6]. More recently, the BSBM theory has been extended to include the case where there is a coupling function (rather than simply a coupling constant) between the charged matter fields and the scalar field driving changes in α [7] and where that scalar field possesses a self-interaction potential [8]. These theories are the analogues of the Jordan-Brans-Dicke theories for varying G [9].

In [10] it was shown how theories of this type could produce singularity-free homogeneous and isotropic cosmologies which displayed stable oscillations around an Einstein static universe because the effect of variations in

the scalar field driving variations in α is to introduce a negative “ghost” density. Barrow and Tsagas [11] considered a broader context for these solutions and showed how the inclusion of simple anisotropic expansion can modify the results because the anisotropy can diverge just as quickly as a bounce-producing ghost scalar field on a approach to the singularity. In this paper we will consider more general closed anisotropic cosmologies with anisotropic three-curvature in this same context.

Matter bounces introduced by the presence of ghost fields are not a new discovery (for a detailed examination see [12]). However, in BSBM models the ghost field is an effective manifestation of underlying physics, not a new matter source introduced by hand. In such models quantum effects are ignored because of the prevailing attitude that ghost fields should not be quantized (and are in fact ill behaved when quantized, with negative probability states). Furthermore, any coupling between a ghost field and a nonghost field would allow an infinite amount of energy to be transferred from the ghost field.

We simply take the view that BSBM models can serve as test models for bouncing cosmologies. The idea of a “phoenix” universe within relativity is almost as old as big-bang models themselves, and goes back to Tolman [13] and Lemaître [14]. This classical picture of oscillating closed universes with zero value of the cosmological constant, Λ , painted by Tolman is well known. If there is no entropy production then cycles for the time-evolution of the scale factor are periodic with the same amplitude and total lifetime. If entropy increase is introduced in accord with the second law of thermodynamics then the oscillating cycles become larger and longer to the infinite future. The classical picture for isotropic universes was competed by Barrow and Dąbrowski [15], who showed that if a positive cosmological constant is included then the sequence of growing cycles will always come to an end, no matter how small the value of $\Lambda > 0$. The ensuing behavior will be to approach de Sitter expansion. If the entropy increase from cycle to cycle is small then the asymptotic state will be one in which the

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expansion is very close to a zero-curvature state with comparable energy densities associated with matter and dark energy (i.e., the cosmological constant). The dark energy will necessarily be slightly dominant and the curvature will be positive—not unlike the situation in our observed universe. This scenario was later also considered in flat universes that are made to recollapse by the presence of a negative cosmological constant and using a specific energy-transfer model for the entropy production. This leads to increasing minima and maxima of successive cycles by Biswas and Mazumdar [16]. Barrow and Dąbrowski [15] also considered the evolution of some simple bouncing anisotropic universes of Kantowski-Sachs type, but not in a context that included varying constants. Other interesting studies of evolution of bouncing universes have been made that reveal other effects. The development of asymmetric cycles in bouncing closed universes has been considered by Biswas *et al.* [17] who also study the spectrum of observable fluctuations created by periods of inflation. Other unusual effects were found in string gas cosmologies with a maximum Hagedorn temperature by Greene *et al.* [18], although they confined attention to isotropic universes.

Many current quantum theories of gravity exhibit curvature singularity avoidance, often in the form of a bounce (although they do not necessarily avoid geodesic incompleteness). In Loop Quantum Cosmology, holonomy corrections to the Friedmann equation give rise to a bounce at Planck scales (see [19] for a review). Horava-Lifschitz gravity also introduces higher-order curvature corrections to Einstein's equations which can cause the universe to bounce [20] for some parameter choices. In the latter case the dynamics of an anisotropic solution have also been explored [21].

The aim of this paper is to extend [10,15] to spatially homogeneous models which exhibit local rotational symmetry (LRS) and so are effectively axisymmetric. LRS models exhibit some of the features of full anisotropic model [22], yet the differential equations governing their dynamics can be solved with relative ease using numerical solvers and exact methods. We will begin in Sec. II by setting out the action principle underlying variation of the fine structure “constant,” then in Sec. III we set out the equations of motion for our system and define quantities of physical interest, such as shear and Hubble expansion rates, in terms of metric variables. Section IV deals with two specific solutions to the equations of motion: a static solution and ghost-induced inflation. In particular, we will focus on the role played by anisotropies in both these cases and examine perturbations about isotropic cases.

II. THEORIES OF VARYING ALPHA

Varying “constants” can be described by extensions of the standard model of particle physics and/or general relativity by the promotion of constants to space and time

dependent scalar fields. A well known and much studied example is that of Jordan-Brans-Dicke theory in which general relativity is extended by generalising Newton's constant G to become a field variable [9]. These self-consistent models for the variation of constants necessarily contain conservation equations for the energy and momentum carried by the varying scalar field and the gravitational field equations account for the scalar field's effect on the spacetime geometry. This is in contrast to much of the old literature on varying constants, other than G , which merely “write in” variations of constants into the equations that hold in the theory where the constant does not vary. The existence of a self-consistent theory for the variation of a constant also shows that much discussion about the meaning of the variation of dimensional constants is not relevant because the solution of the second-order conservation equation for the scalar field describing the variation of a traditional constant always produces constants of integration with the same dimensions as the varying constant and a dimensionless combination is trivially available.

Physical models with extra dimensions often exhibit massless or light degrees of freedom which can lead to the variation of such constants [23,24] and there is the possibility for observational bounds to be placed on any shift in the size of extra dimensions over the age of the universe [25,26].

In this paper we shall follow the BSBM model in which the fine structure constant, α is taken to be dynamical. Evidence of a dipolar spatial variation has been recently claimed [6] and therefore it is natural to extend this scenario to consider space and time variations but in this paper we will only discuss time variations so that we can confine attention to ordinary differential equations. Such variations are bounded by terrestrial experiments to have small variation at present [27,28]. However evidence that the variation is small currently does not rule out more significant changes in the past. In particular, in BSBM theories α is not expected to vary during the radiation era, to increase only logarithmically in time during the cold dark matter dominated era, and then to become constant after the universal expansion begins accelerating. Thus laboratory experiments today would not be expected to find evidence for the variation of α found in high-redshift quasar observations (that derive from epochs before the universe began accelerating) even though it has been proved that any cosmological variations in α will be seen in terrestrial experiments [29].

The BSBM model describes the effect of varying the fine structure constant by the introduction of a scalar dielectric field, ψ with evolution of the charge of an electron given by $e = e_0 e^{\psi}$, in which e_0 is the value of the electron charge at some fixed time, for example today. Notice that e_0 is a fundamental constant and e/e_0 is dimensionless. It has been shown [30] that in spite of modifying black hole solutions, the variability of α respects the second law of

thermodynamics. This will be important in Sec. III, as we will assume that all couplings between our fields obey the second law.

The physical action is given by

$$S = \int \sqrt{-g}(\mathcal{L}_g + \mathcal{L}_m + \mathcal{L}_\psi + e^{-2\psi} \mathcal{L}_{\text{em}}), \quad (2.1)$$

where $\mathcal{L}_g = R/16\pi G$ is the usual Einstein-Hilbert Lagrangian, $\mathcal{L}_\psi = -\frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi$ governs the scalar dielectric field, $\mathcal{L}_{\text{em}} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu}$, and \mathcal{L}_m is a matter Lagrangian independent of ψ . Of particular importance to this paper is the constant coupling parameter ω which we shall take to be negative, so rendering ψ an effective ghost scalar field.

More specifically, in the Friedmann equation for the volume expansion dynamics the kinetic energy term for the scalar field enters as $\frac{\omega}{2} \dot{\psi}^2$ and will contribute a ghost field with a stiff equation of state for negative couplings, $\omega < 0$, or periods of negative coupling if $\omega(\psi)$ is not constant. The sign of ω/ς , where $\varsigma \equiv \mathcal{L}_m/\rho_m$ monitors the fraction of matter coupling to the electromagnetic interaction, determines whether α increases (if $\varsigma/\omega < 0$) or decreases (if $\varsigma/\omega > 0$) with time during the cold-dark-matter-dominated era of the universe's evolution. The evolution at this late time is not significantly affected by the kinetic term, $\frac{\omega}{2} \dot{\psi}^2$, regardless of its sign. The effects of varying α on the microwave background [31], quasar spectra [6], or white-dwarf spectra [32] may constrain the sign of ω because they are determined by the sign of ς/ω and ς can be of either sign depending on whether the cold dark matter is dominated by electric (E) or magnetic (H) energy, as $\varsigma = (E^2 - H^2)/(E^2 + H^2)$. If, like baryonic matter (but not, for example, like superconducting cosmic strings which have $\varsigma = -1$) the dark matter is E -dominated then $1 \geq \varsigma > 0$ and α increasing with time requires $\varsigma/\omega < 0$ and hence $\omega < 0$. Typically, pre-2004 quasar data from Keck give a good fit to $\varsigma/\omega = -2.1 \times 10^{-4}$ [5]. We do not consider here the generalized case where $\omega'(\psi) \neq 0$, see [7], or consider all the astronomical data in detail but our brief discussion is designed to highlight how the sign of ω determines observable quantities. From now on it will be convenient to simply consider the effective field, rather than the underlying dielectric. This field is massless, and it is clear from the action that its motion will be monotonic, as will be that of the effective induced fine structure constant. In terms of fluids, this field will appear to be stiff, with equation of state $p_\psi = \rho_\psi < 0$. In a closed anisotropic cosmological model we will allow energy exchange to occur between the ψ field and an equilibrium radiation field with equation of state $3p_r = \rho_r$, to model an entropy increasing nonequilibrium process. It is interesting to compare this with the situation of a negative cosmological constant, which is in effect a ghost field with $p = -\rho > 0$ that always creates recollapse, which has been studied in this connection by Biswas and Mazumdar [16].

III. COSMOLOGICAL EXPANSION

The physical system under consideration will consist of a homogeneous anisotropic cosmology. For simplicity we will examine a system which is locally rotationally symmetric, and use this to gain insight into the more general case. For a concise review of these cosmological models, see [22]. This model is general enough to contain the purely general relativistic ingredient of anisotropic three-curvature, which is missing from the simple anisotropic models of Bianchi types I and V. It includes the closed Bianchi type IX universe but only in the axisymmetric case where no chaotic behavior occurs. The LRS type IX metric is

$$ds^2 = dt^2 - h_{ij} \sigma^i \sigma^j$$

where σ^i are the $SO(3)$ invariant one-forms [33],

$$\begin{aligned} \sigma^1 &= \cos \psi d\theta + \sin \psi \sin \theta d\phi \\ \sigma^2 &= -\sin \psi d\theta + \cos \psi \sin \theta d\phi \\ \sigma^3 &= d\psi + \cos \theta d\phi, \end{aligned}$$

and the LRS condition requires

$$h_{ij} = \text{diag}\{a(t), b(t), b(t)\}.$$

The metric contains two time-dependent scale factors (due to the LRS condition), $a(t)$ and $b(t)$. The energy densities are denoted by ρ_r for radiation, ρ_ψ for the scalar field, and ρ_Λ for the cosmological constant, and the total density and pressure are ρ and p , where

$$\rho = \rho_r + \rho_\psi + \rho_\Lambda.$$

All matter sources have isotropic pressures. The independent variables are the principal three-curvatures,

$$R_1^1 = \frac{a^2}{2b^4}, \quad R_2^2 = \frac{1}{b^2} - \frac{a^2}{2b^4},$$

with the mean Hubble expansion rate defined by

$$H = \frac{1}{3} \left(\frac{\dot{a}}{a} + 2 \frac{\dot{b}}{b} \right),$$

and the expansion shear scalar by

$$\sigma = \frac{1}{3} \left(\frac{\dot{b}}{b} - \frac{\dot{a}}{a} \right).$$

These variables are subject to a constraint equation (the generalized Friedmann equation with $8\pi G = c = 1$),

$$\rho = \frac{1}{b^2} - \frac{a^2}{4b^4} + 2 \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2}. \quad (3.1)$$

The remaining field equations are

$$\frac{\ddot{a}}{a} = -\frac{1}{2b^4} - 2 \frac{\dot{a}\dot{b}}{ab} + \frac{\rho - p}{2}, \quad (3.2)$$

$$\frac{\ddot{b}}{b} = \frac{a^2}{2b^4} - \frac{1}{b^2} - \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}^2}{b^2} + \frac{\rho - p}{2}, \quad (3.3)$$

$$\dot{\sigma} = -3H\sigma + \frac{1}{3}(R_1 - R_2), \quad (3.4)$$

$$\dot{H} = -H^2 - 2\sigma^2 - \frac{1}{6}(\rho + 3p). \quad (3.5)$$

These reduce to the special case of the closed Friedmann universes when $a = b$. The shear does not evolve with $\sigma \propto (ab^2)^{-1}$ as in Bianchi type I models because of the three-curvature anisotropy on the right-hand side of Eq. (3.4). For ease of exposition, let us define the variables $r = \frac{a}{b}$ and $H_b = \dot{b}/b$. The essential field equations then simplify to

$$\frac{\ddot{r}}{r} = \frac{1 - r^2}{b^2} - \frac{3\dot{r}\dot{b}}{rb}, \quad (3.6)$$

$$\sigma = -\frac{\dot{r}}{3r}, \quad (3.7)$$

$$H = \frac{\dot{b}}{b} - 3\sigma = H_b - 3\sigma. \quad (3.8)$$

The continuity equation, which implies constraint conservation, is

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (3.9)$$

This governs the total energy density and pressure. Now, we introduce some energy exchange between the fluids so that they obey

$$\dot{\rho}_\psi + 6H\rho_\psi = s, \quad (3.10)$$

$$\dot{\rho}_r + 4H\rho_r = -s, \quad (3.11)$$

where, s parametrizes the flow of energy between the scalar field and radiation. For our purposes, this will be taken to be of the form

$$s = -\rho_\psi\beta.$$

Thus, β is a bulk viscous coefficient determined by the volume expansion and is present even when the expansion is isotropic (specifically, $\beta = 3\eta$ where η is the bulk viscosity coefficient). In general β can be a function of ρ_ψ or H^2 and we can write

$$\beta = \beta_0 + \beta_H H^2.$$

This is similar to the form dictated by nonrelativistic kinetic theory that is generally employed to model dissipative effects from imperfect fluids that evolve in accord with the second law of thermodynamics, which requires $s \geq 0$ here (see [34] and Sec. 5.2 of [35]). If there was also a shear viscous effect, damping the shear anisotropy by momentum transfer and producing entropy, then there would be further energy exchange, transferring the anisotropy energy density

σ^2 into radiation. For simplicity, we will ignore this effect because we are interested in energy transfer out of the ψ field.

The scalar dielectric field evolves according to

$$\ddot{\psi} + (3H + \beta)\dot{\psi} = 0 \quad (3.12)$$

and so

$$\dot{\psi} \propto a^{-3} \exp\left[-\int \beta dt\right].$$

IV. SOLUTIONS

In what follows we consider the Bianchi IX case with no cosmological constant ($\rho_\Lambda = 0$). If the matter content is a perfect fluid and obeys $\rho + 3p > 0$, then these universes expand from an initial curvature singularity to a maximum size before collapsing back to a future curvature singularity; the spacetime is past and future geodesically incomplete. However, since the effect of varying the fine structure constant is to produce a ghost scalar field with $\rho_\psi < 0$ which will dominate dynamics at small length scales, the energy condition is violated and solutions to the BSBM model exist which have infinite past and future temporal range.

In this section we will examine two particular solutions to the equations of motion. The first is that of a static spacetime, and its behavior under perturbations. The second is that of an spacetime in which the coupling between fields leads to inflationary behavior.

A. The static solution

There exists a static solution of the form

$$\rho_\psi = -\frac{3}{4b^2} \quad \rho_r = \frac{3}{2b^2}, \quad (4.1)$$

for any given value of b . Note that in order to be static the solution must be isotropic ($\sigma = 0$), since from Eq. (3.4) the three-curvatures must match ($R_1^1 = R_2^2$) for the shear to remain constant and Eq. (3.8) requires $\sigma = 0$.

Now consider making a small perturbation about the isotropic solution by introducing a small anisotropy: $r = 1 + \epsilon$ where $r = 1$ represents isotropy. From (3.6), we find that

$$\ddot{\epsilon} = -3\epsilon H_b + \frac{2\epsilon + 3\epsilon^2 + \epsilon^3}{b^2}. \quad (4.2)$$

Without loss of generality, we take the unperturbed static solution to have $b = 1$. If we introduce small parameters $\delta(t)$ and $\eta(t)$ such that $b = 1 + \eta$, and $\rho = 3/2 + \delta$, then to first order in our small parameters,

$$\epsilon(t) = \epsilon_0 \sin(\sqrt{2}t). \quad (4.3)$$

In the case where there is no coupling between the fields ($s = 0$), these oscillations continue endlessly as shown in

Fig. 1. However, once coupling is introduced ($s \neq 0$), the static case becomes unstable because the balance between ρ_ψ and ρ_r is broken, energy is transferred from the ghost field that supports stable oscillations and eventually, after several oscillations, it settles into radiation-dominated expansion, shown in Fig. 2.

From the constraint equation (3.1), we find that to first order, our small parameters are related by $2\delta = -\epsilon - 3\eta$. Furthermore, we can decompose δ into the radiation and scalar field components, δ_r and δ_ψ . In the absence of field couplings, there is a relationship between these fields, due to their coupled equations of motion (3.12) and (3.10),

$$\rho_r \propto \rho_\psi^{3/2}. \quad (4.4)$$

In the static case under consideration, the constant of proportionality is $-\sqrt{6}$. Note that this relationship is broken by introducing a coupling between the fields. For small δ we are therefore led to $\delta_r = 4\delta = -2\epsilon - 6\eta$.

From (3.3), the evolution of η is given by

$$\ddot{\eta} = \epsilon - \eta + \frac{\delta_r}{3} = \frac{\epsilon}{3} - \eta. \quad (4.5)$$

Therefore there is a (more complicated) stable oscillatory behavior for η about the static solution and the evolution of ϵ has already been determined by (4.3). Thus we have an unusual behavior characterized by stable anisotropic oscillations around the isotropic Einstein static universe. This generalizes the simple isotropic oscillations about the static universe that exist in Friedmann universes with a ghost field found in [10,11].

We can now determine further effects of allowing a coupling between the fields. The second law of thermodynamics requires $s \geq 0$. The exact form of s , given by the form of β , will of course influence the exact dynamics. However, it is possible to make progress by assuming only that s is non-negative.

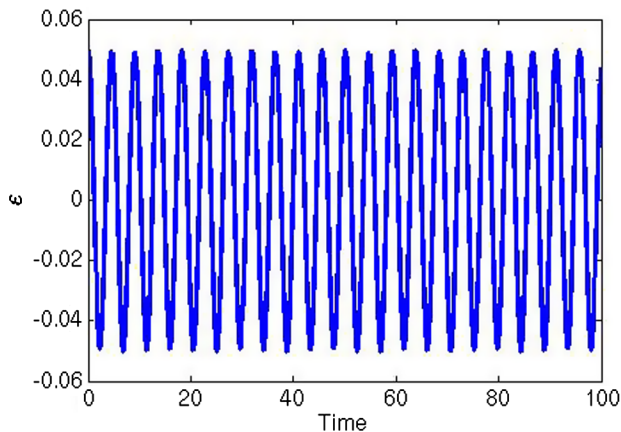


FIG. 1 (color online). ϵ versus time with no coupling ($s = 0$). Initial values: $r = 1.05$, $\sigma = 0$, $H = 0$.

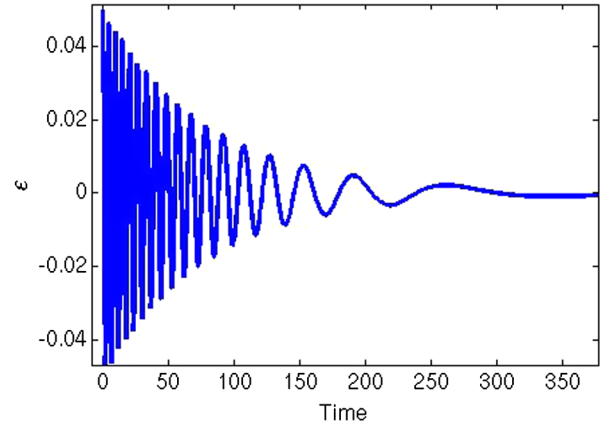


FIG. 2 (color online). ϵ versus time with coupling turned on, $s \neq 0$, showing the system isotropizing. Initial values: $r = 1.05$, $\sigma = 0$, $H = 0$, $\beta_0 = 0.05$, $\beta_H = \beta_\sigma = 0$.

First consider the case of no coupling ($s = 0$). The evolution of the radiation field is determined by

$$\frac{\dot{\rho}_r}{\rho_r} = -\frac{4}{3}H = -\frac{4}{3}\left(\frac{\dot{\epsilon}}{3} + \dot{\eta}\right). \quad (4.6)$$

Due to the cyclic behavior of ϵ and η (and hence of their derivatives), δ_r will also cycle, returning to its initial value, as any integral of the right-hand side of 4.6 across a complete cycle will be zero. However, with a positive coupling between the fields, this relationship is broken, and a term which is always non-negative (and so has a positive integral across cycles) must be added. Hence, across a cycle in η and ϵ , the value of δ_r now increases and we must adjust our equation of motion (4.5) to include this term. We write

$$\ddot{\eta} = \epsilon + \eta + \frac{\delta_r}{3} = \frac{\epsilon}{3} - \eta + \Delta, \quad (4.7)$$

where Δ is a positive term representing the increase in δ_r due to field couplings created by introducing $s > 0$. The variable η no longer cycles about zero, and the system is slowly pushed away from stability, and enters a pseudocyclic phase in which a series of bounces occur with increasing local minima and maxima of the expansion volume, ab^2 , as shown in Fig. 3. The behavior of ϵ (4.2) is affected by this expansion (recall that ϵ is already small). The expansion of $b(t)$ means that H_b is no longer small, and the equation gains a damping term. Similarly, the frequency of oscillations, $\sqrt{2}/b$, is reduced by this expansion in $b(t)$ and the solution takes the approximate form of a damped harmonic oscillator. Note that in the derivation of (4.2) only the smallness of ϵ was used; therefore, this damping behavior is present in all expanding solutions.

B. Ghost-induced inflation

Spacetimes that exhibit inflation are of special interest to cosmologists because inflation can solve a number of

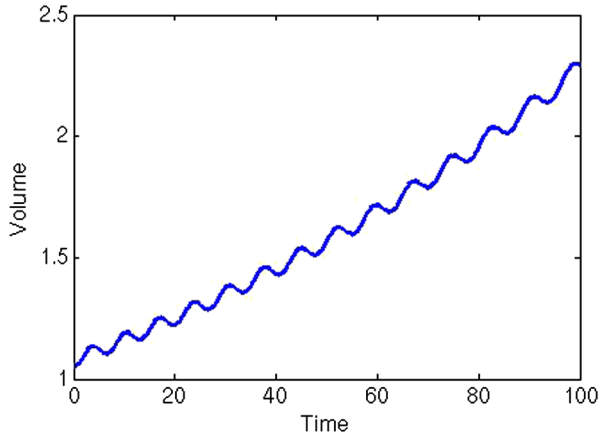


FIG. 3 (color online). Spatial volume versus time with coupling turned on ($s \neq 0$). The system oscillates, but expands from one cycle to the next. Initial values: $r = 1.05$, $\sigma = 0$, $H = 0$, $\beta_0 = 0.01$, $\beta_H = \beta_\sigma = 0$.

well-known puzzles about the universe's structure [36] and make a series of detailed predictions that can be tested by observations of the microwave background radiation [31]. Typical inflationary models exhibit expansion in which the Hubble parameter is (approximately) constant for a finite time interval. In general relativistic cosmology this is usually achieved by introducing a matter content that is (or is equivalent to) a scalar field subject to a self-interaction potential whose contribution to the total energy density is dominant during this expansion, with an equation of state close to that of that produced by an exact cosmological constant with $p_\Lambda = -\rho_\Lambda$.

Inflation induced by ghost fields has been studied as an alternative to the usual slow-roll models [37]. Such models have potentially observable consequences for the microwave background trispectrum [38,39], but require that the translation invariance of the scalar ghost field is broken. In the BSBM models under consideration, however, translation invariance can be preserved, with the field coupling responsible for creating the inflationary energy density.

Let us examine the case of a linear coupling-induced inflation with $s = -\beta_0 \rho_\psi$, for constant $\beta_0 > 0$. When the volume is large there exists an asymptotic solution of the form

$$\rho_r = \frac{9\beta_0^2}{4}, \quad \rho_\psi = -\frac{3\beta_0^2}{2}, \quad H = -\frac{\beta_0}{2}, \quad (4.8)$$

in the isotropic case. This solution is stable, and is approached by dynamical trajectories, as shown in Fig. 4. Under a small perturbation $H = -\beta_0/2 + h$ and $r = 1 + \epsilon$, we find that to first order in the small parameters,

$$\dot{h} = -\beta_0 h \quad (4.9)$$

$$\ddot{\epsilon} = -3\dot{\epsilon}H - 2\epsilon/b. \quad (4.10)$$

In this solution, $b(t)$ is exponentially growing, so the final term in 4.9 quickly becomes negligible. Therefore,

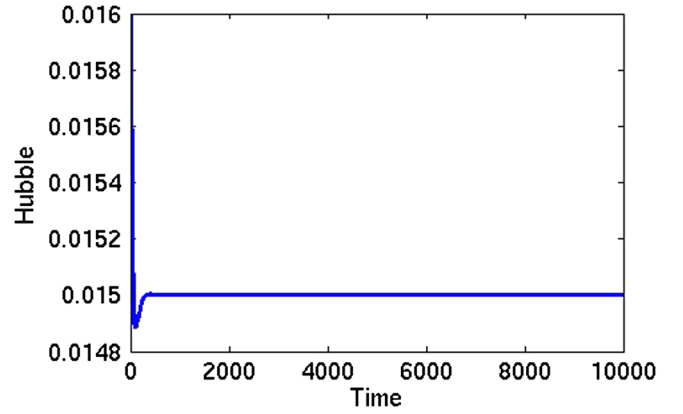


FIG. 4 (color online). Hubble expansion rate versus time for an inflating solution. Initial values: $\beta_0 = 0.03$, $\beta_H = \beta_\sigma = 0$, $r = 1$, $\sigma = 5 \times 10^{-4}$, $a = 10^4$, $H = 1.6 \times 10^{-2}$.

although a shearing expansion may occur, $\dot{\epsilon}$ quickly falls to zero, locking the shear at a fixed value. This inflationary phenomenon is not unique to linear couplings with $\beta_H = 0$, but is simplest to demonstrate in this case. Likewise, there is no requirement for the matter field to consist solely of radiation; introducing more matter fields with couplings whose sign is determined to be in accordance with the second law of thermodynamics yields a system which also exhibits inflation of this type. For most couplings, however, the inflationary phase will end when the dust field becomes dominant.

The evolution of the fine structure constant is shown in Fig. 5. Initially, the solution is like an ascending staircase with rapid changes at each scale factor bounce, see for comparison [10]. Monotonicity of α is ensured since the scalar field cannot have positive energy density; since $\dot{\psi} = \sqrt{\frac{-2\rho_\psi}{w}}$, we have $\dot{\psi} \geq 0$ for all time, and so ψ cannot oscillate through maxima and minima. Across repeated bounces, $\log(\alpha)$ will appear to increase in steps when $|\rho_\psi|$ is small as the relative size of this energy density oscillates greatly within a single cycle. However, as energy is transferred into the radiation field, these steps will become less apparent, eventually approaching a constant gradient once the ghost field reaches the condition for de Sitter inflation to occur. Thus, even though the universe oscillates from cycle to cycle, the fine structure “constant” continues increasing from cycle to cycle, and there will typically only be a finite interval of cycles in which α takes values that allow stable atoms to exist [40,41].

V. DISCUSSION

In this paper, we examined the new effect of introducing anisotropies into the BSBM framework for varying α , although the conclusions have broader applicability to anisotropic cosmologies containing ghost fields and entropy-increasing energy exchanges between fields. In

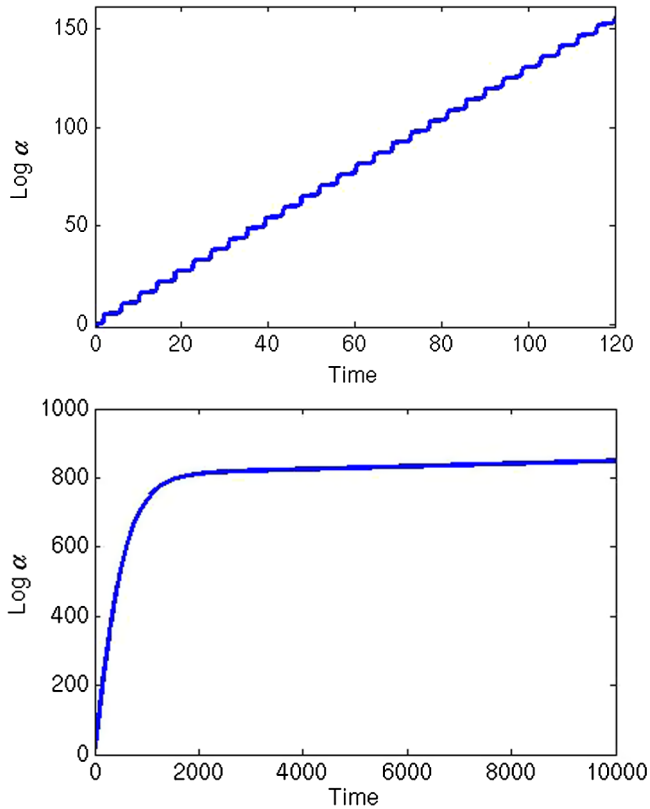


FIG. 5 (color online). Evolution of $\log(\alpha)$ versus time shown over a short timescale (top), and the same evolution over a long timescale (bottom). Initial values: $\beta_0 = 0.01$, $\beta_H = \beta_\sigma = 0$, $r = 1.01$, $\sigma = 0$, $a = 1$, $H = 0$, $\rho_\psi = -0.01$.

particular, we studied the dynamics of locally rotationally symmetric Bianchi IX cosmologies. It was shown that under certain conditions the bouncing behavior observed in isotropic models persists, with the fine structure constant” changing in an almost steplike increasing manner between cycles as time increases. It is apparent from (3.2) that on short scales there is a tension between the shear terms and ghost field, as both scale as the inverse square of the volume. When the anisotropy is small, the contribution

from the ghost field dominates. This leads to a bouncing model, reproducing closely the results seen in [10]. Furthermore, there exists a static solution, perturbations about which lead to a sequence of anisotropic bouncing phases. When there is a coupling between the matter fields, the second law of thermodynamics ensures that this process isotropizes the system by energy exchange.

The resulting dynamics lead to a pseudocyclic universe in which the fine structure constant monotonically increases across bounces, and for small values of the associated dielectric scalar field, this increase is dominated by dynamics near the bounce point. The minimum and maximum volumes of the universe also increase across cycles, with the total energy density decreasing. Eventually, the model reaches a point at which the coupling between fields fixes the energy density to be constant in time, and the universe undergoes a de Sitter phase in which it inflates. Since this model is limited to include only the dielectric field and radiation, there is no transition to the dust-dominated era that one would expect at the end of this phase, and so inflation is endless within the model. It is possible to find solutions with dust in which the system again reaches a point of steady inflation. However, within the space of couplings with $s > 0$ which obey the second law, these solutions are a set of measure zero.

The evolution of α throughout the history of BSBM universes displays interesting traits. At late times, in a large universe it will appear that α has settled to a constant value. In doing so, throughout a series of oscillations the universe will have isotropized greatly, with α stepping up between cycles. As the dynamics are invariant under changing the initial value of α there is no obvious mechanism to determine the constant to which it will approach.

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