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# $Z_c^+(3900)$ decay width in QCD sum rules

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We identify the recently observed charmoniumlike structure $Z_c^{\pm}(3900)$ as the charged partner of the $X(3872)$ state. Using standard techniques of QCD sum rules, we evaluate the three-point function and extract the coupling constants of the $Z_c^+ J/\psi \pi^+$, $Z_c^+ \eta_c \rho^+$ and $Z_c^+ D^+ \bar{D}^{*0}$ vertices and the corresponding decay widths in these channels. The good agreement with the experimental data gives support to the tetraquark picture of this state.

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## I. INTRODUCTION

About ten years after the discovery of the $X(3872)$, the BESIII collaboration has just reported the observation of a charged charmoniumlike structure in the $M(\pi^{\pm} J/\psi)$ mass spectrum of the $Y(4260) \to J/\psi \pi^+ \pi^-$ decay channel [1]. This structure, called $Z_c(3900)$, was also observed at the same time by BELLE [2] and was confirmed by the authors of Ref. [3] using CLEO-c data. During the past decade, as other new nonconventional states were discovered, their internal structure was the subject of intense debate. Definite conclusions have not yet been reached, and some models for these states are still under consideration: the meson molecule [4], tetraquark [5], hadrocharmonium [6], and charmonium-molecule mixture [7]. For a comprehensive review of the theoretical and experimental statuses of these states, we refer the reader to Ref. [8]. In most of these models, it is relatively easy to reproduce the masses of the states. It is, however, much more difficult to reproduce their measured decay widths. In the present case, the $Z_c(3900)$ decay width poses an additional challenge to theorists. Its mass is very close to the $X(3872)$, which may be considered its neutral partner. However, while the $Z_c(3900)$ decay width is in the range 40–60 MeV, the $X(3872)$ width is smaller than 2.3 MeV. A possible reason for this difference is the fact that the $X(3872)$ may contain a significant $|c\bar{c}\rangle$ component [7], which is absent in the $Z_c(3900)$. Probably for this same reason, the $Z_c$ was not observed in $B$ decays, as pointed out in Ref. [9].

In this work, we present a calculation of the $Z_c(3900)$ decay width into $J/\psi \pi^+$, $\eta_c \rho^+$ and $Z_c^+ D^+ \bar{D}^{*0}$.

If the $Z_c$ is a real $D^* - \bar{D}$ molecular state, its decay into $J/\psi \pi^+$ (or $\eta_c \rho^+$) must involve the exchange of a charmed meson. Since the exchange of heavy mesons is a short-range process, when the distance between $D^*$ and the $\bar{D}$ is large it, becomes more difficult to exchange mesons. Using the expression of the decay width obtained with the one boson exchange potential, we can relate the decay width with the effective radius of the state. In Ref. [10], it was shown that, in order to reproduce the measured width, the effective radius must be $\langle r_{\rm eff} \rangle \simeq 0.4$ fm. This size scale is small and pushes the molecular picture to its limit of validity. In another work [11], the new state was again treated as a charged $D^* - \bar{D}$ molecule, in which the interaction between the charm mesons is described by a pionless effective field theory. Introducing electromagnetic interactions through the minimal substitution in this theory, the authors of Ref. [11] were able to study the electromagnetic structure of the $Z_c$ and, in particular, its charge form factor and charge radius, which turned out to be $\langle r^2 \rangle \simeq 0.11$ fm$^2$. Taking this radius as a measure of the spatial size of the state, we conclude that it is more compact than a $J/\psi$, for which $\langle r^2 \rangle \simeq 0.16$ fm$^2$. We take the combined results of Refs. [10,11] as an indication that the $Z_c$ is a compact object, which may be better understood as a quark cluster, such as a tetraquark. Therefore, in this work, we explore this possibility.

As the number of new states increases, a new question arises concerning their grouping in families: which ones belong together? Which ones are ground states, and which are excitations? A possible organization of the charmonium and bottomonium new states was suggested in Ref. [12], and it is summarized in Fig. 1. In the figure, we compare the charm and bottom spectra in the mass region of interest. On the left (right) we show the charm (bottom) states with their mass differences in MeV. The comparison between the two left lines with the two lines on the right emphasizes the similarity between the spectra. In the bottom of the second column, we have now the newly found $Z_c(3900)$. In Ref. [12], there was a question mark in this position. In fact, the existence of a charged partner of the $X(3872)$ was first proposed in Ref. [5]. A few years

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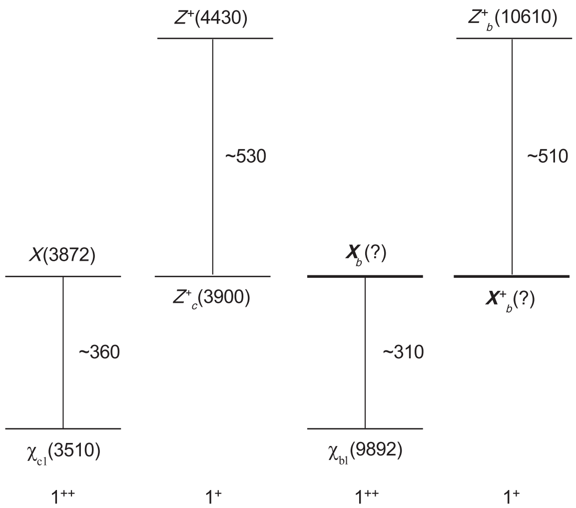


FIG. 1. Charm and bottom energy levels in the mass region of interest. Masses are in MeV. On the two left columns, we show the conjecture presented in Ref. [13]. The $Z_c^+(3900)$ is conjectured to be the charged partner of the $X(3872)$. On the two right columns, we show the conjecture advanced in Ref. [12] for the bottom sector, where the $X_b(?)$ and $X_b^+(?)$ are the proposed states.

later [13], the same group proposed that the $Z^+(4430)$, observed by BELLE [14], would be the first radial excitation of the charged partner of the $X(3872)$. This suggestion was based on the fact that the mass difference corresponding to a radial excitation in the charmonium sector is given by $M_{\Psi(2S)} - M_{\Psi(1S)} = 590$ MeV. This number is close to the mass difference $M_{Z^+(4430)} - M_{X^+(3872)} = 560$ MeV. The very same connection between $Z^+(4430)$ and $Z_c(3900)$ was found in the hadrocharmonium approach [15], in which the former is essentially a $\Psi'$ embedded in light mesonic matter and the latter a $J/\psi$ also embedded in light mesonic matter. In a straightforward extension of this reasoning to the bottom sector, in Ref. [12], it was conjectured that the $Z_b^+(10610)$, observed by the BELLE collaboration in Ref. [16], may be a radial excitation of a yet unmeasured $X_b^+$. The observation of $Z_c^+(3900)$ gives support to this conjecture and should motivate new experimental searches of this bottom charged state and its neutral partner, the only missing states in the diagram.

There are also other suppositions according to which the $Z_c^+(3900)$ should be the charmed partner of the $Z_b^+(10610)$. In this scheme, there should exist another charged state, called $Z_c'$, that would be the charmed partner of the $Z_b^+(10650)$ [15,17,18].

In this work, we use the method of QCD sum rules (QCDSRs) [19–21] to study some hadronic decays of $Z_c(3900)$, considering $Z_c$ as a four-quark state.

## II. $Z_c^+(3900) \to J/\psi\pi^+$ DECAY WIDTH

The QCDSRs were used in Ref. [22] to study the $X(3872)$ meson considered as a $I^G(J^{PC}) = 0^+(1^{++})$ four-quark state, and a good agreement with the experimental mass was obtained. The $Z_c(3900)$ is interpreted here as the isospin 1 partner of the $X(3872)$. As in Refs. [13,17], we assume the quantum numbers for the neutral state in the isospin multiplet to be $I^G(J^{PC}) = 1^+(1^{+-})$. Therefore, the interpolating field for $Z_c^+(3900)$ is given by

$$j_\alpha = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(u_a^T C\gamma_5 c_b)(\bar{d}_d\gamma_\alpha C\bar{c}_e^T) - (u_a^T C\gamma_\alpha c_b)(\bar{d}_d\gamma_5 C\bar{c}_e^T)], \tag{1}$$

where $a, b, c, \ldots$ are color indices, and $C$ is the charge conjugation matrix. Considering $SU(2)$ symmetry, the mass obtained in QCDSRs for the $Z_c$ state is exactly the same one obtained for the $X(3872)$, as it happens in the case of $\rho$ and $\omega$ states. There are also QCDSRs calculations for the $Z_c$ state considered as a $\bar{D}D^*$ molecular state [23,24]. These calculations only confirm the results presented in Refs. [22,25]. Therefore, here, we evaluate only the decay width.

We start with the $Z_c^+(3900) \to J/\psi\pi^+$ decay. The QCDSRs calculation of the vertex $Z_c(3900)J/\psi\pi$ is based on the three-point function, given by

$$\Pi_{\mu\nu\alpha}(p, p', q) = \int d^4x d^4y e^{ip'.x} e^{iq.y} \Pi_{\mu\nu\alpha}(x, y), \tag{2}$$

with $\Pi_{\mu\nu\alpha}(x, y) = \langle 0|T[j_\mu^\psi(x) j_{5\nu}^\pi(y) j_\alpha^\dagger(0)]|0\rangle$, where $p = p' + q$, and the interpolating fields for $J/\psi$ and $\pi$ are given by

$$j_\mu^\psi = \bar{c}_a\gamma_\mu c_a, \tag{3}$$

$$j_{5\nu}^\pi = \bar{d}_a\gamma_5\gamma_\nu u_a. \tag{4}$$

In order to evaluate the phenomenological side of the sum rule, we insert intermediate states for $Z_c$, $J/\psi$ and $\pi$ into Eq. (2). We get

$$\Pi_{\mu\nu\alpha}^{(\text{phen})}(p, p', q) = \frac{\lambda_{Z_c} m_\psi f_\psi F_\pi g_{Z_c\psi\pi}(q^2) q_\nu}{(p^2 - m_{Z_c}^2)(p'^2 - m_\psi^2)(q^2 - m_\pi^2)} \times \left(-g_{\mu\lambda} + \frac{p'_\mu p'_\lambda}{m_\psi^2}\right)\left(-g_\alpha^\lambda + \frac{p_\alpha p^\lambda}{m_{Z_c}^2}\right) + \cdots, \tag{5}$$

where the dots stand for the contribution of all possible excited states. The form factor, $g_{Z_c\psi\pi}(q^2)$, is defined as the generalization of the on-mass-shell matrix element, $\langle J/\psi\pi|Z_c\rangle$, for an off-shell pion:

$$\langle J/\psi(p')\pi(q)|Z_c(p)\rangle = g_{Z_c\psi\pi}(q^2)\varepsilon_\lambda^*(p')\varepsilon^\lambda(p), \tag{6}$$

where $\varepsilon_\alpha(p)$, $\varepsilon_\mu(p')$ are the polarization vectors of the $Z_c$ and $J/\psi$ mesons, respectively. In deriving Eq. (5), we have used the definitions

$$\langle 0|j_\mu^\psi|J/\psi(p')\rangle = m_\psi f_\psi \varepsilon_\mu(p'),$$
$$\langle 0|j_{5\nu}^\pi|\pi(q)\rangle = iq_\nu F_\pi, \tag{7}$$
$$\langle Z_c(p)|j_\alpha|0\rangle = \lambda_{Z_c}\varepsilon_\alpha^*(p).$$

To extract directly the coupling constant, $g_{Z_c\psi\pi}$, instead of the form factor, we can write a sum rule at the pion pole [26], valid only at $Q^2 = 0$, as suggested in Ref. [20] for the pion-nucleon coupling constant. This method was also applied to the nucleon-hyperon-kaon coupling constant [27,28] and to the nucleon $-\Lambda_c - D$ coupling constant [29]. It consists of neglecting the pion mass in the denominator of Eq. (5) and working at $q^2 = 0$. In the operator product expansion (OPE) side, only terms proportional to $1/q^2$ will contribute to the sum rule. Therefore, up to dimension 5, the only diagrams that contribute are the quark condensate and the mixed condensate.

As discussed in Refs. [30,31], large partial decay widths are expected when the coupling constant is obtained from QCDSRs in the case of multiquark states. By multiquark states, we mean that the initial state contains the same number of valence quarks as the number of valence quarks in the final state. This happens because, although the initial current, Eq. (1), has a nontrivial color structure, it can be rewritten as a sum of molecular-type currents with trivial color configuration through a Fierz transformation. To avoid this problem, we follow Refs. [30,31] and consider in the OPE side only the diagrams with nontrivial color structure, which are called color-connected (CC) diagrams. In the present case, the CC diagram that contributes to the OPE side at the pion pole is shown in Fig. 2. Possible permutations (not shown) of the diagram in Fig. 2 also contribute.

The diagram in Fig. 2 contributes only to the structures $q_\nu g_{\mu\alpha}$ and $q_\nu p'_\mu p'_\alpha$ appearing in the phenomenological side. Since structures with more momenta are supposed to give better results, we choose to work with the $q_\nu p'_\mu p'_\alpha$ structure. Therefore, in the OPE side and in the $q_\nu p'_\mu p'_\alpha$ structure, we obtain

$$\Pi^{(\mathrm{OPE})} = \frac{\langle \bar{q}g\sigma.Gq\rangle}{12\sqrt{2}\pi^2}\frac{1}{q^2}\int_0^1 d\alpha \frac{\alpha(1-\alpha)}{m_c^2 - \alpha(1-\alpha)p'^2}. \tag{8}$$

Isolating the $q_\nu p'_\mu p'_\alpha$ structure in Eq. (5) and making a single Borel transformation to both $P^2 = P'^2 \to M^2$, we finally get the sum rule:

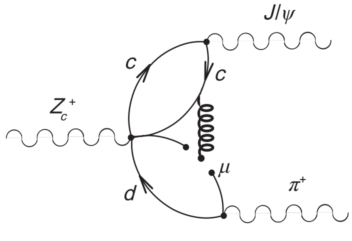


FIG. 2. CC diagram, which contributes to the OPE side of the sum rule.

$$A(e^{-m_\psi^2/M^2} - e^{-m_{Z_c}^2/M^2}) + Be^{-s_0/M^2}$$
$$= \frac{\langle \bar{q}g\sigma.Gq\rangle}{12\sqrt{2}\pi^2}\int_0^1 d\alpha e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}, \tag{9}$$

where $s_0$ is the continuum threshold parameter for $Z_c$,

$$A = \frac{g_{Z_c\psi\pi}\lambda_{Z_c} f_\psi F_\pi(m_{Z_c}^2 + m_\psi^2)}{2m_{Z_c}^2 m_\psi(m_{Z_c}^2 - m_\psi^2)}, \tag{10}$$

and $B$ is a parameter introduced to take into account single pole contributions associated with pole-continuum transitions, which are not suppressed when only a single Borel transformation is done in a three-point function sum rule [30,32–34]. In the numerical analysis, we use the following values for quark masses and QCD condensates [22,35]:

$$m_c(m_c) = (1.23 \pm 0.05)\ \mathrm{GeV},$$
$$\langle \bar{q}q\rangle = -(0.23 \pm 0.03)^3\ \mathrm{GeV}^3, \tag{11}$$
$$\langle \bar{q}g\sigma.Gq\rangle = m_0^2\langle \bar{q}q\rangle,$$
$$m_0^2 = 0.8\ \mathrm{GeV}^2.$$

For the meson masses and decay constants, we use the experimental values [36] $m_\psi = 3.1$ GeV, $m_\pi = 138$ MeV, $f_\psi = 0.405$ GeV, and $F_\pi = 131.52$ MeV. For the $Z_c$ mass, we use the value measured in Ref. [1]: $m_{Z_c} = (3899 \pm 6)$ MeV. The meson-current coupling, $\lambda_{Z_c}$, defined in Eq. (7), can be determined from the two-point sum rule [22]: $\lambda_{Z_c} = (1.5 \pm 0.3) \times 10^{-2}$ GeV$^5$. For the continuum threshold, we use $s_0 = (m_{Z_c} + \Delta s_0)^2$, with $\Delta s_0 = (0.5 \pm 0.1)$ GeV. We evaluate the sum rule in the range $2.0 \leq M^2 \leq 3.0$ GeV$^2$, which is the range in which the two-point function for $X(3872)$ [which is the same for $Z_c(3900)$] shows good OPE convergence and in which the pole contribution is bigger than the continuum contribution [22]. In Fig. 3, we show, through the circles, the rhs of Eq. (9), as a function of the Borel mass.

To determine the coupling constant $g_{Z_c\psi\pi}$, we fit the QCDSRs results with the analytical expression in the lhs of Eq. (9) and find (using $\Delta s_0 = 0.5$ GeV) $A = 1.46 \times 10^{-4}$ GeV$^5$ and $B = -8.44 \times 10^{-4}$ GeV$^5$. Using the definition of $A$ in Eq. (10), the value obtained for the coupling constant is $g_{Z_c\psi\pi} = 3.89$ GeV, which is in excellent agreement with the estimate made in Ref. [17], based on dimensional arguments. Considering the uncertainties given above, we finally find

$$g_{Z_c\psi\pi} = (3.89 \pm 0.56)\ \mathrm{GeV}. \tag{12}$$

The decay width is given by [17]

$$\Gamma(Z_c^+(3900) \to J/\psi\pi^+) = \frac{p^*(m_{Z_c}, m_\psi, m_\pi)}{8\pi m_{Z_c}^2}\frac{1}{3}g_{Z_c\psi\pi}^2\left(3 + \frac{(p^*(m_{Z_c}, m_\psi, m_\pi))^2}{m_\psi^2}\right), \tag{13}$$

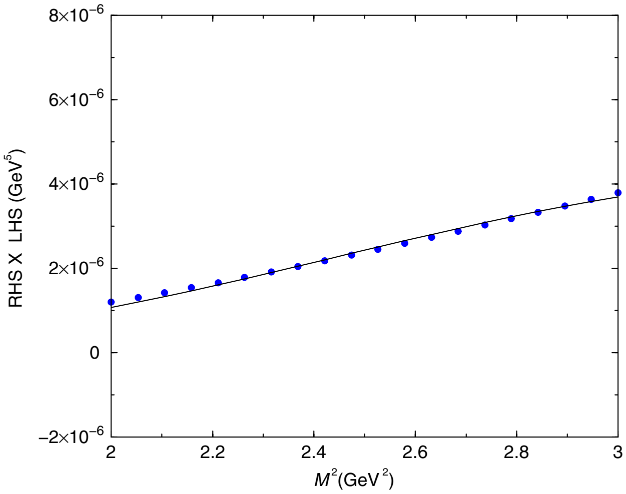


FIG. 3 (color online). Dots: the rhs of Eq. (9), as a function of the Borel mass for $\Delta s_0 = 0.5$ GeV. The solid line gives the fit of the QCDSRs results through the lhs of Eq. (9).

where

$$p^*(a,b,c) = \frac{\sqrt{a^4+b^4+c^4-2a^2b^2-2a^2c^2-2b^2c^2}}{2a}. \tag{14}$$

Therefore, we obtain

$$\Gamma(Z_c^+(3900) \to J/\psi\pi^+) = (29.1 \pm 8.2)\ \text{MeV}. \tag{15}$$

## III. $Z_c^+(3900) \to \eta_c\rho^+$ DECAY WIDTH

Next, we consider the $Z_c^+(3900) \to \eta_c\rho^+$ decay. The three-point function for the corresponding vertex is obtained from Eq. (2) by using

$$\Pi_{\mu\alpha}(x,y) = \langle 0|T[j_5^{\eta_c}(x) j_\mu^\rho(y) j_\alpha^\dagger(0)]|0\rangle, \tag{16}$$

with

$$j_5^{\eta_c} = i\bar{c}_a\gamma_5 c_a, \quad \text{and} \quad j_\mu^\rho = \bar{d}_a\gamma_\mu u_a. \tag{17}$$

In this case, the phenomenological side is

$$\Pi_{\mu\alpha}^{(\text{phen})}(p,p',q) = \frac{-i\lambda_{Z_c} m_\rho f_\rho f_{\eta_c} m_{\eta_c}^2 g_{Z_c\eta_c\rho}(q^2)}{2m_c(p^2-m_{Z_c}^2)(p'^2-m_{\eta_c}^2)(q^2-m_\rho^2)} \times\left(-g_{\mu\lambda}+\frac{q_\mu q_\lambda}{m_\rho^2}\right)\left(-g_\alpha^\lambda+\frac{p_\alpha p^\lambda}{m_{Z_c}^2}\right)+\cdots, \tag{18}$$

where now we have used the definitions

$$\langle 0|j_\mu^\rho|\rho(q)\rangle = m_\rho f_\rho \varepsilon_\mu(q), \qquad \langle 0|j_5^{\eta_c}|\eta_c(p')\rangle = \frac{f_{\eta_c} m_{\eta_c}^2}{2m_c}. \tag{19}$$

In the OPE side, we consider the CC diagrams of the same kind as the diagram in Fig. 2. In the $p'_\alpha q_\mu$ structure, we have

$$\Pi^{(\text{OPE})} = \frac{-im_c\langle\bar{q}g\sigma.Gq\rangle}{48\sqrt{2}\pi^2}\frac{1}{q^2}\int_0^1 d\alpha \frac{1}{m_c^2-\alpha(1-\alpha)p'^2}. \tag{20}$$

Remembering that $p = p' + q$, isolating the $q_\alpha p'_\mu$ structure in Eq. (18), and making a single Borel transformation on both $P^2 = P'^2 \to M^2$, we finally get the sum rule:

$$C(e^{-m_{\eta_c}^2/M^2} - e^{-m_{Z_c}^2/M^2}) + De^{-s_0/M^2} = \frac{Q^2+m_\rho^2}{Q^2}\frac{m_c\langle\bar{q}g\sigma.Gq\rangle}{48\sqrt{2}\pi^2}\int_0^1 d\alpha \frac{e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}}{\alpha(1-\alpha)}, \tag{21}$$

with $Q^2 = -q^2$ and

$$C = \frac{g_{Z_c\eta_c\rho}(Q^2)\lambda_{Z_c} m_\rho f_\rho f_{\eta_c} m_{\eta_c}^2}{2m_c m_{Z_c}^2(m_{Z_c}^2 - m_{\eta_c}^2)}. \tag{22}$$

We use the experimental values for $m_\rho$, $f_\rho$ and $m_{\eta_c}$ [36], and we extract $f_{\eta_c}$ from Ref. [37]:

$$m_\rho = 0.775\ \text{GeV}, \qquad m_{\eta_c} = 2.98\ \text{GeV},$$
$$f_\rho = 0.157\ \text{GeV}, \qquad f_{\eta_c} = 0.35\ \text{GeV}. \tag{23}$$

One can use Eq. (21) and its derivative with respect to $M^2$ to eliminate $D$ from Eq. (21) and to isolate $g_{Z_c\eta_c\rho}(Q^2)$. In Fig. 4, we show $g_{Z_c\eta_c\rho}(Q^2)$ as a function of both $M^2$ and $Q^2$. A good Borel window is determined when the parameter to be extracted from the sum rule is as independent of the Borel mass as possible. Therefore, from Fig. 4, we notice that the Borel window in which the form factor is independent of $M^2$ is in the region of $4.0 \le M^2 \le 10.0$ GeV$^2$. The squares in Fig. 5 show the $Q^2$ dependence of $g_{Z_c\eta_c\rho}(Q^2)$, obtained for $M^2 = 5.0$ GeV$^2$. For other values of the Borel mass, in the range of $4.0 \le M^2 \le 10.0$ GeV$^2$, the results are equivalent. Since the coupling

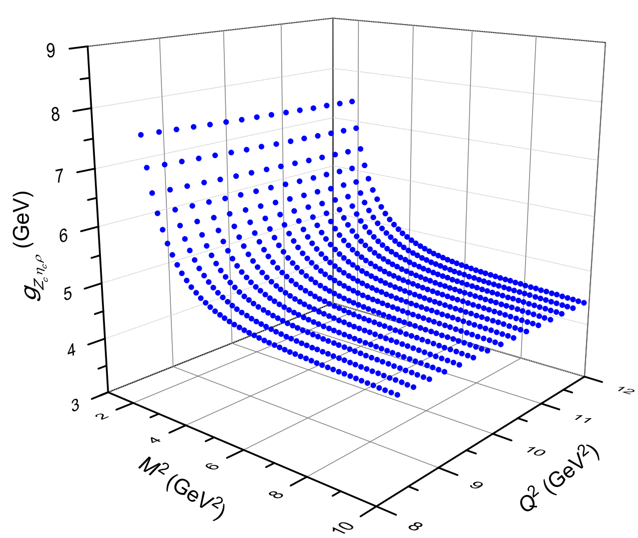


FIG. 4 (color online). QCDSRs results for the form factor $gZ_c\eta_c\rho(Q^2)$ as a function of $Q^2$ and $M^2$ for $\Delta s_0 = 0.5$ GeV.

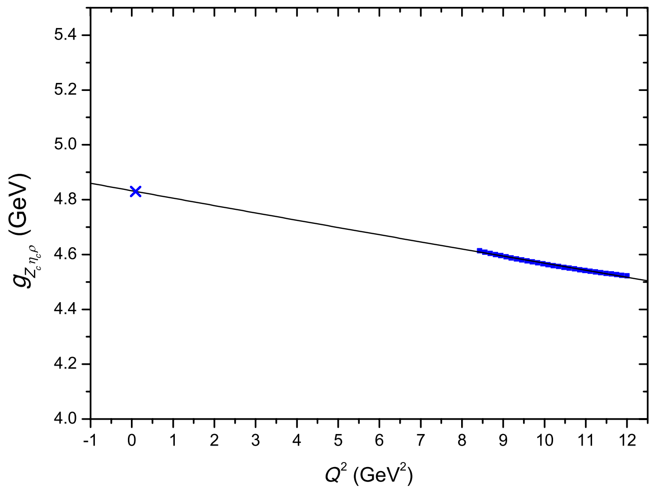


FIG. 5 (color online). QCDSRs results for $g_{Z_c\eta_c\rho}(Q^2)$, as a function of $Q^2$, for $\Delta s_0 = 0.5$ GeV (squares). The solid line gives the parametrization of the QCDSRs results through Eq. (24). The cross gives the value of the coupling constant.

constant is defined as the value of the form factor at the meson pole, $Q^2 = -m_\rho^2$, we need to extrapolate the form factor for a region of $Q^2$ in which the QCDSRs are not valid. This extrapolation can be done by parametrizing the QCDSRs results for $g_{Z_c\eta_c\rho}(Q^2)$ with the help of an exponential form:

$$g_{Z_c\eta_c\rho}(Q^2) = g_1 e^{-g_2 Q^2}, \qquad (24)$$

with $g_1 = 4.83$ GeV and $g_2 = 5.6 \times 10^{-3}$ GeV$^{-2}$. We also show in Fig. 5, through the line, the fit of the QCDSRs results for $\Delta s_0 = 0.5$ GeV, using Eq. (24). The value of the coupling constant, $g_{Z_c\eta_c\rho}$, is also shown in this figure through the cross. We obtain

$$g_{Z_c\eta_c\rho} = g_{Z_c\eta_c\rho}(-m_\rho^2) = (4.85 \pm 0.81)\ \text{GeV}. \qquad (25)$$

The uncertainty in the coupling constant given above comes from variations in $s_0$, $\lambda_{Z_c}$, and $m_c$ in the ranges given above. This value for the coupling is bigger than the estimate presented in Ref. [17]. Inserting this coupling and the corresponding masses into Eq. (13), we find

$$\Gamma(Z_c^+(3900) \to \eta_c\rho^+) = (27.5 \pm 8.5)\ \text{MeV}. \qquad (26)$$

## IV. $Z_c^+(3900) \to D^+\bar{D}^{*0}$ DECAY WIDTH

Finally, we consider the $Z_c^+(3900) \to D^+\bar{D}^{*0}$ decay. In this case, we use in Eq. (2)

$$\Pi_{\mu\alpha}(x,y) = \langle 0|T[j_\mu^{D^*}(x) j_5^D(y) j_\alpha^\dagger(0)]|0\rangle, \qquad (27)$$

where

$$j_5^D = i\bar{d}_a\gamma_5 c_a, \quad \text{and} \quad j_\mu^{D^*} = \bar{c}_a\gamma_\mu u_a. \qquad (28)$$

Using the definitions

$$\langle 0|j_\mu^{D^*}|D^*(p')\rangle = m_{D^*} f_{D^*}\varepsilon_\mu(p'),$$
$$\langle 0|j_5^D|D(q)\rangle = \frac{f_D m_D^2}{m_c}, \qquad (29)$$

the phenomenological side is given by

$$\Pi_{\mu\alpha}^{(\text{phen})}(p,p',q) = \frac{-i\lambda_{Z_c} m_{D^*} f_{D^*} f_D m_D^2 g_{Z_c DD^*}(q^2)}{m_c(p^2 - m_{Z_c}^2)(p'^2 - m_{D^*}^2)(q^2 - m_D^2)} \times \left(-g_{\mu\lambda} + \frac{p'_\mu p'_\lambda}{m_{D^*}^2}\right)\left(-g_\alpha^\lambda + \frac{p_\alpha p^\lambda}{m_{Z_c}^2}\right) + \cdots \qquad (30)$$

In the OPE side, we consider again only the CC diagrams. In the $p'_\alpha p'_\mu$ structure, we have

$$\Pi^{(\text{OPE})} = \frac{-im_c\langle\bar{q}g\sigma.Gq\rangle}{48\sqrt{2}\pi^2}\left[\frac{1}{m_c^2 - q^2}\int_0^1 d\alpha \frac{\alpha(2+\alpha)}{m_c^2 - (1-\alpha)p'^2} - \frac{1}{m_c^2 - p'^2}\int_0^1 d\alpha \frac{\alpha(2+\alpha)}{m_c^2 - (1-\alpha)q^2}\right]. \qquad (31)$$

Isolating the $p'_\mu p'_\alpha$ structure in Eq. (30) and making a single Borel transformation on both $P^2 = P'^2 \to M^2$, we get

$$\frac{1}{Q^2 + m_D^2}[E(e^{-m_{D^*}^2/M^2} - e^{-m_{Z_c}^2/M^2}) + Fe^{-s_0/M^2}] = \frac{m_c\langle\bar{q}g\sigma.Gq\rangle}{48\sqrt{2}\pi^2}\left[\frac{1}{m_c^2 + Q^2}\int_0^1 d\alpha \frac{\alpha(2+\alpha)}{1-\alpha} e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}} - e^{-m_c^2/M^2}\int_0^1 d\alpha \frac{\alpha(2+\alpha)}{m_c^2 + (1-\alpha)Q^2}\right], \qquad (32)$$

with

$$E = \frac{g_{Z_c DD^*}(Q^2)\lambda_{Z_c} f_{D^*} f_D m_D^2}{m_c m_{D^*}(m_{Z_c}^2 - m_{D^*}^2)}. \qquad (33)$$

We use the experimental values for $m_D$ and $m_{D^*}$ [36], and we extract $f_D$ and $f_{D^*}$ from Ref. [26]:

$$m_D = 1.869\ \text{GeV}, \qquad f_D = (0.18 \pm 0.02)\ \text{GeV},$$
$$m_{D^*} = 2.01\ \text{GeV}, \qquad f_{D^*} = (0.24 \pm 0.02)\ \text{GeV}. \qquad (34)$$

In Fig. 6, we show $g_{Z_c DD^*}(Q^2)$, as a function of both $M^2$ and $Q^2$, from where we notice that we get a Borel stability in the region of $2.2 \le M^2 \le 2.8$ GeV$^2$.

Fixing $M^2 = 2.6$ GeV$^2$, we show in Fig. 7, through the squares, the $Q^2$ dependence of the $g_{Z_c DD^*}(Q^2)$ form factor. Again, to extract the coupling constant, we fit the QCDSRs results using the exponential form in Eq. (24) with $g_1 = 1.733$ GeV and $g_2 = 0.076$ GeV$^{-2}$. The line

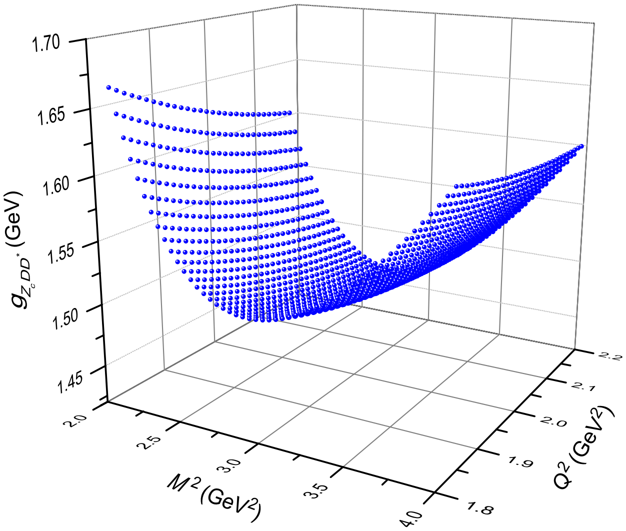


FIG. 6 (color online). QCDSRs results for the form factor $g_{Z_cDD^*}(Q^2)$ as a function of $Q^2$ and $M^2$ for $\Delta s_0 = 0.5$ GeV.

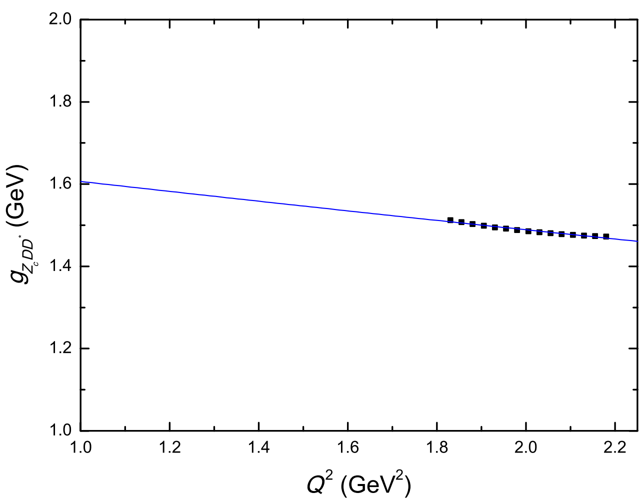


FIG. 7 (color online). QCDSRs results for $g_{Z_cDD^*}(Q^2)$, as a function of $Q^2$, for $\Delta s_0 = 0.5$ GeV (squares). The solid line gives the parametrization of the QCDSRs results through Eq. (24).

in Fig. 7 shows the fit of the QCDSRs results for $\Delta s_0 = 0.5$ GeV, using Eq. (24). We get for the coupling constant

$$g_{Z_cDD^*} = g_{Z_cDD^*}(-m_D^2) = (2.5 \pm 0.3)\ \text{GeV}. \tag{35}$$

TABLE I. Coupling constants and decay widths in different channels.

| Vertex | Coupling constant (GeV) | Decay width (MeV) |
|---|---|---|
| $Z_c^+(3900)J/\psi\pi^+$ | $3.89 \pm 0.56$ | $29.1 \pm 8.2$ |
| $Z_c^+(3900)\eta_c\rho^+$ | $4.85 \pm 0.81$ | $27.5 \pm 8.5$ |
| $Z_c^+(3900)D^+\bar{D}^{*0}$ | $2.5 \pm 0.3$ | $3.2 \pm 0.7$ |
| $Z_c^+(3900)\bar{D}^0D^{*+}$ | $2.5 \pm 0.3$ | $3.2 \pm 0.7$ |

The uncertainty in the coupling constant comes from variations in $s_0$, $\lambda_{Z_c}$, $f_D$, $f_{D^*}$, and $m_c$. This value for this coupling is again in excellent agreement with the estimate presented in Ref. [17]. Using again Eq. (13) with this coupling, the decay width in this channel is

$$\Gamma(Z_c^+ \to D^+\bar{D}^{*0}) = (3.2 \pm 0.7)\ \text{MeV}. \tag{36}$$

## V. CONCLUSIONS

In conclusion, we have used the three-point QCDSRs to evaluate the coupling constants in the vertices $Z_c^+(3900)J/\psi\pi^+$, $Z_c^+(3900)\eta_c\rho^+$, and $Z_c^+(3900)D^+\bar{D}^{*0}$. In the case of the $Z_c^+(3900)J/\psi\pi^+$ vertex, we have used the sum rule at the pion pole, and the coupling was extracted directly from the sum rule. In the cases of $Z_c^+(3900)\eta_c\rho^+$ and $Z_c^+(3900)D^+\bar{D}^{*0}$ vertices, we have extracted the form factors, and the couplings were obtained with a fit of the QCDSRs results. In the three cases, we have only considered the color connected diagrams, since we expect the $Z_c(3900)$ to be a genuine tetraquark state with a nontrivial color structure. The obtained couplings, with the respective decay widths, are given in Table I. We have also included in this table the results for the vertex $Z_c^+(3900)\bar{D}^0D^{*+}$, since it is exactly the same result as in the $Z_c^+(3900)D^+\bar{D}^{*0}$ vertex.

Considering these four decay channels, we get a total width $\Gamma = (63.0 \pm 18.1)$ GeV for $Z_c(3900)$, which is in agreement with the two experimental values: $\Gamma = (46 \pm 22)$ MeV from BESIII [1] and $\Gamma = (63 \pm 35)$ MeV from BELLE [2].

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