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Is the Higgs boson a sign of extra dimensions?

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We introduce a four-dimensional cutoff in the scenario of gauge-Higgs unification to control the ultraviolet behavior. A one-loop effective potential for a Higgs field and the Higgs mass are obtained with the cutoff. We find an *interrelation* between the four-dimensional cutoff and the scale of extra dimensions, which is concretized through the Higgs mass. Combining this interrelation and the recently discovered Higgs boson at the LHC, we obtain an interesting constraint for the four-dimensional cutoff and the extra-dimensional scale.

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I. INTRODUCTION

A higher-dimensional gauge theory is one of the attractive candidates for physics beyond the standard model. Gauge-Higgs unification [1] is one such gauge theory, where the gauge and Higgs fields are unified into a higher-dimensional gauge field. Component gauge fields for compactified extra directions behave like the Higgs fields at low energy.

In the scenario of the gauge-Higgs unification, the gauge symmetry is broken through quantum corrections [2], and the Higgs mass—which is zero at the tree-level due to the higher-dimensional gauge invariance—arises at the quantum level. It has been said that the effective potential for the Higgs field and the Higgs mass do not suffer from ultraviolet divergences. Thanks to this property, the gauge-Higgs unification may solve the gauge hierarchy problem without relying on supersymmetry [3]. The gauge-Higgs unification has been an attractive alternative for the Higgs mechanism. Many attempts to seek phenomenologically viable models with the gauge-Higgs unification have been carried out in the past [4–6]. In addition, various aspects of the gauge-Higgs unification such as the finite-temperature phase transition have also been studied [7–10].

In the gauge-Higgs unification, one needs to evaluate the effective potential for the Higgs field in order to discuss the gauge symmetry-breaking patterns and to calculate the Higgs mass, which is obtained by the second derivative of the potential at the vacuum. In the past, one employed the dimensional regularization for the momentum integration in evaluating the effective potential at the one-loop level. The divergent terms that depend on the order parameter (the Higgs field) do not appear in the effective potential and the Higgs mass. But the dimensional regularization essentially can not account for power divergences.

As stated above, the Higgs mass arises through quantum corrections in extra dimensions, say, Kaluza-Klein modes in the gauge-Higgs unification. It is, however, difficult to obtain the definite quantum effect of the higher-dimensional gauge theory because of the nonrenormalizability. The detailed structure of the effective potential for the Higgs field is unknown as long as one cannot solve the dynamics in higher dimensions. At the moment, it remains unclear how much one should take the quantum correction in the extra dimension into account in order to determine the low-energy physics.

The effective potential we shall compute has the Kaluza-Klein modes and the four-dimensional momentum cutoff which originates from the five-dimensional cutoff because we start with the five-dimensional gauge theory in which there are uncontrollable ultraviolet divergences due to the nonrenormalizability. We would like to keep the shift symmetry [11] which is a remnant of the original gauge symmetry, so that one has to sum up all the Kaluza-Klein modes.[1] Then the five-dimensional ultraviolet divergence reduces to the four-dimensional momentum cutoff.

In a theory like the gauge-Higgs unification—the five-dimensional physics—the Kaluza-Klein mode determines the low-energy physics, such as the Higgs mass. It is important to have the parameter which tells us how much the five-dimensional physics contributes to determining the low-energy physics. Such a parameter can be constructed by using the four-dimensional momentum cutoff and the five-dimensional scale in our case. We shall refer to this parameter as the *interrelation*. It should be noted that the interrelation is not a phenomenological parameter, but it is a theoretical one. It is interesting, however, that if one takes account of the experimental value of a physical observable such as the Higgs mass, one obtains a constraint on the interrelation by which we understand how much the Kaluza-Klein mode should contribute to the Higgs mass.

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[1]When we consider an effective theory of the five-dimensional gauge theory with the cutoff, it is natural to respect the shift symmetry as the four-dimensional theory.

This paper is organized as follows. In the next section, after a brief setup, we present the expression for the one-loop effective potential for the Higgs field and the Higgs mass with the four-dimensional cutoff. The interrelation, which is a key notion, is also explained. We also find that there is a remarkable combination between the periodicity of the Higgs field and an exponential suppression with respect to the interrelation. In Sec. III, we study the interrelation through the Higgs mass in some models with the gauge-Higgs unification. We give a constraint on the four-dimensional cutoff and the scale of the extra dimension by taking account of the result on the Higgs mass at the LHC [12,13]. The final section is devoted to the conclusions. In the Appendix, important formulas used in the text are derived.

II. EFFECTIVE POTENTIAL AND HIGGS MASS WITH FOUR-DIMENSIONAL CUTOFF

Let us consider a nonsupersymmetric SU(3) gauge theory on $M^4 \times S^1/Z_2$, where M^4 is the four-dimensional Minkowski spacetime and S^1/Z_2 is an orbifold.[2] One must specify the boundary conditions of fields for the S^1 direction and the two orbifold fixed points at $y = 0, \pi R$, where R is the radius of the S^1. They are defined by

$$A_{\hat{\mu}}(x^\mu, y + 2\pi R) = U A_{\hat{\mu}}(x^\mu, y) U^\dagger, \tag{1}$$

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x^\mu, y_i - y) = P_i \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x^\mu, y_i + y) P_i^\dagger \qquad (i = 0, 1), \tag{2}$$

where $U = U^\dagger$, $P_i^\dagger = P_i = P_i^{-1}$, and $y_0 = 0$, $y_1 = \pi R$. The coordinate $x^\mu (\mu = 0, \ldots, 3)$ denotes the four-dimensional Minkowski spacetime and y is the coordinate of the extra dimension. The translation U together with the reflection P_1 is equivalent to the reflection P_0, so that there is a relation $U = P_1 P_0$. We take $P_i (i = 0, 1)$ to be fundamental projections.

In the scenario of the gauge-Higgs unification, the zero modes for A_y play an important role and behave like Higgs fields at low energy. If the Higgs field develops a vacuum expectation value, the SU(2) × U(1) gauge symmetry is broken to the electromagnetic $\mathrm{U(1)_{em}}$. One must choose the boundary conditions $P_{0,1}$ in such a way that the zero mode for A_y belongs to the fundamental representation under the SU(2) gauge group. We choose $P_0 = P_1 = \mathrm{diag}(-1, -1, 1)$. Then the SU(3) gauge symmetry is broken down explicitly to SU(2) × U(1) by the orbifolding. The zero modes for the gauge field are read off by Eq. (2) for the boundary condition of $P_{0,1}$.

[2]Notations used in this paper are the same as those in Ref. [14].

The zero modes for A_μ are given by

$$A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix} A_\mu^3 + \frac{A_\mu^8}{\sqrt{3}} & A_\mu^1 - i A_\mu^2 & 0 \\ A_\mu^1 + i A_\mu^2 & -A_\mu^3 + \frac{A_\mu^8}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} A_\mu^8 \end{pmatrix}, \tag{3}$$

by which the residual gauge symmetry is clearly SU(2) × U(1). On the other hand, the zero mode for A_y is found to be

$$A_y^{(0)} = \frac{1}{2} \begin{pmatrix} 0 & 0 & A_y^4 - i A_y^5 \\ 0 & 0 & A_y^6 - i A_y^7 \\ A_y^4 + i A_y^5 & A_y^6 + i A_y^7 & 0 \end{pmatrix}. \tag{4}$$

We observe that

$$\Phi \equiv \sqrt{2\pi R} \frac{1}{\sqrt{2}} \begin{pmatrix} A_y^4 - i A_y^5 \\ A_y^6 - i A_y^7 \end{pmatrix} \tag{5}$$

belongs to the fundamental representation under the SU(2). The adjoint representation of the SU(3) is decomposed under the SU(2) into

$$\mathbf{8} \to \mathbf{3} + \mathbf{2} + \mathbf{2}^* + \mathbf{1}. \tag{6}$$

We understand how the gauge and Higgs fields are embedded into the higher-dimensional gauge field.

By utilizing the SU(2) × U(1) degrees of freedom, the vacuum expectation value for the Higgs field is parametrized by

$$\langle A_y^6 \rangle = \frac{a}{gR}, \tag{7}$$

where g is the five-dimensional gauge coupling and a is a real parameter. The parameter a is related with the Wilson-line phase,

$$\begin{aligned} W &= \mathcal{P} \exp\left(ig \oint_{S^1} dy \langle A_y \rangle \right) \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i \sin(\pi a) \\ 0 & i \sin(\pi a) & \cos(\pi a) \end{pmatrix} \quad (a \bmod 2). \end{aligned} \tag{8}$$

The original gauge invariance, namely concerning the fifth direction, guarantees that the Wilson-line phase is mod 2. The gauge symmetry-breaking patterns of the SU(2)×U(1) are classified by the values of a,

$$\mathrm{SU(2)} \times \mathrm{U(1)} \to \begin{cases} \mathrm{SU(2)} \times \mathrm{U(1)} & \text{for } a = 0, \\ \mathrm{U(1)} \times \mathrm{U(1)}' & \text{for } a = 1, \\ \mathrm{U(1)_{em}} & \text{for otherwise.} \end{cases} \tag{9}$$

The value of a is determined as the global minimum of the effective potential for the Higgs field.

In the scenario of the gauge-Higgs unification, one needs not only the matter fields that satisfy the periodic boundary condition (PBC), but also the ones that satisfy the anti-periodic boundary condition (APBC). They are distinguished by the parameter $\eta(= 1$ for PBC, -1 for APBC$)$ [14]. In addition, we also consider the matter fields belonging to the large representation under the SU(3) gauge group such as the adjoint representation. These are necessary ingredients for a viable model with the gauge-Higgs unification.

In a gauge-Higgs unification scenario, we start with the following five-dimensional effective potential contribution:

$$F_5(Qa, \delta, \Lambda) = \frac{1}{4\pi R} \sum_{n=-\infty}^{\infty} \int_{-\Lambda}^{\Lambda} \frac{d^4 p}{(2\pi)^4} \times \ln\left[p_E^2 + \left(\frac{n + Qa - \frac{\delta}{2}}{R}\right)^2 \right], \quad (10)$$

where $Q = 1,\ 1/2$ for the adjoint, fundamental representation under the SU(2), respectively. The parameter δ takes a value of 0 (1) for the field with the PBC (APBC). We have introduced the four-dimensional ultraviolet cutoff Λ in the momentum integration, which originates in the five-dimensional ultraviolet cutoff because our starting theory is a five-dimensional Yang-Mills theory and has some ultraviolet-divergent quantities owing to the nonrenormalizability. Noting that it is necessary to sum up all the Kaluza-Klein modes in order to keep the shift invariance reflected as five-dimensional gauge invariance, the five-dimensional ultraviolet divergence reduces to the four-dimensional cutoff Λ [Eq. (10)]. The effective potential is given by collecting all the contributions of the fields in the theory,

$$V_{\rm eff} = \sum_{i=\rm fields} (-1)^F N_{\rm deg}^i F_5^i(Qa, \delta). \quad (11)$$

The F stands for the fermion number of the internal loop, and $N_{\rm deg}^i$ is the number of on-shell degrees of freedom for the relevant matter field.

We first sum up all the Kaluza-Klein modes,

$$\sum_{n=-\infty}^{\infty} \frac{2 p_E R^2}{(R p_E)^2 + (n + Qa - \frac{\delta}{2})^2} = L \times \frac{\sinh(L p_E)}{\cosh(L p_E) - \cos(2\pi(Qa - \frac{\delta}{2}))}, \quad (12)$$

where we have defined $L \equiv 2\pi R$ and used the formula

$$\sum_{n=-\infty}^{\infty} \frac{1}{x^2 + (n+a)^2} = \frac{\pi}{x} \frac{\sinh(2\pi x)}{\cosh(2\pi x) - \cos(2\pi a)}. \quad (13)$$

Let us note that summing up all the Kaluza-Klein modes is consistent with the gauge invariance for the direction of the extra dimension. By integrating it with respect to p_E, we immediately have

$$\sum_{n=-\infty}^{\infty} \ln\left[p_E^2 + \left(\frac{n + Qa - \frac{\delta}{2}}{R}\right)^2 \right] = \ln\left[\cosh(L p_E) - \cos\left(2\pi\left(Qa - \frac{\delta}{2}\right)\right)\right]. \quad (14)$$

It can be shown that the integration constant does not depend on the order parameter a, so we have set it to be zero.

Second, we perform the four-dimensional momentum integration,

$$F_5(Qa, \delta, \tilde\Lambda) = \frac{1}{2L^5} \frac{2\pi^2}{\Gamma(2)(2\pi)^4} \int_0^{\tilde\Lambda} d\tilde p_E \tilde p_E^3 \times \ln\left[\cosh \tilde p_E - \cos\left(2\pi\left(Qa - \frac{\delta}{2}\right)\right)\right] \quad (15)$$

$$= \frac{1}{(4\pi)^2 L^5} [-6(\mathrm{Li}_5(e^{2\pi i (Qa - \frac{\delta}{2})}) + \mathrm{c.c.}) + 6(\mathrm{Li}_5(e^{2\pi i (Qa - \frac{\delta}{2}) - \tilde\Lambda}) + \mathrm{c.c.}) + 6\tilde\Lambda(\mathrm{Li}_4(e^{2\pi i (Qa - \frac{\delta}{2}) - \tilde\Lambda}) + \mathrm{c.c.}) + 3\tilde\Lambda^2(\mathrm{Li}_3(e^{2\pi i (Qa - \frac{\delta}{2}) - \tilde\Lambda}) + \mathrm{c.c.}) + \tilde\Lambda^3(\mathrm{Li}_2(e^{2\pi i (Qa - \frac{\delta}{2}) - \tilde\Lambda}) + \mathrm{c.c.})], \quad (16)$$

where the dimensionless integration variable has been defined by $\tilde p_E \equiv L p_E$ in Eq. (15) and $\tilde\Lambda = L\Lambda$, and we have used the polylogarithm defined in Eq. (A4) in the Appendix. Here we have ignored constant terms that do not depend on the order parameter a. The derivation of Eq. (16) is given in the Appendix.

In Eqs. (15) and (16) we have introduced a dimensionless parameter $\tilde\Lambda$ which relates the four-dimensional cutoff Λ and the energy scale of the extra dimension L^{-1} as

$$\tilde\Lambda = L\Lambda = \frac{\Lambda}{1/L} \equiv \xi. \quad (17)$$

The parameter ξ in Eq. (17) plays an important role in the low-energy physics. We call this the *interrelation* between a four-dimensional physics and the extra dimension. Here we notice that $\tilde\Lambda$ stands for not only the cutoff, but also for the contribution of the Kaluza-Klein mode, depending on the scale of Λ. Namely, the latter point of view is crucial for the interrelation (which will be discussed in the Sec. III), so we shall use a different notation ξ when we emphasize the interrelation, such as in the calculation of the Higgs mass.

Originally the five-dimensional dynamics is out of control due to the nonrenormalizabilty. A cutoff must be introduced to define the theory, and it lies in a certain energy scale, though it is unknown where this should be. One does not know how much we should take account of the quantum correction from the Kaluza-Klein mode in order to determine the low-energy physics. At present, the discovery of the Higgs boson has been reported [12,13] and we expect that the consistent cutoff with the LHC result must lie in a certain energy scale. Then the interrelation tells us how much quantum correction from the Kaluza-Klein mode one should take into account in order for the cutoff to be consistent with the LHC result. At the one-loop level, the effective potential is written in terms of the interrelation and—as we will see concretely later—the interrelation becomes manifest through the Higgs mass.

The first term on the right-hand side of Eq. (16) is well known and has been obtained in a previous calculation [15]. One observes that all the terms except for the first term have a remarkable combination of ξ and the order parameter,[3]

$$e^{2\pi i Q a-\xi}. \tag{18}$$

The combination is a result of respecting the gauge invariance for the direction of the extra dimension, that is, the periodicity of the order parameter a and introducing the four-dimensional cutoff in the momentum integration (15). The potentially dangerous order parameter-dependent divergence disappears as $\xi(=\tilde{\Lambda})$ goes to infinity thanks to the exponential damping. Let us note that the exponential behavior of the cutoff (18) never appears in the dimensional regularization.

The combination (18) is traced back to Eq. (12). By setting $\delta = 0$, it is rewritten as

$$\begin{aligned}\sum_{n=-\infty}^{\infty} \frac{2p_E R^2}{(Rp_E)^2+(n+Qa)^2} &= L\times\frac{\sinh(Lp_E)}{\cosh(Lp_E)-\cos(2\pi Qa)}\\ &= L\times\left(1+\left\{\frac{e^{2\pi iQa-\tilde{p}_E}}{1-e^{2\pi iQa-\tilde{p}_E}}+\text{c.c.}\right\}\right).\end{aligned} \tag{19}$$

Then the relevant quantity is obtained by the integral of the form

$$I(\tilde{\Lambda}) \equiv \int_0^{\tilde{\Lambda}} dy f(y) e^{i\bar{a}-y}, \tag{20}$$

where the function $f(y)$ is an nth polynomial, $f(y)=\sum_{k=1}^{n} a_k y^k$. The above integral is evaluated as

$$I(\tilde{\Lambda}) = F(0)e^{i\bar{a}} - F(\tilde{\Lambda})e^{i\bar{a}-\tilde{\Lambda}} = I(\infty) - F(\tilde{\Lambda})e^{i\bar{a}-\tilde{\Lambda}}. \tag{21}$$

Here we have defined

$$F(y) \equiv \sum_{m=0}^{n} f^{(m)}(y). \tag{22}$$

The first term in Eq. (21) corresponds to the well-known finite term obtained in the previous calculation. It is interesting to note that the ultraviolet limit of the function $I(\tilde{\Lambda})$ is evaluated at the infrared point of the integration variable $y=0$ for another function $F(y)$. This is a notable feature in the scenario of the gauge-Higgs unification.

The effective potential is a special quantity in the gauge-Higgs unification because of the combination $e^{2\pi iQa-\xi}$ at least at the one-loop level, which is never observed in the usual quantum field theory. Once we recognize this point, one immediately realizes that a quantity other than this type does not possess such a combination and hence the finiteness. As we will see below, the Higgs mass also has the same combination.

Now let us proceed to the Higgs mass, which is obtained by the second derivative of the effective potential at the vacuum denoted by $a = a_0$,

$$m_H^2 \equiv \frac{\partial^2 V_{\text{eff}}}{\partial \langle A_y^6\rangle^2}\bigg|_{\text{vac}} = (gR)^2 \frac{\partial^2 V_{\text{eff}}}{\partial a^2}\bigg|_{a=a_0}. \tag{23}$$

The structure of the second derivative of the effective potential can be seen from Eq. (16) by

$$\begin{aligned}\frac{\partial^2 V_{\text{eff}}}{\partial a^2} &\propto \frac{\partial^2 F(Qa,\delta,\xi)}{\partial a^2}\\ &\propto -6(\mathrm{Li}_3(e^{2\pi i(Qa-\frac{\delta}{2})})+\text{c.c.})\\ &\quad + 6(\mathrm{Li}_3(e^{2\pi i(Qa-\frac{\delta}{2})-\xi})+\text{c.c.})\\ &\quad + 6\xi(\mathrm{Li}_2(e^{2\pi i(Qa-\frac{\delta}{2})-\xi})+\text{c.c.})\\ &\quad + 3\xi^2(\mathrm{Li}_1(e^{2\pi i(Qa-\frac{\delta}{2})-\xi})+\text{c.c.})\\ &\quad + \xi^3(\mathrm{Li}_0(e^{2\pi i(Qa-\frac{\delta}{2})-\xi})+\text{c.c.}).\end{aligned} \tag{24}$$

As stated before, we confirm that the Higgs mass also possesses the same combination, $e^{2\pi iQa-\xi}$, as that of the effective potential.

If one takes the infinite limit of ξ, only the first finite term in Eq. (16) survives to reproduce the well-known expression for the Higgs mass. In order to make discussions of the interrelation concrete, we need to consider models explicitly, which we will do in the next section.

III. HIGGS AS AN INTERRELATION BETWEEN FOUR AND EXTRA DIMENSIONS

Let us introduce a set of matter. We follow the studies of the gauge-Higgs unification made in the past [7,14,16], in which we have introduced the fermions and bosons satisfying the periodic boundary condition ($\eta = 1$) and antiperiodic boundary condition ($\eta = -1$), and whose representations under the SU(3) gauge group are the adjoint and fundamental ones. We denote their flavor numbers by

$$\begin{aligned}&(N_F^{\text{adj}(+)}, N_F^{\text{fd}(+)}, N_S^{\text{adj}(+)}, N_S^{\text{fd}(+)}),\\ &(N_F^{\text{adj}(-)}, N_F^{\text{fd}(-)}, N_S^{\text{adj}(-)}, N_S^{\text{fd}(-)}).\end{aligned} \tag{25}$$

Here the $N_{F(S)}^{\text{adj(fd)}}$ stands for the number of the fermion (scalar) belonging to the adjoint (fundamental) representation

[3]The boundary condition δ of the field is not essential in this discussion, so we have ignored it.

under the SU(3) gauge group. The $\pm$ sign associated with $N_{F(S)}^{\mathrm{adj(fd)}}$ is the periodicity of the matter field, $\eta = \pm 1$.

Recalling Eq. (11), the effective potential with these types of matter fields is given by

$$\begin{aligned} V_{\mathrm{eff}}^{\mathrm{total}} = \frac{1}{(4\pi)^2 L^5}[&(-1)^0 3 J^{\mathrm{adj}(+)} + (-1)^1 4 N_F^{\mathrm{adj}(+)} J^{\mathrm{adj}(+)} \\ &+ (-1)^1 4 N_F^{\mathrm{fd}(+)} J^{\mathrm{fd}(+)} + (-1)^0 2 N_S^{\mathrm{adj}(+)} J^{\mathrm{adj}(+)} \\ &+ (-1)^0 2 N_S^{\mathrm{fd}(+)} J^{\mathrm{fd}(+)} + (-1)^1 4 N_F^{\mathrm{adj}(-)} J^{\mathrm{adj}(-)} \\ &+ (-1)^1 4 N_F^{\mathrm{fd}(-)} J^{\mathrm{fd}(-)} + (-1)^0 2 N_S^{\mathrm{adj}(-)} J^{\mathrm{adj}(-)} \\ &+ (-1)^0 2 N_S^{\mathrm{fd}(-)} J^{\mathrm{fd}(-)}], \end{aligned} \tag{26}$$

where the first term is the contribution from the gauge bosons, and we have defined

$$\begin{aligned} J^{\mathrm{adj}(+)} \equiv\ & F^\infty(2a, 0) + F^\xi(2a, 0, \xi) \\ &+ 2(F^\infty(a, 0) + F^\xi(a, 0, \xi)), \end{aligned} \tag{27}$$

$$\begin{aligned} J^{\mathrm{adj}(-)} \equiv\ & F^\infty(2a, 1) + F^\xi(2a, 1, \xi) \\ &+ 2(F^\infty(a, 1) + F^\xi(a, 1, \xi)), \end{aligned} \tag{28}$$

$$J^{\mathrm{fd}(+)} \equiv F^\infty(a, 0) + F^\xi(a, 0, \xi), \tag{29}$$

$$J^{\mathrm{fd}(-)} \equiv F^\infty(a, 1) + F^\xi(a, 1, \xi), \tag{30}$$

and

$$F^\infty(x, \delta) = -6(\mathrm{Li}_5(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})}) + \mathrm{c.c.}), \tag{31}$$

$$\begin{aligned} F^\xi(x, \delta, \xi) = 6(&\mathrm{Li}_5(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}) \\ &+ 6\xi(\mathrm{Li}_4(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}) \\ &+ 3\xi^2(\mathrm{Li}_3(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}) \\ &+ \xi^3(\mathrm{Li}_2(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}). \end{aligned} \tag{32}$$

The shape of the effective potential is determined once we fix the number of flavors and ξ. In the limit of $\xi \to \infty$, the $F^\xi(x, \delta, \xi)$ vanishes, and the effective potential is given by the function $F^\infty(x, \delta)$ alone, which is consistent with the results obtained in the previous calculation. The effective potential vanishes at $\xi = 0$, as seen from Eqs. (31) and (32).

Let us also give the second derivative of the effective potential, which is necessary for the calculation of the Higgs mass by Eq. (23),

$$\begin{aligned} \frac{\partial^2 V_{\mathrm{eff}}^{\mathrm{total}}}{\partial a^2} = \frac{(2\pi)^2}{(4\pi)^2 L^5}(-1)[&(-1)^0 3 J_H^{\mathrm{adj}(+)} \\ &+ (-1)^1 4 N_F^{\mathrm{adj}(+)} J_H^{\mathrm{adj}(+)} + (-1)^1 4 N_F^{\mathrm{fd}(+)} J_H^{\mathrm{fd}(+)} \\ &+ (-1)^0 2 N_S^{\mathrm{adj}(+)} J_H^{\mathrm{adj}(+)} + (-1)^0 2 N_S^{\mathrm{fd}(+)} J_H^{\mathrm{fd}(+)} \\ &+ (-1)^1 4 N_F^{\mathrm{adj}(-)} J_H^{\mathrm{adj}(-)} + (-1)^1 4 N_F^{\mathrm{fd}(-)} J_H^{\mathrm{fd}(-)} \\ &+ (-1)^0 2 N_S^{\mathrm{adj}(-)} J_H^{\mathrm{adj}(-)} + (-1)^0 2 N_S^{\mathrm{fd}(-)} J_H^{\mathrm{fd}(-)}], \end{aligned} \tag{33}$$

where we have defined

$$\begin{aligned} J_H^{\mathrm{adj}(+)} \equiv\ & F_H^\infty(2a, 0) + F_H^\xi(2a, 0, \xi) \\ &+ \frac{1}{4} \times 2(F_H^\infty(a, 0) + F_H^\xi(a, 0, \xi)), \end{aligned} \tag{34}$$

$$\begin{aligned} J_H^{\mathrm{adj}(-)} \equiv\ & F_H^\infty(2a, 1) + F_H^\xi(2a, 1, \xi) \\ &+ \frac{1}{4} \times 2(F_H^\infty(a, 1) + F_H^\xi(a, 1, \xi)), \end{aligned} \tag{35}$$

$$J_H^{\mathrm{fd}(+)} \equiv \frac{1}{4}(F_H^\infty(a, 0) + F_H^\xi(a, 0, \xi)), \tag{36}$$

$$J_H^{\mathrm{fd}(-)} \equiv \frac{1}{4}(F_H^\infty(a, 1) + F_H^\xi(a, 1, \xi)), \tag{37}$$

and

$$F_H^\infty(x, \delta) = -6(\mathrm{Li}_3(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})}) + \mathrm{c.c.}), \tag{38}$$

$$\begin{aligned} F_H^\xi(x, \delta, \xi) = 6(&\mathrm{Li}_3(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}) \\ &+ 6\xi(\mathrm{Li}_2(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}) \\ &+ 3\xi^2(\mathrm{Li}_1(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}) \\ &+ \xi^3(\mathrm{Li}_0(\mathrm{e}^{2\pi i(\frac{x}{2}-\frac{\delta}{2})-\xi}) + \mathrm{c.c.}). \end{aligned} \tag{39}$$

The $F_H^\xi(x, \delta, \xi)$ vanishes for $\xi \to \infty$ to reproduce the old results for the Higgs mass, which is given by Eq. (38). At $\xi = 0$, the Higgs mass vanishes, as seen from Eqs. (38) and (39). The Higgs mass is given by[4]

$$\begin{aligned} m_H^2 &= (gR)^2 \frac{\partial^2 V_{\mathrm{eff}}}{\partial a^2}\bigg|_{a=a_0} = \frac{(2\pi g R)^2}{(4\pi)^2 L^5} H(Qa_0, \delta, \xi) \\ &= \frac{g_4^4}{(8\pi^2)^2}\left(\frac{v}{a_0}\right)^2 H(Qa_0, \delta, \xi), \end{aligned} \tag{40}$$

where we have used the relation $v = a_0/(g_4 R)$ following from the weak gauge boson mass $M_W = a_0/(2R)$, and we have defined $H(Qa, \delta, \xi)$ by the expression aside from the

[4]The models of the gauge-Higgs unification in this paper do not predict the correct Weinberg angle, and we implicitly assume that we have used the prescription done, for example, in Ref. [17], so that the four-dimensional gauge coupling becomes a free parameter and that its size is of order of one.

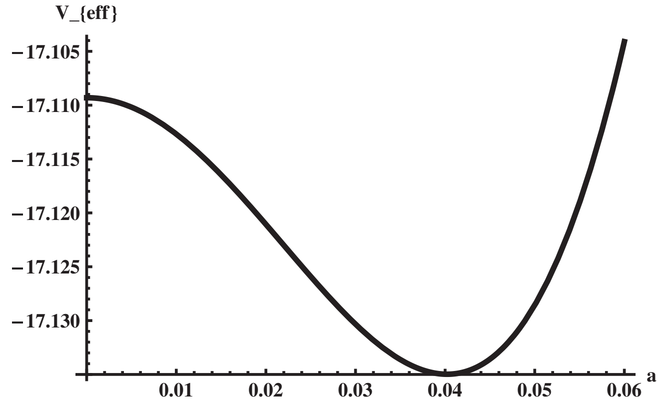

FIG. 1. The shape of the effective potential in the limit of $\tilde{\Lambda} \to \infty$ for the model A. The global minimum is located at $a_0 = 0.0402199$.

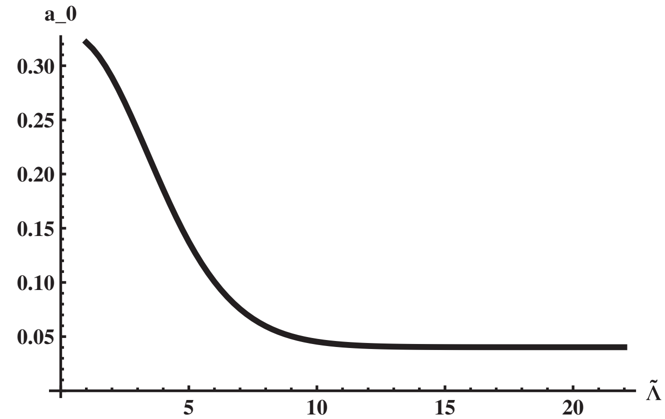

FIG. 2. The behavior of the order parameter a with respect to $\tilde{\Lambda}$ for the model A.

factor $(2\pi)^2/((4\pi)^2 L^5)$ in Eq. (33). The four-dimensional gauge coupling is defined by $g_4 \equiv g/\sqrt{L}$. The value of the Higgs mass is determined by inserting the values of a_0, ξ and the number of flavors.

Now let us first study the case where the matter content is given by

$$\text{model A:} \begin{cases} (N_F^{\mathrm{adj}(+)}, N_F^{\mathrm{fd}(+)}, N_S^{\mathrm{adj}(+)}, N_S^{\mathrm{fd}(+)}) = (2, 2, 0, 0), \\ (N_F^{\mathrm{adj}(-)}, N_F^{\mathrm{fd}(-)}, N_S^{\mathrm{adj}(-)}, N_S^{\mathrm{fd}(-)}) = (2, 2, 0, 3). \end{cases} \tag{41}$$

We first present the typical shape of the effective potential for $\tilde{\Lambda} \to \infty$ in Fig. 1. The global minimum is located at $a_0 = 0.0402199$ and the $SU(2) \times U(1)$ gauge symmetry breaks down to $U(1)_{\mathrm{em}}$. By using the vacuum expectation value a_0, the Higgs mass in the same limit is calculated as $m_H/g_4^2 = 130.222$ GeV. It is known that the matter content is crucial for obtaining the sufficiently heavy Higgs mass [16].

Now we turn on the cutoff $\tilde{\Lambda}(= \xi)$. The shape of the effective potential is changed according to the value of $\tilde{\Lambda}$, so that the position of the global minimum is also changed. We show the behavior of a_0 with respect to $\tilde{\Lambda}$ in Fig. 2. The gauge symmetry is correctly broken, that is, $a_0 \neq 0, 1$ for the range of $\tilde{\Lambda}$ we have studied.[5] The magnitude of $\tilde{\Lambda}$ for $\tilde{\Lambda} \gtrsim 10$ almost saturates the values obtained in the limit of $\tilde{\Lambda} \to \infty$.

Let us next depict the behavior of the Higgs mass with respect to the interrelation $\xi = \frac{\Lambda}{1/L}$ in Fig. 3. We observe that the Higgs mass becomes larger as ξ is larger and for $\xi \gtrsim 10$ the Higgs mass almost saturates the value obtained in the limit of $\xi \to \infty$. On the other hand, for $1 \lesssim \xi \lesssim 8$, the Higgs mass grows almost linearly with respect to ξ. If we take account of the recently reported Higgs mass of 126 GeV at the LHC [12,13], we obtain a bound on ξ. It is given by $\xi = \frac{\Lambda}{1/L} \gtrsim 10$, which implies that the four-dimensional cutoff Λ must satisfy $\Lambda \gtrsim 10 L^{-1}$. The value of the Higgs mass is smoothly connected to zero for $\xi \to 0$ as far as our numerical analyses are concerned.

We can also understand the behavior of the Higgs mass with respect to ξ by the first derivative of $F_H^{\xi}(x, \delta, \xi)$, which essentially controls the Higgs mass. It is given by

$$\frac{\partial F_H^{\xi}(x, \delta, \xi)}{\partial \xi} = -\xi^3 \sum_{n=1}^{\infty} n(e^{2\pi i n(\frac{x}{2} - \frac{\delta}{2}) - n\xi} + \text{c.c.}) = -\xi^3 \left[\frac{e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi}}{(1 - e^{2\pi i(\frac{x}{2} - \frac{\delta}{2}) - \xi})^2} + \text{c.c.} \right]. \tag{42}$$

For a large value of ξ, due to the exponential damping factor, the first derivative vanishes, so that the value of the Higgs mass becomes constant. This corresponds to the flat behavior in Fig. 3. When ξ becomes larger than zero, the ξ^3 starts to control the behavior of the Higgs mass. This gives the almost linear growth of the Higgs mass with respect to ξ in Fig. 3.

Let us discuss the interrelation $\xi = \frac{\Lambda}{1/L}$ which is manifest through the Higgs mass. If the four-dimensional cutoff

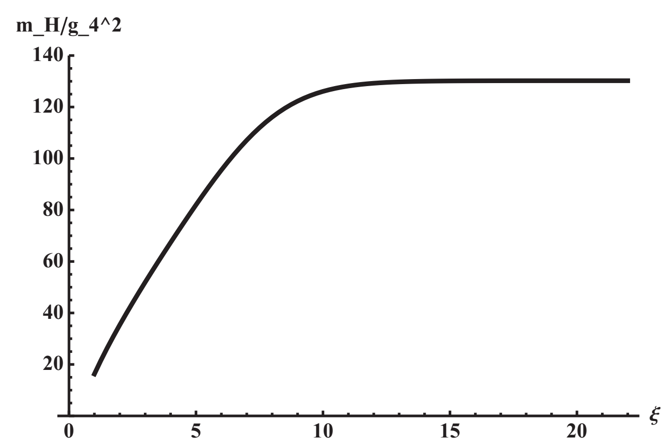

FIG. 3. The behavior of the Higgs mass with respect to $\xi = \frac{\Lambda}{1/L}$ for the model A. The asymptotic value of the Higgs mass is about 130 GeV.

[5] At $\tilde{\Lambda} = 0$, the effective potential vanishes, so that the position of the global minimum in the limit is unclear.

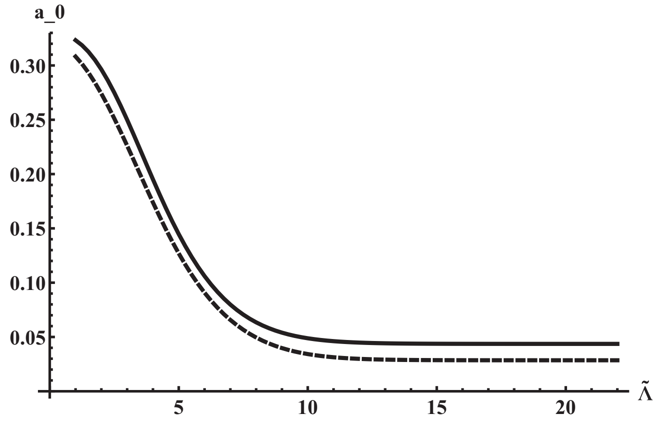

FIG. 4. The behavior of the order parameter a with respect to $\tilde{\Lambda}$. The dotted (solid) line stands for the case of model C (B).

Λ is smaller than the scale of the extra dimension, $\Lambda < L^{-1}$, the Kaluza-Klein modes can not be excited in the four dimensions. Since the Higgs mass is essentially generated by the quantum effect of the Kaluza-Klein mode, the Higgs mass is tiny enough for the region of the scale $\xi \lesssim 1$. As the cutoff Λ becomes larger, the Kaluza-Klein modes can start to excite and contribute to the Higgs mass, so that it gradually becomes heavier. This corresponds to the slope in the region of $1 \lesssim \xi \lesssim 8$. When the Λ becomes larger, $L^{-1} \le \Lambda$, the Kaluza-Klein modes can be excited enough to yield the Higgs mass corresponding to the flat part. The behavior of the Higgs mass clearly shows the interrelation between the effect of the four-dimensional cutoff and the physics in five dimensions, that is, the Kaluza-Klein mode.

As an illustration, let us also consider two more cases, where the matter contents are given by

$$\text{model B:} \begin{cases} (N_F^{\text{adj}(+)}, N_F^{\text{fd}(+)}, N_S^{\text{adj}(+)}, N_S^{\text{fd}(+)}) = (3, 2, 0, 0), \\ (N_F^{\text{adj}(-)}, N_F^{\text{fd}(-)}, N_S^{\text{adj}(-)}, N_S^{\text{fd}(-)}) = (4, 1, 1, 3), \end{cases} \tag{43}$$

$$\text{model C:} \begin{cases} (N_F^{\text{adj}(+)}, N_F^{\text{fd}(+)}, N_S^{\text{adj}(+)}, N_S^{\text{fd}(+)}) = (3, 4, 0, 0), \\ (N_F^{\text{adj}(-)}, N_F^{\text{fd}(-)}, N_S^{\text{adj}(-)}, N_S^{\text{fd}(-)}) = (5, 1, 2, 4). \end{cases} \tag{44}$$

In the limit of $\tilde{\Lambda} \to \infty$, the Higgs mass in the model B (C) is 186.694 (168.096) GeV, where the order parameter at the vacuum is given by $a_0 = 0.0285365$ (0.0436442).

We turn on the cutoff $\tilde{\Lambda}$ and depict the behavior of the order parameter a_0 in Fig. 4 for the models B and C. For the range of $\tilde{\Lambda}$ we have studied the gauge symmetry is broken correctly. In Fig. 5, we show the behaviors of the Higgs mass for the two models. For the model B (C), if we take account of the LHC result of the Higgs mass of 126 GeV, we obtain $\xi = \frac{\Lambda}{1/L} \gtrsim 5.7$ (6.26), which implies $\Lambda \gtrsim 5.7\ (6.26) L^{-1}$.

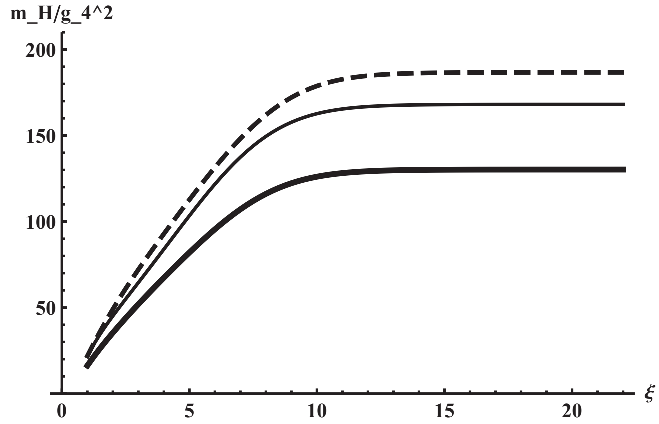

FIG. 5. The behavior of the Higgs mass with respect to $\xi = \frac{\Lambda}{1/L}$. The thick (thin, dashed) line is the case for the model A (C, B). The asymptotic value of the Higgs mass for each model is about 130 (168, 186) GeV for model A (C, B).

IV. CONCLUSIONS

We have evaluated the one-loop effective potential and the Higgs mass in the scenario of the gauge-Higgs unification by introducing the four-dimensional cutoff Λ in order to control the ultraviolet effect. It was clarified how much the Kaluza-Klein mode appearing in four dimensions contributes to the effective potential and the Higgs mass thanks to the cutoff. The effective potential and the Higgs mass depend on both the order parameter a and $\xi \equiv \frac{\Lambda}{1/L}$ through the remarkable combination $e^{2\pi i Q a - \xi}$. Due to the exponential damping, the well-known terms obtained in past calculations are reproduced in the limit of $\xi \to \infty$.

The parameter $\xi = \frac{\Lambda}{1/L}$ stands for the interrelation, which is, in particular, concretized through the Higgs mass. We have presented the three models in order to study the interrelation. We have obtained the behaviors of the Higgs mass with respect to $\xi = \frac{\Lambda}{1/L}$. The behavior shows the interrelation between the four dimensions and the extra dimension. For the smaller cutoff Λ, the Kaluza-Klein excitations are suppressed in four dimensions, so that the Higgs mass, which essentially originates from the quantum effect of the Kaluza-Klein mode, is suppressed as well. As the cutoff Λ becomes larger, the excitations can be allowed to generate the Higgs mass gradually, and for certain large value of Λ the Higgs mass approaches the value obtained in the limit of $\Lambda \to \infty$, which means that the quantum correction in the extra dimension is fully incorporated. The interrelation is manifest through the Higgs mass, which shows that the five-dimensional effect dominates for the large Λ, while the four-dimensional cutoff becomes effective for the smaller Λ.

We have also obtained the bound on ξ by taking account of the LHC result. This, in turn, gives the bound on the ratio between the four-dimensional cutoff Λ and the scale of the extra dimensions $1/L$.

The combination $e^{2\pi i Q a - \xi}$ is remarkable if we think of the usual logarithm and power behaviors with respect to the cutoff in the quantum field theory. The combination shows that the effective potential and Higgs mass are the special quantities in the gauge-Higgs unification. The origin of the combination may be the gauge invariance in the extra dimension. It is interesting to ask whether such a combination still holds beyond the one-loop calculation [18] and to investigate the role of the combination further. It may shed new light on the gauge-Higgs unification from the point of view of quantum field theory. Of course, it is important to study nonperturbatively the five-dimensional gauge theory in view of the interrelation. This will be reported on elsewhere [19].

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APPENDIX

Derivation of Eq. (16)

The momentum integration in Eq. (15) can be performed analytically. It is easy to show that the indefinite integration is carried out as

$$\begin{aligned}\int dy y^3 \ln(\cosh y - \cos\bar{a}) &= \int dy\left(\frac{y^4}{4}\right)' \ln(\cosh y - \cos\bar{a}) \\ &= \frac{y^4}{4}\ln(\cosh y - \cos\bar{a}) - \int dy \frac{y^4}{4}\left(1 + \left\{\frac{e^{i\bar{a}-y}}{1-e^{i\bar{a}-y}} + \text{c.c.}\right\}\right) \\ &= \frac{y^4}{4}\ln[\cosh y - \cos\bar{a}] - \frac{y^5}{20} - \frac{y^4}{4}(\ln(1-e^{i\bar{a}-y}) + \text{c.c.}) \\ &\quad + \int dy y^3(\ln(1-e^{i\bar{a}-y}) + \text{c.c.}). \end{aligned} \tag{A1}$$

It is straightforward to show that the first three terms in Eq. (A1) become

$$\begin{aligned}&\frac{y^4}{4}\ln[\cosh y - \cos\bar{a}] - \frac{y^5}{20} - \frac{y^4}{4}(\ln[1-e^{i\bar{a}-y}] + \text{c.c.}) \\ &= \frac{-y^5}{20} + \frac{y^4}{4}\ln\left(\frac{\cosh y - \cos\bar{a}}{(1-e^{-i\bar{a}-y})(1-e^{i\bar{a}-y})}\right) \\ &= \frac{-y^5}{20} + \frac{y^4}{4}\ln\left(\frac{e^y}{2}\right) = \frac{y^5}{5} - \frac{\ln 2}{4}y^4. \end{aligned} \tag{A2}$$

In the second line of Eq. (A1) we first expand the logarithm by[6]

$$\ln(1-x) = -\sum_{n=1}^{\infty}\frac{x^n}{n}, \tag{A3}$$

and after the partial integration we make use of the polylogarithm,

$$\text{Li}_s(z) \equiv \sum_{n=1}^{\infty}\frac{z^n}{n^s}. \tag{A4}$$

We finally obtain that

$$\begin{aligned}\int dy y^3 \ln[\cosh y - \cos\bar{a}] &= \frac{y^5}{5} - \frac{\ln 2}{4}y^4 + y^3(\text{Li}_2(e^{i\bar{a}-y}) + \text{c.c.}) \\ &\quad + 3y^2(\text{Li}_3(e^{i\bar{a}-y}) + \text{c.c.}) + 6y(\text{Li}_4(e^{i\bar{a}-y}) + \text{c.c.}) \\ &\quad + 6(\text{Li}_5(e^{i\bar{a}-y}) + \text{c.c.}). \end{aligned} \tag{A5}$$

The first and second terms are independent of $\bar{a}$ and are something like the cosmological constant. Equipped with Eq. (A5), the momentum integration (15) is evaluated as Eq. (16).

The momentum integration for the case of $M^{D-1} \times S^1/Z_2$ is also carried out in the same manner. It is given by

$$\begin{aligned}\int dy y^{D-2}\ln(\cosh y - \cos\bar{a}) &= \int dy\left(\frac{y^{D-1}}{D-1}\right)' \ln(\cosh y - \cos\bar{a}) \\ &= \frac{y^{D-1}}{D-1}\ln(\cosh y - \cos\bar{a}) - \int dy \frac{y^{D-1}}{D-1}\left(1 + \left\{\frac{e^{i\bar{a}-y}}{1-e^{i\bar{a}-y}} + \text{c.c.}\right\}\right) \\ &= \frac{y^D}{D} - \frac{\ln 2}{D-1}y^{D-1} + y^{D-2}(\text{Li}_2(e^{i\bar{a}-y}) + \text{c.c.}) + (D-2)y^{D-3}(\text{Li}_3(e^{i\bar{a}-y}) + \text{c.c.}) \\ &\quad + (D-2)(D-3)y^{D-4}(\text{Li}_4(e^{i\bar{a}-y}) + \text{c.c.}) + (D-2)(D-3)(D-4)y^{D-5}(\text{Li}_5(e^{i\bar{a}-y}) + \text{c.c.}) \\ &\quad + \cdots + (D-2)(D-3)\cdots(D-(D-2))(D-(D-1))(\text{Li}_D(e^{i\bar{a}-y}) + \text{c.c.}). \end{aligned} \tag{A6}$$

$D = 5$ is our case (A5).

[6]Note that the mode n in Eqs. (A3) and (A4) is different from the original Kaluza-Klein mode n. We point out that the mode summation (A3) is the same as the one obtained by the Poisson resummation formula.

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