

Relaxing the Landau-pole constraint in the NMSSM with the Abelian gauge symmetriesBumseok Kyae^{1,*} and Chang Sub Shin^{2,†}¹*Department of Physics, Pusan National University, Busan 609-735, Korea*²*Asia Pacific Center for Theoretical Physics, Pohang, Gyeongbuk 790-784, Korea*

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In order to relax the Landau pole constraint on “ λ ,” which is a coupling constant between a singlet S and the minimal supersymmetric standard model (MSSM) Higgs, $\lambda S h_u h_d$ in the next-to MSSM, and also maintains the gauge coupling unification, we consider perturbative U(1) gauge extensions of the next-to MSSM. For relatively strong U(1) gauge interactions down to low energies, we assign U(1) charges only to the Higgs and the third family of the chiral matter among the MSSM superfields. In the U(1)_Z [U(1)_Z × U(1)_X] extension, the low-energy value of λ can be lifted up to 0.85–0.95 [0.9–1.0], depending on the employed charge normalizations, when λ and the new gauge couplings are required not to blow up below the 10^{16} GeV energy scale. The introduction of extra vectorlike superfields can induce the desired Yukawa couplings for the first two families of the chiral matter. We also discuss various phenomenological constraints associated with extra U(1) breaking.

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I. INTRODUCTION

The smallness of the Higgs mass and the gauge coupling unification are the two salient features noted in the minimal supersymmetric standard model (MSSM) [1]. Since the Higgs quartic coupling in the tree-level potential is given by the small gauge couplings in the MSSM, the relatively light Higgs is favored. By supersymmetry (SUSY), the small Higgs mass can be protected up to the fundamental scale. As a result, the standard model (SM) can be naturally embedded in a grand unified theory (GUT) at a very high energy scale. Indeed, the gauge coupling unification inferred from the renormalization group (RG) runnings of the three SM gauge couplings in the MSSM might imply the presence of such a unified theory at the GUT scale ($\approx 2.2 \times 10^{16}$ GeV).

Recently, CMS and ATLAS reported the observations of the SM Higgs-like signals at 126 GeV invariant mass [2,3]. In fact, however, 126 GeV is too heavy as the Higgs mass in the MSSM: the Higgs mass at the tree level in the MSSM is predicted to be lighter even than the Z boson mass, and so excessively large radiative correction to it for explaining the 126 GeV Higgs mass is indispensable. However, it could result in a fine-tuning among the soft parameters [1]. In order to avoid such a fine-tuning, an extension of the MSSM would be necessary such that the tree-level Higgs mass [4–11] or the radiative correction to it is enhanced [12,13].

In the next-to MSSM (NMSSM), the “ μ term” in the MSSM superpotential is promoted to the trilinear term [4],

$$\lambda S h_u h_d, \quad (1)$$

by introducing a singlet S and a dimensionless parameter λ . A vacuum expectation value (VEV) of S can reproduce

the MSSM μ term. Such a trilinear term in the superpotential yields the quartic term in the Higgs potential. It adds a sizable correction to the MSSM Higgs mass at the tree level:

$$m_h^2 = m_Z^2 \cos^2 2\beta + \lambda^2 v_h^2 \sin^2 2\beta + \Delta m_h^2. \quad (2)$$

Here, the first and the last terms indicate the tree-level Higgs mass in the MSSM and radiative corrections, respectively. $v_h (\equiv \sqrt{v_u^2 + v_d^2} = 174 \text{ GeV})$ denotes the Higgs VEV. The second term in Eq. (2) originates from Eq. (1) of the NMSSM, which is very helpful for raising the Higgs mass up to 126 GeV without a serious fine-tuning among the SUSY-breaking soft mass parameters and the Z-boson mass, if λ is of order unity.

The RG analysis shows that the size of λ monotonically increases with energy, and so it eventually meets a Landau pole (LP) at a higher energy scale, if λ is too large at low energy. It is known that λ in the NMSSM should be smaller than 0.7 at the electroweak scale for λ not to blow up below the GUT scale (“LP constraint”) [4,14]. Moreover, λ is required to be larger than 0.6 to achieve 126 GeV Higgs mass with the s-top mass (\tilde{m}_t) much lighter than 1 TeV, which is a necessary condition for the naturalness of the small Higgs mass. The relatively small value of λ pushes $\tan \beta$ toward unity so that $\sin^2 2\beta [= 4 \tan^2 \beta / (1 + \tan^2 \beta)^2]$ in the tree-level correction of Eq. (2) becomes almost *maximized*. Thus, the perturbativity of λ and the naturalness of the Higgs mass restrict the parameter space quite seriously [5,15]:

$$0.6 \lesssim \lambda \lesssim 0.7, \quad 1 \lesssim \tan \beta \lesssim 3, \quad (3)$$

unless the “maximal mixing scenario,” which would also require a fine-tuning, is assumed. However, if the upper bound of λ could somehow be relaxed even slightly to 0.8–1.0 with its perturbativity maintained up to the GUT

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scale, $\tan\beta$ can be remarkably relieved to 4–8 for $\tilde{m}_t = 500\text{--}700$ GeV, yielding the 126 GeV Higgs mass [15].^{1,2}

From the Yukawa term, Eq. (1), the beta function of λ^2 reads as the summation of the anomalous dimensions of S , h_u , and h_d :

$$\beta_{\lambda^2} = \mu \frac{d\lambda^2}{d\mu} = 2\lambda^2(\gamma_S + \gamma_{h_u} + \gamma_{h_d}). \quad (4)$$

While the Yukawa couplings make positive contributions to the anomalous dimensions and the beta function, the gauge couplings make negative ones to them. Thus, one can expect that the LP constraint is relaxed by enhancing the gauge sector in which S , h_u , and h_d are involved. For a simple analysis, we will confine our discussion only on the case of a perturbative gauge interaction. In this paper, we will attempt to relax the LP constraint by introducing new Abelian gauge symmetries, under which the MSSM Higgs $\{h_u, h_d\}$ (and also the singlet S) are charged. Then, the new gauge interactions would resist the blowup of λ at higher energies.³

In this paper, *we intend to maintain the gauge coupling unification at the GUT scale*, which is one of the great achievements in the MSSM. In fact, SU(5) and SO(10) GUTs can provide the frameworks to extend the SM gauge group G_{SM} to a simple group, keeping the gauge coupling unification. Because of the gauge coupling unification and also doublet/triplet splitting in the Higgs sector, however, SU(5) and SO(10) should be broken around the GUT scale. Accordingly, one needs to consider product gauge groups, $G_{\text{SM}} \times G$.

If the new gauge group G is a non-Abelian group, the chiral matter sector of the MSSM as well as the Higgs sector are required to be extended by introducing more *chiral* fields such that they could be accommodated in nontrivial multiplets of G . Of course, the extra chiral matter should somehow be made heavy at low energy. With such other extra matter fields, however, the gauge coupling unification might be difficult to maintain, since

¹A strongly coupled region of λ in energy scales ($\mu_- < \mu < \mu_+$) potentially ruins the gauge coupling unification. However, it could be reversely used to ameliorate the precision of the unification, only if $\gamma_{h_u, h_d} \log(\mu_+/\mu_-)$ are not too large for the strongly coupled region [15]. For other studies on nonperturbative λ (by additional strong gauge interactions), see, e.g., Refs. [16].

²For phenomenological analyses on cases with an order one λ (“ λ -SUSY”), see, e.g., Refs. [17].

³Once a new gauge symmetry, under which the MSSM Higgs doublets are charged, is introduced, the D-term potential associated to it might be used to raise the tree-level Higgs mass. For a sizable effect by it, it turns out that the soft masses squared of the new Higgs, which break the new gauge symmetry, $\{\tilde{m}_\phi^2, \tilde{m}_{\phi^c}^2\}$, should be much heavier than the new gauge boson’s mass squared [18]. Too large mass splittings of them could introduce a fine-tuning associated with the electroweak symmetry breaking [19], which is a nontrivial constraint in this scenario.

extra matter fields would not be guaranteed to compose SU(5) or SO(10) multiplets at all.⁴

Even in the case of a new Abelian gauge symmetry, however, anomaly cancellation conditions often require also the presence of extra matter fields, which have not been yet observed in the laboratories. Thus, the extra matter fields should be vectorlike under the SM gauge symmetries such that they can obtain heavy masses below the breaking scale of G . Moreover, they should compose SU(5) or SO(10) multiplets for the gauge coupling unification. As mentioned above, in this paper, we are interested in gauge extensions with Abelian groups in the NMSSM.

In fact, the extra matter is helpful for relaxing the LP constraint of λ , only if it is embedded in SU(5) or SO(10) multiplets and made heavy at low energies: this would result in quite larger SM gauge couplings at higher energies compared to those in the original MSSM and so enhance its negative contributions to the RG equation of λ . With five pairs of extra $\{\mathbf{5}, \bar{\mathbf{5}}\}$, indeed, the allowed low energy value of λ can be lifted to 0.8, avoiding the LP below the GUT scale [4,14].

Our paper is organized as follows. We will survey promising U(1) gauge symmetries in Sec. II. In Sec. III, we will reanalyze the LP constraint in the presence of new U(1) gauge symmetries under various conditions. In Sec. IV, we will propose the concrete models reflecting the conditions considered in Sec. III. In Sec. V, we will discuss phenomenological constraints on the breaking scale of the extra U(1) introduced in Sec. IV. Section VI will be devoted to a conclusion.

II. U(1) GAUGE SYMMETRIES EMBEDDED IN THE GUTS

In the search for anomaly-free combinations of extra matter under extra U(1) gauge symmetries, it is worthwhile to consider the U(1)s outside the SM gauge group G_{SM} but embedded in the well-known GUTs such as SO(10) and E_6 : it can provide a guide for constructing a consistent model with gauge anomaly-free U(1)s. But we will not discuss the GUTs themselves in this paper; *we just pragmatically employ such U(1)s embedded in the GUTs, particularly for easily obtaining anomaly-free matter contents.*

The minimal GUT containing the SM gauge group is SU(5). Together with SU(5), U(1)_X is embedded in SO(10).

⁴If we give up the gauge coupling unification, one could find examples of the non-Abelian gauge extension. In Ref. [20], a model of $SU(3)_c \times SU(2)_1 \times SU(2)_2 \times U(1)_Y$ is proposed to relax the LP constraint on λ . It is possibly embedded in a product group, $SU(5)_1 \times SU(5)_2$. The $SU(3)_c$ results from spontaneous symmetry breaking by a Higgs of the bifundamental representation under the two SU(3)s embedded in the two different SU(5)s. $SU(2)_L$ can be obtained in a similar way. The third family of the chiral matter and the Higgs are assumed to be charged only under $SU(2)_1$, while the first two families are charged only under $SU(2)_2$. On the other hand, the singlet S remains neutral.

Under the symmetry-breaking $SO(10) \rightarrow SU(5) \times U(1)_X$, the spinor and vector representations of $SO(10)$ are split as

$$\begin{aligned} \mathbf{16} &\rightarrow \mathbf{10}_{1/\sqrt{40}} + \bar{\mathbf{5}}_{-3/\sqrt{40}} + \mathbf{1}_{5/\sqrt{40}}, \\ \mathbf{10} &\rightarrow \mathbf{5}_{-2/\sqrt{40}} + \bar{\mathbf{5}}_{2/\sqrt{40}}, \end{aligned} \quad (5)$$

where the subscripts indicate the $U(1)_X$ charges. The above tensor and (anti)fundamental representations of $SU(5)$ can accommodate the following MSSM matter:

$$\begin{aligned} \mathbf{10}_{1/\sqrt{40}} &= \{u^c, q, e^c\}, & \bar{\mathbf{5}}_{-3/\sqrt{40}} &= \{d^c, l\}, \\ \mathbf{1}_{5/\sqrt{40}} &= \nu^c; & \mathbf{5}_{-2/\sqrt{40}} &= \{D, h_u\}, \\ \bar{\mathbf{5}}_{2/\sqrt{40}} &= \{D^c, h_d\}, \end{aligned} \quad (6)$$

where the notation for the MSSM matter is self-evident. Throughout this paper, we will use the small (capital) letters for the MSSM (extra) superfields. Here, $\{D, D^c\}$, for which the MSSM quantum numbers are opposite of or same as d^c , are absent in the MSSM field spectrum unlike the other matter in Eq. (6). They spoil the gauge coupling unification and possibly lead to too fast a proton decay. Thus, they should be dropped from the field content of the low-energy effective theory. Even without $\{D, D^c\}$, the anomaly-free conditions associated with $U(1)_X$ are still fulfilled since they are exactly vectorlike under $G_{SM} \times U(1)_X$. Not only $\{D, D^c\}$ but also the MSSM Higgs $\{h_u, h_d\}$ carry the opposite charges $\mp 2/\sqrt{40}$. On the other hand, the singlet S in Eq. (1) still remains neutral. Since the Higgs in Eq. (1) are charged under $U(1)_X$, the $U(1)_X$ gauge symmetry could be helpful for relaxing the LP constraint on λ .

Like the $U(1)_X$, the $U(1)_{B-L}$ symmetry also resides between $SO(10)$ and G_{SM} , even if it is not ‘‘orthogonal’’ to G_{SM} . Under $U(1)_{B-L}$, however, the Higgs $\{h_u, h_d\}$ as well as the singlet S are neutral, and so it is not useful for relaxing the LP constraint. The $U(1)$ symmetry embedded in the $SU(2)_R$ of the Pati-Salam gauge group, $SU(4)_c \times SU(2)_L \times SU(2)_R$, is also an interesting gauge group. However, it can be obtained just by a linear combination of $U(1)_X$ and $U(1)_{B-L}$.

Another interesting $U(1)$ symmetry is ‘‘ $U(1)_Z$ ’’ embedded in E_6 together with $SO(10)$. Under the symmetry breaking, $E_6 \rightarrow SO(10) \times U(1)_Z$, the fundamental representation of E_6 , $\mathbf{27}$ is split as follows:

$$\mathbf{27} \rightarrow \mathbf{16}_{1/\sqrt{24}} + \mathbf{10}_{-2/\sqrt{24}} + \mathbf{1}_{4/\sqrt{24}}, \quad (7)$$

where the subscripts denote the charges of $U(1)_Z$. Hence, one $\mathbf{27}$ contains one family of the SM chiral matter, one Higgs doublet pair, extra colored matter $\{D, D^c\}$, and a singlet:

$$\begin{aligned} \mathbf{16}_{1/\sqrt{24}} &= \{u^c, q, e^c; d^c, l; \nu^c\}, \\ \mathbf{10}_{-2/\sqrt{24}} &= \{D, h_u; D^c, h_d\}, & \mathbf{1}_{4/\sqrt{24}} &= S. \end{aligned} \quad (8)$$

As a result, if one introduces three families of $\mathbf{27}$, two more pairs of the Higgs doublets and three $\{D, D^c\}$, as well as

three singlets in total, should be accompanied with the MSSM chiral matter. Note that S in Eq. (8) can be the NMSSM singlet appearing in Eq. (1) since it has the $U(1)_Z$ charge of $4/\sqrt{24}$, while h_u and h_d both carry $-2/\sqrt{24}$. Accordingly, $U(1)_Z$ can also be helpful for avoiding the LP for λ below the GUT scale. According to our analysis, $U(1)_Z$ turns out to be much more efficient than $U(1)_X$ in relaxing the LP constraint. Thus, we will mainly focus on $U(1)_Z$.

For a gauge interaction of G to efficiently seize λ in the perturbative regime, the following conditions should be generically satisfied:

- (1) The gauge coupling associated with G needs to be large enough at the GUT scale.
- (2) The beta-function coefficient of G needs to be small enough.
- (3) The breaking energy scale of G should be low enough.

In order to reflect the condition (1) in the model, one would not require that the gauge couplings of $U(1)_Z$ and $U(1)_X$ are necessarily unified with the SM gauge couplings at the GUT scale: relatively larger $U(1)_{Z,X}$ gauge couplings than those of the MSSM are allowed. Alternatively, we can take the $U(1)_Z$ [$U(1)_X$] charge normalization smaller than the E_6 [$SO(10)$] normalization ‘‘ $\sqrt{24}$ ’’ [‘‘ $\sqrt{40}$ ’’], assuming that the $U(1)_Z$ [$U(1)_X$] gauge coupling is unified with the SM gauge couplings at the GUT scale. We just naively anticipate that such a charge normalization of $U(1)_Z$ and $U(1)_X$ can be supported by a proper UV theory embedding our model. Of course, both yield the same result in lifting λ . Throughout this paper, we take the latter choice.

For fulfilling the condition (2), we will assign the $U(1)_Z$ and $U(1)_X$ charges to only one family of the MSSM chiral matter and one pair of the Higgs doublets together with $\{D, D^c\}$ and S , which compose an anomaly-free combination of the matter. Hence, the other two families of the chiral matter cannot couple to the Higgs because they remain neutral under $U(1)_Z$ and $U(1)_X$, while the Higgs doublets carry the charges. Thus, we should introduce additional vectorlike matter such that the desired Yukawa couplings for them can be generated after $U(1)_Z$ and $U(1)_X$ breakings. Since the extra matter $\{D, D^c\}$ can mediate unwanted too-fast proton decay, we need to introduce a (global) symmetry in order to forbid such a possibility.

If the $U(1)_Z$ breaking scale is too low, the condition (3) can be in conflict with the constraints on flavor-changing neutral current (FCNC) processes, since our $U(1)_Z$ and $U(1)_X$ charge assignments are family dependent. Moreover, a too-low breaking scale of $U(1)_Z$ could affect also the precision tests of the SM gauge interactions associated with the ‘‘ S ’’ and ‘‘ T ’’ parameters. Hence, we take the $U(1)_Z$ and $U(1)_X$ breaking scales of 5–10 TeV.

In principle, models with such modified $U(1)$ gauge symmetries could originate from a GUT defined in a higher-dimensional spacetime such as the heterotic string

theory compactified in an orbifold [21], in which $U(1)_{Z,X}$ gauge symmetries are embedded in $E_8 \times E'_8$. However, the discussion on it would be beyond the scope of this paper.

III. LIFTING THE λ COUPLING CONSTANT

In this paper, we will consider only the SM gauge group G_{SM} and the $U(1)_Z$ and $U(1)_X$, which are motivated by the $U(1)$ s embedded in E_6 and $SO(10)$. For small beta-function coefficients $b_{Z,X}$, we assign the $U(1)_{Z,X}$ charges only to one family of the MSSM chiral matter (the third family of the quarks, leptons, and their superpartners) and the Higgs, introducing the extra colored matter $\{D, D^c\}$ and a singlet S . In Table I, we present their charges of the global $U(1)_R$ as well as the gauged $U(1)_Z$ and $U(1)_X$. The charges for the third family of the MSSM chiral matter in Table I just follow the charge assignments of Eqs. (8) and (6). Concerning the E_6 and $SO(10)$ charge normalizations, refer to, e.g., Ref. [22]. Since they compose a **27** of E_6 , all the gauge anomalies must be canceled out. Note that $\{h_u, h_d\}$ and $\{D, D^c\}$ are vectorlike under $G_{\text{SM}} \times U(1)_X$ but not under $U(1)_Z$. Even if the $U(1)_Z$ and $U(1)_X$ embedded in the GUTs are introduced, we do not follow the charge normalization determined when they are embedded in the GUTs. Thus, n_Z and n_X are not rigorously required to be unity in our case. For the SM gauge coupling unification, two lepton doublets $\{H_u, H_d\}$, which do not have any charges of $U(1)_Z \times U(1)_X$, should be supplemented as seen in Table I.

The charge assignments of $U(1)_Z \times U(1)_X$ and $U(1)_R$ in Table I permit the Yukawa couplings for the third family of the chiral matter and the Higgs doublets:

$$W_3 = \lambda S h_u h_d + y_a q_3 h_u u_3^c + y_b q_3 h_d d_3^c + y_\tau l_3 h_d e_3^c, \quad (9)$$

in which the MSSM μ term is promoted to the trilinear λ coupling. Throughout this paper, we suppose that y_b and y_τ are relatively smaller than λ and y_a . Note that ν_3^c cannot couple to the lepton doublet l_3 and the Higgs h_u due to the $U(1)_R$ symmetry. We assume that it develops a VEV of the order of the TeV scale, breaking $U(1)_Z$ completely.

The mass squared of h_u is assumed to be negative at low energies, and so it leads to a nonzero VEV of the Higgs, triggering the electroweak symmetry breaking. Note that we have an additional quartic term $\lambda^2 |h_u h_d|^2$ in the scalar potential apart from the quartic potential coming from the

“D term.” So there is no D-flat direction in the D-term potential of the Higgs unlike the MSSM. The “A term” corresponding to the λ term in Eq. (9) provides a tadpole of \tilde{S} , and so a VEV of \tilde{S} can also be developed. It could induce the MSSM “ μ ” parameter ($\mu_{\text{eff}} \equiv \lambda \langle \tilde{S} \rangle$), which is a SUSY mass parameter of $\{h_u, h_d\}$.

If $\langle \tilde{S} \rangle$ is the main source of the $U(1)_Z$ breaking, it should be large enough to avoid low-energy constraints on an extra $U(1)$. Then, λ should be small enough to ensure the small Higgs and Higgsino masses. However, a small λ cannot enhance the quartic coupling of the Higgs potential, and so a sizable fine-tuning becomes unavoidable [10]. Hence, we will assume $\langle \tilde{S} \rangle \sim \mathcal{O}(1)$ TeV or lower and introduce another main breaking source of $U(1)_Z$ separately.

Since there is no S^3 term in the superpotential Eq. (9) unlike the ordinary NMSSM, one might think that there exists an accidental Pecci-Quinn (PQ) symmetry. However, such a global symmetry, under which S carries a nonzero charge, is gauged to $U(1)_Z$ in this case. Even below the $U(1)_Z$ breaking scale, which is assumed to be higher than $\langle S \rangle$, we will show later the absence of such an accidental PQ symmetry.

At the moment, $\{D, D^c\}$ and $\{H_u, H_d\}$ remain massless because of the $U(1)_R$. We will explain how they get their masses in Sec. IV. As will be seen later, they and other vectorlike fields play important roles to induce the ordinary Yukawa couplings for the first and second families of the chiral matter.

Although we have not yet proposed a full model with $U(1)_Z \times U(1)_X$ charge assignments, we first attempt to perform a relatively model-independent analysis on how much the λ coupling in Eq. (9) can be lifted at low energy. We will discuss the cases of $U(1)_Z$ [case I] and $U(1)_Z \times U(1)_X$ [case II].

The solution of the RG equation for the three MSSM gauge couplings are given by

$$g_k^2(t) = \frac{g_U^2}{1 + \frac{g_U^2}{8\pi^2} b_k(t_0 - t)} \quad \text{for } k = 3, 2, 1, \quad (10)$$

where t parametrizes the renormalized mass scale, $t = \log(\mu/M_{\text{GUT}})$. b_k ($k = 3, 2, 1$) denotes the beta function coefficients of the gauge couplings for $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$. In the presence of the extra ν pairs of $\{\mathbf{5}, \bar{\mathbf{5}}\}$, they are given by $b_k = (-3 + \nu, 1 + \nu, 33/5 + \nu)$, where $\nu = 0$

TABLE I. Charge assignments of the gauged $U(1)_Z$, $U(1)_X$, and the global $U(1)_R$. A singlet, the MSSM Higgs doublets, the third family of the chiral matter, and an extra vectorlike pair $\{D, D^c\}$ carry the charges. They compose anomaly-free matter contents under $G_{\text{SM}} \times U(1)_Z \times U(1)_X$. $\{H_u, H_d\}$ are necessary for the Yukawa couplings for the first and second families of the chiral matter and also gauge coupling unification.

Superfields	S	h_u	h_d	u_3^c	q_3	e_3^c	d_3^c	l_3	ν_3^c	D	D^c	H_u	H_d
$U(1)_Z (\times n_Z/\sqrt{24})$	4	-2	-2	1	1	1	1	1	1	-2	-2	0	0
$U(1)_X (\times n_X/\sqrt{40})$	0	-2	2	1	1	1	-3	-3	5	-2	2	0	0
$U(1)_R$	-2	2	2	-1	1	1	-1	-1	2	1	-1	0	0

corresponds to the case of the MSSM. For the matter content of Table I ($\nu = 1$), the unified gauge coupling g_U^2 is estimated as 0.62. With the one more extra pair of $\{\mathbf{5}, \bar{\mathbf{5}}\}$ ($\nu = 2$), g_U^2 is lifted to 0.82. For $\nu = 3, 4$, and 5 , g_U^2 are given by 1.18, 2.13, and 11.19, respectively.

Similarly, the solution to the RG equations of the $U(1)_Z$ and $U(1)_X$ gauge couplings are

$$g_Z^2(t) = \frac{g_{Z0}^2}{1 + \frac{g_{Z0}^2}{8\pi^2} b_Z(t_0 - t)} \quad \text{for } t > t_Z; \quad (11)$$

$$g_X^2(t) = \frac{g_{X0}^2}{1 + \frac{g_{X0}^2}{8\pi^2} b_X(t_0 - t)} \quad \text{for } t > t_X,$$

where $b_{Z,X}$ indicates the beta-function coefficient of $U(1)_{Z,X}$, and $t_{Z,X}$ parametrizes the $U(1)_{Z,X}$ breaking scale $M_{Z,X}$ [$t_{Z,X} \equiv \log(M_{Z,X}/M_{\text{GUT}})$]. In particular, case I corresponds to the case of setting $g_{X0}^2 = 0$, which turns off the $U(1)_X$ gauge interaction. Only with the field contents in Table I, the beta-function coefficients $b_{Z,X}$ are given by $3n_{Z,X}^2$. As will be discussed later, however, some additional vectorlike superfields charged under $U(1)_Z$ [and $U(1)_X$] are necessary in order to induce the desired Yukawa couplings for the first two families of the matter in case I [case II]. Then, b_Z becomes $(4 + \frac{1}{12})n_Z^2$ in case I [$(4 + \frac{1}{2})n_Z^2$ in case II]. With such additional matter in case II, b_X would increase to $(4 + \frac{3}{10})n_X^2$. Note that $b_{Z,X}$ in both cases are quite smaller than those in which all the three families of the matter and additional Higgs are charged, $3 \times 3n_{Z,X}^2$. Since we do not consider the GUT of E_6 or $SO(10)$, the normalization factors $n_{Z,X}$ in Table I and in $b_{Z,X}$ of Eq. (11) do not have to be unity in our case. Instead, we set $g_{Z0}^2 = g_{X0}^2 = g_U^2$. In the perturbative regime, $(n_{Z,X}g_{Z,X})^2$, which are

effective gauge couplings, should be quite smaller than $4\pi^2$.

With the charge assignments in Table I and the Yukawa interactions in Eq. (9), one can obtain the one-loop anomalous dimensions for S , h_u , h_d , q_3 , and u_3^c in the standard manner:

$$16\pi^2\gamma_S = 2\lambda^2 - \frac{4}{3}(n_Zg_Z)^2, \quad (12)$$

$$16\pi^2\gamma_{h_u} = \lambda^2 + 3y_t^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 - \frac{1}{3}(n_Zg_Z)^2 - \frac{1}{5}(n_Xg_X)^2, \quad (13)$$

$$16\pi^2\gamma_{h_d} = \lambda^2 - \frac{3}{2}g_2^2 - \frac{3}{10}g_1^2 - \frac{1}{3}(n_Zg_Z)^2 - \frac{1}{5}(n_Xg_X)^2, \quad (14)$$

$$16\pi^2\gamma_{q_3} = y_t^2 - \frac{8}{3}g_3^2 - \frac{3}{2}g_2^2 - \frac{1}{30}g_1^2 - \frac{1}{12}(n_Zg_Z)^2 - \frac{1}{20}(n_Xg_X)^2, \quad (15)$$

$$16\pi^2\gamma_{u_3^c} = 2y_t^2 - \frac{8}{3}g_3^2 - \frac{8}{15}g_1^2 - \frac{1}{12}(n_Zg_Z)^2 - \frac{1}{20}(n_Xg_X)^2, \quad (16)$$

where we ignored the contributions by the Yukawa couplings of y_b and y_τ . The $(n_Zg_Z)^2$ and $(n_Xg_X)^2$ terms in the above anomalous dimensions, which are all negative, result from the $U(1)_Z$ and $U(1)_X$ gauge interactions, respectively. Then, it is straightforward to write down the RG equation for the λ and y_t couplings:

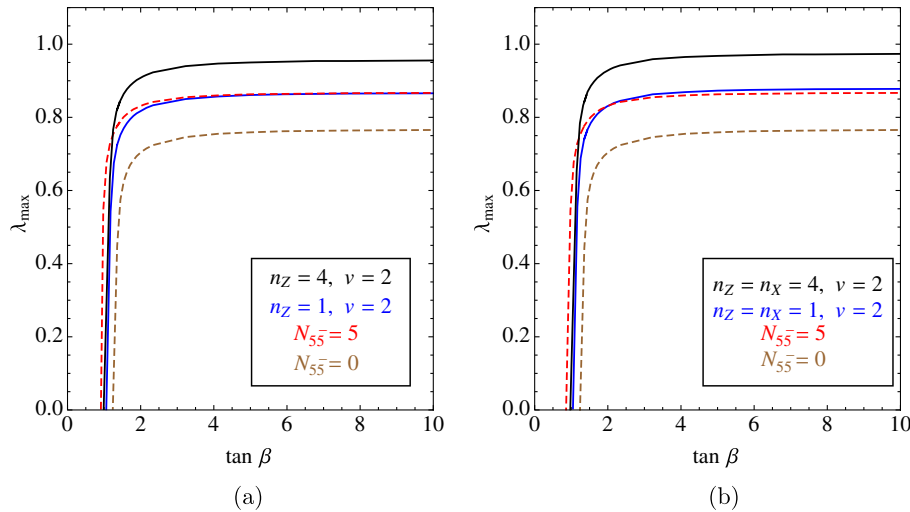


FIG. 1 (color online). λ_{max} vs $\tan\beta$ for (a) the $U(1)_Z$ extension (case I) and (b) the $U(1)_Z \times U(1)_X$ extension (case II) of the NMSSM. The $n_Z = 1$ [$n_X = 1$] corresponds to the case that the charge normalization of $U(1)_Z$ [$U(1)_X$] determined by E_6 [$SO(10)$] is employed. ν stands for the number of the extra $\{\mathbf{5}, \bar{\mathbf{5}}\}$, which are charged under $U(1)_Z$ [and also $U(1)_X$], while N_{55} stands for the total numbers of extra $\{\mathbf{5}, \bar{\mathbf{5}}\}$ in the absence of the extra $U(1)$ s. In both cases, the $U(1)$ breaking scales are set to be 5 TeV. In case II, λ_{max} slightly more increases compared to case I.

$$\frac{d\lambda^2}{dt} = \frac{\lambda^2}{8\pi^2} \left[4\lambda^2 + 3y_i^2 - 3g_2^2 - \frac{3}{5}g_1^2 - 2(n_Z g_Z)^2 - \frac{2}{5}(n_X g_X)^2 \right], \quad (17)$$

$$\frac{dy_i^2}{dt} = \frac{y_i^2}{8\pi^2} \left[\lambda^2 + 6y_i^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 - \frac{1}{2}(n_Z g_Z)^2 - \frac{3}{10}(n_X g_X)^2 \right]. \quad (18)$$

Based on the analyses with Eqs. (17) and (18), we display the upper bounds of λ ($\equiv \lambda_{\max}$) at low energy for case I in Fig. 1(a) and case II in Fig. 1(b), respectively. In these analyses, we naively assume that the gauged U(1)s' breaking scales are around 5 TeV, and the masses of the extra multiplets are 5–10 TeV. Below 5 TeV, thus, we ignored the $g_{Z,X}$'s contributions to Eqs. (17) and (18).

As seen in Fig. 1(a), λ_{\max} in the absence of extra U(1)s and matter is given by about 0.7. If five pairs of $\{\mathbf{5}, \bar{\mathbf{5}}\}$ are added, λ_{\max} increase up to 0.8 in the absence of the extra U(1)s [14]. We note the similar result $\lambda_{\max} \approx 0.8$ can be achieved also in case I [i.e., when only U(1)_Z is turned on] with the two pairs of $\{\mathbf{5}, \bar{\mathbf{5}}\}$, even if the E_6 normalization ($n_Z = 1$) is employed. For $n_Z = 4$, which is almost the maximal possibility in the perturbative regime [$(n_Z g_U)^2 \approx (3.62)^2$], λ_{\max} can reach 0.95 for $\tan\beta > 3$. When both U(1)_Z and U(1)_X are turned on, λ_{\max} can be slightly more lifted: Fig. 1(b) shows that it becomes close to 1.0 (0.9) for $\tan\beta > 3$ with $n_Z = n_X = 4$ ($n_Z = n_X = 1$).

In fact, two-loop corrections start being sizable in the existence of three or more pairs of $\{\mathbf{5}, \bar{\mathbf{5}}\}$ [23]. Moreover, the case with five pairs of $\{\mathbf{5}, \bar{\mathbf{5}}\}$ is the perturbatively marginal case since the expansion parameter associated with the MSSM gauge couplings, $g_U^2/4\pi$, becomes 0.89 at the GUT scale. Even in the case of two pairs of $\{\mathbf{5}, \bar{\mathbf{5}}\}$ we considered in Fig. 1, g_Z and λ reach the perturbatively marginal values at the GUT scale. In order to get the upper bound on λ in the absence or presence of the extra U(1)s, however, we considered such extreme cases in Fig. 1. For more precise results, more rigorous estimations including two-loop corrections would be needed.

IV. THE MODELS

While the Yukawa couplings for the third family of the MSSM chiral matter are allowed as seen in Eq. (9), the first and second families of the MSSM matter fields cannot yet couple to the ordinary MSSM Higgs doublets $\{h_u, h_d\}$ since

they do not carry U(1)_Z and U(1)_X charges unlike the Higgs doublets. Moreover, $\{D, D^c\}$ and $\{H_u, H_d\}$ should have mass terms. Thus, we need to introduce some vectorlike superfields in order to induce all the desired Yukawa couplings and mass terms. Now, we propose the two models as examples.

A. Model of U(1)_Z

In the first model, we consider only the gauge U(1)_Z and the global U(1)_R symmetries. Thus, we ignore the U(1)_X charges of Table I in the first model. The charge assignments for the first and second families of the MSSM chiral superfields and other needed vectorlike matter are listed in Table II. X , $\{Y, Y^c\}$, and $\{Z, Z^c\}$ in Table II are the MSSM singlet superfields. $\{D', D^c\}$ and $\{L, L^c\}$ are extra SU(3)_c triplets and SU(2)_L doublets. Since $\{D', D^c\}$ and $\{L, L^c\}$ are embedded in a pair of $\{\mathbf{5}, \bar{\mathbf{5}}\}$, the gauge coupling unification can still be maintained. With the field contents in Tables I and II, the beta function coefficient of U(1)_Z is estimated as $b_Z = (4 + \frac{1}{12})n_Z^2$. It was used for analyses of the $\nu = 2$ cases in Fig. 1(a).

The relevant superpotential for the first two families of the MSSM chiral matter and $\{Z, Z^c\}$ are written as follows:

$$W_{1,2} = \sum_{i,j=1,2} (y_u^{ij} q_i u_j^c H_u + y_d^{ij} q_i d_j^c H_d + y_\nu^{ij} l_i \nu_j^c H_u + y_e^{ij} l_i e_j^c H_d + M_\nu^{ij} \nu_i^c \nu_j^c) + Z(y_{h1} H_u h_d + y_{h2} H_d h_u + y_{D1} D D^c + y_{D2} D^c D') + y_\nu \nu_3^c Y Z^c, \quad (19)$$

where we assume the Yukawa coupling constants y_{h1} and y_{h2} are small enough to guarantee the light enough Higgs mass. We will discuss later how Eq. (19) should be modified when U(1)_X is also introduced. The soft SUSY breaking A term corresponding to the y_ν term in the scalar potential ($\equiv y_\nu A_\nu \tilde{\nu}_3^c \tilde{Y} Z^c$) provides tadpoles of ν_3^c , Y , and Z^c , and so it can generate nonzero VEVs of them.

Indeed, the VEVs of $\{\tilde{\nu}_3^c, \tilde{Y}, \tilde{Z}^c\}$ can be of order m_{soft}/y_ν . With a relatively smaller y_ν , thus, we get higher energy scale VEVs for them than the typical soft mass scale. It is possible basically because the quartic terms in the scalar potential, which makes the scalar potential bounded from below, come from the y_ν term in Eq. (19) with the coefficient of $|y_\nu|^2$. With the y_ν term in the superpotential Eq. (19) and the D-term potential ($\equiv \frac{g_Z^2}{2} D_Z^2$), one can show that $\{\tilde{\nu}_3^c, \tilde{Y}, \tilde{Z}^c\}$ should satisfy the following conditions at the minimum:

TABLE II. Charge assignments for the gauge U(1)_Z and global U(1)_R for the extra vectorlike fields and the first and second families of the chiral matter. Here, the U(1)_Z charge normalization, $n_Z/\sqrt{24}$, is omitted for a simple presentation. The first and second families of the ordinary chiral matter do not carry U(1)_Z charges. Since $\{D', D^c\}$ and $\{L, L^c\}$ compose $\{\mathbf{5}, \bar{\mathbf{5}}\}$, they maintain the gauge coupling unification. X , $\{Y, Y^c\}$, and $\{Z, Z^c\}$ are inert under the SM gauge interactions.

Superfields	X	Y	Y^c	Z	Z^c	D'	D^c	L	L^c	$u_{1,2}^c$	$q_{1,2}$	$e_{1,2}^c$	$d_{1,2}^c$	$l_{1,2}^c$	$\nu_{1,2}^c$
U(1) _Z	0	1	-1	2	-2	0	0	2	-2				0		
U(1) _R	0	0	0	0	0	3	1	-1	1				1		

$$\begin{aligned} \tilde{Y}^c &= 0, & 2g_Z^2 D_Z &= 2g_Z^2(|\tilde{\nu}_3^c|^2 + |\tilde{Y}|^2 - |\tilde{Y}^c|^2 + 2|\tilde{Z}|^2 - 2|\tilde{Z}^c|^2) = -\tilde{m}_Z^2, \\ y_V^2(|\tilde{\nu}_3^c|^2 + |\tilde{Y}|^2 + |\tilde{Z}^c|^2) &= \frac{4\cot^2\theta(\tilde{m}_Z^2 + \tilde{m}_{Z^c}^2) - \sin^2 2\phi(\tilde{m}_{\tilde{\nu}_3^c}^2 + \tilde{m}_{\tilde{Y}}^2 - \tilde{m}_Z^2)}{(1 + \cos^2\theta)\sin^2 2\phi - 4\cos^2\theta}. \end{aligned} \quad (20)$$

Here, $\tilde{m}_{\tilde{\nu}_3^c}^2$, $\tilde{m}_{\tilde{Y}}^2$, \tilde{m}_Z^2 , and $\tilde{m}_{Z^c}^2$ indicate the soft masses squared of $\tilde{\nu}_3^c$, \tilde{Y} , \tilde{Z} , and \tilde{Z}^c , respectively, and θ and ϕ parametrize $\tilde{\nu}_3^c$, \tilde{Y} , and \tilde{Z}^c as follows:

$$\begin{aligned} |\tilde{\nu}_3^c| &\equiv R \sin \theta \cos \phi, & |\tilde{Y}| &\equiv R \sin \theta \sin \phi, \\ |\tilde{Z}^c| &\equiv R \cos \theta, \end{aligned} \quad (21)$$

where $R \equiv \sqrt{|\tilde{\nu}_3^c|^2 + |\tilde{Y}|^2 + |\tilde{Z}^c|^2}$. With smaller y_V , hence, the VEVs of $\{\tilde{\nu}_3^c, \tilde{Y}, \tilde{Z}^c\}$ can be larger than the typical sizes of the soft mass parameters. Note that the sign of \tilde{m}_Z^2 can be negative at low energies if $y_{D1,2}$ in Eq. (19) is of order unity. (Since $\{D, D^{c'}; D^c, D'\}$ are colored particles, $y_{D1,2}$ of order one would not incur another LP problem.) We suppose that the VEVs for $\{\tilde{\nu}_3^c, \tilde{Y}, \tilde{Z}^c\}$ are around 5–10 TeV, breaking $U(1)_X$ at that scale, even if the typical soft masses are assumed to be of the order of TeV. For instance, we have $\tilde{\nu}_3^c \approx 6.4$ (11.8) TeV, $\tilde{Y} \approx 5.7$ (9.8) TeV, $\tilde{Z} \approx 7.1$ (6.9) TeV, $\tilde{Z}^c \approx 9.3$ (12.9) TeV, for the given parameters $y_V = 0.1$, $g_Z^2 = 0.3$, $\phi = 41.6^\circ$ (39.7°), $\theta = 42.5^\circ$ (50.1°), $A_V = -2.6$ (-3.4) TeV, and $\sqrt{\tilde{m}_{\tilde{\nu}_3^c}^2 - \frac{1}{2}\tilde{m}_Z^2} = 1$ TeV, $\sqrt{\tilde{m}_{\tilde{Y}}^2 - \frac{1}{2}\tilde{m}_Z^2} = 1.2$ (1.5) TeV, $\sqrt{\tilde{m}_{Z^c}^2 - \frac{1}{2}\tilde{m}_Z^2} = 0.6$ (0.8) TeV.

$\{H_u, H_d\}$ are assumed to be relatively (say, about five times) heavier than other superpartners of the SM chiral fermions. It is possible because a μ term for them, $\mu_H H_u H_d$, can be induced in the superpotential, e.g., via the Giudice-Masiero mechanism [24], as will be shown later. $\{H_u, H_d\}$ can be integrated out to yield the effective SM Yukawa couplings, since they are much heavier than the SM chiral fermions and Higgs. After integrating out $\{H_u, H_d\}$, thus, the Yukawa couplings for the masses of the first and second families of the SM chiral fermions can be generated, which are estimated as $y_{d,e,u,\nu}^{ij} \langle \tilde{Z} \rangle / \mu_H$. Note that they are still perturbatively consistent for small enough dimensionless couplings $y_{d,e,u,\nu}^{ij}$ ($i, j = 1, 2$) and $y_{h1,2}$, even if $\langle \tilde{Z} \rangle / \mu_H \sim \mathcal{O}(1)$.

Once \tilde{Z} and $\{h_u, h_d\}$ develop VEVs, $\{H_u, H_d\}$ also acquire VEVs through the A term corresponding to the y_{h1} and y_{h2} terms in Eq. (19). $\langle H_{u,d} \rangle$ are estimated as $y_{h1,2} m_{3/2} \langle \tilde{Z} \rangle \langle h_{d,u} \rangle / \mu_H^2$, which are more suppressed than $\langle h_{u,d} \rangle$. One should note here that the VEVs, $\langle H_{u,d} \rangle$, are along the directions of $\langle h_{d,u} \rangle$. Accordingly, $\langle H_{u,d} \rangle$ keep intact the electromagnetic $U(1)$ gauge symmetry. All the charged components in $\{H_u, H_d\}$ get heavy masses from the soft mass terms, the $\mu_H H_u H_d$ term, and its corresponding “ $B\mu$ term.”

As mentioned in Sec. III, we assume that $\langle \tilde{S} \rangle$ is small enough [$\mathcal{O}(1)$ TeV or lower ($\ll \langle \tilde{Z} \rangle$)] that the effective

μ ($\equiv \lambda \langle \tilde{S} \rangle$) is not very large. Hence, one might expect an accidental PQ symmetry below the scale of $\langle \tilde{Z} \rangle$, which is regarded as the dominant $U(1)_Z$ breaking source. After $\{H_u, H_d\}$ decouples, however, a bare μ term is also induced,

$$\left(y_{h1} y_{h2} \frac{\langle \tilde{Z} \rangle^2}{\mu_H} \right) h_u h_d, \quad (22)$$

and so there does not remain an accidental PQ symmetry below the $U(1)_Z$ breaking scale.

Note that the new vectorlike colored particles $\{D', D^{c'}\}$ introduced in Table II and Eq. (19) couple to $\{D, D^c\}$ of Table I and get masses when \tilde{Z} develops a VEV of the order of the 10 TeV scale. In fact, the quantum numbers of $d_{1,2}^c$ are the same as those of $D^{c'}$ [neglecting $U(1)_X$], and so $d_{1,2}^c$ could also couple to D and Z like $D^{c'}$. However, we have only one D , and so two of the mass eigenstates from $\{D^{c'}, d_1^c, d_2^c\}$ remain light. We redefine them as the d-type quarks appearing in the MSSM.

Were it not for the $U(1)_R$ symmetry, the following terms would be admitted in the superpotential:

$$\begin{aligned} u_i^c d_j^c D^{c'}, & \quad D u_3^c e_3^c; & \quad q_i q_j D', \\ D^c q_3 l_3 & \quad (i, j = 1, 2), \end{aligned} \quad (23)$$

which are deduced to the operators leading to the dimension-five proton decay, $u_i^c u_3^c d_j^c e_3^c$ and $q_i q_j q_3 l_3$, after integrating out $\{D, D^{c'}\}$ and $\{D', D^c\}$ of the y_{D1} and y_{D2} terms in Eq. (19).⁵ Although $U(1)_R$ is broken to the Z_2 symmetry, these are still forbidden because all the superfields appearing in Eq. (23), including $\{D, D^c\}$ and $\{D', D^{c'}\}$, carry only the odd parity.

Although the SM fermions can get their masses through the Yukawa couplings in Eqs. (9) and (19), the mixings between the third and the first two families of them, and also the CP phase in the SM quark sector are absent only with the fields discussed above. Thus, we need one more vectorlike lepton pair $\{L, L^c\}$, which is also assumed to be relatively heavier than other MSSM matter fields. How a mass term of the type $\mu_L L L^c$ can be obtained will be explained later. The mixings between the third and the first two families in the mass matrices of the d-type quarks and the charged leptons can be generated from the following Yukawa interactions after integrating out $\{D, D^{c'}\}$, $\{L, L^c\}$, and $\{H_u, H_d\}$, because ν_3^c develops a VEV:

⁵Only with the $U(1)_Z$ [and $U(1)_X$] gauge interactions, the dimension-six operators leading to proton decay are not induced. In this model, the dimension-six proton decay is possible only through gravity interactions and is still safe at the moment.

$$W_{\text{mix}} = \sum_{i=1,2} (y_{Dd} \nu_3^c d_3^c D + y_q^i D^{c'} H_d q_i + y_l \nu_3^c l_3 L^c + y_{Le}^i L h_d e_i^c), \quad (24)$$

which fills the $(3, i)$ entries of the mass matrices of the d-type quarks ($\equiv [M_D]_{3,i}$) and the $(i, 3)$ of the charged leptons ($\equiv [M_E]_{i,3}$). They are estimated as $(y_{Dd} y_q^i / y_{D1}) \times \langle \tilde{\nu}_3^c \rangle / \mu_H$ and $y_l y_{Le}^i \langle \tilde{\nu}_3^c \rangle / \mu_L$, respectively. Even if $[M_D]_{i,3}$ and $[M_E]_{3,i}$ ($i = 1, 2$) are still zero, $M_D^\dagger M_D$ and $M_E^\dagger M_E$ are fully general Hermitian matrices. Accordingly, the unitary matrices $U_L^{(d)}$ and $U_L^{(e)}$, which diagonalize $M_D^\dagger M_D$ and $M_E^\dagger M_E$, respectively, are also general enough to reproduce the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices.

Because of the $U(1)_Z$ and $U(1)_R$ symmetries, the right-handed neutrino ν_3^c cannot obtain the Dirac and Majorana masses. Hence, the seesaw mechanism should be implemented only with the two heavy right-handed neutrinos ν_i^c ($i = 1, 2$) in Eq. (19). As shown in Ref. [25], the seesaw mechanism and also the leptogenesis can still work even with two heavy right-handed neutrinos. The mixings of the observed neutrinos can come from the charged lepton sector.

Finally, let us present various μ terms in this model, $\mu_H H_u H_d$, $\mu_L L L^c$, $\mu_Y Y Y^c$, and $\mu_Z Z Z^c$, for which the generations are associated with the SUSY-breaking effect in the Kähler potential [24]:

$$K = \frac{X^\dagger}{M_P} (\kappa_H H_u H_d + \kappa_L L L^c + \kappa_Y Y Y^c + \kappa_Z Z Z^c) + \text{H.c.}, \quad (25)$$

in which X is a SUSY-breaking source: its F-term component ($\equiv F_X$), which carries the $U(1)_R$ charge of -2 , is assumed to develop a VEV of order $m_{3/2} M_P$. Thus, the VEV of F_X breaks the $U(1)_R$ symmetry to Z_2 , which can be interpreted as the matter parity in the MSSM, since the ordinary matter [and also $\{D^{(l)}, D^{c(l)}\}$, $\{L, L^c\}$] except ν_3^c carries the $U(1)_R$ charges of odd integers. On the other hand, the superfields S , $\{h_d, h_u\}$, ν_3^c , $\{Y, Y^c\}$, and $\{Z, Z^c\}$, for which the scalar components can develop VEVs, carry $U(1)_R$ charges of even integers. Accordingly, the remaining Z_2 symmetry forbids the R -parity violating couplings including the operators leading to dimension-four proton decay and also can guarantee the existence of the lightest supersymmetric particle dark matter. Note that the μ parameters in $\mu_H H_u H_d$, $\mu_L L L^c$, and $\mu_Y Y Y^c$, which are all generated from Eq. (25), can be regarded as spurion fields carrying the 2 charge of $U(1)_R$ since they originate from the VEV of F_X^* .

As mentioned above, the A term corresponding to the λ term in Eq. (9) provides a tadpole of S in the scalar potential, when $\{h_u, h_d\}$ develop VEVs, and it eventually leads to a nonzero VEV of S . Such an A term is induced by $W \supset \lambda X S h_u h_d / M_P$. Since $U(1)_R$ as well as $U(1)_Z$ are broken, e.g., additional tadpole terms of S could be potentially induced. Indeed, an additional tadpole of

S is generated in the scalar potential at one loop, $\sim (\lambda y_{h1}^* y_{h2}^* / 16\pi^2) \mu_H \tilde{S} (\tilde{Z}^*)^2 + \text{H.c.}$, in which the fermionic components of $\{h_u, h_d\}$ and $\{H_d, H_u\}$ circulate in the loop. Thus, small enough $y_{h1,2}$ ($\lesssim 0.1$) leave intact our previous discussion on the sizes of $\langle \tilde{S} \rangle$ and $\langle \tilde{Z} \rangle$. Similarly, e.g., the A terms of the $y_{h1,2}$ terms in Eq. (19) and the $B\mu$ term of the κ_H term in Eq. (25) are generated from $W \supset y_{h1,2} X^\dagger Z H_{u,d} h_{d,u} / M_P$ and $K \supset \kappa_H X^\dagger X H_u H_d / M_P^2 + \text{H.c.}$, respectively.

The presence of $\kappa_{Y,Z}$ terms in Eq. (25) can affect our earlier discussion on the VEVs of $\{\tilde{\nu}_3^c, \tilde{Y}, \tilde{Y}^c, \tilde{Z}, \tilde{Z}^c\}$. Only if $|\kappa_{Y,Z}| \lesssim 1$, however, their VEVs determined without the $\kappa_{Y,Z}$ would be just slightly modified. As mentioned earlier, we require that $\kappa_{H,L}$ is relatively large (~ 5). Throughout this paper, we assume that all the soft parameters at low energies are not heavier than $\mu_{H,L}$. $\{H_u, H_d\}$ make contribute to the radiative correction to the masses of ordinary Higgs $\{h_u, h_d\}$ via the $y_{h1,2}$ terms in Eq. (19). If they were too heavy, hence, they could radiatively destabilize the electroweak scale. However, the mixings between $\{H_u, H_d\}$ and $\{h_u, h_d\}$, i.e., $y_{h1,2}$ in Eq. (19), are small enough ($\lesssim 0.3$), and so the electroweak scale still remains radiatively stable even with relatively large $\kappa_{H,L}$.

In this model, the four fermionic components of $\{Z, Z^c, S\}$ and the $U(1)_Z$ gaugino, (Z, Z^c, S, λ_Z) , are mixed to each other via the mass matrix,

$$\begin{pmatrix} 0 & \mu_Z & 0 & \sqrt{2} g_Z q_z \langle \tilde{Z} \rangle \\ \mu_Z & 0 & 0 & -\sqrt{2} g_Z q_z \langle \tilde{Z}^c \rangle \\ 0 & 0 & 0 & \sqrt{2} g_Z q_s \langle \tilde{S} \rangle \\ \sqrt{2} g_Z q_z \langle \tilde{Z} \rangle & -\sqrt{2} g_Z q_z \langle \tilde{Z}^c \rangle & \sqrt{2} g_Z q_s \langle \tilde{S} \rangle & M_\lambda \end{pmatrix}, \quad (26)$$

where $q_z = 2$, $q_s = 4$, and M_λ denotes the $U(1)_Z$ gaugino mass. Here, we ignored $\lambda S h_u h_d$ and $y_\nu \nu_3^c Y Z^c$ couplings for a moment. For the case of $\langle \tilde{S} \rangle \ll \langle \tilde{Z} \rangle \sim \langle \tilde{Z}^c \rangle \sim \mu_Z$, the mass of S -like (singlinolike) fermion is approximately given by

$$M_S \approx \frac{2g_Z^2 q_s^2 \langle \tilde{S} \rangle^2}{M_\lambda + 4g_Z^2 q_z^2 \langle \tilde{Z} \tilde{Z}^c \rangle / \mu_Z} \sim \frac{\langle \tilde{S} \rangle^2}{\langle \tilde{Z} \rangle} \sim \mu_{\text{eff}} \left(\frac{\mu_{\text{eff}}}{M_Z} \right), \quad (27)$$

where M_Z stands for the $U(1)_Z$ symmetry breaking scale and μ_{eff} the effective μ parameter ($= \lambda \langle \tilde{S} \rangle$). The order of magnitude of Eq. (27) would still be the same even if we include also the fermionic components of $\{\nu_3^c, Y, Y^c\}$ in the mass matrix. Thus, we get $M_S \sim 100$ GeV for a relatively large μ_{eff} , e.g., $\mu_{\text{eff}} \sim 1$ TeV and $M_Z \sim 10$ TeV. As a result, the invisible decay of the Higgs to the two singlinos is kinematically forbidden.

On the other hand, if $\mu_{\text{eff}} = \mathcal{O}(100)$ GeV, the singlino's mass given by Eq. (27) becomes lighter than 10 GeV. In this case, the singlino mass would be dominantly given by the superpotential term $\lambda S h_u h_d$:

$$M_{\tilde{g}} \approx \frac{\lambda^2 v_h^2 \sin 2\beta}{\mu_{\text{eff}} + \lambda^2 v_h^2 / \mu_{\text{eff}}}. \quad (28)$$

The upper bound of Eq. (28) is achieved when $\mu_{\text{eff}} \sim \lambda v_h$. Thus, $M_{\tilde{g}} \leq (\lambda v_h \sin 2\beta)/2$. In the ‘‘minimal’’ NMSSM case [without the $U(1)_Z$ gauge symmetry and extra matter], the perturbativity of λ up to the GUT scale (< 0.7) constrains the mass of the light singlino as $M_{\tilde{g}} < m_h/2 \approx 63$ GeV. It opens the invisible decay of the Higgs to the two singlinos [8]. In our model, however, the upper bound of λ is relaxed, and so the singlino mass can be heavier than 63 GeV, maintaining the perturbativity of λ , e.g., $\lambda \approx 0.88$ for $n_Z = 4$, $\tan \beta \approx 2$, and $\mu_{\text{eff}} \approx 150$ GeV. Thus, the invisible decay of the Higgs can still be kinematically forbidden even for $\mu_{\text{eff}} = \mathcal{O}(100)$ GeV.

B. Model of $U(1)_Z \times U(1)_X$

For the case that $U(1)_X$ is also considered, the first and second families of the MSSM chiral matter are still assumed to be neutral under $U(1)_X$ as well as $U(1)_Z$, carrying unit $U(1)_R$ charges. We need more fields in the $U(1)_Z \times U(1)_X$ case, $\{Z', Z'^c\}$ and $\{N, N^c\}$. The charge assignments for the extra fields are displayed in Table III. With the field contents in Tables I and III, the beta-function coefficients of $U(1)_Z$ and $U(1)_X$ are estimated as $b_Z = (4 + \frac{1}{2})n_Z^2$ and $b_X = (4 + \frac{3}{10})n_X^2$, respectively. They were used for analyses of the $\nu = 2$ cases in Fig. 1(b).

For the $U(1)_Z \times U(1)_X$ case, the y_{h2} , y_{D1} , and y_V terms in Eq. (19) and the y_i^j term in Eq. (24) should be modified with the new superfields:

$$\begin{aligned} y_{h2} Z H_d h_u &\rightarrow y_{h2} Z' H_d h_u, & y_{D1} Z D D^{c'} &\rightarrow y_{D1} Z' D D^{c'}, \\ y_V \nu_3^c Y Z^c &\rightarrow y_V \nu_3^c Y Z^{c'}, & y_i^j \nu_3^c l_3 L^c &\rightarrow y_i^j N^c l_3 L^c. \end{aligned} \quad (29)$$

Instead of Z^c , thus, Z'^c obtains a VEV together with ν_3^c and Y from the A term of y_V . From the D-term potentials, Z' and Y^c can also get VEVs. With the new superfields, the superpotential allows the following term:

$$W \supset y_N Y Z^c N^c, \quad (30)$$

for which the A term can induce VEVs of Z^c and N^c as well as Y . Then, the D-term potentials can yield VEVs of Z and N . The Kähler potential, Eq. (25), is supplemented with

$$K \supset \frac{X^\dagger}{M_P} (\kappa_{Z'} Z' Z^{c'} + \kappa_N N N^c) + \text{H.c.}, \quad (31)$$

which generates the μ terms for $\{Z', Z'^c\}$ and $\{N, N^c\}$.

V. BREAKING SCALE OF EXTRA $U(1)$

Now, let us discuss the $U(1)_Z$ breaking scale. Because of the family-dependent charge assignment of $U(1)_Z$, the flavor-violating process can be induced through the exchange of the $U(1)_Z$ gauge boson Z'_μ . We will estimate $\langle \tilde{Z}^c \rangle$, which determines the $U(1)_Z$ breaking scale, following the formulation driven by Ref. [26]. A similar estimation would be applicable to the model of $U(1)_Z \times U(1)_X$.

The most important constraint on the mixing angle θ_Z between Z_μ and Z'_μ vector bosons come from the coherent μ - e conversion rate in nuclei. The branching ratio is [27,28]

$$\begin{aligned} \text{Br}(\mu \mathcal{N} \rightarrow e \mathcal{N}^*) &= \frac{\alpha_{\text{em}}^3 |F_p|^2 G_F^2 m_\mu^5 Z_{\text{eff}}^4}{2\pi^2 \Gamma_{\text{cap}} Z_{\text{atm}}} (|Q_{12}^{e_L}|^2 + |Q_{12}^{e_R}|^2) \\ &\times \left| x_1 \left(\frac{Z_{\text{atm}} - N_{\text{ncl}}}{2} - 2Z_{\text{atm}} \sin^2 \theta_W \right) \right. \\ &\left. + x_2 (Z_{\text{atm}} + 2N_{\text{ncl}}) (Q_{11}^{d_L} + Q_{11}^{d_R}) \right|^2, \end{aligned} \quad (32)$$

where F_p is the nuclear form factor, Γ_{cap} is the muon capture rate, Z_{eff} is the effective charge of the nuclei for the muon, and Z_{atm} (N_{ncl}) is the atomic number (the neutron number) of the nucleus \mathcal{N} . $Q_Z^{d_L}$ and $Q_Z^{e_L}$ are defined with the CKM and PMNS matrices, $(Q_Z^{d_L})_{ij} \equiv q_m (V_{\text{CKM}})_{3i}^* (V_{\text{CKM}})_{3j}$ and $(Q_Z^{e_L})_{ij} \equiv q_m (V_{\text{PMNS}})_{i3} (V_{\text{PMNS}})_{j3}^*$, where q_m denotes the electromagnetic charge of relevant SM matter. On the other hand, $(Q_Z^{d_R})_{ij}$ and $(Q_Z^{e_R})_{ij}$ are rather model dependent. We assume that $(Q_Z^{d_R})_{ij}$ and $(Q_Z^{e_R})_{ij}$ are the same order with $(Q_Z^{d_L})_{ij}$ and $(Q_Z^{e_L})_{ij}$, respectively. For a small mixing ($\theta_Z \ll 1$), x_1 and x_2 are given by

$$x_1 \approx \frac{g_Z \theta_Z}{\sqrt{g^2 + g'^2}}, \quad x_2 \approx \frac{v_h^2}{2(M_{Z'}^2/g_Z^2)}, \quad (33)$$

where $M_{Z'}$ denotes the mass of the $U(1)_Z$ gauge boson. From the SINDRUM II collaboration, we get the most serious bound from $\mathcal{N} = \text{Au}$ [29],

TABLE III. The charge assignments for the extra vectorlike fields under the gauge $U(1)_Z \times U(1)_X$ and the global $U(1)_R$ in case II. Here, the $U(1)_Z$ and $U(1)_X$ charge normalizations, $n_Z/\sqrt{24}$ and $n_X/\sqrt{40}$, are omitted for simple presentations. The extra vectorlike fields listed in Table II should carry also $U(1)_X$ charges. In this case, more fields $\{Z', Z'^c\}$ and $\{N, N^c\}$ are needed. As in case I, however, the first two families of the chiral matter do not carry charges of $U(1)_Z \times U(1)_X$ but do carry a unit charge of $U(1)_R$.

Superfields	X	Y	Y^c	Z	Z^c	D'	D'^c	L	L^c	Z'	Z'^c	N	N^c
$U(1)_Z$	0	1	-1	2	-2	0	0	2	-2	2	-2	-1	1
$U(1)_X$	0	-3	3	-2	2	0	0	-2	2	2	-2	-1	1
$U(1)_R$	0	0	0	0	0	3	1	-1	1	0	0	-2	2

$$\text{Br}(\mu\text{Au} \rightarrow e\text{Au}) < 7 \times 10^{-13}, \quad (34)$$

which yields the constraint on x_1 as

$$x_1 \leq 2 \times 10^{-6}. \quad (35)$$

If the kinetic mixing between $U(1)_Y$ and $U(1)_Z$ is negligible, $x_1 \approx 0$ at tree level since \tilde{Z}^c does not carry the SM gauge quantum number. Thus, it can be easily satisfied in this model.

The constraint on x_2 from the quark sector is rather mild. Since the $U(1)_Z$ charges for the first and second families of the SM chiral fields are zero, the flavor-violating effects are proportional to the off-diagonal components of the CKM matrix. However, there is no such large suppression in the lepton sector. The most important constraint originates from the flavor-changing muon decay modes: $\mu^- \rightarrow e^- \gamma$ and $\mu^- \rightarrow e^- e^+ e^-$. The branching ratios for the processes are estimated as

$$\begin{aligned} \text{Br}(\mu^- \rightarrow e^- \gamma) & \approx \frac{48\alpha_{\text{em}}}{\pi} (q_m^2 x_2)^2 \left| (V_R^e)_{31} (V_{\text{PMNS}})_{23} \frac{[M_E]_{3,3}}{m_\mu} \right|^2, \\ \text{Br}(\mu^- \rightarrow e^- e^+ e^-) & \approx 4x_2^2 [2|(Q_Z^{eL})_{11} (Q_Z^{eL})_{12}|^2 + 2|(Q_Z^{eR})_{11} (Q_Z^{eR})_{12}|^2 \\ & \quad + |(Q_Z^{eL})_{11} (Q_Z^{eR})_{12}|^2 + |(Q_Z^{eR})_{11} (Q_Z^{eL})_{12}|^2] \end{aligned} \quad (36)$$

in the limit of a small $Z_\mu - Z'_\mu$ mixing, which is already constrained by Eq. (35). Here, V_R^e indicates the unitary matrix diagonalizing $M_E M_E^\dagger$ (rather than $M_E^\dagger M_E$), which does not contribute to the PMNS matrix. $[M_E]_{3,3}$ means the (3,3) components in the mass matrix of the charged leptons ($\equiv y_\tau v_h \cos \beta$). The present experimental bounds for such processes [30,31] are found in Table IV, which provide the most stringent constraints on x_2 or $\langle \tilde{Z}^c \rangle$:

$$\langle \tilde{Z}^c \rangle \geq 5.6 \text{ TeV} \times \left(\frac{|y_\tau|}{10^{-3}} \right)^{1/2} \left(\frac{\cos \beta}{0.2} \right)^{1/2} \quad (37)$$

and $\langle \tilde{Z}^c \rangle \geq 3.1 \text{ TeV}$, respectively. Thus, the VEV of \tilde{Z}^c discussed in Sec. IV ($\approx 5\text{--}10 \text{ TeV}$) is consistent with these bounds. Other less severe constraints on x_2 [32] are also listed in Table IV.

The upper bound on x_2 will possibly be further lowered by future experiments. Then, this model will be testable. Especially for $\mu \rightarrow 3e$, the sensitivity is planned to reach $\text{Br}(\mu \rightarrow 3e) = 10^{-16}$ [33]. Concerning the μ - e conversion, many experiments are proposed to explore $\text{Br}(\mu\text{Al} \rightarrow e\text{Al}) = 10^{-16}$ [34,35] and $\text{Br}(\mu\text{Ti} \rightarrow e\text{Ti}) = 10^{-18}$ [36].

Other potential flavor-violation effects in this model are the processes mediated by the heavy Higgs $\{H_u, H_d\}$. The constraints by them would be mild compared to those by Z' boson mediations because small Yukawa couplings are also involved there. In the mass eigenbasis, the neutral flavor-changing couplings can be written as

TABLE IV. Constraints on $\langle \tilde{Z}^c \rangle$ from various experimental bounds. m_μ denotes the muon mass, and $[M_E]_{3,3}$ means the (3,3) component in the mass matrix of the charged leptons ($\equiv y_\tau v_h \cos \beta$).

Experimental bounds	$q_m^2 x_2 (= v_h^2 / 16 \langle \tilde{Z}^c \rangle^2)$
$\text{Br}(\mu^- \rightarrow e^- \gamma) < 5.7 \times 10^{-13}$	$< 0.2 \frac{m_\mu}{ [M_E]_{3,3} } \times 10^{-4}$
$\text{Br}(\mu^- \rightarrow e^- e^+ e^-) < 1.0 \times 10^{-12}$	$< 2 \times 10^{-4}$
$\Delta m_{B_s^0} = (117.0 \pm 0.8) \times 10^{-13} \text{ GeV}$	$< 10^{-3}$
$\Delta m_{B^0} = (3.337 \pm 0.033) \times 10^{-13} \text{ GeV}$	$< 10^{-3}$
$\Delta m_{K^0} = (3.484 \pm 0.006) \times 10^{-15} \text{ GeV}$	$< 10^{-2}$
$\epsilon_K = (2.233 \pm 0.015) \times 10^{-3}$	$< 4 \times 10^{-4}$
$\text{Br}(B^0 \rightarrow \mu^+ \mu^-) < 1.4 \times 10^{-9}$	$< 10^{-2}$
$\text{Br}(K_L^0 \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$	$< 1.5 \times 10^{-3}$
$\text{Br}(K_L^0 \rightarrow e^+ e^- \pi^0) < 2.8 \times 10^{-10}$	< 0.4
$\text{Br}(K_L^0 \rightarrow \mu^+ \mu^- \pi^0) < 3.8 \times 10^{-10}$	< 0.1
$\text{Br}(K_L^0 \rightarrow \mu^+ e^- \pi^0) < 7.6 \times 10^{-11}$	< 0.1

$$W_{\text{FCNC}} = \sum_{i,j=1,2,3} (\xi_{ij}^d \hat{d}_i \hat{d}_j^c + \xi_{ij}^e \hat{e}_i \hat{e}_j^c) H_d^0, \quad (38)$$

where $\xi_{ij}^{d,e}$ parametrize (Yukawa couplings) \times (mixing angles), and so they are of the order of the Yukawa couplings or smaller. Here, H_d^0 denotes the neutral component of H_d . After integrating out the heavy Higgs $\{H_u, H_d\}$, Eq. (38) provides flavor-violating four-fermion interactions suppressed by μ_H^2 from the Kähler potential. Unlike H_d^0 , H_u^0 does not give rise to flavor violations. Note that H_u^0 has only the 2×2 block-diagonal couplings of the first and second generations of the SM chiral matter, while H_u^0 couples only to the third generation.

Because of the hierarchical structure of the masses for the quarks and charged leptons, we employ the parametrization of Cheng and Sher [37] as

$$\xi_{ij}^d = \lambda_{ij}^d \frac{\sqrt{m_i^d m_j^d}}{v_h}, \quad \xi_{ij}^e = \lambda_{ij}^e \frac{\sqrt{m_i^e m_j^e}}{v_h}, \quad (39)$$

where m_i^q (m_i^e) denotes the mass of the d -type quark (charged lepton) of the generation i . For $\lambda^{d,e} \sim \mathcal{O}(1)$, thus, $\xi^{d,e}$ become the same order of magnitude with the average of the relevant Yukawa couplings. If $\lambda^{d,e}$ turn out to be of order unity, hence, the couplings in Eq. (38) can be regarded as being natural. On the other hand, if the experimental bounds requires the too small $\lambda^{d,e}$, the couplings in Eq. (38) should be finely tuned.

For the quark sector, the most serious bound comes from the neutral meson ($M^0 = q_i \bar{q}_j$) mixing. The mass splitting is calculated as [38]

$$\frac{\Delta m_{M^0}}{m_{M^0}} \approx \frac{(\xi_{ij}^d)^2 B_{M^0} F_{M^0}^2}{3\mu_H^2} \left[1 + \frac{6m_{M^0}^2}{(m_i^q + m_j^q)^2} \right], \quad (40)$$

where F_{M^0} is the meson decay constant, and B_{M^0} is the vacuum insertion parameters defined in Ref. [39]. The

experimental bounds on $\Delta m_{B_s^0}$, Δm_{B^0} , and Δm_{K^0} in Table IV constrain the parameters as

$$(\lambda_{12}^d, \lambda_{31}^d, \lambda_{32}^d) \left(\frac{5 \text{ TeV}}{\mu_H} \right) \lesssim (4.2, 2.5, 2.5). \quad (41)$$

The less severe constraint is from $\text{Br}(B_s^0 \rightarrow \mu^+ \mu^-) < 6.4 \times 10^{-9}$ [32], which yields

$$\sqrt{\lambda_{32}^d \lambda_{22}^e} \left(\frac{5 \text{ TeV}}{\mu_H} \right) \lesssim 37. \quad (42)$$

For the lepton sector, the most important constraint is from the $\mu \rightarrow e\gamma$ bound. The decay rate is given by [40]

$$\Gamma(\mu \rightarrow e\gamma) \approx \left[\frac{\alpha_{\text{em}} (\xi_{13}^e)^2 (\xi_{23}^e)^2}{4\pi^4} \right] \left[\frac{m_\tau^4 m_\mu}{\mu_H^4} \right] \left[\ln \frac{\mu_H}{m_\tau} \right]^2. \quad (43)$$

So the resulting bound is estimated as

$$\sqrt{\lambda_{13}^e \lambda_{23}^e} \left(\frac{5 \text{ TeV}}{\mu_H} \right) \lesssim 3.3. \quad (44)$$

Therefore, all the flavor violations associated with the heavy Higgs, $\{H_u, H_d\}$ can be sufficiently suppressed only if $\mu_H \gtrsim 5 \text{ TeV}$.

VI. CONCLUSION

We considered the perturbative U(1) gauge extensions of the NMSSM to relieve the LP constraint on the λ coupling, maintaining the gauge coupling unification. They are closely

associated with raising the tree-level mass of the Higgs. To minimize the beta-function coefficient(s) of $U(1)_Z$ or $U(1)_Z \times U(1)_X$, which is necessary for U(1) gauge interaction(s) relevant down to low energies, we assign U(1) charges only to the Higgs doublets and the third family of the chiral matter among the MSSM field contents. In the $U(1)_Z [U(1)_Z \times U(1)_X]$ case, the low-energy value of λ can be lifted up to 0.85–0.95 [0.9–1.0], depending on the normalization of the charges, when the gauge coupling g_Z [and also g_X] and λ are constrained not to blow up below the GUT scale. We also discussed how to induce the Yukawa couplings for the first and second families of the quarks and leptons and the resulting phenomenological constraints associated with flavor violations.

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