

Transverse single spin asymmetry in hadronic $\eta_{c,b}$ production

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We study the transverse single spin asymmetry in $\eta_{c,b}$ production in polarized hadron collisions, employing the collinear twist-3 approach in combination with the color singlet model. Our main focus lies on the contribution from the twist-3 Efremov-Teryaev-Qiu-Sterman function. By extrapolating the derived spin-dependent cross section to the small transverse momentum region, consistency between the collinear twist-3 approach and the transverse-momentum-dependent factorization approach is confirmed. As a byproduct of this work, we identify a term contributing to the scale evolution of trigluon correlations in the flavor-singlet case which was originally missed, see also V. M. Braun, A. N. Manashov, and B. Pirnay [Phys. Rev. D **80**, 114002 (2009); **86**, 119902(E) (2012)].

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I. INTRODUCTION

Attempts to understand the large transverse single spin asymmetries (SSAs) observed in the various high-energy scattering processes presently form an active field of research. The experimental observations are a great theoretical challenge as the collinear leading twist contribution to the asymmetries is proven to be proportional to the quark mass [1,2] and thus very small. However, during the past few decades, remarkable theoretical progress was achieved by following mainly two approaches—one approach based on transverse-momentum-dependent (TMD) factorization [3–8] and the other on collinear twist-3 factorization [9–17]. In TMD factorization, a naive time reversal odd TMD distribution, known as the Sivers function [3], describing the correlation between the parton intrinsic transverse momentum and hadron transverse spin vector, is responsible for the asymmetries. In the collinear twist-3 approach the SSAs arise from twist-3 quark gluon correlators, so-called Efremov-Teryaev-Qiu-Sterman functions (ETQS) $T_F(x, x)$ [9,10]. These two frameworks have their own kinematic regions of validity and were shown to produce the same results in the overlap of these kinematic regions [14–16,18].

The fundamental property of both mechanisms is that the imaginary part necessary for nonvanishing SSAs is dynamically generated through initial/final state interactions. In the case of TMD factorization, the effect of these initial/final state interactions is encoded in the process-dependent gauge link appearing in the matrix element defining the TMD distributions. Due to different structures of these gauge links in different processes, naive time reversal odd TMD distributions like the Sivers function possess a very unique, modified universality property, namely, that the Sivers functions in deep inelastic scattering (DIS) and the Drell-Yan process differ by a minus sign [6,7].

While SSAs in DIS have been measured by HERMES [19] and COMPASS [20], a polarized Drell-Yan measurement is not yet available, such that this prediction could not

yet be tested. Alternatively, one can simply fit the ETQS function related to the k_T moment of the Sivers function [21] using input from HERMES and COMPASS [22] and compare it with a direct extraction of T_F from the SSA observed for pion production in polarized pp collisions [13,23–25]. Promising early results along these lines have, unfortunately, been caused by a sign error, such that the observed discrepancies are now referred to as “sign mismatch” of these two processes [26]. Our recent work [27] also indicated that this procedure gives the wrong sign for SSA in inclusive DIS off a neutron target (for a complete theoretical treatment of this asymmetry, see Ref. [28]) if the quark gluon correlation T_F fitted to data for the process $p \uparrow p \rightarrow \pi X$ were used as the input for the twist-3 quark photon correlation. This suggests that the SSAs observed in processes like $p \uparrow p \rightarrow \pi X$ are not mainly caused by the Sivers mechanism as described in the twist-3 collinear approach [11,13]. A possible explanation is suggested by the observation [29] that the Collins effect [5,30], describing SSA as caused by parton fragmentation, can contribute significantly. This observation has been confirmed by a more recent analysis [31].

In order to better understand the Sivers mechanism in hadronic collisions, it is desirable to investigate SSA for cases of particle production in polarized pp collisions for which the Collins effect is absent. Possible options are the SSA in direct photon or jet production $p \uparrow p \rightarrow \gamma(\text{Jet})X$ [10,32–34]. Another option which was initially proposed in Ref. [35] is the SSA in heavy quarkonium production. The author of the paper [35] analyzed SSA in heavy quarkonium production following the general arguments of nonrelativistic QCD (NRQCD) [36,37]. In the framework of NRQCD, the heavy quarkonium can be produced at short distance not only in the color singlet but also the color octet configuration. The transition from heavy quark and antiquark pair to a quarkonium state is treated as a nonperturbative process and encoded in the long-distance universal matrix elements which are characterized according to the velocity expansion of NRQCD. As demonstrated

in Ref. [35], the SSA in heavy quarkonium production offers a unique way to investigate its production mechanisms since the SSA crucially depends on the final state interactions which differ significantly for heavy quarkonium production in the color singlet and the color octet channel. Recently, a SSA in $p \uparrow p \rightarrow J/\psi X$ has been measured at PHENIX and found to be sizeable [38]. According to the analysis of Ref. [35], this hints at a dominance of a color singlet mechanism at low transverse momentum and at a nonzero gluon Sivers effect. Also, in Ref. [39], the color evaporation model in combination with TMD factorization was used to study the SSA for J/ψ production in $ep \uparrow$ collisions.

Following this research line, we apply the collinear twist-3 approach to compute the SSA for heavy quarkonium production in polarized pp collisions. In the present work, the earlier pioneering analysis [35] is extended and refined in the sense that we compute the transverse-momentum-dependent behavior of the spin asymmetry, and also take into account the hard gluon pole contribution. We only focus on $\eta_{c,b}$ production in this paper, though our formalism can be easily extended to SSAs for the production of other C-even quarkonia, such as χ_{c0} , χ_{b0} , χ_{c2} and χ_{b2} . In leading order, a C-even quarkonium can be produced through two-gluon fusion, which makes it a promising way to access both unpolarized and linearly polarized gluon TMD distributions [40–44]. The application of TMD factorization in these processes has been justified by a recent NLO calculation [45]. Experiments measuring such $\eta_{c,b}$ asymmetries could be performed at RHIC and a proposed fixed target experiment at LHC(AFTER) [40–42]. LHC would allow us to study the gluon polarization effect in a deep saturation regime [46] due to the very high energy of LHC and relative low mass of C-even charmonium. In Refs. [47,48], the promising channels to detect η_b were discussed.

The analysis [36] based on NRQCD suggested that for C-even quarkonium production, the color octet contribution is suppressed, while the color singlet contribution dominates, especially at low transverse momentum [49], though complications may arise at large transverse momentum [50,51]. Moreover, C-even bottomonium production in the color octet configuration can certainly be neglected [40,41,47], since it is strongly suppressed in the velocity expansion. On the other hand, the SSA in C-even heavy quarkonium production receives a contribution not only from ETQS function but also from the trigluon correlation [52]. Nevertheless, we restrict ourselves to color singlet production, i.e. the T_F contribution, and leave the color octet part and trigluon correlation for a future study, as we are primarily aiming at establishing a formalism that combines collinear twist-3 techniques and NRQCD factorization to describe SSA in heavy quarkonium production in the present paper.

Our calculation is carried out in the covariant gauge closely following the techniques outlined in Refs. [10,11,13] in

which the SSAs for pion and direct photon production in hadronic collisions were computed. Recently, in analogy to the SSAs, the double spin asymmetry A_{LT} in the same processes was also studied within the collinear twist-3 framework [53]. Unlike the SSA in pion or photon production, TMD factorization could be applied in the kinematical region where the transverse momentum of the heavy quarkonium is much smaller than its mass. We extrapolate the complete collinear twist-3 result to the small transverse momentum region and find that it is consistent with the result obtained from TMD factorization. The key step in establishing this connection is to derive the large transverse momentum gluon Sivers function and relate it to the ETQS function. Doing so, we find that the hard gluon pole contribution to the gluon Sivers function was overlooked in [35]. We will also comment on the scale evolution of trigluon correlations for the flavor singlet case in the following section.

The paper is organized as follows: in Sec. II, we first briefly review $\eta_{c,b}$ production in unpolarized hadron collisions. Next we derive the spin-dependent differential cross section for $\eta_{c,b}$ production in hadronic collisions, presenting the result in the limits of high and low transverse momentum. In particular, we discuss the matching between TMD factorization and the collinear twist-3 approach at small transverse momentum. The paper is summarized in Sec. III.

II. CALCULATION OF THE UNPOLARIZED AND POLARIZED CROSS SECTIONS

We start by introducing the relevant kinematical variables. For the process under consideration,

$$A(P, \vec{S}_\perp) + B(P') \rightarrow \eta_{c,b}(l) + X, \quad (1)$$

we define the 4-momenta and polarization vector of the incoming nucleons A , B and outgoing heavy quarkonium $\eta_{c,b}$ as indicated. The Mandelstam variables are $S = (P + P')^2$, $T = (P' - l)^2$ and $U = (P - l)^2$. The corresponding Mandelstam variables on the partonic level are given by $\hat{s} = (xP + x'P')^2$, $\hat{t} = (x'P' - l)^2$ and $\hat{u} = (xP - l)^2$, where x , x' are the longitudinal momentum fractions carried by the partons from nucleon A and B , respectively. The squared invariant mass of the heavy quarkonium is $M^2 = l^2$ and $M = 2M_Q$ up to small relativistic corrections, where M_Q is the heavy quark mass. In the unpolarized case, to lowest order $O(\alpha_s^2)$, one only has the gluon fusion process shown in Fig. 1(a). This production mechanism dominates at small transverse momentum, where the mean produced quarkonium transverse momentum is of order Λ_{QCD} . At order $O(\alpha_s^3)$, there are many more contributions from the two-by-two scattering processes illustrated in Figs. 1(b) and 1(c). In these processes, the large heavy quarkonium transverse momentum is generated by recoiling against an outgoing gluon or quark. Note that heavy

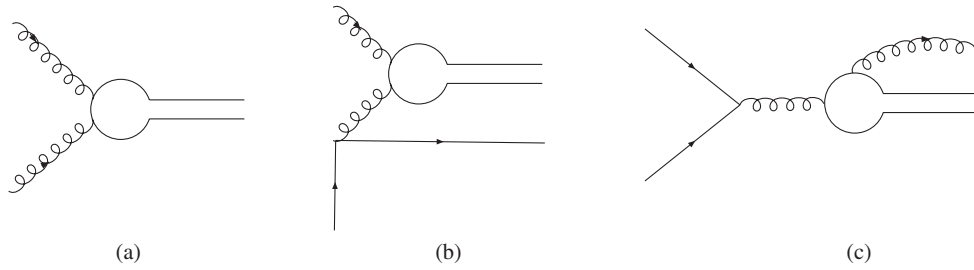


FIG. 1. Diagrams contributing to the unpolarized cross section at order $O(\alpha_s^2)$ and $O(\alpha_s^3)$. Gluon-gluon scattering at order $O(\alpha_s^3)$ is not shown. Diagrams with permutations of the gluon lines are also not shown.

quarkonium production at $O(\alpha_s^3)$ from gluon-gluon scattering is not shown in the Fig. 1.

Within the so-called color singlet model [54–56], the amplitude for the processes discussed has the general form,

$$\mathcal{M} = \int \frac{d^4 q}{(2\pi)^4} \text{Tr}[O(l, q)\phi(l, q)], \quad (2)$$

where $2q$ is the relative momentum of the heavy quarks. Here, $\phi(l, q)$ is the Bethe-Salpeter wave function of the produced bound state. $O(l, q)$ is the perturbative part of the diagrams shown in Fig. 1 with the heavy quark legs cut off. Due to the nonrelativistic nature of heavy quarkonium, the relative momentum is assumed to be small with respect to the quark mass $M/2$. For the production of S-wave states, one can simply neglect the dependence of $O(l, q)$ on the relative momentum. This leads to

$$\mathcal{M} = \frac{1}{\sqrt{4\pi}} R_0(0)[O(l, 0)\mathcal{P}_{SS_z}(0)]. \quad (3)$$

Here, $R_0(0)$ is the value of the radial S-wave function at the origin, and $\mathcal{P}_{SS_z}(0)$ is the spin projection operator. According to the color singlet model, the heavy quark pair produced in the partonic scattering can evolve into a quarkonium nonperturbatively only if they have the same quantum numbers as the corresponding quarkonium. This implies that the spin projector operator associated with $\eta_{c,b}$ production has the form

$$\mathcal{P}_{SS_z}(0) = \frac{1}{4M^{3/2}} (\not{I} + M)\gamma^5(\not{I} + M). \quad (4)$$

With these calculational recipes, one derives the unpolarized differential cross section for the gluon fusion channel [56],

$$\frac{d\sigma}{dyd^2l_\perp} = \sigma_0 G_1(z)G_1(z'), \quad (5)$$

where y is the rapidity of the produced bound state. $G_1(z)$, $G_1(z')$ are the unpolarized collinear gluon distribution functions of two incoming hadrons. The longitudinal momentum fractions are constrained by kinematics, $z = \frac{M}{\sqrt{s}} e^y$, $z' = \frac{M}{\sqrt{s}} e^{-y}$. In Eq. (5), σ_0 is given by

$$\sigma_0 = \frac{\pi^2 R_0^2 \alpha_s^2}{3M^3 S}. \quad (6)$$

For the quark gluon scattering channel, we have [56]

$$\begin{aligned} \frac{d\sigma}{dyd^2l_\perp} &= \sigma_0 C_F \frac{\alpha_s}{2\pi^2} M^2 \sum_a \int \frac{dx}{x} \frac{dx'}{x'} f_1^a(x) G_1(x') \\ &\times \frac{(\hat{t} - M^2)^2 - 2\hat{s}\hat{u}}{(-\hat{t})(\hat{t} - M^2)^2} \delta(\hat{s} + \hat{t} + \hat{u} - M^2), \end{aligned} \quad (7)$$

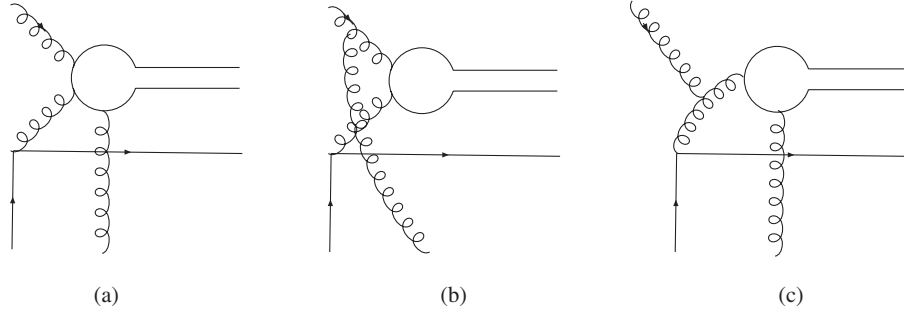
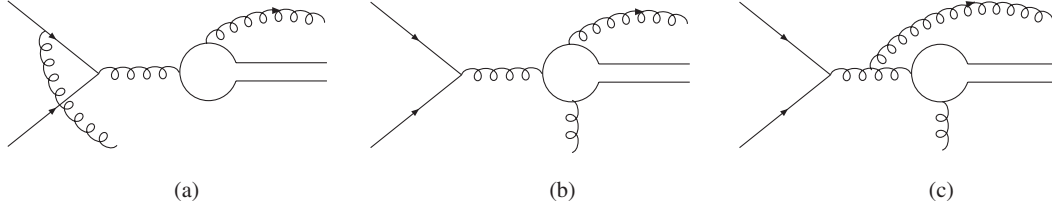
where $f_1^a(x)$ is the unpolarized collinear quark distribution. The index a runs over all quark flavors. For the $q\bar{q}$ channel, the unpolarized differential cross section reads [56]

$$\begin{aligned} \frac{d\sigma}{dyd^2l_\perp} &= \sigma_0 C_F \frac{-8}{3} \frac{\alpha_s}{2\pi^2} M^2 \sum_a \int \frac{dx}{x} \frac{dx'}{x'} f_1^a(x) \bar{f}_1^a(x') \\ &\times \frac{(\hat{s} - M^2)^2 - 2\hat{t}\hat{u}}{-\hat{s}(\hat{s} - M^2)^2} \delta(\hat{s} + \hat{t} + \hat{u} - M^2). \end{aligned} \quad (8)$$

One notices that apart from the color factor, the hard coefficient appearing in the $q\bar{q}$ channel can be obtained from those of the quark gluon channel by crossing $\hat{t} \leftrightarrow \hat{s}$. The result for the gluon-gluon scattering channel can also be found in Ref. [56]. Next-to-leading order corrections to C-even hadronic quarkonium production have been calculated in Refs. [57,58]. Now let's proceed to derive the polarized cross section. To generate the spin asymmetry, one additional gluon must be exchanged between the active partons and the remanent part of the nucleon target as shown in Figs. 2 and 3. The hard part, if an additional gluon is attached, can be calculated perturbatively, while the nonperturbative part describes the relevant three parton correlations. As stated in the Introduction, we only focus on the contribution in which the quark-gluon correlation in the transversely polarized nucleon enters. This is precisely the ETQS function that is defined as

$$\begin{aligned} \int \frac{d\xi^-}{2\pi} \frac{d\xi'^-}{2\pi} e^{ix_1 P^+ \xi^-} e^{i(x-x_1)P^+ \xi'^-} \langle P, S_\perp | \bar{\psi}_\beta(0) g F_\perp^{+\mu}(\xi^-) \\ \times \psi_\alpha(\xi'^-) | P, S_\perp \rangle = \frac{M_N}{2} T_F(x, x_1) \epsilon_\perp^{\mu\nu} S_{\perp\mu} \not{n}_{\alpha\beta}, \end{aligned} \quad (9)$$

where we have suppressed Wilson lines and have indicated the nucleon mass by M_N . We also introduced the light-cone

FIG. 2. Sample diagrams giving rise to the SSA in hadronic $\eta_{c,b}$ production from quark gluon channel.FIG. 3. Sample diagrams giving rise to the SSA in hadronic $\eta_{c,b}$ production from the quark-antiquark channel.

vector $n = (1^+, 0^-, \vec{0}_\perp)$, whose conjugate vector is $\bar{n} = (0^+, 1^-, \vec{0}_\perp)$. Note that our definition of the ETQS function differs by a factor $2\pi M_N$ from the conventions used in Refs. [14,15]. This ETQS function plays an important role for SSA phenomenology. Its scale evolution has been derived in Refs. [59–66].

Similar to the SSA in the Drell-Yan process, the strong interaction phase factor necessary for having a nonvanishing spin asymmetry arises from the interference between an imaginary part of the partonic scattering amplitude with an extra gluon, as shown in Fig. 2 for the quark-gluon scattering channel, and the real scattering amplitude without a gluon attachment. The imaginary part is due to the pole of the parton propagator associated with the integration over the gluon momentum fraction x_g . This effectively implies that one of the internal parton lines goes on shell. To isolate the imaginary part of such poles, the distribution identity: $\frac{1}{x \pm i\epsilon} = \text{PV} \frac{1}{x} \mp i\pi\delta(x)$ was used. Depending on which propagator's pole contributes, the amplitude may get contributions from $x_g = 0$ (“soft pole”) and $x_g \neq 0$ (“hard pole”) [14,15]. Both types of gluon poles show up in our calculation.

As the reader can find the relevant technical details for our calculation in the literature [10–15], we sketch here only some key steps of such twist-3 calculations.

As mentioned before, we carry out the calculation in the covariant gauge, in which the leading contribution of the exchanged gluon is the “plus” component A^+ . The gluon's momentum is dominated by the $x_g P + k_{g\perp}$, where x_g is the longitudinal momentum fraction with respect to the polarized proton. In order to calculate consistently with twist-3 accuracy, one has to expand the hard parts in the gluon transverse momentum and keep the terms linear in $k_{g\perp}$. Then the $k_{g\perp}$ factor can be combined with A^+ to yield $\partial^\perp A^+$, which is an element of the field strength tensor F^{∂^+} . After adding the term proportional to $\partial^+ A^\perp$ of the same tensor [12,67], the soft part can be rewritten in the form of the ETQS function. Making use of the ingredients described above, the calculation is straightforward. However, the derived complete results for the spin-dependent cross sections is rather lengthy. To highlight some interesting features of our results, in the following we will only present the expression in the limit of large transverse momentum ($l_\perp \gg M$) and small transverse momentum ($\Lambda_{\text{QCD}} \ll l_\perp \ll M$) instead of the full expression. At large transverse momentum, after dropping all terms suppressed by powers of M/l_\perp , one ends up with the polarized differential cross section for the qg channel,

$$\begin{aligned} \frac{d\sigma}{dy d^2l_\perp} \approx & \sigma_0 \epsilon^{\mu\nu} S_{\perp\mu} l_{\perp\nu} \frac{\alpha_s}{\pi} M^2 M_N \sum_a \int \frac{dx}{x} \frac{dx'}{x'} \frac{1}{-\hat{t}} G_1(x') \delta(\hat{s} + \hat{t} + \hat{u}) \left\{ \left[2T_F^a(x, x) - x \left(\frac{d}{dx} T_F^a(x, x) \right) \right] \frac{N_c}{2} \frac{\hat{s}^2 + \hat{u}^2}{-\hat{t}^3} \right. \\ & \left. - T_F^a(x, x) \left(\frac{N_c}{2} \frac{2\hat{s}^2 + \hat{t}\hat{s}}{\hat{t}^3} - \frac{1}{N_c} \frac{\hat{s}}{2\hat{u}\hat{t}} \right) - T_F^a \left(x, x \frac{\hat{s}}{\hat{s} - \hat{t}} \right) \frac{N_c}{2} \frac{2\hat{s}^3 + \hat{s}^2\hat{t} - \hat{t}^3}{\hat{u}\hat{s}\hat{t}^2} \right\} \end{aligned} \quad (10)$$

and for the $q\bar{q}$ channel,

$$\begin{aligned} \frac{d\sigma}{dyd^2l_\perp} \approx & \sigma_0 \left(\frac{-8}{3} \right) \epsilon^{\mu\nu} S_{\perp\mu} l_{\perp\nu} \frac{\alpha_s}{\pi} M^2 M_N \sum_a \int \frac{dx}{x} \frac{dx'}{x'} \frac{1}{-\hat{t}} \bar{f}_1^a(x') \delta(\hat{s} + \hat{t} + \hat{u}) \left\{ \left[2T_F^a(x, x) - x \left(\frac{d}{dx} T_F^a(x, x) \right) \right] \frac{-1}{2N_c} \frac{\hat{t}^2 + \hat{u}^2}{-\hat{s}^3} \right. \\ & \left. - T_F^a(x, x) \left(\frac{N_c}{2} \frac{2\hat{t}^2 + \hat{t}\hat{s}}{\hat{s}^3} - \frac{1}{N_c} \frac{\hat{t}}{2\hat{u}\hat{s}} \right) - T_F^a \left(x, x \frac{-\hat{t}}{\hat{s} - \hat{t}} \right) \frac{N_c}{2} \frac{2\hat{t}^3 + \hat{t}^2\hat{s} - \hat{s}^3}{-\hat{u}\hat{t}\hat{s}^2} \right\}. \end{aligned} \quad (11)$$

A few remarks on these analytical results are in order. First, as usual, the SSA is suppressed by the factor M_N/l_\perp at large transverse momentum. Second, one notices that the derivative term and nonderivative term from the soft gluon pole contribution cannot be combined into a compact form as it can be done for the polarized cross section for pion production in hadron collisions [13]. Also, let us note that the hard gluon pole contribution survives even though the heavy quarkonium mass is neglected, in contrast to the cases of pion and direct photon production for which the hard gluon pole contribution is absent. Finally, it is observed that except for the different color

factors, the soft gluon contribution in the quark gluon channel can be obtained from the quark-antiquark channel by crossing $\hat{s} \leftrightarrow \hat{t}$, while the hard coefficients associated with the hard gluon pole contributions for the qg and $q\bar{q}$ channels differ by a minus sign after crossing $\hat{s} \leftrightarrow \hat{t}$.

Now let us discuss our results in the low transverse moment limit. In this limit, we neglect all terms suppressed by the power of l_\perp^2/M^2 , while keeping those enhanced by the factor $1/l_\perp^3$. In order to extrapolate our results to the small transverse momentum region, one has first to expand the delta function in l_\perp ,

$$\delta(\hat{s} + \hat{t} + \hat{u} - M^2) = \delta(\hat{s}(1 - \xi)(1 - \xi') - l_\perp^2) = \frac{1}{\hat{s}} \left[\frac{\delta(1 - \xi)}{(1 - \xi')_+} + \frac{\delta(1 - \xi')}{(1 - \xi)_+} + \delta(1 - \xi') \delta(1 - \xi) \ln \frac{M^2}{l_\perp^2} \right], \quad (12)$$

where $\xi = z/x$ and $\xi' = z'/x'$. The ‘‘plus’’ prescription is defined in the standard way. With this expansion, the spin-dependent cross section in the kinematical region $\Lambda_{\text{QCD}} \ll l_\perp \ll M$ is given by

$$\begin{aligned} \frac{d\sigma}{dyd^2l_\perp} \approx & \sigma_0 \frac{N_c}{2} \frac{\alpha_s}{\pi} \frac{M_N}{l_\perp^4} \epsilon^{\mu\nu} S_{\perp\mu} l_{\perp\nu} G_1(z') \int \frac{dx}{x} \left\{ \left(x \frac{\partial}{\partial x} T_F(x, x) \right) \frac{\xi - 1}{\xi} [1 + (1 - \xi)^2] - T_F(x, x) 2(1 - \xi)^2 \right. \\ & \left. - T_F(x, x - z) \frac{2 - \xi}{\xi} \right\} \\ = & \sigma_0 \frac{N_c}{2} \frac{\alpha_s}{\pi} \frac{M_N}{l_\perp^4} \epsilon^{\mu\nu} S_{\perp\mu} l_{\perp\nu} G_1(z') \int \frac{dx}{x} \left\{ T_F(x, x) \frac{1 + (1 - \xi)^2}{\xi} - T_F(x, x - z) \frac{2 - \xi}{\xi} \right\}. \end{aligned} \quad (13)$$

In the second step of the derivation of the above formula, we carried out the integration over x by parts. It is interesting to note that the hard coefficient associated with the soft gluon matrix element is the well-known splitting kernel \mathcal{P}_{gq} . Here, it is worthwhile to point out that no leading power contribution at small l_\perp comes from the quark-antiquark channel.

On the other hand, when $l_\perp \ll M$, transverse-momentum-dependent factorization can be applied. In the TMD factorization approach, the SSA in $\eta_{c,b}$ production is generated by the gluon Sivvers function $G_{\text{IT,DY}}^\perp$ through the gluon-gluon fusion channel,

$$\begin{aligned} \frac{d\sigma}{dyd^2l_\perp} = & \sigma_0 \frac{\epsilon^{\mu\nu} S_{\perp\mu} l_{\perp\nu}}{M_N} \int d^2k_\perp d^2k'_\perp \frac{k_\perp \cdot l_\perp}{l_\perp^2} \\ & \times \delta^2(l_\perp - k_\perp - k'_\perp) G_1(z', k'_\perp) G_{\text{IT,DY}}^\perp(z, k_\perp), \end{aligned} \quad (14)$$

where $G_{\text{IT,DY}}^\perp(z, k_\perp)$ denotes the gluon Sivvers function. The subscript ‘‘DY’’ indicates that the gluon Sivvers function

contains a past-pointing gauge link built up through initial state interactions, similar to that in the Drell-Yan process.

When k_\perp is of the order of Λ_{QCD} , the gluon Sivvers function is an entirely nonperturbative object. However, in the kinematic region $\Lambda_{\text{QCD}} \ll k_\perp \ll M$, TMD factorization still holds and at the same time the function $G_{\text{IT,DY}}^\perp(z, k_\perp)$ can be calculated in terms of the twist-3 parton correlation function within perturbative QCD. It can receive contributions from both the ETQS function and trigluon correlations. In the current case, we are interested only in the former one. Our perturbative calculation follows a similar procedure as in [14,15], resulting in

$$\begin{aligned} G_{\text{IT,DY}}^\perp(z, l_\perp) = & \frac{N_c}{2} \frac{\alpha_s}{\pi} \frac{M_N^2}{l_\perp^4} \int \frac{dx}{x} \left\{ T_F(x, x) \frac{1 + (1 - \xi)^2}{\xi} \right. \\ & \left. - T_F(x, x - z) \frac{2 - \xi}{\xi} \right\}, \end{aligned} \quad (15)$$

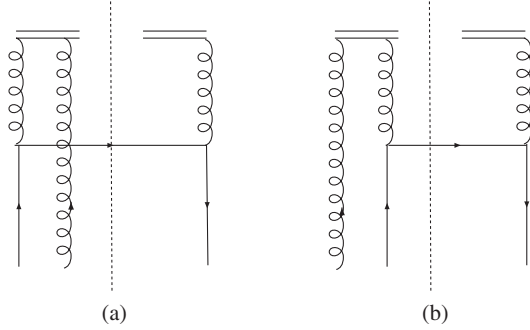


FIG. 4. Diagrams contributing to the gluon Sivvers function at large transverse momentum in the flavor singlet case. (a) Soft gluon pole contribution; (b) hard gluon pole contribution.

where the soft gluon pole contribution is generated by the diagram shown in the Fig. 4(a), and the hard gluon pole contribution arises from Fig. 4(b). This result was first derived in Ref. [35], though the second term was missed there. At this point, we would like to mention that the hard gluon pole term also contributes to the scale evolution of the trigluon correlation for the flavor singlet case, which was originally overlooked in [59] (see also the erratum to [62]). In addition, due to the existence of diagram Fig. 4(b), the chiral partner of the ETQS function, $\tilde{T}_F(x, x_1)$ (notation used in Ref. [16]) also contributes to the scale evolution of trigluon correlations [68].

By inserting Eq. (15) into Eq. (14) and making the approximation $\delta^2(l_\perp - k_\perp - k'_\perp) \approx \delta^2(l_\perp - k_\perp)$, we reproduce the result obtained in the collinear twist-3 approach. Therefore, for the observable under consideration, we have obtained a unified picture in the kinematical region, where TMD factorization and the collinear twist-3 approach both apply, as was found in many other cases [14–16].

To end, let us briefly discuss a possible future numerical study. In order to estimate the size of SSAs for $\eta_{c,b}$ production, we must determine the input for the ETQS functions including the diagonal and off-diagonal contributions, as the polarized cross section depends on both soft and hard gluon pole contributions. Unfortunately, the off-diagonal contributions to T_F needed for this SSA observable are not as well determined as the diagonal pieces that can be related to moments of the quark Sivvers function. In Ref. [59], a Gaussian form was assumed for $T_F(x, x_1)$ with a maximum at $x = x_1$. This study was done in the context of the evolution of $T_F(x, x)$. In Ref. [69], an analysis of higher-twist functions was conducted using light-cone wave functions that include $qqqg$ Fock states. In contrast to [59], this study found that $T_F(x, x_1)$ reaches its maximum when $x \neq x_1$ and some of its lowest values when $x = x_1$. These completely different behaviors clearly demonstrate our lack of knowledge for $T_F(x, x_1)$. Nevertheless, it would be useful to determine the potential impact of $T_F(x, x_1)$ on the size of SSAs in $\eta_{c,b}$ hadronic production.

III. SUMMARY

In summary, we have calculated the transverse single spin asymmetry in hadronic $\eta_{c,b}$ production by employing the collinear twist-3 approach in combination with the color singlet model. We discussed the behavior of the spin asymmetry at high and low transverse momentum. In particular, at low transverse momentum, a match between the TMD factorization approach and the collinear twist-3 formalism has been found after reexamining the derivation of the gluon Sivvers function at large transverse momentum. As a byproduct of this work, we identified an additional term, which contributes also to the scale evolution of the trigluon correlation for the flavor singlet case. In addition, we have briefly outlined a plan for future numerical studies. Let us mention that it would be feasible to measure this asymmetry at RHIC and a proposed fixed target experiment at LHC(AFTER) [40–42].

We emphasize that the transverse spin physics and the heavy quarkonium production physics could mutually benefit from studying this observable as explained in the following. First of all, due to the sign mismatch issue [26], a doubt concerning the validity of the collinear twist-3 approach in hadron collisions has arisen. SSA in heavy quarkonium production provides a clean way to test this approach, as compared to that in pion production where the Collins mechanism could dominate the asymmetry [29]. Furthermore, one notices that the spin asymmetry is independent of the radial wave function at the origin R_0 and only sensitive to the heavy quarkonium production mechanism. Therefore, measuring this observable would provide us a unique chance to pin down the ratio between the values of the color singlet long-distance matrix element and the color octet matrix element.

There are a number of directions in which our work could be extended. First, the color octet contribution to SSA should be taken into account as it plays a role at large transverse momentum. Second, it is natural and straightforward to study the SSAs for other C-even heavy quarkonium production in the framework outlined in this paper. It is also possible to study SSAs for hadronic J/ψ , Υ production. To do so, we have to compute the SSAs generated from the trigluon correlation since it is the only contribution that appears at the nontrivial leading order in J/ψ , Υ production. Finally, one can calculate the SSAs for photonic/electronic J/ψ , Υ production, using the collinear twist-3 approach in combination with the NRQCD framework. These would be the relevant observables at the future EIC [70,71].

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