PHYSICAL REVIEW D **88,** 013012 (2013)

Muonium annihilation into $\nu_e \bar{\nu}_\mu$ and $\nu_e \bar{\nu}_\mu \gamma$

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(Received 11 June 2013; published 19 July 2013)

We calculate in detail the annihilation of muonium (Mu) into $\nu_e \bar{\nu}_\mu$ and $\nu_e \bar{\nu}_\mu \gamma$ states. For $\mathrm{Mu} \to \nu_e \bar{\nu}_\mu$, we obtain the branching ratio $\mathrm{Br} = 6.6 \times 10^{-12}$, and for $\mathrm{Mu} \to \nu_e \bar{\nu}_\mu \gamma$ in the limit of high-energy photons, we obtain $\mathrm{Br} = 4.3 \times 10^{-11}$.

DOI: 10.1103/PhysRevD.88.013012 PACS numbers: 13.35.Bv, 14.60.Ef

I. INTRODUCTION

In the light of new possibilities of experimental verification of the Standard Model, one needs to reconcile the different theoretical predictions. The discrepancy takes place, e.g., in the case of muonium (Mu) system decay. The dominant decay channel occurs by muon beta decay, $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$, but there is another possible channel: muonium annihilation, $\mathrm{Mu} \to \nu_e \bar{\nu}_\mu$. The corresponding decay rate is rather small but detectable in the planned experiments [1] as an invisible decay of muonium. The branching ratio of this process was first estimated in Refs. [2,3], and then was calculated in some detail and found to be $\sim 10^{-12}$ in Refs. [4,5] and $\sim 10^{-10}$ in Ref. [6]. To recheck these results, we provide here the full calculations and also estimate the full decay width and a photon energy spectrum of the reaction $\mathrm{Mu} \to \nu_e \bar{\nu}_\mu \gamma$.

II. Mu → $\nu_e \bar{\nu}_\mu$ ANNIHILATION

A. $\mu^+ e^- \to \nu_e \bar{\nu}_\mu$ process

We start with a calculation of the amplitude of the $\mu^+ e^- \to \nu_e \bar{\nu}_\mu$ process assuming both μ^+ and e^- to be free states. In our case, particles have small energies in comparison with the masses of the weak bosons, so the amplitude has the form

$$M_0 = \frac{G_F}{\sqrt{2}} j_1^\mu j_{\mu 2},$$

where

$$j_1^\mu = \bar{v}_\mu^s (p') O^\mu v_{\nu_\mu}^{s'} (k'), \qquad j_{\mu 2} = \bar{u}_{\nu_e}^r (k) O_\mu u_e^{r'} (p).$$

Here u and v are the solutions to the Dirac equation with positive and negative frequencies, respectively. p, p', k, k' are the 4-momenta of e, μ, ν_e, and ν_μ, respectively, and $O^\mu = \gamma^\mu (\gamma^5 + 1)$. For the square of this matrix element neglecting the neutrino masses, we have

$$|M_0|^2 = 128 G_F^2 (k \cdot p')(p \cdot k').$$

Since electrons and muons are supposed to be nonrelativistic, and both neutrinos are ultrarelativistic, we have

$$p = (m_e, \vec{p}), \qquad k = (m_\mu/2, m_\mu/2), \tag{1}$$

$$p' = (m_\mu, -\vec{p}), \qquad k' = (m_\mu/2, -m_\mu/2). \tag{2}$$

So, the final expression for the matrix element squared is

$$|M_0|^2 \approx 32 G_F^2 (m_\mu^3 m_e + m_\mu^2 |\vec{p}| \cos\theta (m_\mu + |\vec{p}| \cos\theta)),$$

where θ is the angle between the muon μ and the electron neutrino ν_e. Hereafter, we will use the leading part of this expression, assuming that $|\vec{p}| \approx 0$ and $m_e \ll m_\mu$, so the accuracy of our calculations has order m_e/m_μ.

B. Bound states

The amplitude associated with the bound state is expressed in terms of the amplitude of the free process as

$$M = \sqrt{2m} \int \frac{d^3 q}{(2\pi)^3} \hat{\psi}^*(\vec{q}) \frac{1}{\sqrt{2m_\mu}} \frac{1}{\sqrt{2m_e}} M_0,$$

where m is the mass of the bound state, which in our case is equal to m_μ within our accuracy, and $\hat{\psi}(\vec{q})$ is a Fourier transform of the Schrödinger wave function of the bound state which we set to be equal to the S ground state. Hence, the full angular momentum of the system is determined by the summary spin of μ^+ and e^-. The multipliers $(1/\sqrt{2m_e})$, $(1/\sqrt{2m_\mu})$ provide the integral normalization to unity, and the factor $\sqrt{2m}$ in front of the integral is necessary for accordance with the cross section formula [7]. Since M_0 does not contain any dependency on $\vec{q}$, this expression simplifies, so for its square we have

$$|M|^2 = \frac{1}{2m_e} |M_0|^2 \cdot |\psi(0)|^2.$$

The wave function of the ground state of muonium is the same as for hydrogen:

$$\psi(r) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a},$$

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where a is the muonium Bohr radius $a^{-1} = |\vec{p}| = \alpha m_{\rm rd}$, and $m_{\rm rd} = m_e m_\mu/(m_e + m_\mu) \approx m_e$ with our precision. Therefore,

$$|\psi(0)|^2 = \frac{m_e^3\alpha^3}{\pi}. \tag{3}$$

C. Decay width

The decay width of muonium is written in the usual way (again, $m \approx m_\mu$):

$$\Gamma = \frac{1}{2m_\mu}\int |M|^2 d\Pi_2.$$

A two-particle phase volume $d\Pi_2$ for $2 \to 2$ reactions, which are symmetrical with respect to the collision axis, can be reduced to the form

$$d\Pi_2 = \frac{1}{16\pi} d\cos\theta \frac{2|\vec{k}|}{E_{\rm CM}},$$

where $E_{\rm CM} \approx m_\mu$ is the energy of particles in the center-of-mass system and $|\vec{k}|$ is the momentum of the outgoing particles, $|\vec{k}| \approx m_\mu/2$. The fact that the dependence on the polar angle θ pertains only to $|M_0|^2$ allows us to write

$$\Gamma = \frac{1}{4m_e m_\mu}|\psi(0)|^2 \int |M_0|^2 d\Pi_2. \tag{4}$$

Integration over the phase volume and substitution of the explicit expression for $|\psi(0)|^2$ from Eq. (3) leads to the expression

$$\Gamma = \frac{G_F^2\alpha^3 m_e^3 m_\mu^2}{\pi^2}.$$

Averaging over the initial polarizations gives

$$\frac{G_F^2\alpha^3 m_e^3 m_\mu^2}{4\pi^2} = 48\pi\left(\frac{\alpha m_e}{m_\mu}\right)^3 \Gamma_{\mu\to e\nu_\mu\bar{\nu}_e},$$

so the branching ratio is

$$\mathrm{Br} = \Gamma\tau = 6.6 \times 10^{-12}. \tag{5}$$

III. Mu $\to \nu_e\bar{\nu}_\mu\gamma$ ANNIHILATION

To the leading order in α, there are three diagrams which contribute to the Mu $\to \nu_e\bar{\nu}_\mu\gamma$ process. They are presented in Fig. 1.

The first of them contains two virtual W-boson propagators, and hence its amplitude is suppressed significantly. Naively, the leading-order part of the matrix element squared of the $\mu^+e^- \to \nu_e\bar{\nu}_\mu\gamma$ process involving the term from this diagram is proportional to

$$|M_0|^2 \sim \frac{e^2 G_F^2}{m_W^2 m_e^2},$$

and for its contribution to the full decay width of Mu one can obtain

$$\Gamma \sim \frac{\alpha^4 G_F^2 m_\mu^7}{m_W^2}\left(1 + O\left(\frac{m_e}{m_\mu}\right)\right).$$

The corresponding branching ratio has an order $\mathrm{Br} \sim 10^{-15}$ and turns out to be small even in comparison with the next-order loop corrections. The second diagram is also suppressed by a factor $\sim m_e/m_\mu$, and we will not consider it in accordance with the discussion above. The last diagram describes the emission of a photon from the external electron line. Its amplitude is equal to

$$\begin{aligned} M_{0\gamma} = \frac{G_F}{\sqrt{2}}\frac{i}{-2p\cdot q}&(\bar{u}^r_{\nu_e}(k)\gamma_\mu(1+\gamma_5) \\ &\times(\gamma_\rho(p^\rho - q^\rho) - m_e)\gamma_\sigma\epsilon^{\sigma*}(q)u_e^s(p)) \\ &\times(\bar{v}_\mu^{s'}(p')\gamma^\mu(1+\gamma_5)v^{r'}_{\nu_\mu}(k')). \end{aligned}$$

Here q is the photon momentum. Conjugation, production and averaging over the initial polarizations leads to the following expression:

$$\begin{aligned} |M_{0\gamma}|^2 = \frac{8e^2G_F^2}{(p\cdot q)^2}&[2(p\cdot l)(k'\cdot l)(k\cdot p') \\ &- l^2(k'\cdot p)(k\cdot p')], \end{aligned} \tag{6}$$

where $l = p - q$. For 4-momenta of the particles, we have

$$p = (m_e, \vec{p}), \qquad p' = (m_\mu, -\vec{p}), \qquad k = (\omega_1, \vec{k}_1),$$
$$k' = (\omega_2, \vec{k}_2), \qquad q = (\omega_\gamma, \vec{k}_\gamma).$$

For the differential decay width, one can write, similarly to Eq. (4),

$$\Gamma_\gamma = \frac{1}{2m_\mu}\int |M_\gamma|^2 d\Pi_3 = \frac{|\psi(0)|^2}{4m_e m_\mu}\int |M_{0\gamma}|^2 d\Pi_3, \tag{7}$$

where $d\Pi_3$ is now a differential three-particle phase volume:

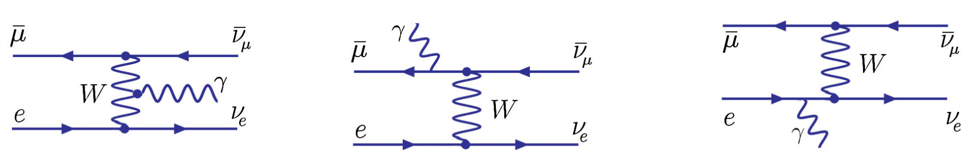

FIG. 1 (color online). The leading-order diagrams for the free $\mu^+e^- \to \nu_e\bar{\nu}_\mu\gamma$ process.

$$d\Pi_3 = (2\pi)^4 \delta(\omega_1 + \omega_2 + \omega_\gamma - m_\mu) \times \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_\gamma) \frac{d^3\vec{k}_1}{(2\pi)^3 2\omega_1} \frac{d^3\vec{k}_2}{(2\pi)^3 2\omega_2} \frac{d^3\vec{k}_\gamma}{(2\pi)^3 2\omega_\gamma}.$$

One can integrate over the 3-momenta $\vec{k}_1$ and $\vec{k}_2$ using the equality [8]

$$\int k_\alpha k'_\beta \frac{d^3\vec{k}_1}{\omega_1} \frac{d^3\vec{k}_2}{\omega_2} \delta^4(k + k' - Q) = \frac{\pi}{6}(Q^2 g_{\alpha\beta} + 2Q_\alpha Q_\beta), \tag{8}$$

where $Q = p + p' - q$ in our case. Noting that $d^3\vec{k}_\gamma = 4\pi\omega_\gamma^2 d\omega_\gamma$, one can reduce the differential decay width to the form

$$\frac{d\Gamma_\gamma}{d\omega_\gamma} = \frac{G_F^2 \alpha^4}{12\pi^3} m_e m_\mu^4 F(x), \tag{9}$$

where $F(x) = x(3 - 4x)$ is a photon spectrum function, $x = \omega_\gamma / m_\mu$, $x \leq 0.5$. In accordance with our assumption, this expression is valid for $\omega_\gamma \gg m_e$, i.e., for $x \gg 5 \times 10^{-3}$. Integration over ω_γ leads to the final expression:

$$\Gamma_\gamma = \frac{5G_F^2 \alpha^4 m_e m_\mu^4}{288\pi^3}. \tag{10}$$

The appropriate branching ratio is $\mathrm{Br} = \Gamma_\gamma \tau = 4.3 \times 10^{-11}$.

IV. RESULTS

The branching ratio [Eq. (5)] for the reaction $\mathrm{Mu} \to \nu_e \bar{\nu}_\mu$ is in agreement with Refs. [4,5] but differs from the one obtained in Ref. [6]. Thus, its value lies in the detectable range of an experiment proposed in Ref. [1]. The value [Eq. (10)] of the full decay width in the $\mathrm{Mu} \to \nu_e \bar{\nu}_\mu \gamma$ reaction differs slightly from the result in Ref. [5]. The differential decay width [Eq. (9)] is valid for the range of photon energies $\omega_\gamma \sim 5$–50 MeV, as we discussed above, and can be also verified in the new experiments.

ACKNOWLEDGMENTS

The author thanks D. S. Gorbunov for useful discussions. The work was partially supported by the Grant No. NS-5590.2012.2.

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